# Stress tests for lapse risk: correlation and contagion among policyholders' behaviours

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Xavier Milhaud, ISFA
Joint work with F. Barsotti and Y. Salhi

# A word on the lapse risk

What is the **lapse risk** ?
→ Due to death, maturity, change of premium level, surrender, ...

Why so much interest ?

1. Among the 3 major risks in life insurance ;
2. Understand the behaviours, and design new products ;
3. Predictions : segmentation and risk management (ALM).

## Lapse classification in life insurance

The 2 main historical explanations for surrenders are ([Out90])

- *liquidity needs* → idiosynchratic → structural surrenders ;
- *economic distress* → environment → temporary surrenders.

**Current context** : never experienced such low interest rates ⇒ impact on the underwriting of new business...

**Threat** : massive (temporary) surrenders due to ↗ of interest rate.

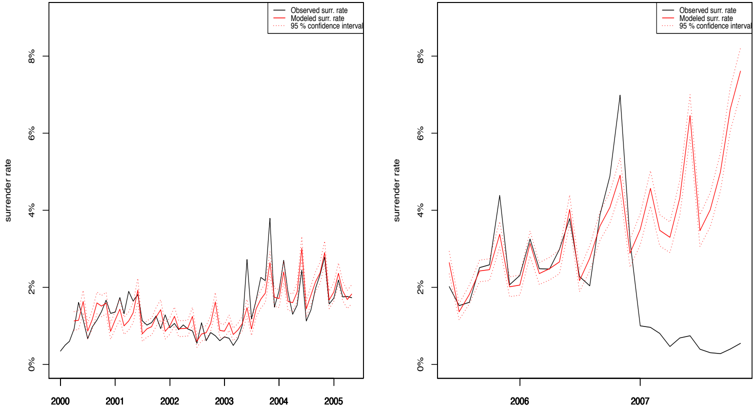
# Estimation of structural lapses

Tables with profiles, e.g. yearly structural lapse rates (LR) for 9 segments :

| | Bank | Agent | Direct |
|---|---|---|---|
| $[0, 4]$ years | 5% | 8% | 15% |
| $[4, 8]$ years | 3% | 10% | 15% |
| $> 8$ years | 10% | 19% | 20% |

→ Segment the population with a priori discriminant risk factors.
→ Empirical estimation / model-based estimates (GLM, ...).
→ **Associated important assumptions**...

# Model-based example : logit on spanish Endowments




Crisis not captured (despite financial covariates).

→ GLM / survival models missed something....

## Regulation : QIS 5 (EIOPA)

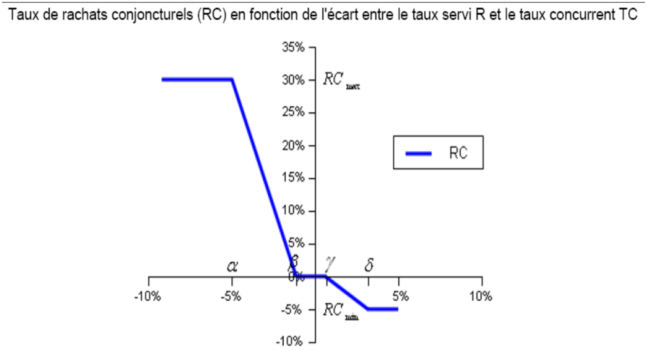
Compute the SCR in 2 steps, and keep the max. b/w (1) and (2).

**Step (1) : shocks applied to structural LR (misestimation).**
$LR_{up} = \min(100\%, 150\% \times LR)$ → our context !
$LR_{down} = \min(0, \max(50\% \times LR, LR - 20\%))$.

**Step (2) : mass lapse event, ∼ "bank run".**
*30%-loss of the sum of positive surrender strain over portfolio* ;
→ Empirics to calibrate mass lapse event is poor ;
→ Should be adjusted to the type of life insurance policy...

*"Steps (1) AND (2) incorporate temporary lapses"*

## Internal model and practical approach (S-shaped)




At the end... $LR_{shocked} = \min(1, \max(0, RS + RC)).$

## Still some issues to deal with

+ Pros :
  - easy-to-understand, easy-to-implement,
  - integrates (artificially) copycat behaviours → correlation risk ;

- Cons :
  - not fully realistic ;
  - this is a static model...(does not depend on time $t$),
  - does not consider the contagion between policyholders...

⇒ We'd like to introduce a model that copes with both correlation and contagion risks to define extreme scenarios.

# Intensity models - Hawkes process

Intensity models are often used in mortgage prepayments (most of them of Cox-type). **Focus here on Hawkes-type processes** [HO74].

Counting process s.t.

$$\lambda_t = \lambda_\infty + (\lambda_0 - \lambda_\infty)e^{-\beta t} + \alpha \int_0^t e^{-\beta(t-s)} dN_s,$$

→ Path-dependent stochastic intensity ;
→ Piecewise deterministic : “internal” source of excitation ;
→ No correlation...

## Dynamic contagion process for the lapse intensity

→ Extended Hawkes, [DZ11].
→ $(N_t)_{t\geq 0}$ : counting process of lapses over the whole portfolio.

$$\lambda_t = \lambda_\infty + (\lambda_0 - \lambda_\infty) e^{-\beta t} + \sum_{i\geq 1} X_i \, e^{-\beta(t-T_i)} 1_{T_i \leq t} + \sum_{j\geq 1} Y_j \, e^{-\beta(t-\hat{T}_j)} 1_{\hat{T}_j \leq t} \tag{1}$$

where lapses occur at $T_i$, and $X_i$, $Y_j$ are magnitudes of jumps.

1. Structural surrender forces $\lambda_0$, and $\lambda_\infty$ (constant here),
2. Temporary surrenders, with
   - endogenous shocks : **contagion**, internal ;
   - exogenous shocks : history of $\hat{N}_t$ → dynamic dependence, source of **correlation** in our setting (to be defined later).

## Cause of correlation : interest rate movements

→ Consider a contract with

- guaranteed return $R^g > 0$ : minimum profitability,
- **credited rate** $R_t^c$ : encompasses $R^g +$ potential profit benefit.

At the contract inception, $R^g \simeq 0$ for most of life insurers in 2015. Moreover, we have $R_0^c = R^g$.

→ Let $(r_t)_{t \geq 0}$ be the interest rate with GBM dynamics $(\mu, \sigma)$.

Q : in critical scenarios, how the surrender decision could be affected by the level of $r_t$ ?

Look at the following standardized spread :

$$RG_t^0 := \frac{r_t - R_0^c}{R_0^c}$$

→ Makes sense to $\nearrow$ the propensity to lapse when $RG_t^0 \nearrow$ ;

→ Say that policyholders would exercise their option to surrender at time $\hat{T}_1$ being the first time $RG_t^0$ hits a constant barrier $B > 0$.

→ Assume that the company can then adjust the credited rate $R_t^c$ depending on the interest rate level (to avoid massive lapses).

This defines the new standardized spread $RG_t^1$, given by

$$RG_t^1 = \frac{r_t - R^c_{\hat{T}_1}}{R^c_{\hat{T}_1}} = \frac{r_t - r_{\hat{T}_1}}{r_{\hat{T}_1}}, \qquad \hat{T}_1 \leq t < \infty.$$

Next adjustment will be operated as soon as $RG_t^1 = B$, and so on...

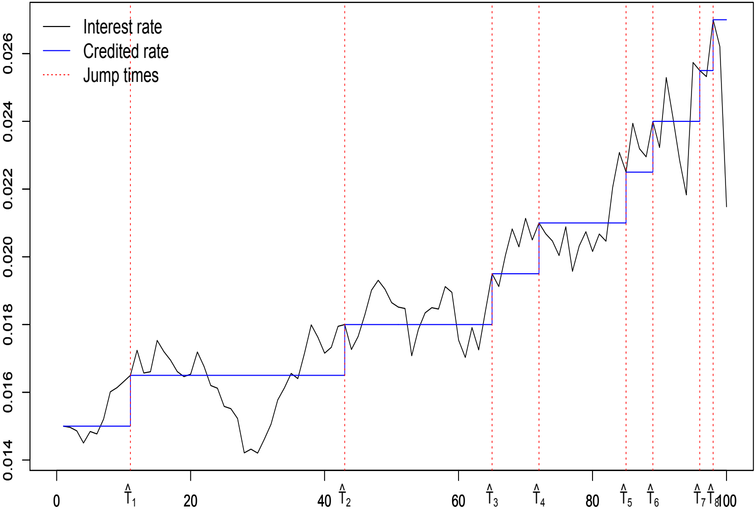
⇒ **These events thus characterize the sequence** $(\hat{T}_j)_{j=0,1,\ldots}$ s.t.

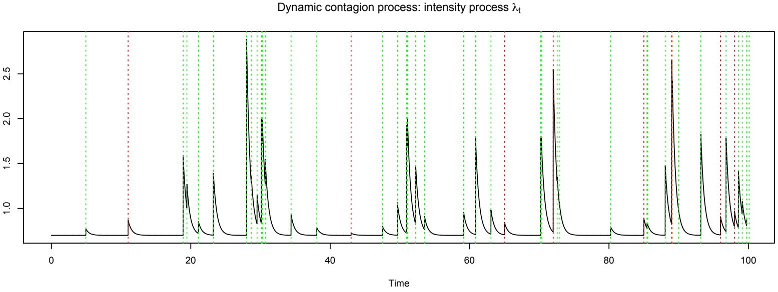
$$\hat{T}_{j+1} \quad = \quad \inf\{t > 0,\ RG_t^j = B\}, \tag{2}$$

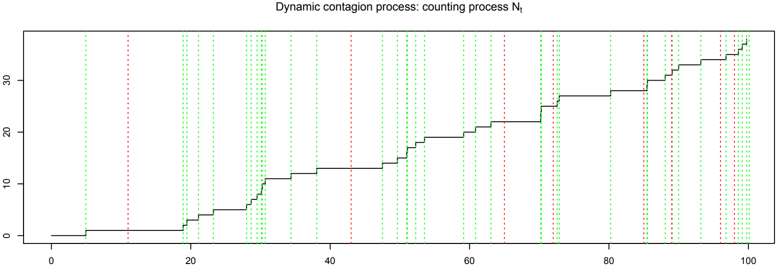
with $\hat{T}_0 = 0$ for convenience.

($\hat{N}_t = \sum_{j \geq 1} 1_{\hat{T}_j \leq t}$ : counting process associated to such events.)

# Adjustments of the credited rate




Dynamic contagion process: intensity process $\lambda_t$
2.5
2.0
1.5
1.0
0
20
40
60
80
100
Time


Dynamic contagion process: counting process $N_t$
30
20
10
0
0
20
40
60
80
100

## About external jumps at $\hat{T}_j$

$\rightarrow (r_t)_{t\geq 0}$ follows a GBM ($\log(r_t/r_0) = (\mu - \sigma^2/2)t + \sigma W_t$).
$\rightarrow (\hat{T}_j)_{j=0,1,\dots}$ are hitting times of the process $RG_t$.

$\Rightarrow$ The events $\hat{T}_j$ **can also be characterized as follows**

$$\hat{T}_j \quad = \quad \hat{T}_{j-1} \; + \; \inf\{t \geq 0, \quad \mu t + \sigma W_t = \log(1+B)\}.$$

$\Rightarrow$ Inter-arrival times $\Delta \hat{T}_j = \hat{T}_j - \hat{T}_{j-1}$ are i.i.d., with distribution $F$ ([SK91]) :

$$\Delta \hat{T}_j \sim \mathcal{IG}(\theta_1, \theta_2),$$

with $\theta_1 = 2\log(1+B)/(2\mu - \sigma^2)$, $\theta_2 = \log(1+B)^2/\sigma^2$.

$\rightarrow \lambda_t, (\lambda_t, N_t), (\lambda_t, N_t, \widehat{N}_t)$ not Markovian.

$\rightarrow \hat{T}_j = \sum_{k=1}^{j} \Delta \hat{T}_k$.

Introduce $P(\hat{T}_j \leq t) = F^{j*}(t)$.

$\rightarrow$ Denote by $h(t) = E[\hat{N}_t] = \sum_{j=0}^{\infty} P(\hat{N}_t \geq j)$, thus

$$h(t) = \sum_{j=0}^{\infty} F^{j*}(t). \tag{3}$$

The CDF $F^{j*}$ is still $\mathcal{IG}$, **with mean** $j\theta_1$ **and shape** $j\theta_2$.

## Moments of the lapse intensity

→ Let

$$m(t,\theta) = E[e^{\theta\lambda_t}],$$

and $m^{(n)}(t,\theta)$ : $n^{\text{th}}$ derivative of $m$ with respect to $\theta$.
We have

$$m^{(n)}(t,0) = E[\lambda_t^n].$$

→ Denote respectively $\xi(t,\theta)$ and $\widehat{\xi}(t,\theta)$ the m.g.f. of

$$Z_t = \sum_{i=1}^{N_t} X_i e^{\beta T_i} \quad \text{and} \quad \widehat{Z}_t = \sum_{j=1}^{\widehat{N}_t} Y_j e^{\beta \widehat{T}_j}. \tag{4}$$

$Z_t, \widehat{Z}_t$ are discounted compound renewal processes ([LGFW10]).

Similarly, $\xi^{(n)}(t,\theta)$ and $\widehat{\xi}^{(n)}(t,\theta)$ refer to the $n^{\text{th}}$ derivative $\xi(t,\theta)$ and $\widehat{\xi}(t,\theta)$ with respect to $\theta$.

→ $\lambda_t$ can be written in the following form

$$\lambda_t = (\lambda_\infty + (\lambda_0 - \lambda_\infty)e^{-\beta t}) + e^{-\beta t} Z_t + e^{-\beta t}\widehat{Z}_t.$$

⇒ We can then derive

1. the m.g.f. of $Z_t$ and $\widehat{Z}_t$ ;
2. the m.g.f. of $\lambda_t$ in function of those of $Z_t$ and $\widehat{Z}_t$ ;

⇒ At the end, we obtain a recursive formula.

## (1) Moment generating functions of $Z_t$ and $\widehat{Z}_t$

The m.g.f. $\xi$ and $\widehat{\xi}$ of $Z_t$ and $\widehat{Z}_t$ are given by **recursive formulas** :

$$E[e^{\theta Z_t}] = \xi(t,\theta) = ... + \int_0^t ... \xi(t-u, \theta e^{\beta u})\, m^{(1)}(u,0)\, du \quad (5)$$

$$E[e^{\theta \widehat{Z}_t}] = \widehat{\xi}(t,\theta) = ... + \int_0^t ... \widehat{\xi}(t-u, \theta e^{\beta u})\, dh(u), \quad (6)$$

→ We can derive the moments of the renewal processes ;
→ The first moment of the intensity $\lambda_t$ is key (self-excited) ;
→ Recall that $h(t) = E[\hat{N}_t] = \sum_{j=0}^{\infty} F^{j*}(t)$.

## (2) Moment generating function of $\lambda_t$

**Proposition.** For $n > 1$, the $n^{\text{th}}$ derivative of the surrender intensity m.g.f. is given recursively :

$$\begin{aligned} m^{(n)}(t,\theta) &= K(t,\lambda_0,\lambda_\infty)\, m^{(n-1)}(t,\theta) \\ &\quad + \sum_{i=0}^{n-1} G(i,n)\Big(I_i(t,\theta) + \widehat{I}_i(t,\theta)\Big)\, m^{(i)}(t,\theta), \end{aligned} \tag{7}$$

with $I_k$ and $J_k$ for $\{k = 1, 2, \dots\}$ given by

$$\begin{aligned} I_k(t,\theta) &= I_{k-1}^{(1)}(t,\theta) + H(I_{k-1}(t,\theta))\, \xi^{(1)}(t,\theta e^{-\beta t}), \\ \widehat{I}_k(t,\theta) &= \widehat{I}_{k-1}^{(1)}(t,\theta) + H'(\widehat{I}_{k-1}(t,\theta))\, \widehat{\xi}^{(1)}(t,\theta e^{-\beta t}). \end{aligned}$$

## Application : expected intensity process

→ The expectation $E[\lambda_t]$ is given by

$$m^{(1)}(t,0) = \left(\lambda_0 - \frac{\beta\lambda_\infty}{\beta - 1/\gamma}\right) e^{-(\beta-\frac{1}{\gamma})t} + \frac{\beta\lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta}\int_0^t e^{-(\beta-\frac{1}{\gamma})(t-s)} h'(s)ds.$$

**Remark** : $m^{(1)}(t,0)$ comprises an infinite series associated with the external jumps component ($h'(s) = \sum_{j=0}^{\infty} f^{j*}(s)$).

Trick to get closed-form expressions : convolution of exponential and inverse gaussian r.v.

$$\rightarrow E[N_t] = E\left[\int_0^t \lambda_s ds\right] = \int_0^t m^{(1)}(s,0)ds.$$

## Limiting behaviour of the lapse intensity

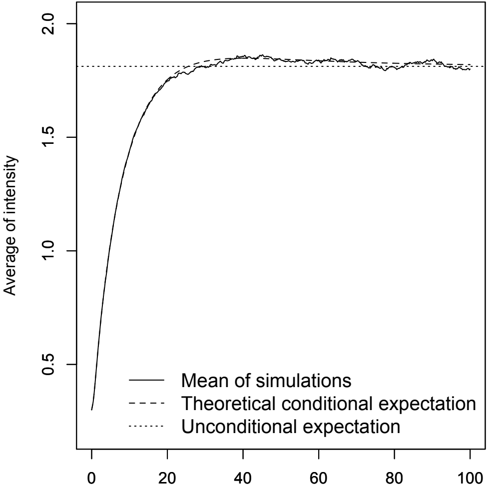
We can also compute the limit of this expectation :

$$\lim_{t \to \infty} E[\lambda_t] = \frac{\beta \lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta \theta_1 (\beta - 1/\gamma)}. \tag{8}$$

→ The limiting behavior of the lapse intensity first moment strongly depends on the limit of the last term in the previous result.

→ Serfozo [2009] for such results.

## Mean intensity process (# simu : 20 000)



## Application to risk management - calibration

Some parameters can be calibrated from practitioners' knowledge :

- $\lambda_0$ (initial force of lapse) is a constant.
  $\Rightarrow$ Exponential underlying lifetime distribution before lapse.
- $\lambda_\infty$ can be fixed by the risk managers as their goal...
  $\Rightarrow$ When the time horizon is given, this can be easily fixed.
- $B$ depicts the sensitivity of PH to opportunities (experts).

Some parameters (e.g. GBM) should be calibrated from empirical data / history whenever possible.
Others relate to the management : $\beta$ (ability to reassure the PH), $\gamma, \delta$ tie in with the mean size (SI) of lapsed contracts...

# Stress tests : comparison with SII and S-shaped

→ **Within the Solvency II framework** : run-off, 1-year horizon.
→ **With regard to financial context** : focus on the upper-shock.
→ Risk measures under consideration : *VaR* and *TVaR*.

| | | **Solvency II Standard formula** | | **S-shaped curve (ONC)** | | **Hawkes counting process** | | | **Dynamic contagion process** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Parameters | | Risk level | Shocks | Risk level | Shocks | $E[N_t]$ | $VaR_\alpha$ | $TVaR_\alpha$ | $E[N_t]$ | $VaR_\alpha$ | $TVaR_\alpha$ |
| | 10% | | | | | | | | 455 | 1028 | 1142 |
| $B$ | 30% | 75 | 112 | 75 | 375 | 291 | 776 | 837 | 312 | 818 | 930 |
| | 50% | | | | | | | | 293 | 778 | 886 |
| | 0.1 | | | | | | | | 2461 | 4286 | 4559 |
| $\delta$ | 0.5 | 75 | 112 | 75 | 375 | 291 | 776 | 837 | 702 | 1460 | 1594 |
| | 1.5 | | | | | | | | 455 | 1028 | 1142 |

TABLE: Impact of contagion and correlation on $VaR_\alpha(N_t)$, $TVaR_\alpha(N_t)$ at level $\alpha = 99.5\%$, in a 1-year time horizon ($t = 250$).

## Conclusion on stress tests

→ The shock in SII looks neither consistent nor realistic.
→ Stress tests in most of companies seem to be **underestimated**.

→ OK for extreme scenarios (reserving), not so realistic in classical regime (pricing).

→ PH' sensitivity to IR movements is obviously not linear...

→ External component has a limited impact, provided that mean size of the external jumps is low ⇒ portfolio composition is crucial !

## Key messages

Integrate only main risk factors + **correlation** + **contagion**.

***Perspectives :***

1. Calibration on a real-life portfolio ;
2. Use a martingale approach to retrieve the whole distribution of lapses $N_t$,
3. Extend this approach with an adapted interest rate model.

# References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