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Robust online aggregation of ensemble forecasts with applications to the forecasting of electricity consumption and of exchange rates

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The framework of this talk

Sequential and worst-case deterministic prediction of time series based on ensemble forecasts Framework

A time series $y_1, y_2, \ldots \in \mathbb{R}^d$ is to be predicted

Ensemble forecasts are available, e.g., given by some stochastic or machine-learning models (for us: black boxes)



At each instance t, forecasting black-box $j \in \{1, \dots, N\}$ outputs

$$f_{j,t} \equiv f_{j,t} \left(y_1^{t-1} \right)$$

Observations and predictions are made in a sequential fashion:

The prediction \hat{y}_t of y_t is determined based

- on the past observations $y_1^{t-1} = (y_1, \dots, y_{t-1})$,
- and the current and past ensemble forecasts $f_{j,s}$, where $s \in \{1, \ldots, t\}$ and $j \in \{1, \ldots, N\}$



Typical solution: convex (or linear) combinations of the ensemble forecasts, with adaptive weights $\mathbf{p}_t = (p_{1,t}, \dots, p_{N,t})$

Aggregated forecasts:

$$\widehat{y}_t = \sum_{j=1}^N p_{j,t} f_{j,t}$$

The observations y_t will not be considered stochastic anymore at this stage; thus the performance criterion will be a relative one

Given a convex loss function $\ell : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$, e.g., the square loss $\ell(x, y) = ||x - y||^2$:

The cumulative losses of the statistician and of the constant convex combinations $\mathbf{q} = (q_1, \dots, q_N)$ of the forecasts equal

$$\widehat{L}_{\mathcal{T}} = \sum_{t=1}^{T} \ell\left(\sum_{j=1}^{N} p_{j,t} f_{j,t}, y_t\right) \quad \text{and} \quad L_{\mathcal{T}}(\mathbf{q}) = \sum_{t=1}^{T} \ell\left(\sum_{j=1}^{N} q_j f_{j,t}, y_t\right)$$

Framework

The regret R_T is defined as the difference

$$\widehat{L}_{\mathcal{T}} - \min_{\mathbf{q}} L_{\mathcal{T}}(\mathbf{q}) = \sum_{t=1}^{\mathcal{T}} \ell\left(\sum_{j=1}^{N} p_{j,t} f_{j,t}, y_t\right) - \min_{\mathbf{q}} \sum_{t=1}^{\mathcal{T}} \ell\left(\sum_{j=1}^{N} q_j f_{j,t}, y_t\right)$$

We are interested in aggregation rules with (uniformly) vanishing per-round regret,

$$\limsup_{\mathcal{T}\to\infty} \quad \frac{1}{\mathcal{T}} \, \sup\left\{\widehat{\mathcal{L}}_{\mathcal{T}} - \min_{\mathbf{q}} \mathcal{L}_{\mathcal{T}}(\mathbf{q})\right\} \leqslant 0$$

The supremum is over all possible sequences of observations and of ensemble forecasts (not just over most of these sequences!)

Remarks:

- Hence the name "prediction of individual sequences" (or robust aggregation of ensemble forecasts)
- The best convex combination q^{*} is known in hindsight whereas the statistician has to predict in a sequential fashion

This framework leads to a meta-statistical interpretation:

- ensemble forecasts are given by some statistical forecasting methods, each possibly tuned with a different given set of parameters
- these ensemble forecasts relying on some stochastic model are then combined in a robust and deterministic manner

The cumulative loss of the statistician can be decomposed as

$$\widehat{L}_{\mathcal{T}} = \min_{\mathbf{q}} L_{\mathcal{T}}(\mathbf{q}) + R_{\mathcal{T}}$$

In words:

cumulative loss = approximation error + sequential estimation error

Disclaimer

We could also consider batch learning methods to aggregate forecasts, like

- BMA (Bayesian model averaging),
- CART (classification and regression trees),
- random forests, etc.,

or even selection methods, and apply them online, by running a batch analysis at each step

 \rightarrow We instead resort to "real" online techniques that, in addition, come up with theoretical guarantees even in non-stochastic scenarios

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First study

Forecasting of the electricity load

Data source: EDF R&D Authors: Pierre Gaillard and Yannig Goude Reference: Proceedings of WIPFOR '2013



Some characteristics of one among the studied data sets:

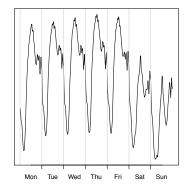
- January 1, 2008 August 31, 2011 as a training data set
- September 1, 2011 June 15, 2012 (excluding some special days) as testing set
- Electricity demand for EDF clients, at a half-hour step
- Typical values: median = 43 496 MW maximum = 78 922 MW
- Three forecasters: GAM, CLR, KWF

 \rightarrow Instead of trusting only one model/base forecaster ("selection"), we proceed in a more greedy way and consider ensemble forecasts, which we combine sequentially ("aggregation")

This leads to more accurate and more stable (meta-)predictions

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Data looks like...



Convex loss functions considered:

- square loss
$$\ell(x,y) = (x-y)^2 \longrightarrow \mathsf{RMSE}$$

– absolute percentage of error $\ell(x,y) = |x-y|/|y| \rightarrow \mathsf{MAPE}$

Operational constraint:

One-day ahead prediction at a half-hour step, i.e., 48 aggregated forecasts

Ensemble forecasters:

- GAM / generalized additive models (see Wood 2006; Wood, Goude, Shaw 2014)
- CLR / curve linear regression (see Cho, Goude, Brossat, Yao 2013, 2014)
- KWF / functional wavelet-kernel approach (see Antoniadis, Paparoditis, Sapatinas 2006; Antoniadis, Brossat, Cugliari, Poggi 2012, 2013)

Framework

Two strategies / First study

RMSE and MAPE on the testing set (with no warm-up period):

$$\sqrt{\frac{1}{T}\sum_{t=1}^{T} (y_t - \widehat{y}_t)^2} \quad \text{and} \quad \frac{1}{T}\sum_{t=1}^{T} |y_t - \widehat{y}_t|$$

How good are our building blocks? See the "oracles" below

	Uniform mean	Best forecaster	Best convex p	Best linear u
RMSE (MW)	725	744	629	629
MAPE (%)	1.18	1.29	1.06	1.06

In this article the focus is to create more base forecasting methods and to improve the oracles (and in turn the performance of the aggregation methods) accordingly

A strategy to pick convex weights

Let's do some maths!

Conclusion

Reminder of the aim and setting:

Given a loss function $\ell:\mathbb{R}^d\times\mathbb{R}^d\to\mathbb{R}$

Choose sequentially the convex weights $p_{j,t}$

To uniformly bound the regret with respect to all sequences of observations y_t and ensemble forecasts $f_{j,t}$:

$$\sum_{t=1}^{T} \ell\left(\sum_{j=1}^{N} p_{j,t} f_{j,t}, y_{t}\right) - \min_{\mathbf{q}} \sum_{t=1}^{T} \ell\left(\sum_{j=1}^{N} q_{j} f_{j,t}, y_{t}\right)$$

When ℓ is convex and differentiable in its first argument: For all $x, y \in \mathbb{R}^d$,

$$\forall x' \in \mathbb{R}^d, \qquad \ell(x,y) - \ell(x',y) \leqslant \nabla \ell(x,y) \cdot (x-x')$$

Assumption OK for RMSE, MAE, MAPE, etc.

Framework

To uniformly bound the regret with respect to all convex weight vectors \mathbf{q} , we write

$$\begin{aligned} \max_{\mathbf{q}} \sum_{t=1}^{T} \ell\left(\sum_{j=1}^{N} p_{j,t} f_{j,t}, y_{t}\right) &- \sum_{t=1}^{T} \ell\left(\sum_{j=1}^{N} q_{j} f_{j,t}, y_{t}\right) \\ \leqslant \max_{\mathbf{q}} \sum_{t=1}^{T} \nabla \ell\left(\sum_{k=1}^{N} p_{k,t} f_{k,t}, y_{t}\right) \cdot \left(\sum_{j=1}^{N} p_{j,t} f_{j,t} - \sum_{j=1}^{N} q_{j} f_{j,t}\right) \\ = \max_{\mathbf{q}} \sum_{t=1}^{T} \left(\sum_{j=1}^{N} p_{j,t} \widetilde{\ell}_{j,t} - \sum_{j=1}^{N} q_{j} \widetilde{\ell}_{j,t}\right) \\ = \sum_{t=1}^{T} \sum_{j=1}^{N} p_{j,t} \widetilde{\ell}_{j,t} - \min_{i=1,\dots,N} \sum_{t=1}^{T} \widetilde{\ell}_{i,t} \end{aligned}$$

where we denoted

$$\widetilde{\ell}_{j,t} = \nabla \ell \left(\sum_{k=1}^{N} p_{k,t} f_{k,t}, y_t \right) \cdot f_{j,t}$$

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Uncertainty measures

Considering the (signed) pseudo-losses

the regret is smaller than

$$\widetilde{\ell}_{j,t} = \nabla \ell \left(\sum_{k=1}^{N} p_{k,t} f_{k,t}, y_t \right) \cdot f_{j,t}$$
$$\sum_{t=1}^{T} \sum_{j=1}^{N} p_{j,t} \widetilde{\ell}_{j,t} - \min_{i=1,\dots,N} \sum_{t=1}^{T} \widetilde{\ell}_{i,t}$$

Exponentially weighted averages [also called AFTER]: $p_{i,1} = 1/N$ then

$$p_{j,t} = \frac{\exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{j,s}\right)}{\sum_{k=1}^{N} \exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{k,s}\right)}$$

ensure that if all $\widetilde{\ell}_{j,t} \in [m,M]$, then

$$\sum_{t=1}^{T}\sum_{j=1}^{N}p_{j,t}\widetilde{\ell}_{j,t} - \min_{i=1,\dots,N}\sum_{t=1}^{T}\widetilde{\ell}_{i,t} \leqslant \frac{\ln N}{\eta} + \eta \frac{(M-m)^2}{8}T$$

References: Vovk '90; Littlestone and Warmuth '94

Proof by mere calculus

Hoeffding's lemma: for all convex weights (p_1, \ldots, p_N) and all numbers u_1, \ldots, u_N with range [b, B], N

$$\ln \sum_{j=1}^{N} p_{j} e^{u_{j}} \leq \frac{(B-b)^{2}}{8} + \sum_{j=1}^{N} p_{j} u_{j}$$

For all t = 1, 2, ...,

$$-\eta \sum_{j=1}^{N} p_{j,t} \widetilde{\ell}_{j,t} = -\eta \sum_{j=1}^{N} \frac{\exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{j,s}\right)}{\sum_{k=1}^{N} \exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{k,s}\right)} \widetilde{\ell}_{j,t}$$
$$\geqslant \ln \frac{\sum_{j=1}^{N} \exp\left(-\eta \sum_{s=1}^{t} \widetilde{\ell}_{j,s}\right)}{\sum_{k=1}^{N} \exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{k,s}\right)} - \frac{\eta^{2}}{8} (M-m)^{2}$$

A telescoping sum appears and leads to

$$\sum_{t=1}^{T} \sum_{j=1}^{N} p_{j,t} \widetilde{\ell}_{j,t} \leqslant \underbrace{-\frac{1}{\eta} \ln \frac{\sum_{j=1}^{N} \exp\left(-\eta \sum_{s=1}^{T} \widetilde{\ell}_{j,s}\right)}{N}}_{\leqslant \min_{i=1,\dots,N} \sum_{t=1}^{T} \widetilde{\ell}_{i,t} + \frac{\ln N}{\eta}} + \eta \frac{(M-m)^2}{8} T$$

Obtained regret bound optimized over η :

$$R_T \leq \min_{\eta>0} \left\{ \frac{\ln N}{\eta} + \eta \frac{(M-m)^2}{8} T \right\} = (M-m) \sqrt{\frac{T}{2} \ln N}$$

for the (theoretical) optimal choice

$$\eta^{\star} = \frac{1}{M-m} \sqrt{\frac{8\ln N}{T}}$$

Issue: the parameters T and [m, M] not always known beforehand Solutions:

- "doubling trick"
- adaptive learning rates η_t , picked according to some theoretical formulas

However, these theoretically satisfactory solutions would not work well in practice!

This is what we do instead. (It is very different from techniques like cross-validation: we exploit the sequential fashion.)

The exponentially weighted average strategy \mathcal{E}_η with fixed learning rate η picks

$$p_{j,t}(\eta) = \frac{\exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{j,s}\right)}{\sum_{k=1}^{N} \exp\left(-\eta \sum_{s=1}^{t-1} \widetilde{\ell}_{k,s}\right)}$$

We denote its cumulative loss $\widehat{L}_t(\eta) = \sum_{s=1}^{t} \ell\left(\sum_{j=1}^{N} p_{j,s}(\eta) f_{j,s}, y_s\right)$

Based on the family of the \mathcal{E}_{η} , we build a data-driven metastrategy which at each instance $t \ge 2$ resorts to

$$\mathbf{p}_{t+1}(\eta_t)$$
 where $\eta_t \in \operatorname*{arg\,min}_{\eta>0} \widehat{L}_t(\eta)$

Reference: An idea of Vivien Mallet

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Other natural variants:

Focus on the most recent losses

Moving sums (with window of size *H*)

$$p_{j,t} = \frac{\exp\left(-\eta \sum_{s=\max\{1,t-H\}}^{t-1} \widetilde{\ell}_{j,s}\right)}{\sum_{k=1}^{N} \exp\left(-\eta \sum_{s=\max\{1,t-H\}}^{t-1} \widetilde{\ell}_{k,s}\right)}$$

Regret is $\geq \Box T$ in the worst case

Discounted losses (with discounts given by a sequence $\beta_t \searrow 0$)

$$p_{j,t} = \frac{\exp\left(-\eta_t \sum_{s=1}^{t-1} (1+\beta_{t-s})\widetilde{\ell}_{j,s}\right)}{\sum_{k=1}^{N} \exp\left(-\eta_t \sum_{s=1}^{t-1} (1+\beta_{t-s})\widetilde{\ell}_{k,s}\right)}$$

Sublinear regret bounds hold for suitable sequences (β_t) and (η_t) : $t\eta_t \longrightarrow 0$ and $\eta_t \sum \beta_s \longrightarrow 0$

 $s \leq t$

(We often take $\beta_s = \Box/s^2$ in the experimental studies.)

A strategy to pick linear weights

You all know it in a stochastic setting!

Linear combinations: Ridge regression (and the LASSO?)

Ridge regression — introduced in the 70s by Hoerl and Kennard:

$$\mathbf{v}_t \in \underset{\mathbf{u} \in \mathbb{R}^N}{\operatorname{arg\,min}} \left\{ \lambda \|\mathbf{u}\|_2^2 + \sum_{s=1}^{t-1} \left(y_s - \sum_{j=1}^N u_j f_{j,s} \right)^2 \right\}$$

It also exhibits a sublinear regret against individual sequences: for all $y_t \in [-B, B]$ and $f_{j,t} \in [-B, B]$, for all $\mathbf{u} \in \mathbb{R}^N$

$$\sum_{t=1}^{T} \left(y_t - \sum_{j=1}^{N} v_{j,t} f_{j,t} \right)^2 - \sum_{t=1}^{T} \left(y_t - \sum_{j=1}^{N} u_j f_{j,t} \right)^2$$
$$\leq \lambda \left\| \mathbf{u} \right\|_2^2 + 2NB^2 \left(1 + \frac{NTB^2}{\lambda} \right) \ln \left(1 + \frac{TB^2}{N\lambda} \right)$$

References: Vovk '01; Azoury and Warmuth '01; Gerchinovitz '11

The bound can be $O(\sqrt{T} \ln T)$ with λ of the order of $1/\sqrt{T}$ Same comments as before when T is unknown We do not know any such regret bounds for the LASSO (yet?)

These methods can compensate for biases in either direction (the weights do not need to sum up to 1)

Can/should even be used as a pre-treatment on each single forecaster:

- turn it into a forecaster with predictions $\gamma_t f_{j,t}$
- performing on average almost as well as the best forecaster of the form $\gamma f_{j,t}$ for some constant $\gamma \in \mathbb{R}$

This would improve greatly the predictions if there existed, for instance, an almost constant multiplicative bias of $1/\gamma$

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First study, continued

Prediction of electricity load

Benchmark and oracles

(RMSE of the ensemble forecasts and of fixed combinations thereof)

Uniform	Best	Best	Best
mean	forecaster	convex p	linear u
725	744	629	629

VS.

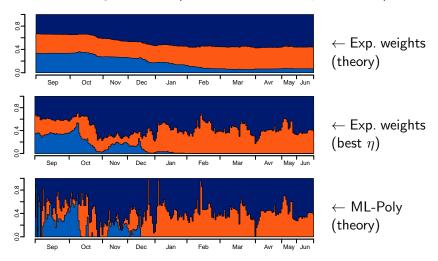
Aggregated forecasts with convex weights (No discount considered)

Exp. weights (best η for theory)	644
Exp. weights (best η on data)	619
Exp. weights (η_t tuned on data)	625

ML-Poly (tuned according to theory) 626



No focus on a single member! (See also the numerical performance.)



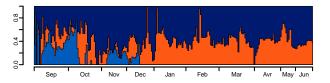
Weights change quickly and significantly over time and do not converge (illustrates that the performance of forecasters varies over time)

Are all forecasters useful? ... Definitely yes!

3 forecasters \rightarrow only best 2

ML-Poly	626	\rightarrow	646
Exp. weights	625	\rightarrow	644

Forecasters not considered anymore can come back to life if needed



$$\leftarrow$$
 ML-Poly

Benchmark and oracles

(RMSE of the ensemble forecasts and of fixed combinations thereof)

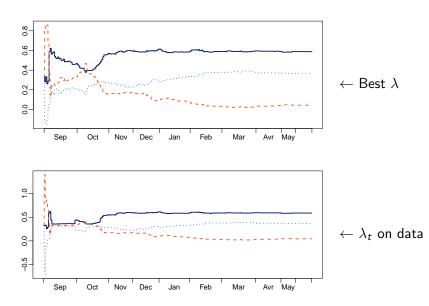
Uniform	Best	Best	Best
mean	forecaster	convex p	linear u
725	744	629	629

VS.

Aggregated forecasts with linear weights (No discount considered)

Ridge (best λ on data)	636
Ridge (λ_t tuned on data)	638
Ridge pre-treatment	744 ightarrow 745
on the forecasters	$887 \to 873$
(tuned on data)	$1287 \rightarrow 1293$

Weight vectors chosen by ridge regression



This was only a small glimpse into the work performed by Pierre Gaillard, Yannig Goude, and others, at EDF R&D

Other data sets studied include the forecasting of

- Slovakian demand for clients of an EDF subbranch
- GEFCom '2014 electricity price
- GEFCom '2014 electricity load
- Heat load of an Ukrainian co-generation plant
- Electricity demand of sub-groups of EDF clients
- \longrightarrow Universality of the aggregation methods!

Reference: Pierre Gaillard's Ph.D. dissertation, July 2015

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Methodological summary

Methodological summary

- Build the N base forecasters, possibly on a training data set, and pick another data set for the evaluation, with T instances
- Occupie to the some benchmarks and some reference oracles
- Evaluate our strategies when run with fixed parameters (i.e., with the best parameters in hindsight)
- The performance of interest is actually the one of the data-driven meta-strategies

We typically expect $T \ge 5N$ or even $T \ge 10N$

Hope arises when the oracles are 10% or 20% better than the methods used so far (e.g., the best ensemble forecast when the latter is known in advance)

This usually requires the ensemble forecasters to be as different as possible!

Second study

Forecasting of exchange rates



Data source: IMF / Fed

Authors: Christophe Amat, Tomasz Michalski, Gilles Stoltz

Reference: SSRN-2448655, April 2015, revised October 2015

Predict 1-month ahead monthly averages r_{t+1} of exchange rates

Based on 5×2 macro-economic indicators describing the state of each country in month *t*:

- inflation rates (Infl)
- short-term interest rates (IR)
- changes in monetary mass (Mon)
- changes in industrial production (Prod)
- changes in interest rates (IR.Diff)

Difficult to improve on the no-change (NC) prediction, i.e., on forecasting r_{t+1} by r_t

Reference: Meese and Rogoff '83

Some (limited) results as well for end-of-month values

Framework 00000000000

Convex or linear combinations of the 5 \times 2 macro-economic indicators for countries *A* and *B*:

$$\ln \hat{r}_{t+1} - \ln r_t = \sum_{j=1}^{5} (u_{j,t+1}^A x_{j,t}^A - u_{j,t+1}^B x_{j,t}^B)$$

Evaluation through RMSE with a short training period of $t_0 = 30$:

$$\sqrt{\frac{1}{T-t_0+1}\sum_{t=t_0}^{T} \left(\ln \widehat{r}_t - \ln r_t\right)^2}$$

Data-driven meta-strategies based on discounted versions of:

- − Exponential weights (no gradient) ← interpretable weights
- Ridge regression \leftarrow pushes in favor of no-change

Some orders of magnitude for the prediction problems at hand are indicated below.

Time intervals Period	Every month March 1973 – December 2014
Time instances <i>T</i> Size <i>N</i> of ensemble	about 500 $10~(=5 imes2)$
USD / GBP Median of the $\ln r_{t+1} - \ln r_t$ Maximum of the $ \ln r_{t+1} - \ln r_t $	$1.48 imes 10^{-2}$ $11.08 imes 10^{-2}$
$\begin{array}{c} JPY \ / \ USD \\ Median \ of \ the \ ln \ r_{t+1} - ln \ r_t \\ Maximum \ of \ the \ \ ln \ r_{t+1} - ln \ r_t \end{array}$	$1.57 imes 10^{-2}$ $10.52 imes 10^{-2}$

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Results for USD / GBP

Pairs of indicators	RMSE	Oracle	RMSE
NC	2.4410×10^{-2}	Best member	$2.4400 imes 10^{-2}$
Infl	2.4561×10^{-2}	Best convex	2.4315×10^{-2}
IR	2.4620×10^{-2}	Best linear	2.3453×10^{-2}
Mon	2.5037×10^{-2}		
Prod	2.4390×10^{-2}		
IR.Diff	2.4400×10^{-2}		

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Rolling OLS	2.5960×10^{-2}	(worse!)	
Exp. weights	2.3777×10^{-2}	(-2.51%)	\rightarrow P-value: 3.3%
Ridge	2.3512×10^{-2}	(-3.68%)	\rightarrow P-value: 2.7%

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Results for JPY / USD

Pairs of indicators	RMSE	Oracle	RMSE
NC	2.7042×10^{-2}	Best member	2.7003×10^{-2}
Infl	2.7003×10^{-2}	Best convex	2.6751×10^{-2}
IR	2.7203×10^{-2}	Best linear	2.6411×10^{-2}
Mon	2.7551×10^{-2}		
Prod	2.7406×10^{-2}		
IR.Diff	2.7038×10^{-2}		

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Rolling OLS	2.8189×10^{-2}	(worse!)	
Exp. weights	2.6125×10^{-2}	(-3.39%)	\rightarrow P-value: 0.5%
Ridge	2.6031×10^{-2}	(-3.74%)	\rightarrow P-value: 0.2%

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Quantile prediction

Uncertainty measures in this deterministic setting

Pinball loss:
$$\ell_{\alpha}$$
: $(x, y) \mapsto (y - x)(\alpha - \mathbb{I}_{\{y < x\}})$

Quantile of order α of the law of Y as a minimizer:

$$q_lpha\inrgmin_{x\in\mathbb{R}}\mathbb{E}ig[\ell_lpha(x,Y)ig]$$

 \longrightarrow Substitute $\ell(x, y) = (x - y)^2$ or $\ell(x, y) = |x - y|/|y|$ with ℓ_{α} to predict an α -quantile \hat{y}_t^{α} for y_t

I.e., control a per-round regret of the form

$$\frac{1}{T}\sum_{t=1}^{T}\ell_{\alpha}(\widehat{y}_{t}^{\alpha}, y_{t}) - \frac{1}{T}\min_{x}\sum_{t=1}^{T}\ell_{\alpha}(x, y_{t})$$

The \hat{y}_t^{α} are based on forecasts $f_{j,t}$ of central tendencies or of α -quantiles

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Strategy: exponential weights

(+ trick from Kivinen and Warmuth '97 to compete with linear combinations)

Does it work well? Ask Pierre Gaillard, Yannig Goude, Raphaël Nedellec: Winners of the two GEFCom'2014 competitions (demand + price) Two strategies / First study

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Other empirical studies

- Forecasting of air quality (INRIA and INERIS)
- Forecasting of the production data of oil reservoirs (IFP-EN)



maîtriser le risque pour un développement durable





 \rightarrow Universality, versatility and efficiency!

But time is over...

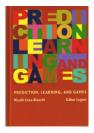
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Reference for theory

The so-called "red bible!"



Prediction, Learning, and Games Nicolò Cesa-Bianchi and Gábor Lugosi