A LINEAR-TIME ALGORITHM FOR THE IDENTIFYING CODE PROBLEM ON BLOCK GRAPHS

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Outline

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1. Identifying codes in graphs
2. The identifying code problem on block graphs
3. Concluding Remarks

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DEFINITION

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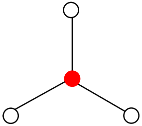

IDENTIFYING CODES IN GRAPHS

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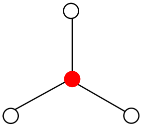

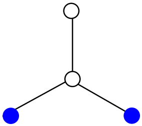

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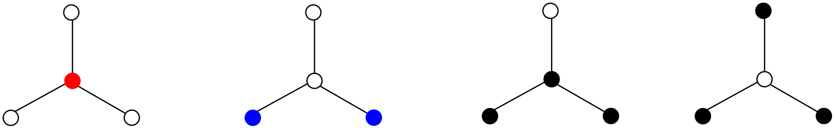

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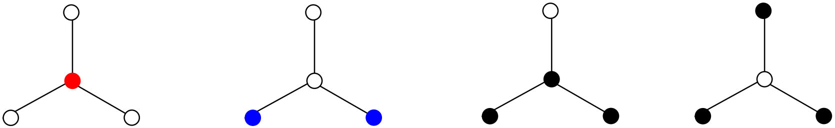

REMARK

G is **identifiable** if and only if it has no true twins, i.e., two nodes $i \neq j$ with $N[i] = N[j]$ [Karpovsky et al. 1998].

THE IDENTIFYING CODE PROBLEM

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Study the identifying code problem of **block graphs**.

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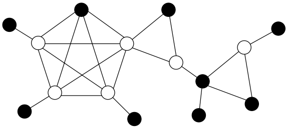

A block graph B (the black vertices form an identifying code of B).

OUTLINE

Previous work on the identifying code problem for trees

Known results

- A tree is a particular case of a block graph.
- Trees have been addressed in the context of the identifying code problem [Auger 2014, Bertrand et al. 2005, Blidia et al. 2007, Karpovsky et al. 1998].
- There is a linear-time algorithm that solves the identifying code problem on trees [Auger 2014].

Let $G = (V, E)$ be a graph and $v \in V$.

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Let call P either any of the properties above or $\overline{\text{ID}}$, $\overline{\text{CO}}$, $\overline{\text{ADJ}}$ and $\overline{\text{FN}}$.

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Let $\Gamma_{\mathsf{P}_1,\ldots,\mathsf{P}_k}(v,G)$ the function that returns the minimum size of a $\{v\}$-almost ID code in G satisfying P_i with $i = 1,\ldots,k$ or ∞ if no such code exists.

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It can be proved that for any graph G and $v \in V(G)$ it holds:

$$\Gamma_{\mathsf{ID}}(G) = min \begin{cases} \Gamma_{\mathsf{ID,CO,ADJ,FN}}(v,G) \\ \Gamma_{\mathsf{ID,CO,ADJ,\overline{FN}}}(v,G) \\ \Gamma_{\mathsf{ID,CO,\overline{ADJ}}}(v,G) \\ \Gamma_{\mathsf{ID,\overline{CO},ADJ}}(v,G) \end{cases}$$

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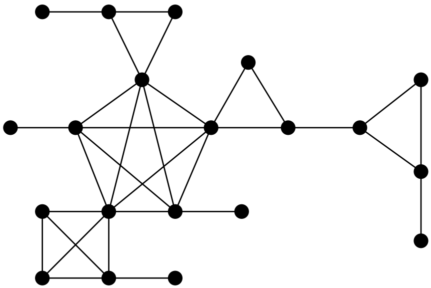

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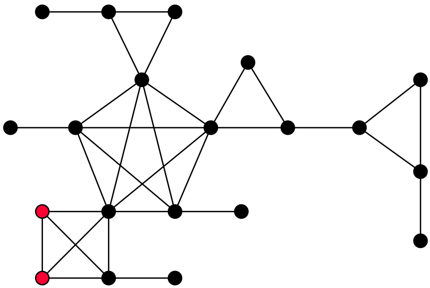

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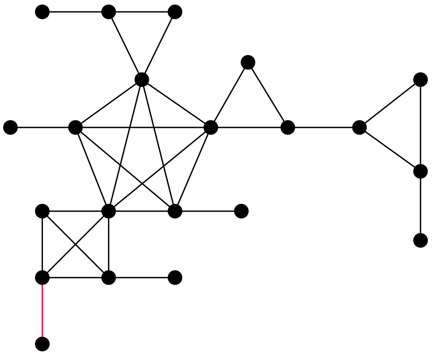

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- If B is a block graph, $v_1 \in V(B)$ and K is a maximal clique with $V(K) = \{v_1, v_2, \ldots, v_k\}$ then if we delete all the edges in K we obtain k block subgraphs, say $B_1, B_2, \ldots, B_k$ containing $v_1, v_2, \ldots, v_k$ respectively.

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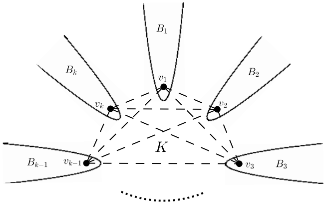

THE IDENTIFYING CODE PROBLEM ON BLOCK GRAPHS

THEOREM

Let B be a block graph and $v_1, v_2, \ldots, v_k$ the vertices of the maximal clique K. Let $B_1, B_2, \ldots, B_k$ be the block graphs, containing $v_1, v_2, \ldots, v_k$ respectively, obtained from B by deletion of the edges in K. Let C be a code in B and $C_i = C \cap V(B_i)$ for all $i \in \{1, 2, \ldots, k\}$.

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Let B be a block graph, K a maximal clique $V(K) = \{v_1, v_2, \ldots, v_k\}$. Let C_i be a v_i-almost ID code in B_i, $\forall\, i \in \{1, 2, \ldots, k\}$ and $C = \bigcup_{i=1}^{k} C_i$, then

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- *If $u, v \in V' = V(B) - V(K)$, they are dominated and separated by C.*
- *Let $v \in V'$ and $v_j \in V(K)$ such that $d(v, v_j) = 1$. Then v and v_j are dominated and separated by C if there is $i \in \{1, 2, \ldots, k\}$ $i \neq j$ such that C_i is CO.*

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- *Let $v \in V'$ and $v_j \in V(K)$ such that $d(v, v_j) = 2$. Then v and v_j are dominated and separated by C if C_j satisfies CO or there is $i \in \{1, 2, \ldots, k\}$ $i \neq j$ such that C_i is CO and $v \notin V(B_i)$.*

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- *If $v_i, v_j \in V(K)$ with $i \neq j$ then v_i and v_j are dominated and separated by C if either C_i is ADJ or C_j is ADJ.*

Remind that given $v \in V(B)$,

$$\Gamma_{\mathsf{ID}}(B) = min \begin{cases} \Gamma_{\mathsf{ID,CO,ADJ,FN}}(v,B) \\ \Gamma_{\mathsf{ID,CO,ADJ,\overline{FN}}}(v,B) \\ \Gamma_{\mathsf{ID,CO,\overline{ADJ}}}(v,B) \\ \Gamma_{\mathsf{ID,\overline{CO},ADJ}}(v,B) \end{cases}$$

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How to compute $\Gamma_{\mathsf{ID,CO,ADJ,FN}}(v,B)$?

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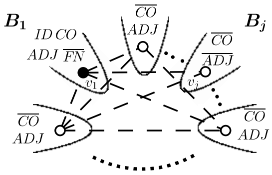

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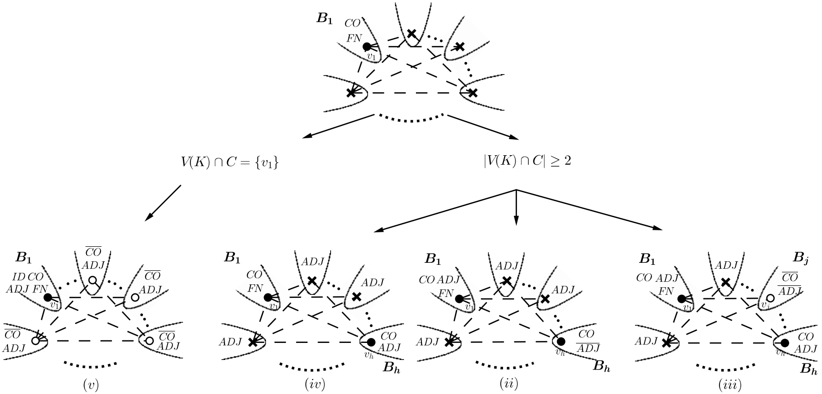

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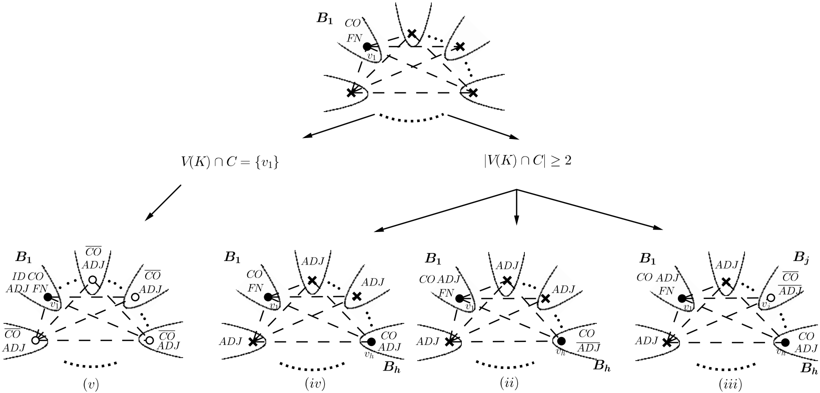
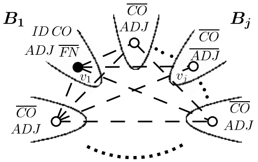


Conversely, C_i is a v_i-almost ID of B_i $\forall i \in \{1, \ldots, k\}$ satisfying any of $(i), (ii), (iii), (iv), (v)$ then C satisfies the properties ID, CO, ADJ and FN.

Hence, if C_i is a v_i-almost ID of B_i $\forall i \in \{1,\ldots,k\}$ satisfying (i), then

- C_1 satisfies ID, CO, ADJ, $\overline{\text{FN}}$,
- $\exists j \in \{2,3,\ldots,k\}$ such that C_j satisfies $\overline{\text{CO}}$, $\overline{\text{ADJ}}$,
- $\forall i \in \{2,3,\ldots,k\}$, $i \neq j$, C_i satisfies $\overline{\text{CO}}$, ADJ.



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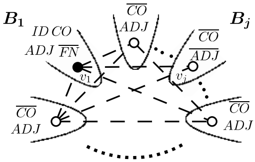

In order to obtain $\Gamma_{\text{ID,CO,ADJ,FN}}(v,B)$ we need to compute:

- $\Gamma_{\text{ID,CO,ADJ,}\overline{\text{FN}}}(v_1,B_1)$
- $\Gamma_{\overline{\text{CO}},\overline{\text{ADJ}}}(v_j,B_j)$ for only one $j \in \{1,\ldots,k\}$
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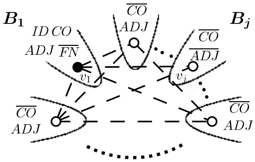

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- $\Gamma_{\overline{\text{CO}},\text{ADJ}}(v_i,B_i)$ for all $i \neq j$

And all functions corresponding to the cases $(ii),(iii),(iv),(v)$

Name	Function	Name	Function
f_1	$\Gamma_{\text{ID,CO,ADJ,FN}}$	f_6	$\Gamma_{\text{CO,ADJ,}\overline{\text{FN}}}$
f_2	$\Gamma_{\text{ID,CO,ADJ,}\overline{\text{FN}}}$	f_7	$\Gamma_{\text{CO,}\overline{\text{ADJ}}\text{,FN}}$
f_3	$\Gamma_{\text{ID,CO,}\overline{\text{ADJ}}}$	f_8	$\Gamma_{\text{CO,}\overline{\text{ADJ}}\text{,}\overline{\text{FN}}}$
f_4	$\Gamma_{\text{ID,}\overline{\text{CO}}\text{,ADJ}}$	f_9	$\Gamma_{\overline{\text{CO}}\text{,ADJ}}$
f_5	$\Gamma_{\text{CO,ADJ,FN}}$	f_{10}	$\Gamma_{\overline{\text{CO}}\text{,}\overline{\text{ADJ}}}$

List of main functions

Name	Function	Name	Function
f_1	$\Gamma_{\text{ID,CO,ADJ,FN}}$	f_6	$\Gamma_{\text{CO,ADJ,}\overline{\text{FN}}}$
f_2	$\Gamma_{\text{ID,CO,ADJ,}\overline{\text{FN}}}$	f_7	$\Gamma_{\text{CO,}\overline{\text{ADJ}}\text{,FN}}$
f_3	$\Gamma_{\text{ID,CO,}\overline{\text{ADJ}}}$	f_8	$\Gamma_{\text{CO,}\overline{\text{ADJ}}\text{,}\overline{\text{FN}}}$
f_4	$\Gamma_{\text{ID,}\overline{\text{CO}}\text{,ADJ}}$	f_9	$\Gamma_{\overline{\text{CO}}\text{,ADJ}}$
f_5	$\Gamma_{\text{CO,ADJ,FN}}$	f_{10}	$\Gamma_{\overline{\text{CO}}\text{,}\overline{\text{ADJ}}}$

List of main functions

Name	Function	Name	Function
f_{11}	$\Gamma_{\text{CO,}\overline{\text{ADJ}}} = min\{f_7, f_8\}$	f_{15}	$\Gamma_{\overline{\text{CO}}} = min\{f_9, f_{10}\}$
f_{12}	$\Gamma_{\text{CO,ADJ}} = min\{f_5, f_6\}$	f_{16}	$\Gamma_{\text{ADJ}} = min\{f_5, f_6, f_9\}$
f_{13}	$\Gamma_{\text{CO,FN}} = min\{f_5, f_7\}$	f_{17}	$\Gamma_{\text{ID,CO,ADJ}} = min\{f_1, f_2\}$
f_{14}	$\Gamma_{\text{CO,}\overline{\text{FN}}} = min\{f_6, f_8\}$		

List of auxiliary functions

$$f_1(v_1,B) = \Gamma_{\text{ID,CO,ADJ,FN}}(v_1,B) =$$

$$= min \begin{cases} f_2(v_1,B_1) + \min\limits_{j=2,\dots,k} \left\{ f_{10}(v_j,B_j) + \sum\limits_{\substack{i=2 \\ i\neq j}}^{k} f_9(v_i,B_i) \right\} \\ f_5(v_1,B_1) + \min\limits_{h=2,\dots,k} \left\{ f_{11}(v_h,B_h) + \sum\limits_{\substack{i=2 \\ i\neq h}}^{k} f_{16}(v_i,B_i) \right\} \\ f_5(v_1,B_1) + \min\limits_{\substack{j,h=2,\dots,k \\ j\neq h}} \left\{ f_{10}(v_j,B_j) + f_{12}(v_h,B_h) + \sum\limits_{\substack{i=2 \\ i\neq j,h}}^{k} f_{16}(v_i,B_i) \right\} \\ f_{13}(v_1,B_1) + \min\limits_{h=2,\dots,k} \left\{ f_{12}(v_h,B_h) + \sum\limits_{\substack{i=2 \\ i\neq h}}^{k} f_{16}(v_i,B_i) \right\} \\ f_1(v_1,B_1) + \sum\limits_{i=2}^{k} f_9(v_i,B_i) \end{cases}$$

Given $v \in V(B)$ and according to the given functions,

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$$\Gamma_{\mathsf{ID}}(B) = min\{f_1(v,B), f_2(v,B), f_3(v,B), f_4(v,B)\}.$$

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Algorithm ICB

Input: a connected block graph B and its list of maximal cliques.
Output: $\Gamma_{\mathsf{ID}}(B)$.

1: randomly select a vertex v_1 and call RICB(v_1, B);
2: return $\Gamma_{\mathsf{ID}}(v_1, B) = min\{f_1(v_1,B), f_2(v_1,B), f_3(v_1,B), f_4(v_1,B)\}$.

Given $v \in V(B)$ and according to the given functions,

$$\Gamma_{\mathsf{ID}}(B) = min\{f_1(v,B), f_2(v,B), f_3(v,B), f_4(v,B)\}.$$

Algorithm ICB
Input: a connected block graph B and its list of maximal cliques. **Output:** $\Gamma_{\mathsf{ID}}(B)$. 1: randomly select a vertex v_1 and call RICB(v_1, B); 2: return $\Gamma_{\mathsf{ID}}(v_1, B) = min\{f_1(v_1,B), f_2(v_1,B), f_3(v_1,B), f_4(v_1,B)\}$.

Algorithm ICB randomly selects a vertex v_1 in block graph B and calls Algorithm RICB(v_1, B) that computes the values of all 10 functions in a recursive manner in smaller and smaller block graphs.

Initialization: List L when $V(B) = \{v\}$

Name	Function	$f_j(v,\{v\})$	Name	Function	$f_j(v,\{v\})$
f_1	$\Gamma_{\text{ID,CO,ADJ,FN}}$	∞	f_6	$\Gamma_{\text{CO,ADJ},\overline{\text{FN}}}$	∞
f_2	$\Gamma_{\text{ID,CO,ADJ},\overline{\text{FN}}}$	∞	f_7	$\Gamma_{\text{CO},\overline{\text{ADJ}},\text{FN}}$	∞
f_3	$\Gamma_{\text{ID,CO},\overline{\text{ADJ}}}$	1	f_8	$\Gamma_{\text{CO},\overline{\text{ADJ}},\overline{\text{FN}}}$	1
f_4	$\Gamma_{\text{ID},\overline{\text{CO}},\text{ADJ}}$	∞	f_9	$\Gamma_{\overline{\text{CO}},\text{ADJ}}$	∞
f_5	$\Gamma_{\text{CO,ADJ,FN}}$	∞	f_{10}	$\Gamma_{\overline{\text{CO}},\overline{\text{ADJ}}}$	0

THE LINEAR-TIME ALGORITHM FOR THE IDENTIFYING CODE PROBLEM ON BLOCK GRAPHS

Initialization: List L when $V(B) = \{v\}$

Name	Function	$f_j(v, \{v\})$	Name	Function	$f_j(v, \{v\})$
f_1	$\Gamma_{\text{ID,CO,ADJ,FN}}$	∞	f_6	$\Gamma_{\text{CO,ADJ,}\overline{\text{FN}}}$	∞
f_2	$\Gamma_{\text{ID,CO,ADJ,}\overline{\text{FN}}}$	∞	f_7	$\Gamma_{\text{CO,}\overline{\text{ADJ}}\text{,FN}}$	∞
f_3	$\Gamma_{\text{ID,CO,}\overline{\text{ADJ}}}$	1	f_8	$\Gamma_{\text{CO,}\overline{\text{ADJ}}\text{,}\overline{\text{FN}}}$	1
f_4	$\Gamma_{\text{ID,}\overline{\text{CO}}\text{,ADJ}}$	∞	f_9	$\Gamma_{\overline{\text{CO}}\text{,ADJ}}$	∞
f_5	$\Gamma_{\text{CO,ADJ,FN}}$	∞	f_{10}	$\Gamma_{\overline{\text{CO}}\text{,}\overline{\text{ADJ}}}$	0

Algorithm RICB

Input: a block graph B, its list of maximal cliques and $v_1 \in V(B)$.
Output: the list L of the values of the ten functions f_j on (v_1, B).

if v_1 has degree 0 in B **then**
  initialize L;
else
  let K be a maximal clique with $V(K) = \{v_1, \ldots, v_k\}$ and delete its edges;
  let $B_1, \ldots, B_k$ be the remaining block graphs, resp., containing $v_1, \ldots, v_k$;
  let $L_i = RICB(v_i, B_i)$ for all $i \in \{1, \ldots, k\}$;
  compute the ten functions on (v_1, B) from L_i for all $i \in \{1, \ldots, k\}$.
end if
return the list L of the values of the ten functions f_j on (v_1, B).

The linear-time algorithm for the identifying code problem on block graphs

Theorem

For each of the ten functions f_j, we can compute $f_j(v_1,B)$ from $L_i(v_i,B_i)$ for all $i \in \{1,\ldots,k\}$ in time $O(k)$.

The linear-time algorithm for the identifying code problem on block graphs

Theorem

For each of the ten functions f_j, we can compute $f_j(v_1, B)$ from $L_i(v_i, B_i)$ for all $i \in \{1, \ldots, k\}$ in time $O(k)$.

Theorem

Algorithm ICB computes in linear time $\Gamma_{ID}(B)$ of an identifiable block graph B (or returns ∞ if no identifying code exists in B).

THE LINEAR-TIME ALGORITHM FOR THE IDENTIFYING CODE PROBLEM ON BLOCK GRAPHS

COROLLARY

If B is a vertex-weighted block graph, the ICB can be easily modified in order to return the minimum weighted identifying code number.

The linear-time algorithm for the identifying code problem on block graphs

Corollary

If B is a vertex-weighted block graph, the ICB can be easily modified in order to return the minimum weighted identifying code number.

Corollary

Algorithm ICB could be modified in order to obtain an identifying code of minimum size.

Outline

1. Identifying codes in graphs
2. The identifying code problem on block graphs
3. Concluding Remarks

The Identifying code Problem (ICP):

- Find $C \subseteq V$ of minimum size such that $C \cap N[i] \neq \emptyset$ and $C \cap N[i] \neq C \cap N[j]$, $i,j \in V$.

Future work

The Identifying code Problem (ICP):

- Find $C \subseteq V$ of minimum size such that $C \cap N[i] \neq \emptyset$ and $C \cap N[i] \neq C \cap N[j]$, $i,j \in V$.

Problems related to the Identifying Code Problem in graphs:

- Locating dominating Problem (LDP)
  Find $L \subseteq V$ of minimum size such that $L \cap N[i] \neq \emptyset$ and $L \cap N(i) \neq L \cap N(j)$, $i,j \in V - L$.

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- Open locating dominating Problem (OLDP)
  Find $O \subseteq V$ of minimum size such that $O \cap N(i) \neq \emptyset$ and $O \cap N(i) \neq O \cap N(j)$, $i,j \in V$.

Future work

The Identifying code Problem (ICP):

- Find $C \subseteq V$ of minimum size such that $C \cap N[i] \neq \emptyset$ and $C \cap N[i] \neq C \cap N[j]$, $i,j \in V$.

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- Locating dominating Problem (LDP)
  Find $L \subseteq V$ of minimum size such that $L \cap N[i] \neq \emptyset$ and $L \cap N(i) \neq L \cap N(j)$, $i,j \in V - L$.
- Open locating dominating Problem (OLDP)
  Find $O \subseteq V$ of minimum size such that $O \cap N(i) \neq \emptyset$ and $O \cap N(i) \neq O \cap N(j)$, $i,j \in V$.

Our future goals:

- Adapt the linear-time algorithm for ICP to LDP and OLDP.
- Extend the results to graphs whose 2-connected components are cycles or complete bipartite graphs.

Thanks