b-colorings: an structural overview

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LAGOS 2017

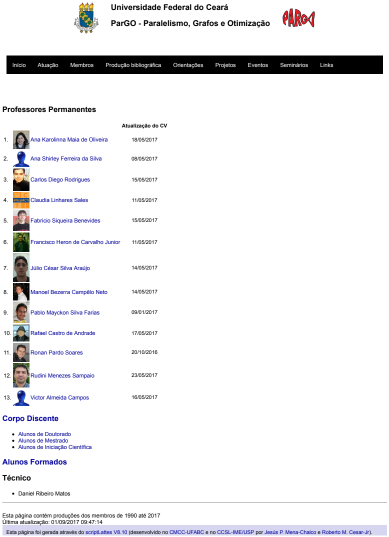
Figure: Photo by Yoshiko Wakabayashi

Thanks very much Frédérique, Flavia, Lionel, Mario and Juan Carlos for this wonderful conference!

PS: so far ;-)

Audience of a regular ParGO's seminar (September 1st)

ParGO's people



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Corpo Discente

- Alunos de Doutorado
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- Daniel Ribeiro Matos

Esta página contém produções dos membros de 1990 até 2017
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- 22 PhD students
- 10 MSc students
- 17 undergraduate students

Jayme's tribe

Summary

1. Preliminaries
2. Vertex coloring heuristics
3. Complexity aspects
4. Structural aspects
5. *b*-homomorphism

Preliminaries

- A (proper) *k*-coloring of a graph $G = (V, E)$ is a function $c : V(G) \rightarrow \{1, 2, \ldots, k\}$ that satisfies $c(u) \neq c(v)$, for every $(u, v) \in E$.

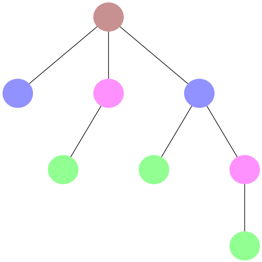
- A k-coloring of G induces a k-partition $\{V_1, \ldots, V_k\}$ of $V(G)$ such that each part V_i, which contains the vertices colored i, is called a color class and induces stable set of G.

- The chromatic number of G, denoted by $\chi(G)$, is the least integer k such that G admits a proper k-coloring.

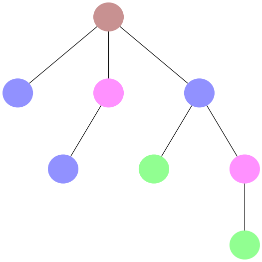
- Determining $\chi(G)$ is *NP*-hard (k-colorability is *hard* even for triangle-free graphs [Maffray & Preissman, 1996])

F. Maffray and M. Preissmann. *On the NP-completeness of the k-colorability problem for triangle-free graphs*. Discrete Math., v. 162, p. 313–317, 1996.

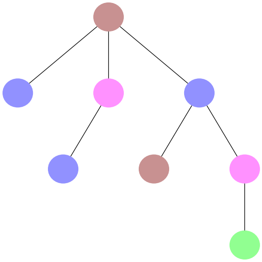
Vertex Coloring Heuristics: The *b* one



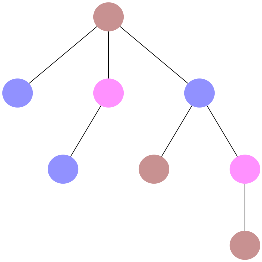
Vertex Coloring Heuristics – The *b* one



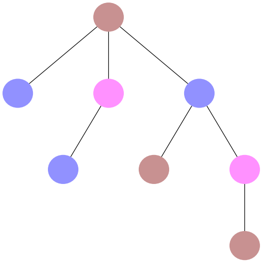
Vertex Coloring Heuristics – The *b* one



Vertex Coloring Heuristics – The *b* one



Vertex Coloring Heuristics – The *b* one



When you cannot apply this procedure anymore, you get a *b*-coloring.

The b-chromatic number

Given a k-coloring of a graph G, a vertex v colored i is a ***b*-vertex** (of color i), if v has a neighbor in all the other color classes.

b-coloring

Given a graph $G = (V, E)$, a b-coloring of G with k colors is a proper coloring of G such that every color class has a b-vertex.

b-chromatic number

The ***b*-chromatic number** of G, denoted by $\chi_b(G)$, is the largest integer k such that G admits a b-coloring with k colors.

R.W. Irving and D.F. Manlove, *The b-chromatic number of a graph*, Discrete Appl. Math. 91, 127–141, 1999.

The ancestral coloring heuristic: the greedy one

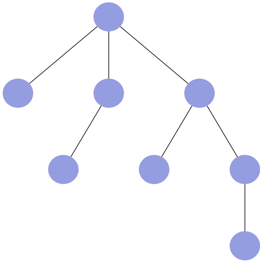
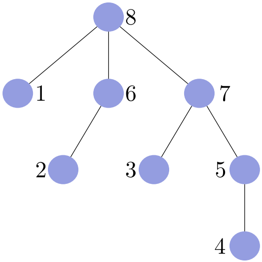
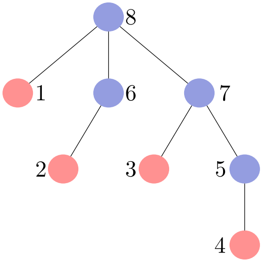


Figure: Binomial tree T_4

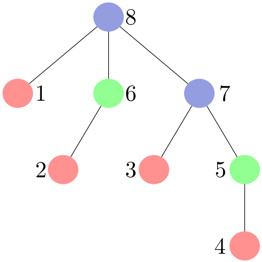
Greedy coloring heuristic



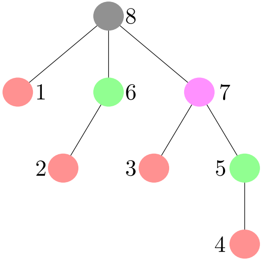
Greedy coloring heuristic



Greedy coloring heuristic



Greedy coloring heuristic



We have obtained a **Grundy** coloring.

Given a k-coloring of a graph G, a vertex v colored i is a **Grundy vertex** (of color i), if v has a neighbor in every color class j, $j < i$.

Grundy coloring

Given a graph $G = (V, E)$, a **Grundy coloring** of G with k colors is a proper coloring of G with colors $\{1, \ldots, k\}$ such that every vertex is a Grundy vertex.
The Grundy number of G, denoted by $\Gamma(G)$, is the largest integer k such that G admits a Grundy coloring with k colors.

Partial Grundy coloring

Given a graph $G = (V, E)$, a **partial** Grundy coloring of G with k colors is a proper coloring of G with colors $\{1, \ldots, k\}$ if every color class has a Grundy vertex.
The **Partial Grundy number** of G, denoted by $\partial\Gamma(G)$, is the largest integer k such that G admits a Partial Grundy coloring with k colors.

P. M. Grundy, *Mathematics and games*, Eureka, 2: 6–8. 1939.

C.A. Christen, S.M. Selkow, *Some perfect coloring properties of graphs*, Journal of Combinatorial Theory, Series B, 27 (1): 49–59. 1979.

P. Erdös, S. T. Hedetniemi, R. C. Laskarc and G.C.E. Prins. *On the equality of the partial Grundy and upper ochromatic numbers of graphs*. Discrete Mathematics, v. 272, p. 53–64, 2003.

The ancestral of the *b* heuristic: the *a* one

a-coloring

Given a graph $G = (V, E)$, an ***a*-coloring** of G with k colors is a proper coloring of G such that there is no two distinct color classes whose union induces a stable set.

The achromatic number

The **achromatic number** of G, denoted by $\chi_a(G)$, is the largest integer k such that G admits an *a*-coloring with k colors.

F. Harary, S.T. Hedetniemi, G. Prins, *An interpolation theorem for graphical homomorphisms*, Portugal Math. 26 (1967) 453-462.

Bounds:

- General bounds
- Refined bounds
- Nordhaus–Gaddum inequalities

Bounds

$$\begin{aligned}\chi_b(G) &\leq \Delta(G)+1\\ \Gamma(G) &\leq \Delta(G)+1\\ \partial\Gamma(G) &\leq \Delta(G)+1\end{aligned}$$

$$\chi(G) \leq \chi_b(G) \leq \partial\Gamma(G) \leq \chi_a(G)$$

$$\chi(G) \leq \Gamma(G) \leq \partial\Gamma(G) \leq \chi_a(G)$$

$\chi_b(G)$ and $\Gamma(G)$ are incomparable

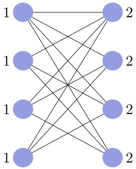


Figure: $\chi_b(G) = 2$

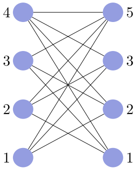


Figure: $\Gamma(G) = 5$

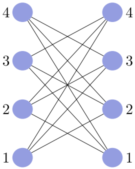


Figure: $\chi_b(G) = \Gamma(G)$

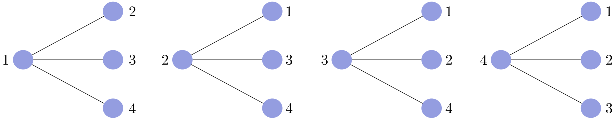


Figure: $\chi_b(G) = 4$ and $\Gamma(G) = 2$

Refined bounds

If G has a b-coloring with k colors, so G has at least k vertices (the b-vertices) of degree at least $k-1$.

The *m-degree* of G is the value $m(G) = \max\{k|$ there exist k vertices of degree $\geq k-1\}$.

$$\chi_b(G) \leq m(G)$$

If G has a k-partial Grundy coloring, so G has a sequence of vertices (the Grundy vertices) $\{v_1, \ldots, v_k\}$ such that v_i has at least $i-1$ neighbors in $G \setminus \{v_{i+1}, \ldots, v_k\}$. This sequence is called a feasible sequence.

The *∂-degree* of G is the maximum k such that G has a feasible sequence of size k.

$$\partial\Gamma(G) \leq \partial(G)$$

Both parameters can be computed in polynomial time.

Nordhaus–Gaddum Equations

$$\chi(G) + \chi(\bar{G}) \leq n + 1$$

$$\chi_a(G) + \chi_a(\bar{G}) \leq n + 1$$

$$\Gamma(G) + \Gamma(\bar{G}) \leq \lfloor \frac{5n+2}{4} \rfloor$$

$$\chi_b(G) + \chi_b(\bar{G}) \leq n + 1$$

$$\partial\Gamma(G) + \partial\Gamma(\bar{G}) \leq n + 1 + \Delta(G) - \delta(G)$$

Question: Are there better bounds for $\partial\Gamma(G) + \partial\Gamma(\bar{G})$, also considering special classes of graphs?

E.A. Nordhaus and J.W. Gaddum, *On Complementary Graphs*, Amer. Math. Monthly 63, 175-177, 1963.

F. Harary and S. Hedetniemi, *The achromatic number of a graph*, J. of Comb. Theory Ser. B, v. 8(2), p. 154–161, 1970.

E.J. Cockayne and A.G. Thomason, Ordered colorings of graphs, J. Combin. Theory Ser. B, v. 27, p. 286–292, 1982.

M. Kouider and M. Mahéo, *Some bounds for the b-chromatic number of a graph*, Discrete Mathematics, v. 256, p. 267–277, 2002.

P. Erdös, S. T. Hedetniemi, R. C. Laskarc and G.C.E. Prins. *On the equality of the partial Grundy and upper ochromatic numbers of graphs*. Discrete Mathematics, v. 272, p. 53–64, 2003.

Complexity aspects

Complexity of b-coloring

The b-chromatic number problem is NP-Complete [Irving and Manlove, 99]

The k-b-coloring problem is NP-complete for **bipartite graphs** [Kratochvíl *et al.*, 2002] and for **chordal graphs** [Havet, Linhares Sales and Sampaio, 2011].

The problem is polynomial solvable for **trees** [Irving and Manlove, 99], **cacti** [Campos, Linhares Sales and Silva, 2011] and P_4**-sparse graphs** [Bonomo *et al.*, 2009].

It does not have an approximation with factor $O(n^{\frac{1}{4}-\epsilon})$ unless $P = NP$ [Galčik and Katrenič, 2013].

It is W[1]-hard with parameter k ($\chi_b(G) \geq k$?) [Panolan *et al.*, 2016].

The **dual** problem "$\chi_b(G) \geq n - k$?" admits an FPT algorithm with parameter k [Havet and Sampaio, 2011].

Question: Does the problem "$\chi_b(G) \geq k$?" admit an FPT algorithm with any other parameter than k?

The Complexity of Grundy coloring

The Grundy number problem is *NP*-Complete, even for **co-bipartite graphs**. [Zaker, 2006]

The problem is polynomial solvable for **trees** [Beyer *et al.*,1982] and P_4**-sparse graphs** [Araújo and Linhares Sales, 2009].

It does not have an approximation with constant factor c unless $NP \subseteq RP$ [Kortsarz,2007].

It is XP with parameter k ($\Gamma(G) \geq k$?) [Zaker,2006].

The **dual** problem "$\Gamma(G) \geq n - k$?" admits an FPT algorithm with parameter k [Havet and Sampaio, 2011].

Question: Does the problem "$\Gamma(G) \geq k$?" admit an FPT algorithm with parameter k?

The Complexity of partial Grundy coloring

The partial Grundy coloring problem is *NP*-Complete, even for **chordal graphs**. [Shi *et al.*,2005]

The problem is polynomial (linear) solvable for **trees** [Shi *et al.*,2005] and **split graphs** [Panda and Verna, 2016].

It is XP with parameter k ($\partial\Gamma(G) \geq k$?) [Effantin *et al.*, 2016].

Questions: Does the problem "$\partial\Gamma \geq k$? or the dual "$\partial\Gamma \geq n - k$?" admit an FPT algorithm with parameter k or any other parameter? Does it admit an approximation? For which more classes of graphs it is polynomial ?

The Complexity of *a*-coloring

The achromatic number problem is *NP*-Complete, even for **trees** [Cairniel and Edwards, 1997], **cographs** and **interval graphs** [Bodlaender, 1989].

It can be approximated with factor $O(n \lg n \lg n / \lg n)$ [Kortsarz and Krauthgamer, 2001].

Others aspects:

- Girth
- Interpolation theorems
- Graph products

Girth

The **girth** of a graph G, denoted by *girth*(G), is the size of a smallest cycle of G.

High girth means that locally the graph looks like a tree?

Girth

The **girth** of a graph G, denoted by $girth(G)$, is the size of a smallest cycle of G.

High girth means that locally the graph looks like a tree?

Theorem (Erdös,1959)

Given any $k, g \in N$, there is a graph G such that $\chi(G) > k$ and $girth(G) > g$.

What is the relationship with the upper bound $\Delta(G)$?

Girth

The **girth** of a graph G, denoted by $girth(G)$, is the size of a smallest cycle of G.

High girth means that locally the graph looks like a tree?

Theorem (Erdös,1959)

Given any $k, g \in N$, there is a graph G such that $\chi(G) > k$ and $girth(G) > g$.

What is the relationship with the upper bound $\Delta(G)$?

Theorem (Bollobás, 1978)

Given any $k \geq 4$, there exists a Δ (a function of k) such that for every g there is a graph G with $\Delta(G) \leq \Delta$, $girth(G) \geq g$ and $\chi(G) \geq k$.

Girth versus X_b and $\partial\Gamma$

Recall:

$$\chi_b(G) \leq m(G)$$

$$\partial\Gamma(G) \leq \partial(G)$$

Theorem (Shi *et al.*, 2005)

For a graph G with girth$(G) > 8$, $\partial\Gamma(G) = \partial(G)$.

Theorem (Campos *et al.*, 2012)

For a graph G with girth$(G) \geq 7$, $\chi_b(G) \geq m(G) - 1$.

Interpolation Theorems

Theorem (Harary *et al.*, 1967)

Given a graph $G = (V, E)$, G has an a-coloring with k colors, for every k, $\chi(G) \leq k \leq \chi_a(G)$.

Theorem (Erdös *et al.*,2003)

Given a graph $G = (V, E)$, G has a Grundy coloring with k colors, for every k, $\chi(G) \leq k \leq \Gamma(G)$.

Theorem (Balakrishnan and Kavaskar, 2013)

Given a graph $G = (V, E)$, G has a partial Grundy coloring with k colors, for every k, $\chi(G) \leq k \leq \partial\Gamma(G)$.

Negative for *b*-colorings

Counterexample: $K_{r,r} - M$, where M is perfect matching, has b-colorings only with χ and χ_b colors.

b-Continuity

Definition (b-continuity)

A graph G is b-continuous if and only if G has a b-coloring by k colors for every integer k satisfying $\chi(G) \leq k \leq \chi_b(G)$.

b-Continuity

Definition (*b*-continuity)

A graph G is b-continuous if and only if G has a b-coloring by k colors for every integer k satisfying $\chi(G) \leq k \leq \chi_b(G)$.

Theorem (Faik, 2005)

Given a graph G, it is NP-complete to decide whether G is b-continuous, even if G is bipartite.

The b-spectrum S_b of a graph G is the set of integers k such G has a b-coloring with k colors.

Complexity and general results of *b*-Continuity

The *b*-continuity problem is polynomial for:

- Chordal graphs [iFaik, 2004];
- Cacti [Sampaio, 2009];
- Hypercubes Q_n, with $n \neq 3$ [Faik & Scale, 2003];
- P_4-tidy graphs [Velasquez *et al.*, 2010]; and
- k-regular graphs with girth at least 7 [Balakrishnana & Kavaskarb, 2012]

Theorem (Linhares Sales and Silva, 2016)

Let $G = (V, E)$ be a graph. If girth$(G) \geq 10$, then G is b-continuous.

Question: Are there any other structural properties related with *b*-continuity?

The parameters on Graphs Products

Given graphs $G = (V, E)$ and $H = (V, E)$, the lexicographic product $G[H]$, the cartesian product $G\Box H$, the direct product $G \times H$ and the strong product $G \boxtimes H$ have set of vertices $V(G) \times V(H)$ and the following set of edges:

$$\begin{aligned}
E(G[H]) &= \{(a,x)(b,y) | ab \in E(G), \text{ or } a = b \text{ e } xy \in E(H)\}; \\
E(G \times H) &= \{(a,x)(b,y) | ab \in E(G), \text{ and } xy \in E(H)\}; \\
E(G\Box H) &= \{(a,x)(b,y) | a = b \text{ and } xy \in E(H) \text{ or } ab \in E(G) \text{ and } x = y\}; \\
E(G \boxtimes H) &= E(G \times H) \cup E(G\Box H).
\end{aligned}$$

Observe that $E(G \boxtimes H) \subseteq E(G[H])$.

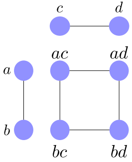


Figure: Cartesian product $G \square H$

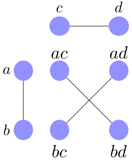


Figure: Direct product $G \times H$

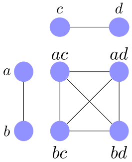


Figure: Strong product $G \boxtimes H$

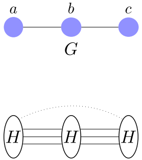


Figure: Lexicographic product $G[H]$

Products and Colorings

Given graphs $G = (V, E)$ and $H = (V, E)$, where $\chi(H) = m$:

$$
\begin{aligned}
\chi(G\Box H) &= \mathtt{max}\{\chi(G), \chi(H)\} && (1)\\
\texttt{Is } \chi(G \times H) &= \mathtt{min}\{\chi(G), \chi(H)\} \texttt{ ?} && (2)\\
\chi(G[H]) &= \chi(G[K_m]) && (3)
\end{aligned}
$$

(1) Sabidussi, 1957
(2) Conjecture of Hedetniemi, 1966
(3) Geller and Stahl, 1975

Observe that $G[K_m] \cong G \boxtimes K_m$, $m \geq 1$, then they have the same upper bounds [Geller and Stahl, 1975].

a-chromatic number of product of graphs

Given graphs $G = (V, E)$ with $\chi_a(G) = m$ and $H = (V, E)$ with $\chi_a(H) = n$:

$$\chi_a(G \times H) \geq \chi_a(G) + \chi_a(H) \texttt{ unless } (\star) \quad (4)$$
$$\chi_a(G \Box H) \geq \chi_a(K_m \Box K_n) \quad (5)$$
$$\chi_a(G[H]) \geq \chi_a(G).\chi_a(H) \quad (6)$$

(*) $\chi_a(H) = 3$ and $\chi_a(G) \leq 5$ or $\chi_a(H) = 2$

(4) Hell and Miller, 1992
(5) Chiang and Fu, 1992
(6) Geller and Stahl, 1975

b-chromatic number of product of graphs

Given graphs $G = (V, E)$ and $H = (V, E)$:

$$\begin{aligned}
\chi_b(G \times H) &\geq \texttt{max}\{\chi_b(G), \chi_b(H)\} && (7)\\
\chi_b(G \square H) &\geq \texttt{max}\{\chi_b(G), \chi_b(H)\} && (8)\\
\chi_b(G[H]) &\geq \chi_b(G).\chi_b(H) && (9)\\
\chi_b(G \boxtimes H) &\geq \chi_b(G).\chi_b(H) && (10)
\end{aligned}$$

$$\begin{aligned}
\chi_b(G \square H) &\leq \chi_b(G)(|V(H)| + 1) + \Delta(H) + 1 && (11)\\
\chi_b(G \boxtimes H) &\leq \Delta(G).\Delta(H) + \Delta(G) + \Delta(H) + 1 && (12)
\end{aligned}$$

[8,11] Kouider and Mahéo, 2002 e 2007
[7,9,10,12] Jakovac and Peterin, 2011

b-continuity and product of graphs

If G and H are b-continuous, is their product b-continuous ?

False for Cartesian product: $K_2 \square C_4 \simeq Q_3$;

False for direct product: $K_2 \times K_n \simeq K_{n,n} - L$, $n \geq 4$, where L is a perfect matching;

Open for the lexicographic product;

It was proved true for some classes of b-continuous graphs G and H;
It was found a strong relationship between the b-spectrum of $G[K_m]$ and $G[H]$, when H admits a m-b-coloring and $G[K_m]$ is b-continuous.

Open for the strong product, but recall that $G[K_m] \cong G \boxtimes K_m$, $m \geq 1$.

C. Linhares Sales, L. Sampaio and R. Vargas, *b*-continuity and the lexicographic product of graphs. In: Latin American Graphs and Optimization Symposium, 2015, Beberibe. Electronic Notes in Discrete Mathematics, 2015.

C. Linhares Sales, L. Sampaio and A. Silva, The lexicographic product of some chordal graphs and of cographs are b-continuous. In Latin 2017.

Grundy number of product of graphs

Given graphs $G = (V, E)$ and $H = (V, E)$:

$$\begin{aligned}
\Gamma(G \square H) &\geq \texttt{max}\ \{\Gamma(G), \Gamma(H)\} && (13)\\
\Gamma(G[H]) &\geq \Gamma(G).\Gamma(H) && (14)\\
\Gamma(G \times H) &\geq \Gamma(G) + \Gamma(H) - 2 && (15)\\
\Gamma(G[H]) &\leq 2^{\Gamma(G)-1}(\Gamma(H) - 1) + \Gamma(G) && (16)\\
\Gamma(G \square H) &\leq \Delta(G).2^{\Gamma(H)-1} + \Delta(H) && (17)\\
\Gamma(G \square H) &\leq \Gamma(G[K_{\Delta(H)+1}]) && (18)
\end{aligned}$$

[13-17] Asté, Havet and Linhares Sales, 2010
[18] Campos, Gyárfas, Havet, Linhares Sales and Maffray, 2012.

There are no upper bounds for $\Gamma(G \square H)$ [Asté et al, 2010], $\Gamma(G \times H)$ and $(\Gamma(G \boxtimes H)$ on $\Gamma(G)$ and $\Gamma(H)$ if both are greater than 3. Moreover, the last two products cannot also be bounded as function of Γ and Δ except in special cases [Campos *et al.*, 2012].

Question: How the parameter $\partial\Gamma$ behaves with products?

b-homomorphisms

b-homomorphisms

Join work with Pavol Hell and Bojan Mohar.

Definition (b-homomorphism)

Given graphs G and H, G is b-homomorphic to H if G has a homomorphism f to H such that:

1. for every vertex $u \in V(H)$, $f^{-}(u)$ is non-empty;
2. for every vertex $u \in V(H)$, $f^{-}(u)$ has a vertex $v \in V(G)$, such that $N_G(v) \cap f^{-}(w) \neq \emptyset$ for every $w \in N_H(u)$.

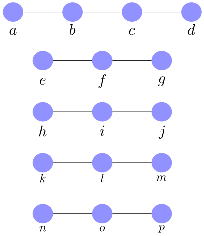


Figure: G

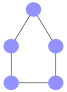


Figure: H

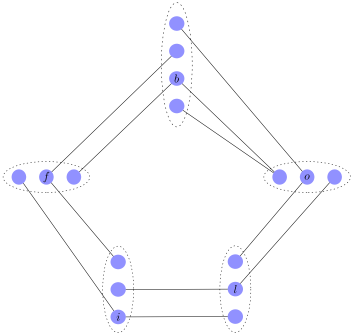


Figure: A b-homomorphism $G \mapsto H$

b-homomorphism

Observe that:

1. A b-coloring with n colors is a b-homomorphism of G to K_n;
2. Every b-homomorphism is surjective;
3. Deciding if G has a surjective homomorphism to a general graph H is NP-complete (It follows from the Hell-Nešetřil dichotomy theorem [2004]);
4. It is an open problem when H is bipartite.

Observation

Given G and H, deciding if G has a b-homomorphism to H is NP-complete.

Question: In which cases b-homomorphism is polynomial?

The case $H \cong C_k$, $k \geq 3$

Observation

Let G be a **non-bipartite graph**. *Then every C_k-homomorphism of G is a b-homomorphism.*

So, we are let with the case when G **is bipartite**.
Let $diam(G)$ be the **diameter** of a graph G.

Theorem

Let G be a **forest**. *Then, G has C_k-b-coloring if and only if G has a subset of trees T, each one with diameter at least 2, such that $\Sigma_{T_j \in T}(diam(T_j) - 1) \geq k$.*

Question: what is the complexity of determining if any bipartite graph G has a b-homomorphism to C_k?

Partial positive answer: G has such b-homomorphism whenever $girth(G) = k$ or $girth(G) \geq 2k$ or $diam(G) \geq k + 2$.

The case $k = 3$

Theorem (Kratochvíl *et al.*,2002)

Let G be a planar bipartite graph. Then it can be decided in polynomial time if G has a 3-b-coloring.

Proposition

If a bipartite graph G with diam(G) ≤ 4 has a b-homomorphism to C_3, then it has a homomorphism to F_6 or to the extended F_6.

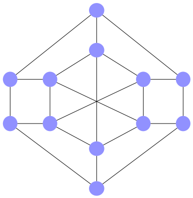


Figure: F_6

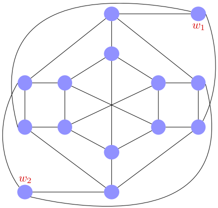


Figure: extended F_6

Partial result for $k = 3$

Theorem

Let G be a bipartite graph. If $diam(G)$ is at least 5 or G admits a **special** *homomorphism to F_6, then G has a 3-b-coloring.*

Question: What is the complexity of determining if a non-planar bipartite graph G has 3-b-coloring?

Thanks for your attention!

Aknowledgments: I would like to thanks the last two elected Presidents of Brazil for their effort to support research and education in Brazil. This thanks is specially relevant nowadays, when the 5th largest university in Brazil, the University of the State of Rio de Janeiro (UERJ), has its existence threatened.