On Generalizations of the Parking Permit Problem and Network Leasing Problems

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LAGOS 2017
September 11-15, 2017

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Introduction

Parking Permit Problem

Johnny goes to work every day

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He lives close to his job

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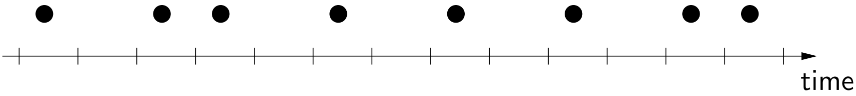
How can Johnny save money for a trip to Marseille?

Parking Permit Problem (2)



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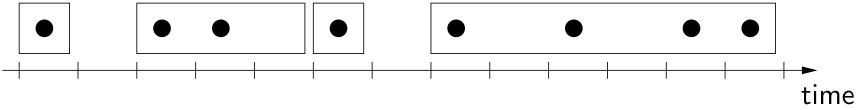

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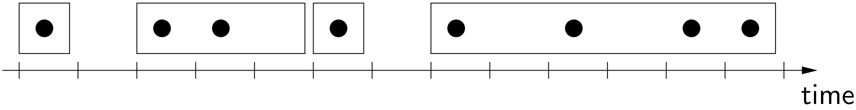

Solvable via dynamic programming (Meyerson, 2005)

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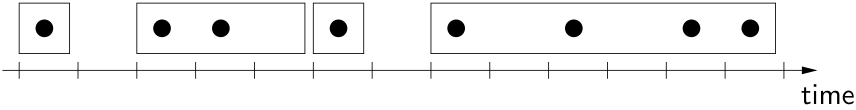

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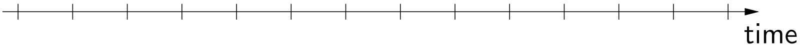
Not very realistic

- Johnny does not know the future!
- weather forecast → unreliable for long-term

Online Parking Permit Problem

length	cost
1	2
2	3
4	4



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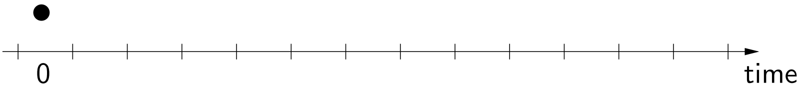

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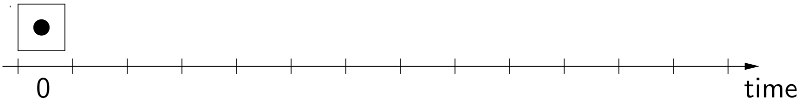

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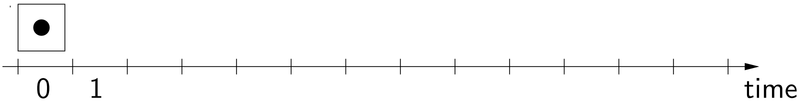

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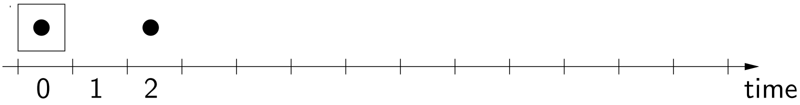

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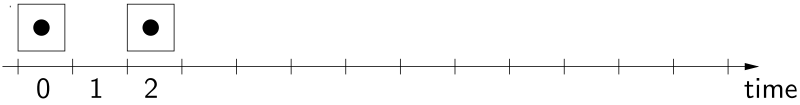

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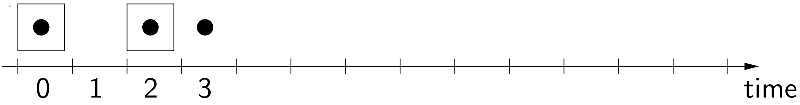

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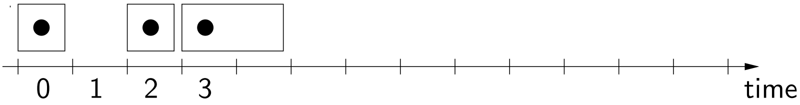

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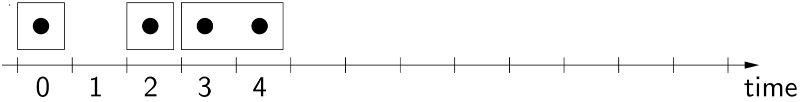

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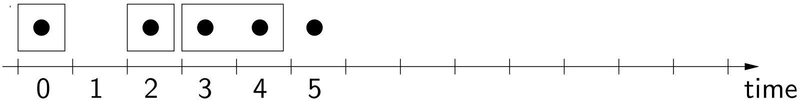

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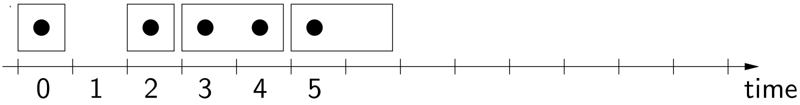

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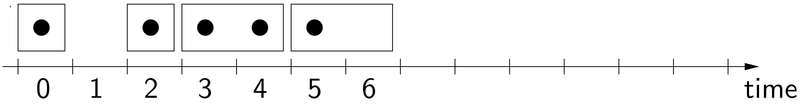

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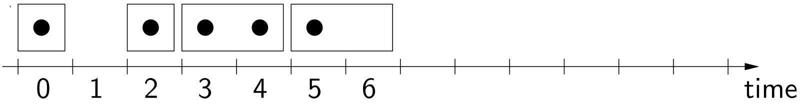

cost: 10

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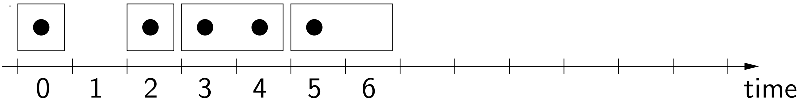

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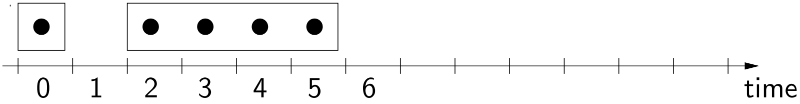

optimum costs 6

Online Parking Permit Problem (2)

(Meyerson, 2005)

- deterministic $\mathrm{O}(K)$-competitive algorithm
- deterministic $\Omega(K)$ lower bound

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Seminal problem for **leasing optimization**

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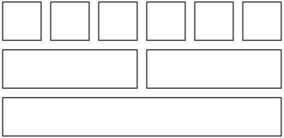
Imply results for **Steiner Leasing Problem** (Meyerson, 2005)

- approximation by tree metrics
  (Bartal, 1996; Fakcharoenphol *et al.*, 2004)

Interval Model

Interval Model

Permit lengths divide each other



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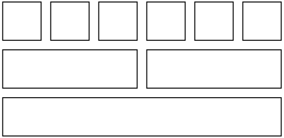
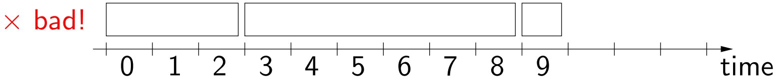


Permits start on multiples of their lenghts



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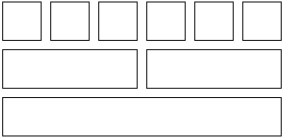

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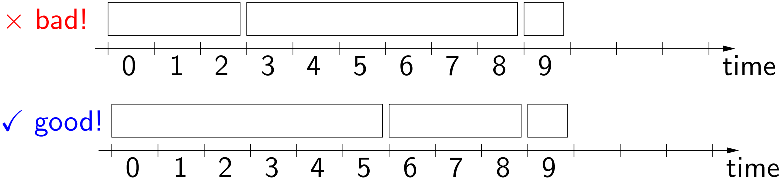

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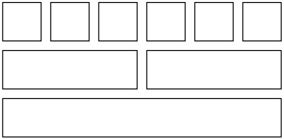

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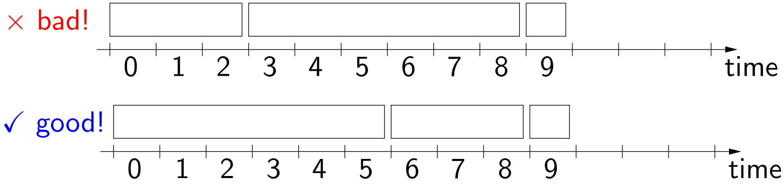

α-competitive under IM $\rightarrow$ 4α-competitive in general

Multi Parking Permit Problem

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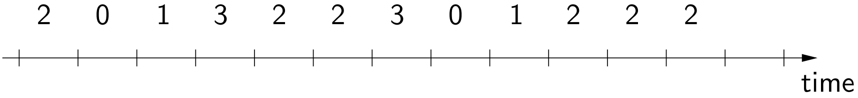

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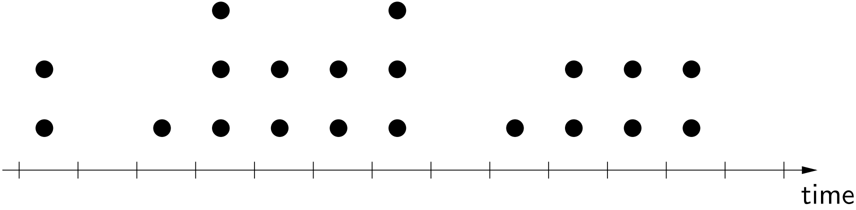

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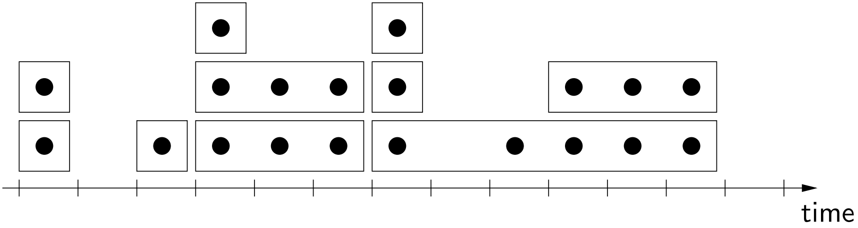
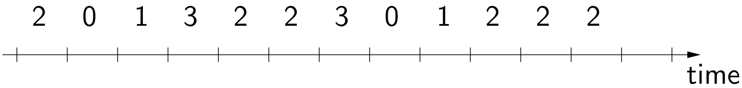


Hanoi Tower Property



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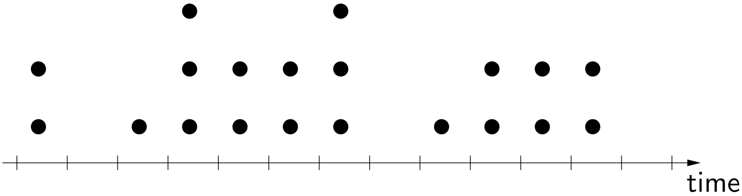

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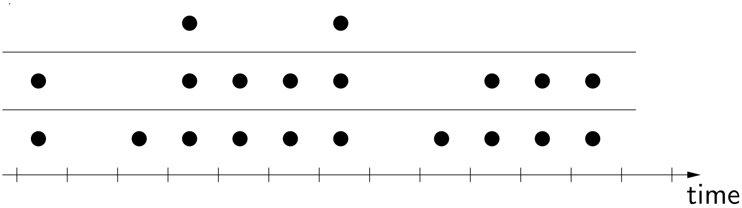

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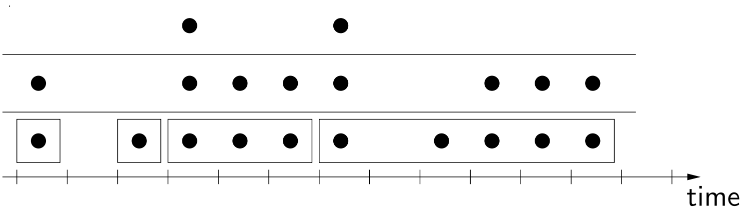

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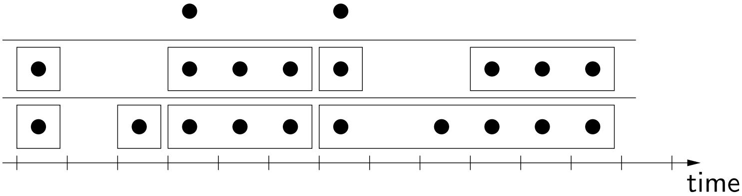

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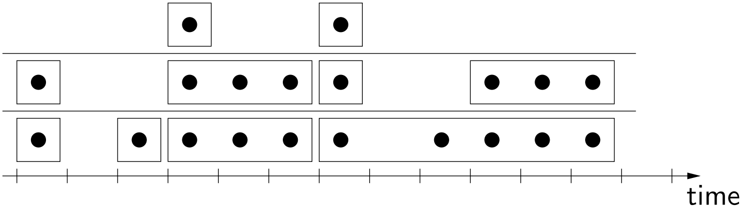

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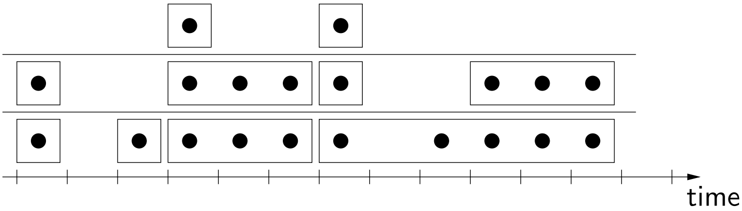

opt on each level $\rightarrow$ opt for the whole input

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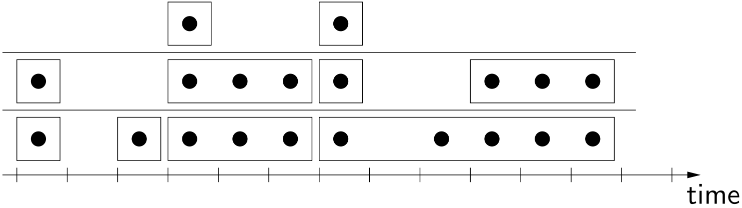

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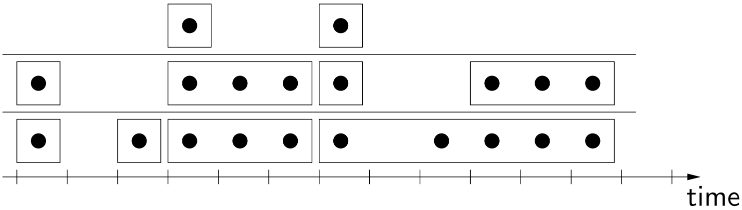

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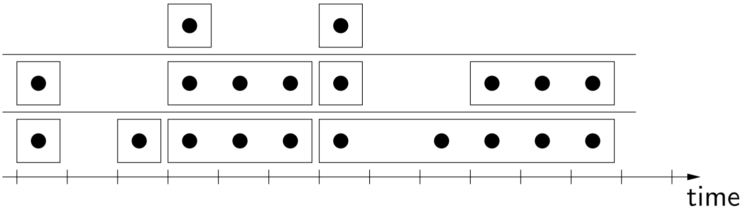

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Relies on Interval Model

Hanoi Tower Property (2)

No Interval Model $\rightarrow$ no Hanoi Tower Property!

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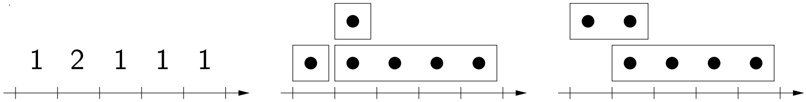

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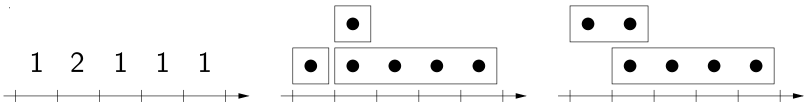

We conjecture that the problem is NP-hard if we do not assume the Interval Model.

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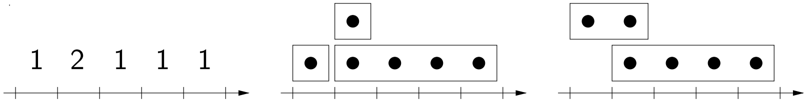

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Under Interval Model we have:

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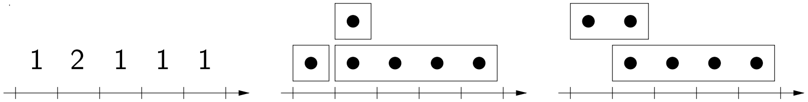

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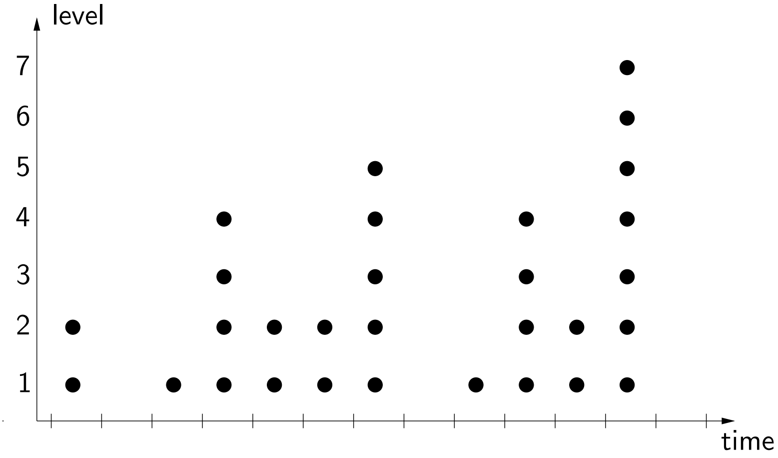
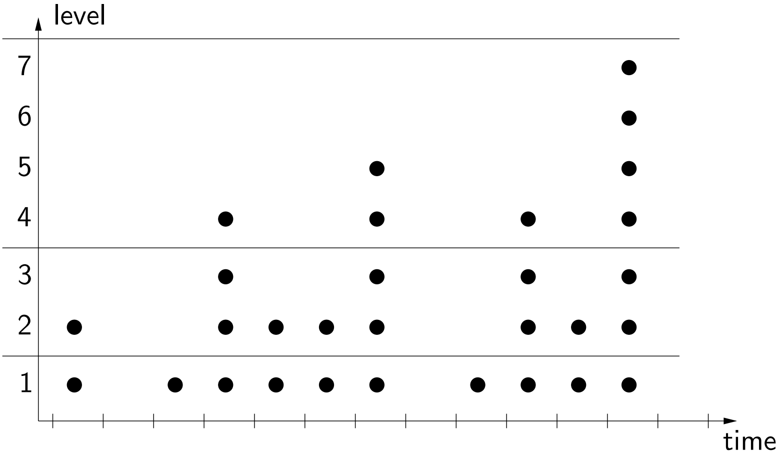


Poly-time approximation-preserving reduction



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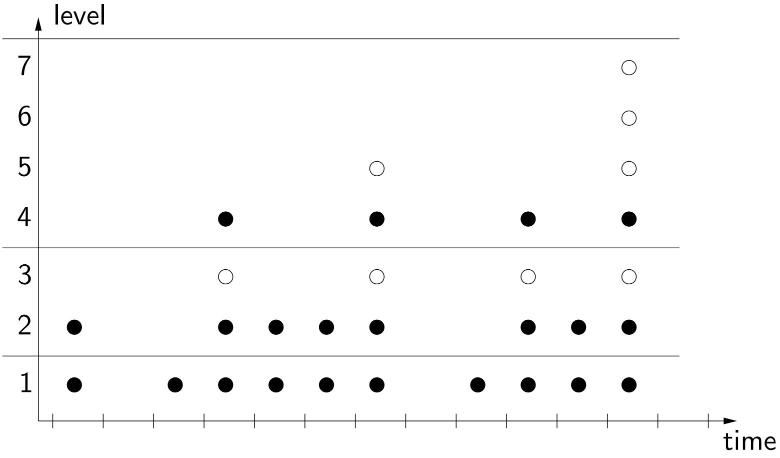

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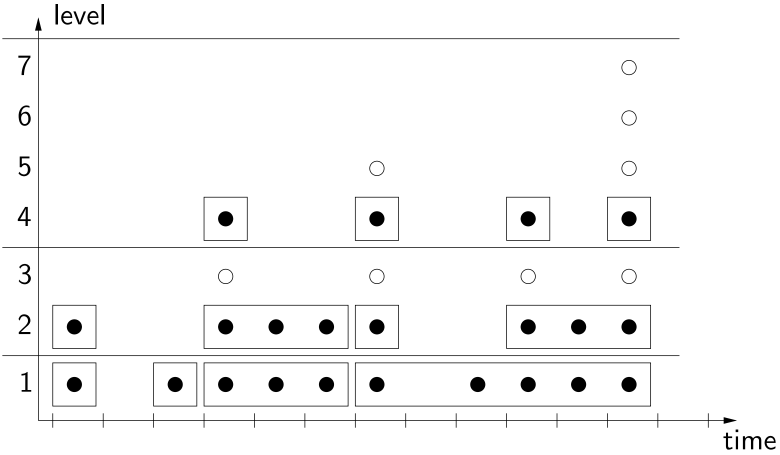

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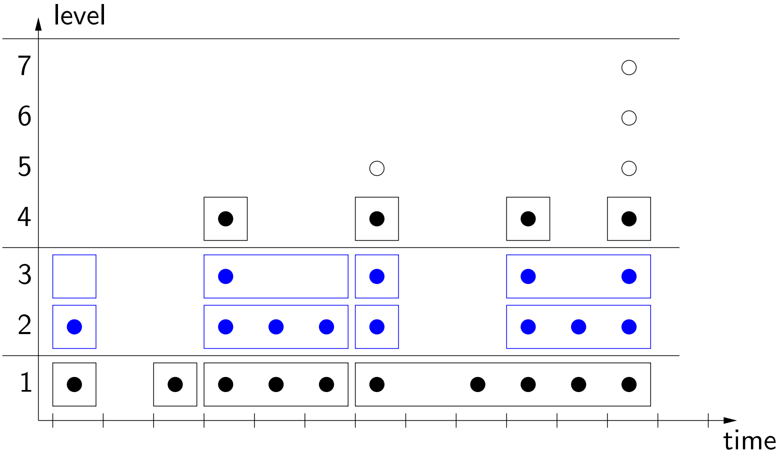

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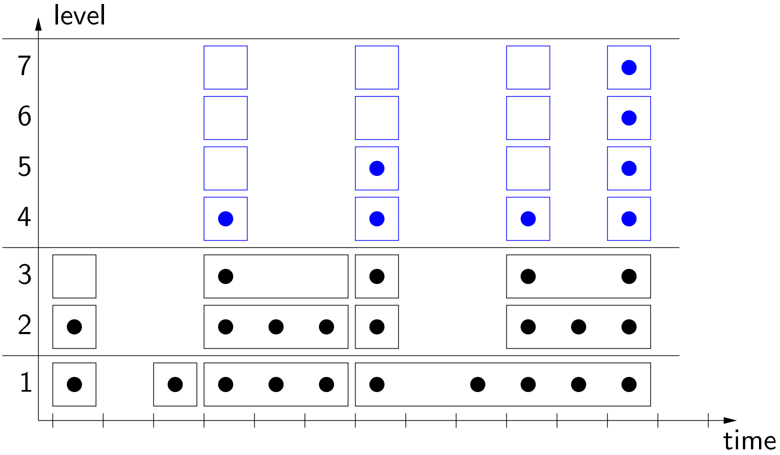

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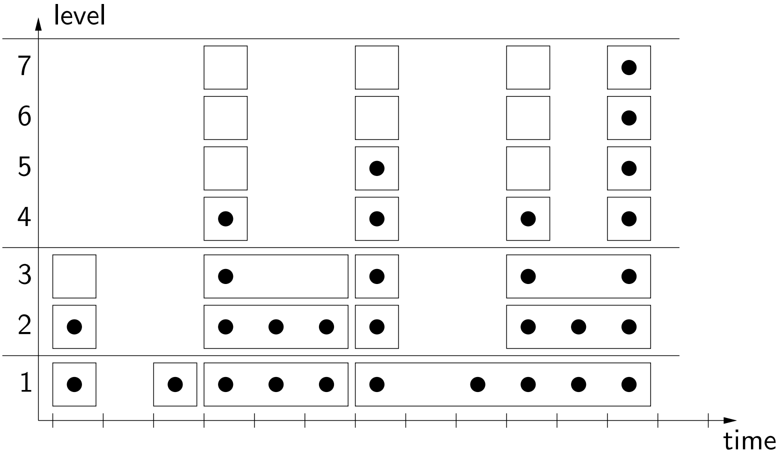

α-competitive Parking Permit $\rightarrow$ 2α-comp. Multi Parking Permit

Poly-time approximation-preserving reduction (2)

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- 2-approximation under Interval Model (8-approx. in general)

Poly-time approximation-preserving reduction (2)

α-competitive Parking Permit $\rightarrow$ 2α-comp. Multi Parking Permit

- 2-approximation under Interval Model (8-approx. in general)
- $\mathrm{O}(K)$-competitive deterministic online algorithm
- $\mathrm{O}(\lg K)$-competitive randomized online algorithm

Poly-time exact algorithm

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We represent multiset of permits $\rightarrow$ set of tuples: (t, ℓ, k, q)

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- $t \rightarrow$ starting time

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We represent multiset of permits $\rightarrow$ set of tuples: (t, ℓ, k, q)

- $t \rightarrow$ starting time
- $\ell \rightarrow$ first level covered
- $k \rightarrow$ permit type

Poly-time exact algorithm

We represent multiset of permits $\rightarrow$ set of tuples: (t, ℓ, k, q)

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- $\ell \rightarrow$ first level covered
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Poly-time exact algorithm

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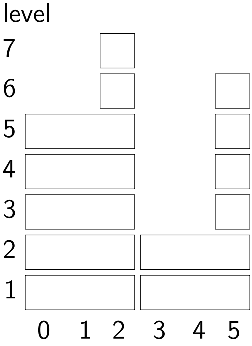

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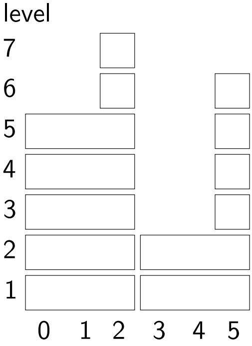

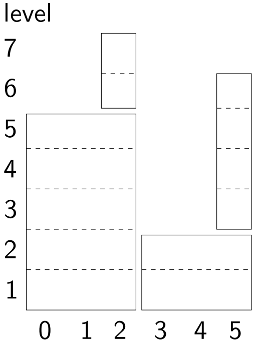

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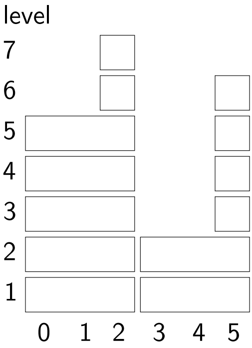

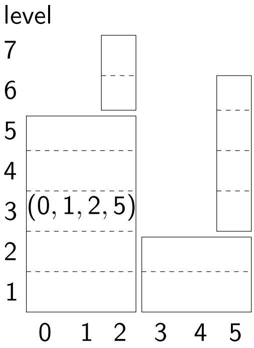

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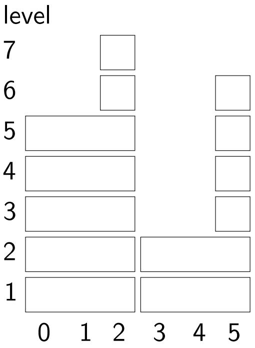

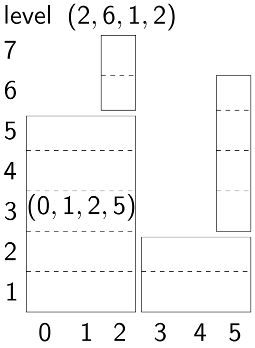

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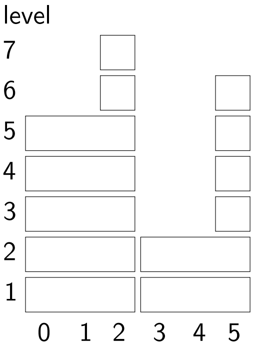

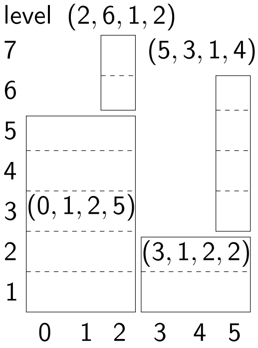

Poly-time exact algorithm (2)

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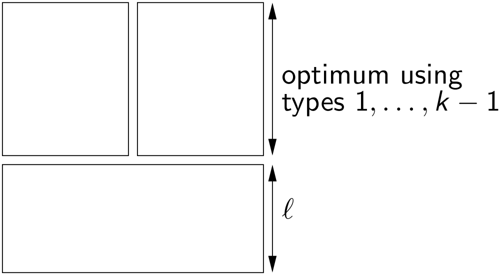

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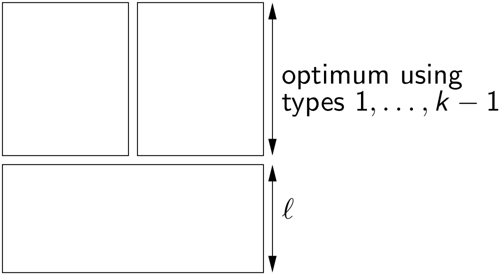

How do we find ℓ?

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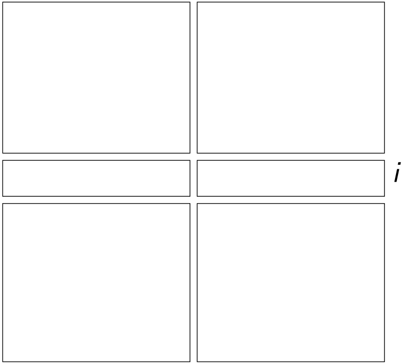

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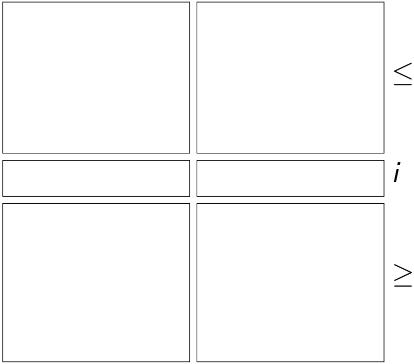

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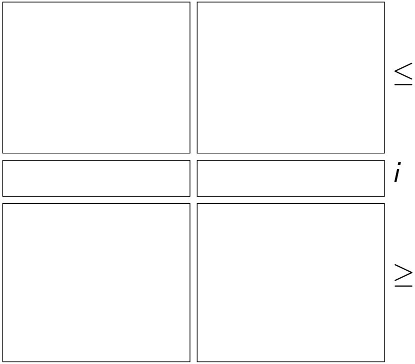

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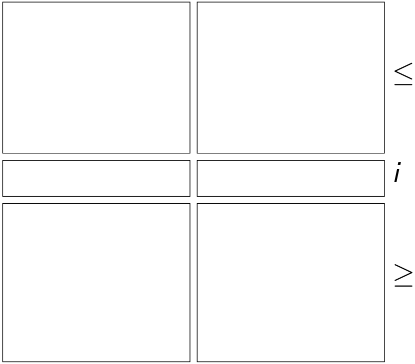

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ℓ: highest level s.t. optimum using types $1, \ldots, k-1$ is worse than permit of type k

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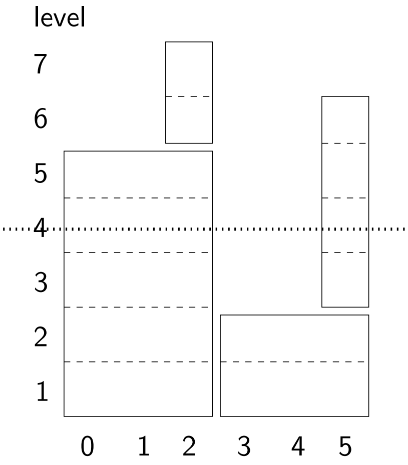

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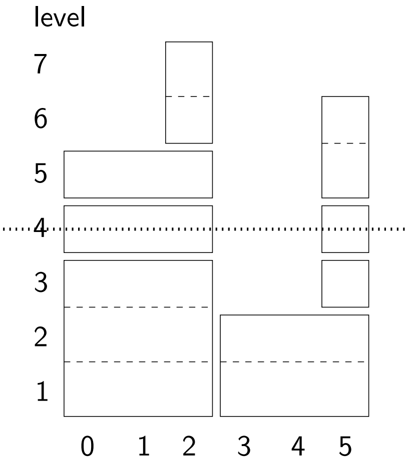

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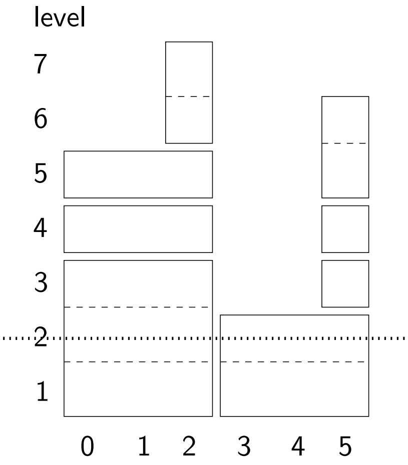

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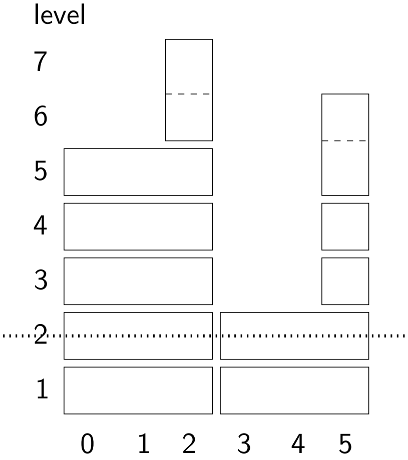

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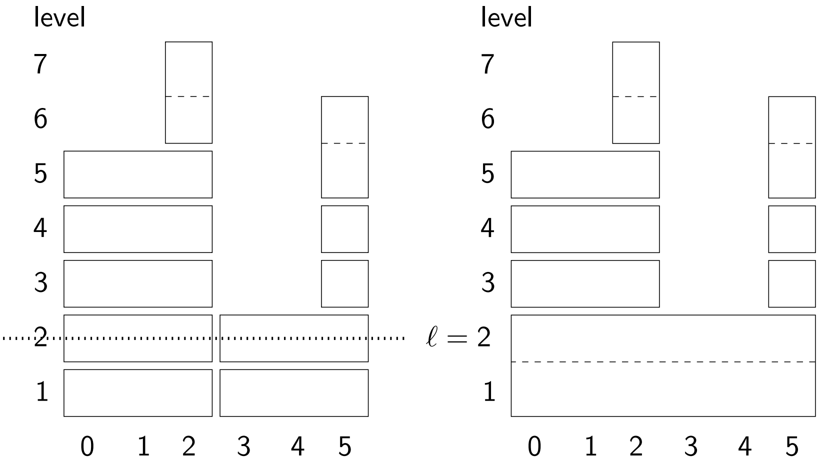

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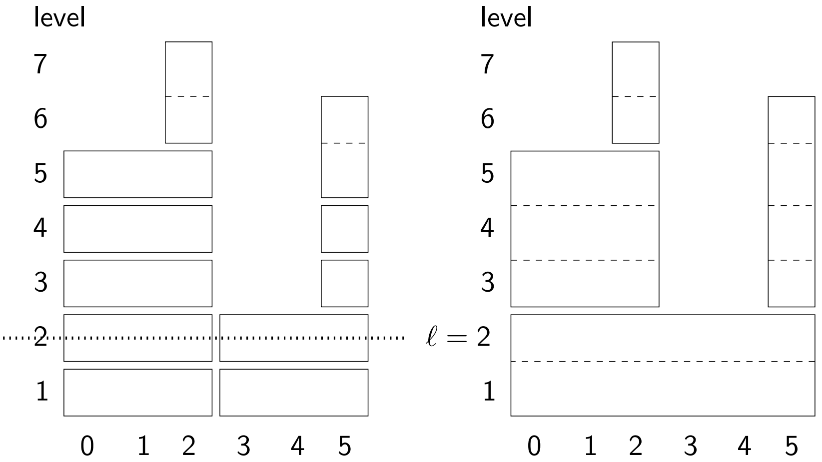

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2D Parking Permit $\rightarrow$ Leasing Buy-at-Bulk Network Design Problem

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Thank you!