Pushforwards of p-adic differential equations

Velibor Bojković

Department of mathematics Tullio Levi-Civita, University of Padova
Joint work with Jérôme Poineau

Luminy
March, 2017

Problem

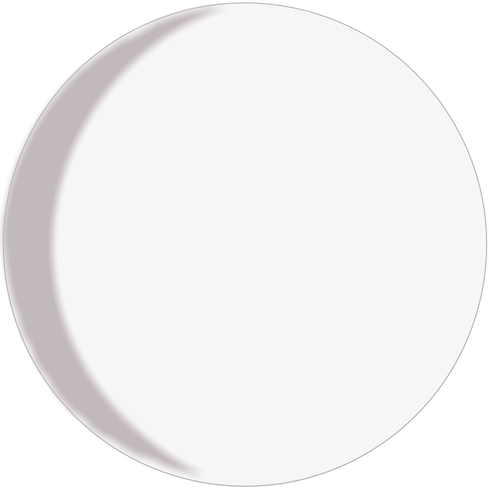
Problem

Given a finite étale morphism $\varphi : Y \to X$*, a differential equation* $(\mathcal{E}, \nabla)$ *on* Y*, a point* $x \in X$ *and* $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$*, find* $\mathcal{MR}(x, (\mathcal{F}, \nabla))$ *in* ***terms of morphism*** φ ***and*** $\mathcal{MR}(y_i, (\mathcal{E}, \nabla))$*, where* $\varphi^{-1}(x) = \{y_1, \ldots, y_l\}$.

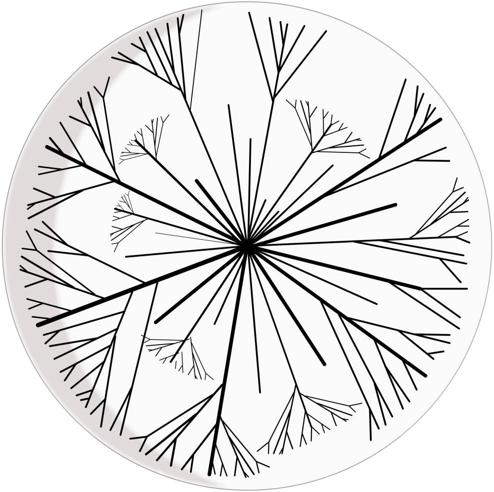
Assumptions

- The base field k is algebraically closed, complete with respect to a nontrivial and nonarchimedean valuation and of characteristic 0
- All the curves are quasi-smooth k-analytic (in the sense of Berkovich)

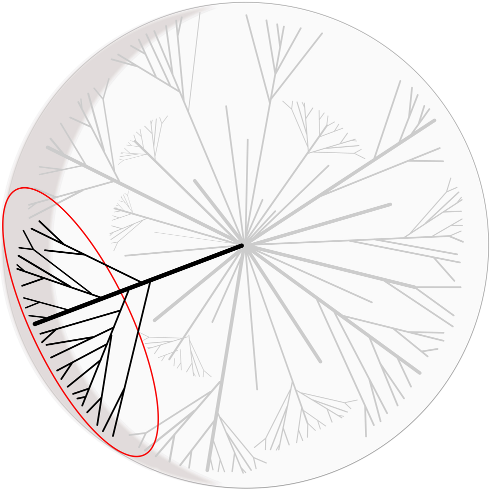
Projective k-algebraic line



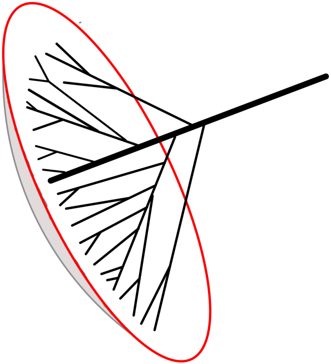
Projective k-analytic line



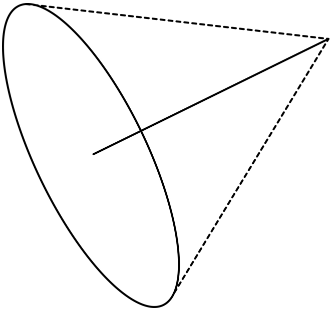
Open disc



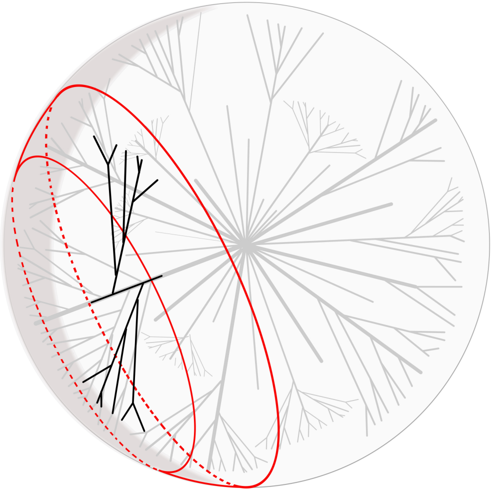
Open disc



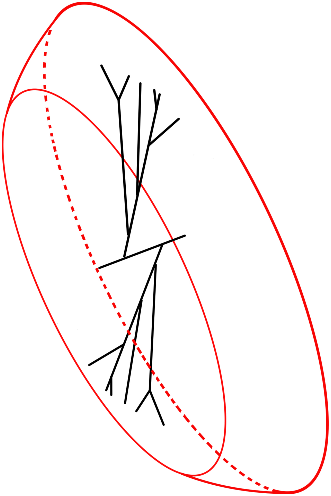
Open disc



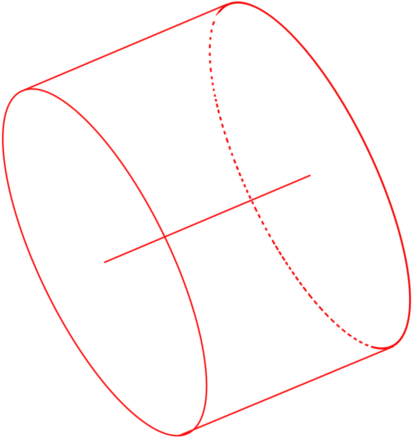
Open annulus



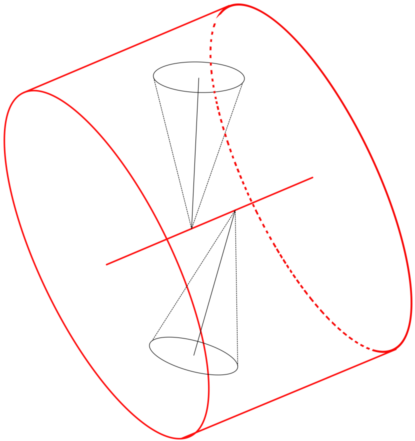
Open annulus



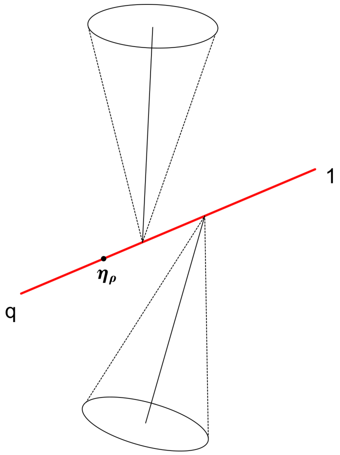
Open annulus



Open annulus



Open annulus



Radial morphisms of open discs

Let $\varphi : D(0,1^-) \to D(0,1^-)$ be a finite morphism of degree d that maps 0 to 0.

Then, we have $\varphi(\eta_{0,r}) = \eta_{0,f(r)}$, and we obtain a function

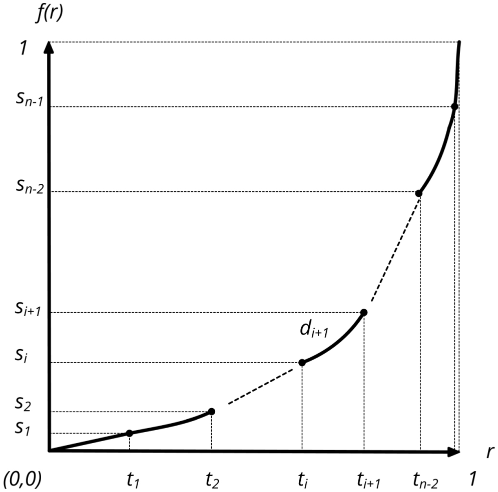
$$f : [0,1] \to [0,1].$$

Definition

φ is radial with the profile function f if $\forall a \in D(0,1^-)(k)$ and $r \in (0,1)$, $\varphi(\eta_{a,r}) = \eta_{\varphi(a),f(r)}$ (i.e. f does not depend on the chosen coordinates).

If φ is radial then it is étale.

Profile function graph



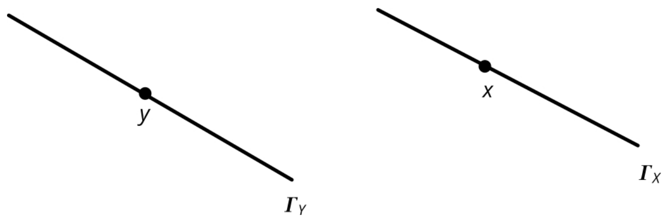
More generally...

Suppose $\varphi : Y \to X$ is a finite étale morphism of quasi-smooth k-analytic curves and let $\Gamma = (\Gamma_Y, \Gamma_X)$ be a skeleton of φ (i.e. $\Gamma_Y = \varphi^{-1}(\Gamma_X)$).

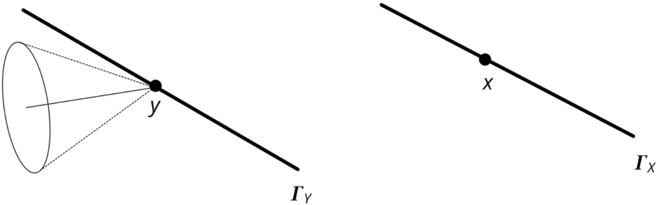
Definition

We say that φ is radial with respect to Γ if restriction of φ to any open disc D in $Y \setminus \Gamma_Y$ is radial and the profile function only depends on the endpoint of D in Γ_Y.

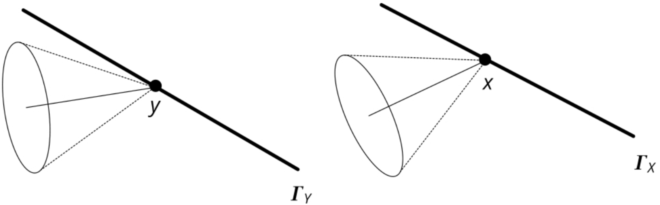
More generally: Picture



More generally: Picture



More generally: Picture



Radializing skeleton

Theorem (M. Temkin)

Given $\varphi : Y \to X$, $\Gamma = (\Gamma_Y, \Gamma_X)$, as before. Then, there exists a skeleton $\Gamma' = (\Gamma'_Y, \Gamma'_X)$ which contains Γ and which is radializing for φ.

To every type 2 point $y \in Y$ we can assign a profile function f_y. But, the procedure can be extended nicely to all the points in Y^{hyp}.

Another construction

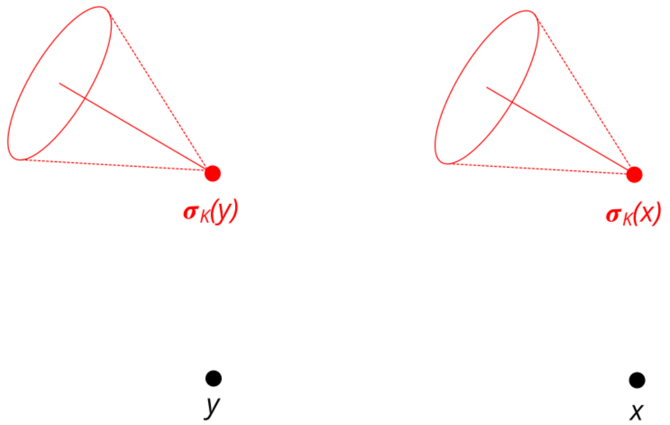
Let $y \in Y^{\mathrm{hyp}}$, $K/\mathscr{H}(y)$ a complete valued extension. Then, we have two maps $\pi_K : Y_K \to Y$ and $\sigma_K : Y \to Y_K$.

Theorem (B-P)

Let $K/\mathscr{H}(y)$ be a complete valued extension, let $y_K \in Y_K(K)$ be "above y" ($\pi_K(y_K) = y$) and let D be a connected component of $\pi_K^{-1}(y) \setminus \{\sigma_K(y)\}$. Then, $\varphi_K : D \to \varphi_K(D)$ is a radial morphism of open discs with the profile that coincides with f_y.

Another construction

Another construction



Another construction

Let $y \in Y^{\mathrm{hyp}}$, $K/\mathscr{H}(y)$ a complete valued extension. Then, we have a two maps $\pi_K : Y_K \to Y$ and $\sigma_K : Y \to Y_K$.

Theorem (B-P)

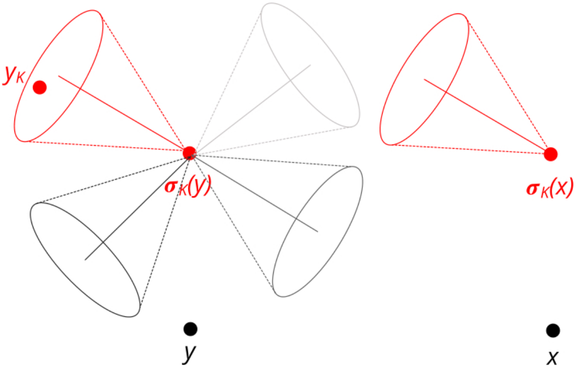
Let $K/\mathscr{H}(y)$ be a complete valued extension, let $y_K \in Y_K(K)$ be a rational point above y ($\pi_K(y_K) = y$) and let D be the maximal connected component of $Y_K \setminus \{\sigma_K(y)\}$. Then, $\varphi_K : D \to \varphi_K(D)$ is a radial morphism of open discs with the profile that coincides with f_y.

Definition

The separable residual degree of φ at y is the number $\mathfrak{s} := \deg(\varphi, y)/\deg(\varphi_{K|D})$.

Similarly, we define residual inseparable degree of φ at y as $\mathfrak{i} := \deg(\varphi_{K|D}) = \deg(\varphi, y)/\mathfrak{s}$.

Another construction



Multiradius of convergence

Let $(\mathcal{E}, \nabla)$ be a p-adic differential equation on Y of rank r.

- Suppose that $Y = D(0,1^-)$. The multiradius of $(\mathcal{E}, \nabla)$ at $a \in D(0,1^-)(k)$ is the r-tuple

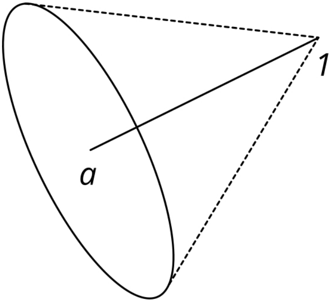
$$\mathcal{MR}(a, (\mathcal{E}, \nabla)) := (\mathcal{R}_1, \ldots, \mathcal{R}_r),$$

  where $\mathcal{R}_i = \sup\{s \in (0,1) \mid \dim_k H^0(D(a,s^-), (\mathcal{E}, \nabla)) = r - i + 1\}$
- For Y general and $y \in Y^{\mathrm{hyp}}$ we proceed as follows:
    - Extend scalars by $K = \mathscr{H}(y)$
    - Let $y_K \in Y_K$ be any rational point above y, let D be a connected component (open disc) in $\pi_K^{-1}(y) \setminus \{\sigma_K(y)\}$ that contains y_K. Then,

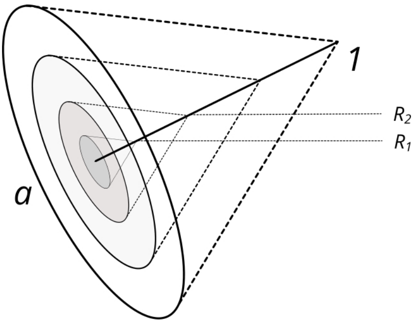
$$\mathcal{MR}(y, (\mathcal{E}, \nabla)) := \mathcal{MR}(y_K, \pi_K^*(\mathcal{E}, \nabla)_{|D}).$$

    - $\mathcal{MR}(y, (\mathcal{E}, \nabla))$ does not depend on the choice of y_k.

Multiradius of convergence: first case

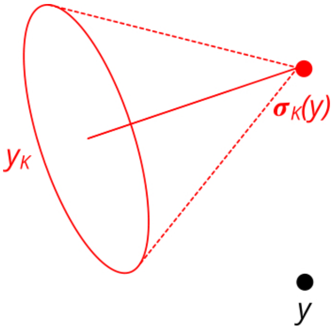


Multiradius of convergence: first case

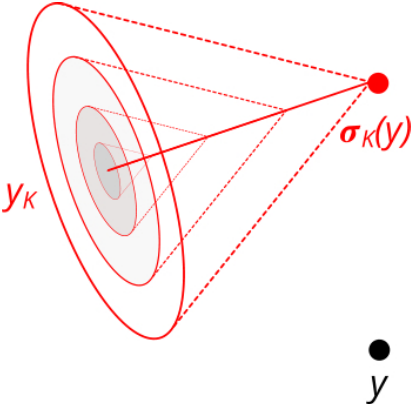


Multiradius of convergence: general case

Multiradius of convergence: general case



Multiradius of convergence: general case



Problem

Problem

Given a finite étale morphism $\varphi : Y \to X$, a differential equation $(\mathcal{E}, \nabla)$ on Y, a point $x \in X$ and $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$, find $\mathcal{MR}(x, (\mathcal{F}, \nabla))$ in terms of morphism φ and $\mathcal{MR}(y_i, (\mathcal{E}, \nabla))$, where $\varphi^{-1}(x) = \{y_1, \dots, y_l\}$.

Two functions: N_x and Φ_x

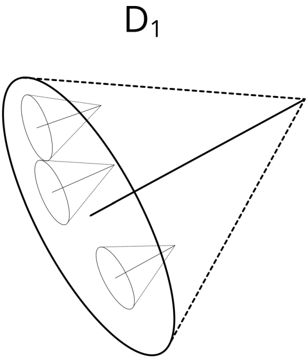
Let $\varphi : D_1 \to D_2$ be a finite étale morphism of open unit discs, and let $x \in D_2(k)$

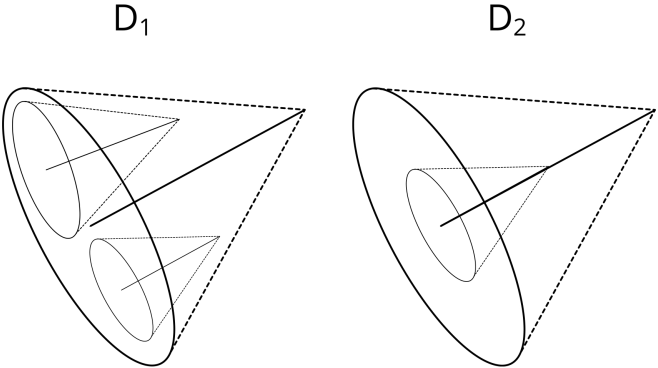
- We define function $N_x : [0,1] \to [0,1]$ by

$$N_x(t) := \#\{\varphi^{-1}(D(x,t^-))\}$$

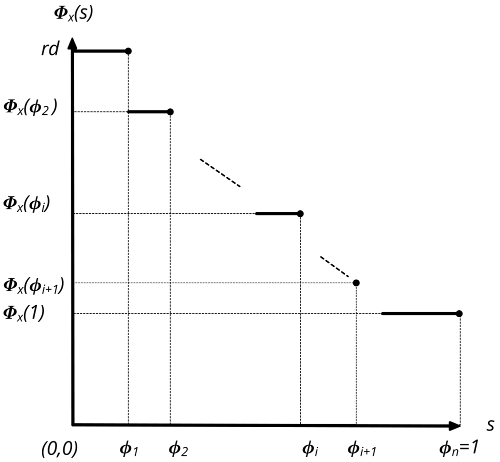
- Let $(\mathcal{E}, \nabla)$ be a p-adic differential equation of rank r on D_1 and let $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$. We define the function $\Phi_x = \Phi(x, (\mathcal{E}, \nabla)) : [0,1] \to [0, rd]$ by

$$\Phi_x(t) := \sum_{i=1}^{N_x(t)} \dim_k H^0(D_t^i, (\mathcal{E}, \nabla))$$


D1
D2

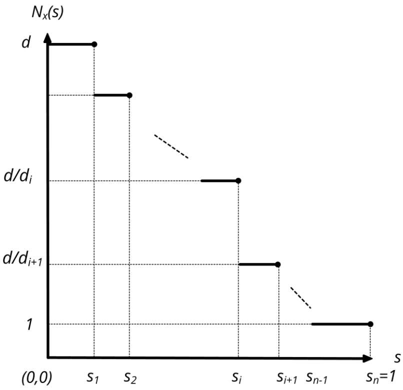

D_1
D_2

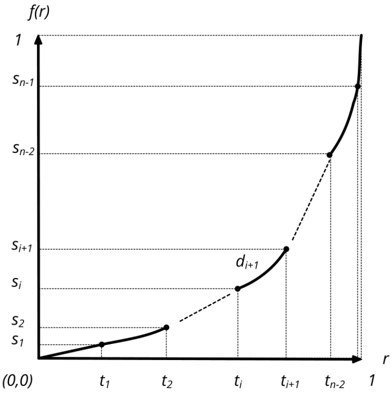
$\Phi_x(s, (\mathcal{E}, \nabla))$



Comparison of N_x and f

$\varphi : D_1 \to D_2$ is radial.





Pushforward over open discs

$\varphi : D_2 \to D_1$, $(\mathcal{E}, \nabla)$ on D_1, $(\mathcal{F}, \nabla) = \varphi_*(\mathcal{E}, \nabla)$ on D_2, and $x \in D_1(k)$

Theorem (B-P)

Let $x \in D_1$. We have

$$\#\{i \in \{1, \ldots, rd\} \mid \mathcal{R}_i(x, (\mathcal{F}, \nabla)) = 1\} = \Phi_x(1)$$

and, for each $R \in (0, 1)$,

$$\#\{i \in \{1, \ldots, rd\} \mid \mathcal{R}_i(x, (\mathcal{F}, \nabla)) = R\} = \Phi_x(R) - \Phi_x(R^+).$$

In other words, the multiradus $\mathcal{MR}(x, (\mathcal{F}, \nabla))$ consists of break-points of the function Φ_x and each break-point s_i appears $\Phi_x(s_i) - \Phi_x(s_i^+)$ - times.

Proof

$$\#\{i \in \{1,\ldots,rd\} \mid \mathcal{R}_i(x,(\mathcal{F},\nabla)) = R\} = \Phi_x(R) - \Phi_x(R^+)$$

Proof.

$$\mathcal{R}_i(x,(\mathcal{F},\nabla)) = \sup\left\{s \in (0,1) \mid \dim_k H^0(D(x,s^-),(\mathcal{F},\nabla)) \geq rd - i + 1\right\}$$

□

Proof

$$\#\{i \in \{1,\dots,rd\} \mid \mathcal{R}_i(x,(\mathcal{F},\nabla)) = R\} = \Phi_x(R) - \Phi_x(R^+)$$

Proof.

$$\mathcal{R}_i(x,(\mathcal{F},\nabla)) = \sup\left\{s \in (0,1) \mid \dim_k H^0(D(x,s^-),(\mathcal{F},\nabla)) \geq rd - i + 1\right\}$$

$$= \sup\left\{s \in (0,1) \mid \dim_k H^0(\varphi^{-1}(D(x,s^-)),(\mathcal{E},\nabla)) \geq rd - i + 1\right\}$$

$\square$

Proof

$$\#\{i \in \{1, \ldots, rd\} \mid \mathcal{R}_i(x, (\mathcal{F}, \nabla)) = R\} = \Phi_x(R) - \Phi_x(R^+)$$

Proof.

$$\mathcal{R}_i(x, (\mathcal{F}, \nabla)) = \sup \left\{ s \in (0,1) \mid \dim_k H^0(D(x, s^-), (\mathcal{F}, \nabla)) \geq rd - i + 1 \right\}$$

$$= \sup \left\{ s \in (0,1) \mid \dim_k H^0(\varphi^{-1}(D(x, s^-)), (\mathcal{E}, \nabla)) \geq rd - i + 1 \right\}$$

$$= \sup \left\{ s \in (0,1) \mid \Phi_x(s) \geq rd - i + 1 \right\}.$$

$\square$

Proof

$$\#\{i \in \{1, \dots, rd\} \mid \mathcal{R}_i(x, (\mathcal{F}, \nabla)) = R\} = \Phi_x(R) - \Phi_x(R^+)$$

Proof.

$$\mathcal{R}_i(x, (\mathcal{F}, \nabla)) = \sup\left\{s \in (0,1) \mid \dim_k H^0(D(x, s^-), (\mathcal{F}, \nabla)) \geq rd - i + 1\right\}$$

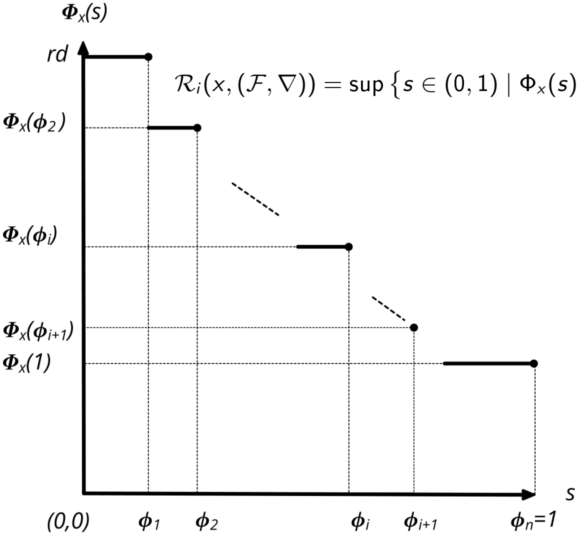
$$= \sup\left\{s \in (0,1) \mid \dim_k H^0(\varphi^{-1}(D(x, s^-)), (\mathcal{E}, \nabla)) \geq rd - i + 1\right\}$$

$$= \sup\left\{s \in (0,1) \mid \Phi_x(s) \geq rd - i + 1\right\}.$$

So $\mathcal{R}_i(x, (\mathcal{F}, \nabla)) = R$ is a break-point of Φ_x and it will appear in multiradius exactly $\Phi_x(R) - \Phi_x(R^+)$ - times.

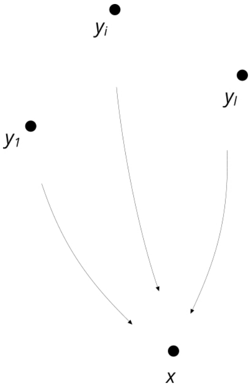
□

Proof/visual

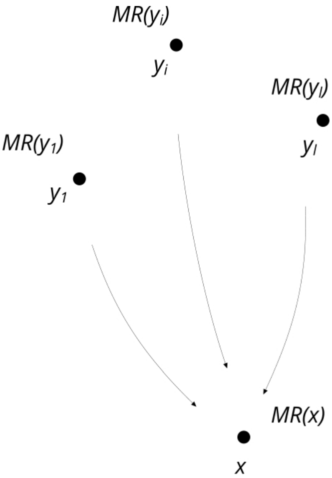


$$\mathcal{R}_i(x,(\mathcal{F},\nabla)) = \sup\left\{s \in (0,1) \mid \Phi_x(s) \geq rd - i + 1\right\}$$

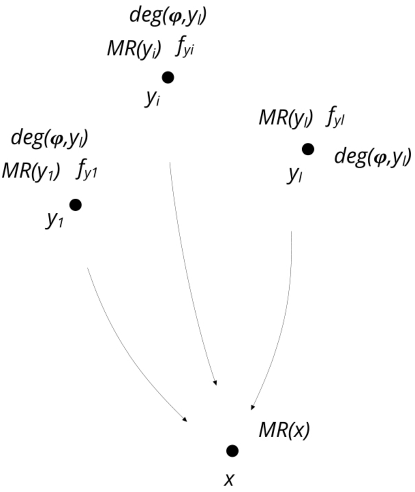
Pushforward: general case



Pushforward: general case



Pushforward: general case



Pushforward: general case

Theorem (B-P)

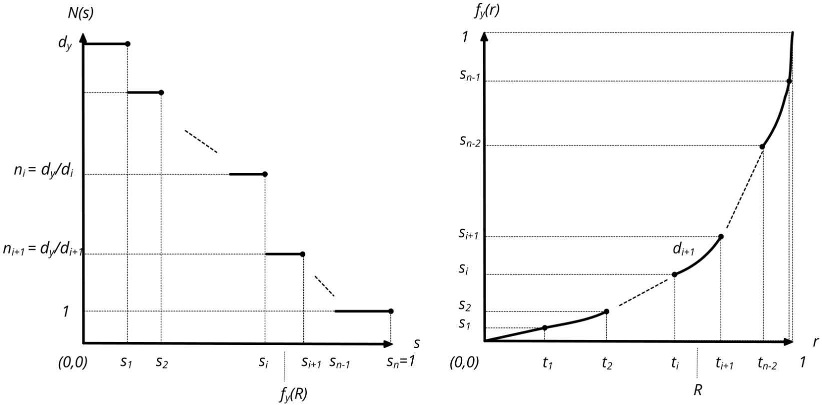
Let $y \in \varphi^{-1}(x)$, let R be a component of $\mathcal{MR}(y, (\mathcal{E}, \nabla))$ and let $t_1, \ldots, t_n$ be the break-points of the function f_y, $s_j := f_y(t_j)$, and $n_i := N(s_i)$. Then, every R in $\mathcal{MR}(y, (\mathcal{E}, \nabla))$ and $t_i < R \leq t_{i+1}$ we have

$$R \quad \longleftrightarrow \quad \underbrace{(s_1, \ldots, s_1, s_2, \ldots, s_2, s_i, \ldots, s_i, f_y(R), \ldots, f_y(R))}_{\text{repeated } \mathfrak{s}_y \text{ times}},$$

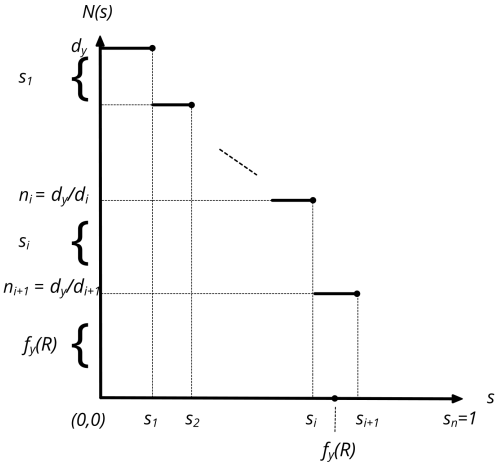
where each s_j appears $n_j - n_{j+1}$ times and $f_y(R)$ appears n_{i+1} times.

All the radii in $\mathcal{MR}(x, (\mathcal{F}, \nabla))$ appear in this way (by varying R and $y \in \varphi^{-1}(x)$).

Pushforward: general case

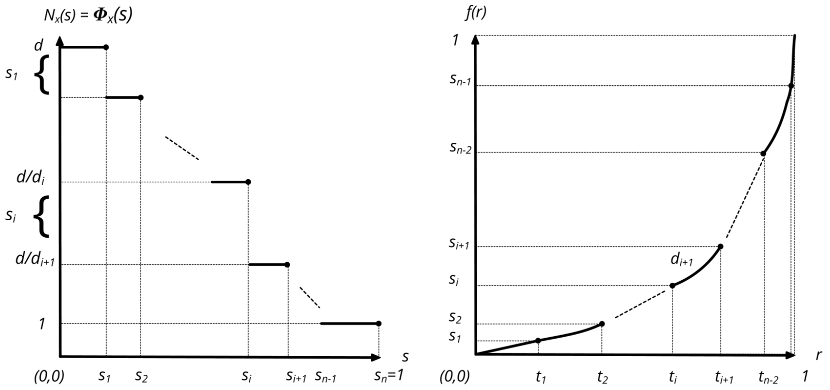


Pushforward: general case



Pushforward of the constant connection

$\varphi : D_1 \to D_2$ radial of degree d; $(\mathcal{E}, \nabla) = (\mathcal{O}, d)$



Residually separable morphisms

- As before, we have $\varphi : Y \to X$ finite étale of degree d, $(\mathcal{E}, \nabla)$ on Y of rank r and $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$ on X of rank rd, $y \in Y^{\mathrm{hyp}}$ and $x = \varphi(y) \in X^{\mathrm{hyp}}$;

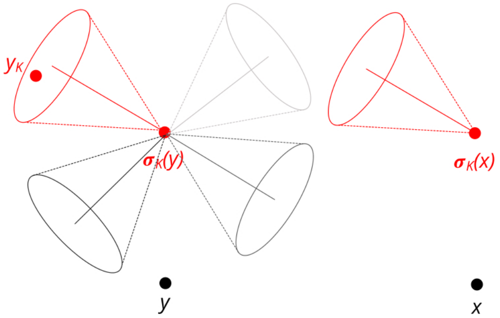
Residually separable morphisms

- As before, we have $\varphi : Y \to X$ finite étale of degree d, $(\mathcal{E}, \nabla)$ on Y of rank r and $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$ on X of rank rd, $y \in Y^{\mathrm{hyp}}$ and $x = \varphi(y) \in X^{\mathrm{hyp}}$;
- We also have f_y-the profile function at y, $\mathfrak{s}$-the residual separable degree at y;

Residually separable morphisms

- As before, we have $\varphi : Y \to X$ finite étale of degree d, $(\mathcal{E}, \nabla)$ on Y of rank r and $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$ on X of rank rd, $y \in Y^{\text{hyp}}$ and $x = \varphi(y) \in X^{\text{hyp}}$;
- We also have f_y-the profile function at y, $\mathfrak{s}$-the residual separable degree at y;
- Suppose that $\mathfrak{s} = d$.

Residually separable morphism



Residually separable morphisms

- As before, we have $\varphi : Y \to X$ finite étale of degree d, $(\mathcal{E}, \nabla)$ on Y of rank r and $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$ on X of rank rd, $y \in Y^{\text{hyp}}$ and $x = \varphi(y) \in X^{\text{hyp}}$;
- We also have f_y-the profile function at y, $\mathfrak{s}$-the residual separable degree at y;
- Suppose that $\mathfrak{s} = d$. Then

$$\mathcal{MR}(y, (\mathcal{E}, \nabla)) \longleftarrow \quad R \longleftrightarrow \underbrace{(R, \dots, R)}_{\mathfrak{s}} \quad \longrightarrow \mathcal{MR}(x, (\mathcal{F}, \nabla)).$$

Residually separable morphisms

- As before, we have $\varphi : Y \to X$ finite étale of degree d, $(\mathcal{E}, \nabla)$ on Y of rank r and $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$ on X of rank rd, $y \in Y^{\mathrm{hyp}}$ and $x = \varphi(y) \in X^{\mathrm{hyp}}$;
- We also have f_y-the profile function at y, $\mathfrak{s}$-the residual separable degree at y;
- Suppose that $\mathfrak{s} = d$. Then

$$\mathcal{MR}(y, (\mathcal{E}, \nabla)) \longleftarrow \quad R \longleftrightarrow \underbrace{(R, \dots, R)}_{\mathfrak{s}} \quad \longrightarrow \mathcal{MR}(x, (\mathcal{F}, \nabla)).$$

- This could also be proved directly.

Frobenius pushforward

- Suppose that residue field $\tilde{k}$ is of characteristic $p > 0$ and that $\varphi : A(0; q, 1) \to A(0; q^p, 1)$ is the Frobenius morphism, acting on rational points as $\alpha \mapsto \alpha^p$. Let $(\mathcal{E}, \nabla)$ and $(\mathcal{F}, \nabla)$ as before;

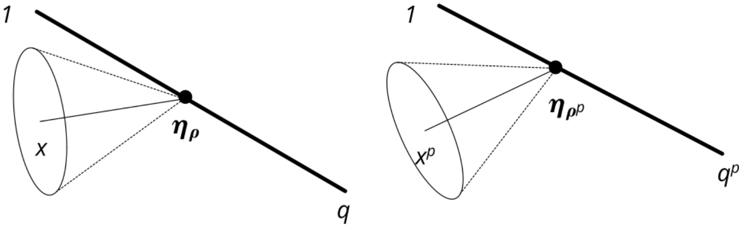
Frobenius pushforward

- Suppose that residue field $\tilde{k}$ is of characteristic $p > 0$ and that $\varphi : A(0; q, 1) \to A(0; q^p, 1)$ is the Frobenius morphism, acting on rational points as $\alpha \mapsto \alpha^p$. Let $(\mathcal{E}, \nabla)$ and $(\mathcal{F}, \nabla)$ as before;
- Let $\rho \in (q, 1)$ so that $\eta_\rho \mapsto \eta_{\rho^p}$;

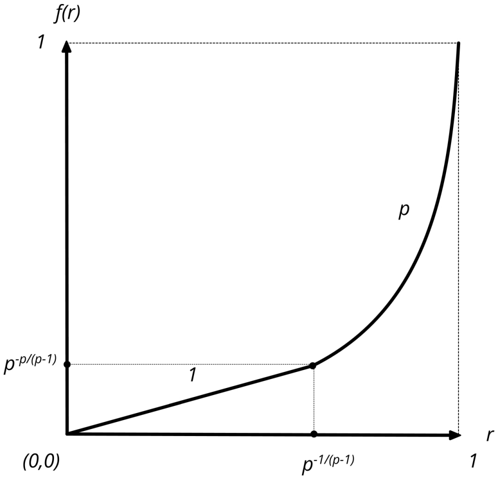
Frobenius pushforward

- Suppose that residue field $\tilde{k}$ is of characteristic $p>0$ and that $\varphi : A(0;q,1) \to A(0;q^p,1)$ is the Frobenius morphism, acting on rational points as $\alpha \mapsto \alpha^p$. Let $(\mathcal{E},\nabla)$ and $(\mathcal{F},\nabla)$ as before;
- Let $\rho \in (q,1)$ so that $\eta_\rho \mapsto \eta_{\rho^p}$;
- We want to compare $\mathcal{MR}(\eta_\rho,(\mathcal{E},\nabla))$ and $\mathcal{MR}(\eta_{\rho^p},(\mathcal{F},\nabla))$.

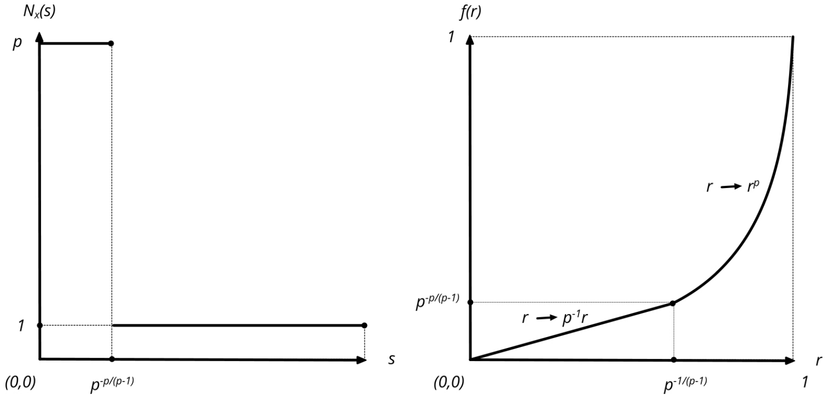
Visual presentation of Frobenius morphism 1



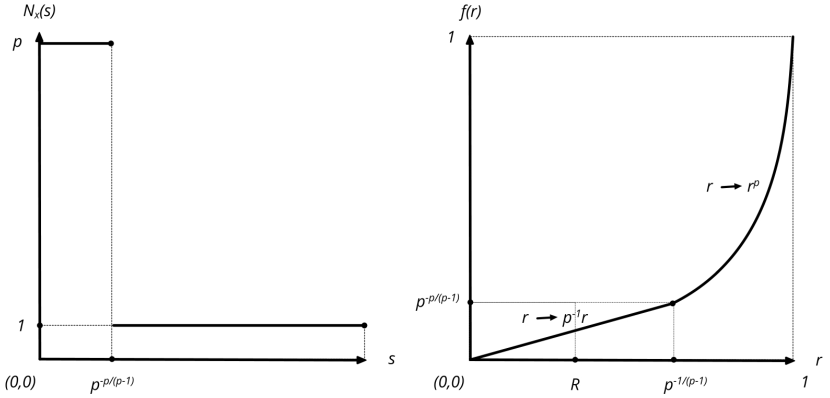
The profile function of Frobenius morphism



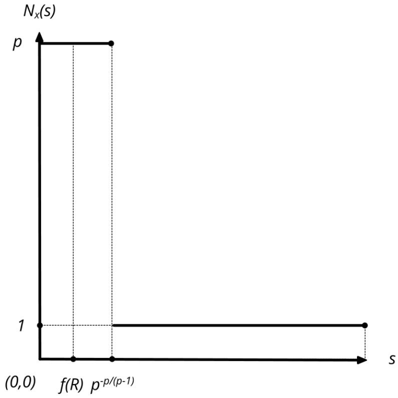
Frobenius N_x and profile

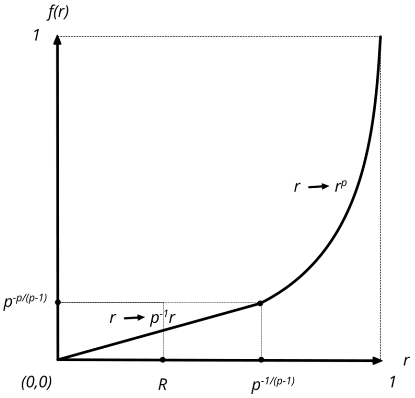


Frobenius pushforward: first case



Frobenius pushforward: first case



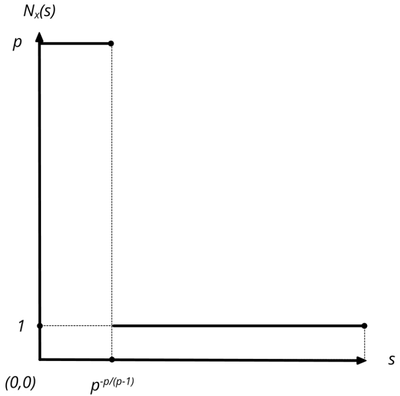


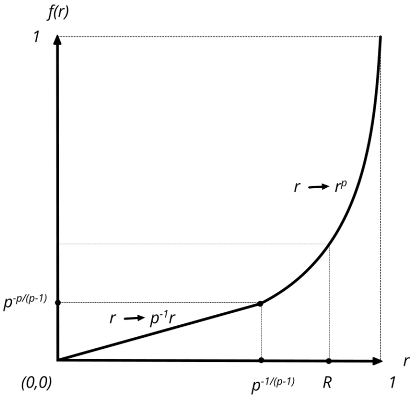
Frobenius pushforward: first case

$$\mathcal{MR}(\eta_\rho, (\mathcal{E}, \nabla)) \quad \longleftrightarrow \quad \mathcal{MR}(\eta_{\rho^p}, (\mathcal{F}, \nabla))$$

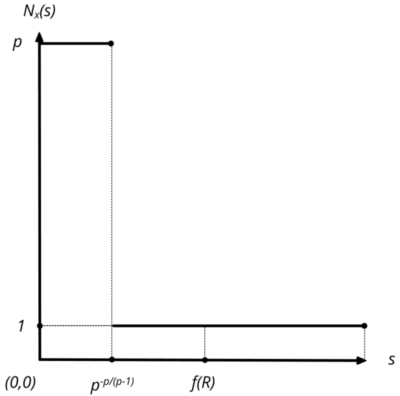
- $R \leq p^{-1/(p-1)} \quad \longleftrightarrow \quad p^{-1}R$: p-times

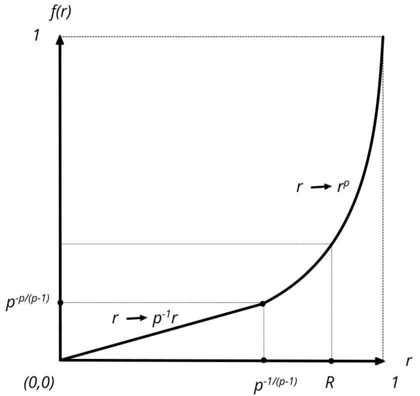
Frobenius pushforward: second case





Frobenius pushforward: second case





Frobenius pushforward

$$\mathcal{MR}(\eta_\rho, (\mathcal{E}, \nabla)) \quad \longleftrightarrow \quad \mathcal{MR}(\eta_{\rho^p}, (\mathcal{F}, \nabla))$$

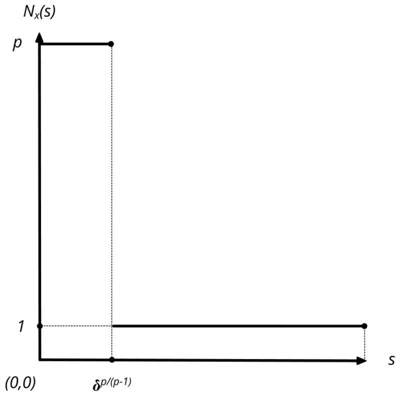
- $R \leq p^{-1/(p-1)} \quad \longleftrightarrow \quad p^{-1}R$: p-times
- $R > p^{-1/(p-1)} \quad \longleftrightarrow \quad p^{-p/(p-1)}$: $(p-1)$-times and R^p.

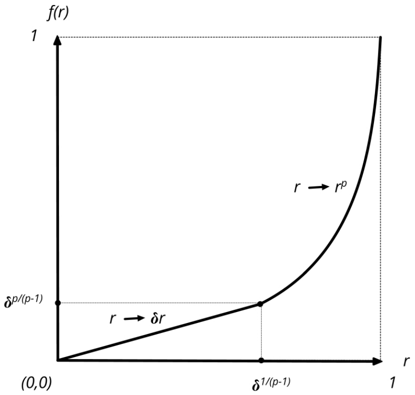
Generalized Frobenius pushforward

We come back to general setting and assume that $y = \varphi^{-1}(x)$ and that φ is residually purely inseparable of degree p at y and that the different of the extension $\mathscr{H}(y)/\mathscr{H}(x)$ is δ.

(The different of $\mathscr{H}(\eta_\rho)/\mathscr{H}(\eta_{\rho^p})$ was p^{-1})

Profile and N_x of generalized Frobenius pushforward





Correspondence for generalized Frobenius pushforward

$$\mathcal{MR}(y,(\mathcal{E},\nabla)) \quad \longleftrightarrow \quad \mathcal{MR}(x,(\mathcal{F},\nabla))$$

- $R \leq \delta^{1/(p-1)} \quad \longleftrightarrow \quad \delta \cdot R$: p-times
- $R > \delta^{1/(p-1)} \quad \longleftrightarrow \quad \delta^{p/(p-1)}$: $(p-1)$-times and R^p.

Relation with ramification of one-dimensional analytic fields

- $\varphi : Y \to X$, $y \in Y^{\mathrm{hyp}}$ and $x = \varphi(y)$ as before and assume that $\mathrm{char}\tilde{k} = p > 0$.

Relation with ramification of one-dimensional analytic fields

- $\varphi : Y \to X$, $y \in Y^{\mathrm{hyp}}$ and $x = \varphi(y)$ as before and assume that $\mathrm{char}\tilde{k} = p > 0$.
- Suppose that $\mathscr{H}(y)/\mathscr{H}(x)$ is Galois with the group G, and let $\sigma \in G$; Then, one defines (at least when y is of type 2) $i(\sigma) := \sup_{a \in \mathscr{H}^\circ(y)} |\sigma(a) - a|$ and

$$\varphi_{y/x} : [0,1] \to [0,1], \quad s \mapsto \prod_{\sigma \in G, \sigma \neq \mathrm{Id}} \max(i(\sigma), s);$$

Relation with ramification of one-dimensional analytic fields

- $\varphi : Y \to X$, $y \in Y^{\mathrm{hyp}}$ and $x = \varphi(y)$ as before and assume that $\mathrm{char}\tilde{k} = p > 0$.
- Suppose that $\mathscr{H}(y)/\mathscr{H}(x)$ is Galois with the group G, and let $\sigma \in G$; Then, one defines (at least when y is of type 2) $i(\sigma) := \sup_{a \in \mathscr{H}^\circ(y)} |\sigma(a) - a|$ and

$$\varphi_{y/x} : [0,1] \to [0,1], \quad s \mapsto \prod_{\sigma \in G, \sigma \neq \mathrm{Id}} \max(i(\sigma), s);$$

- Function i gives a lower ramification filtration on G, their images by $\varphi_{y/x}$ give jumps in the upper ramification filtration on G.

Theorem (M. Temkin)

The Herbrand's function $\varphi_{y,x}$ coincides with the profile function of φ at y.

Relation with pushforward of the constant connection

Theorem (B-P)

Assume that $\varphi^{-1}(x) = \{y\}$ *and that the extension* $\mathscr{H}(y)/\mathscr{H}(x)$ *is Galois with group* G. *Then, the radii of convergence of the differential equation* $\varphi_*(\mathcal{O}, d)$ *at* x *coincide with the non-zero upper ramification jumps of the extension* $\mathscr{H}(y)/\mathscr{H}(x)$.

More precisely, if we denote by $v_{-1} = 1 > v_0 > \cdots > v_n > v_{n+1} = 0$ *the jumps of the upper ramification filtration of the extension* $\mathscr{H}(y)/\mathscr{H}(x)$, *then we have*

$$\mathcal{MR}(x, \varphi_*(\mathcal{O}, d)) = (v_n, \ldots, v_n, \ldots, v_0, \ldots, v_0, 1),$$

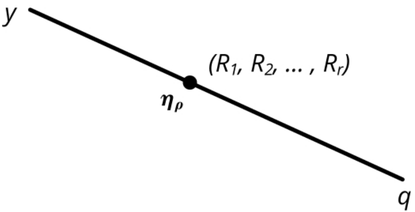
where, for each $j = 0, \ldots, n$, v_j *appears* $(G : G^{v_{j+1}}) - (G : G^{v_j})$ *times.*

Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$

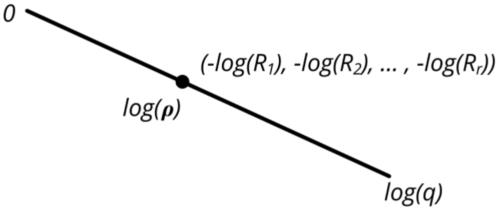
Let $y \in Y^{\mathrm{hyp}}$ of type 2 (for simplicity) and let $A(0; q, 1)$ be a small open annulus "attached" to y (corresponds to an element $\vec{t}$ in the tangent space $T_y Y$).

***

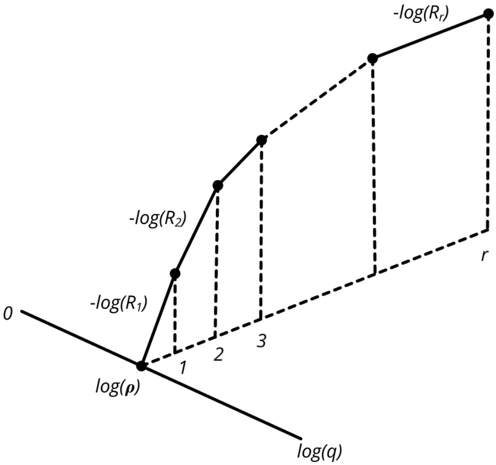
Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$



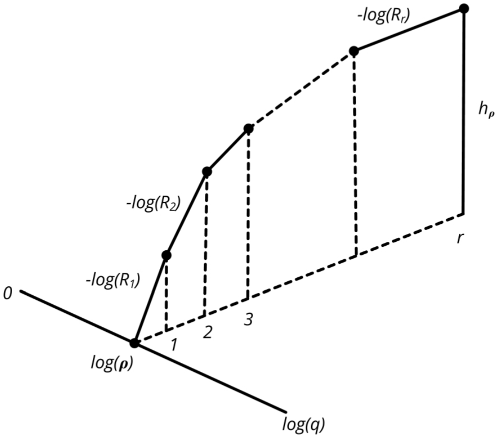
Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$



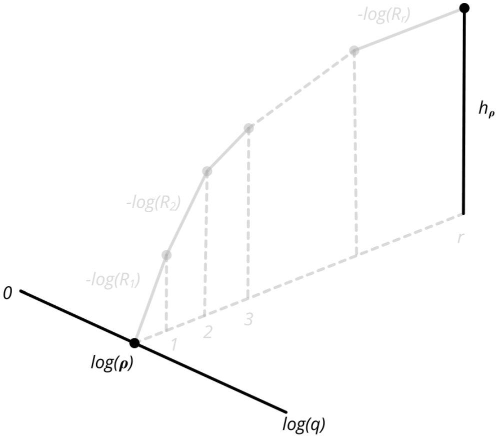
Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$



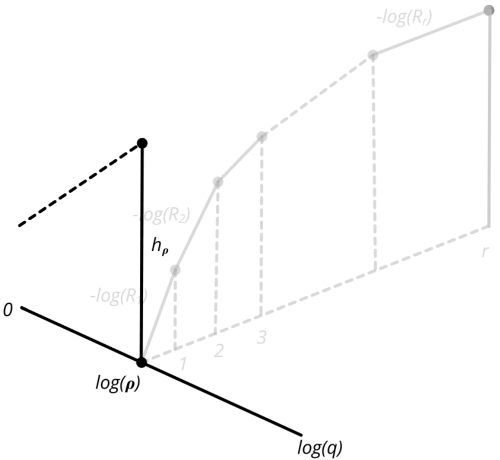
Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$



Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$



Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$



Irregularity $\mathrm{Irr}_{\vec{t}}(y,(\mathcal{E},\nabla))$

Let $y \in Y^{\mathrm{hyp}}$ of type 2 (for simplicity) and let $A(0;q,1)$ be a small open annulus "attached" to y (corresponds to an element $\vec{t}$ in the tangent space $T_y Y$).

***

$\mathrm{Irr}_{\vec{t}}(y,(\mathcal{E},\nabla))$

Comparison of $\mathrm{Irr}_{\vec{t}}(y,(\mathcal{E},\nabla))$ and $\mathrm{Irr}_{\vec{v}}(x,(\mathcal{F},\nabla))$

Let $(\mathcal{E},\nabla)_{\vec{t}}$ be the restriction of $(\mathcal{E},\nabla)$ on $A_{\vec{t}}$, let $(\mathcal{F},\nabla)_{\vec{t}}$ be the pushforward of $(\mathcal{E},\nabla)_{\vec{t}}$ by $\varphi_{\vec{t}} := \varphi_{|A_{\vec{t}}}$ (it will be a connection on a small open annulus $A_{\vec{v}} = A(0;q^d,1)$ attached to $x=\varphi(y)$).
In some coordinates we have $S=\varphi_{\vec{t}}(T) = T^d(1+h(T))$ and $\frac{dS}{dT} = aT^{\sigma}(1+g(T))$. We put $\nu := \sigma - d + 1$.

Irregularity $\mathrm{Irr}_{\vec{t}}(y, (\mathcal{E}, \nabla))$

Let $(\mathcal{E}, \nabla)_{\vec{t}}$ be the restriction of $(\mathcal{E}, \nabla)$ on $A_{\vec{t}}$, let $(\mathcal{F}, \nabla)_{\vec{t}}$ be the pushforward of $(\mathcal{E}, \nabla)_{\vec{t}}$ by $\varphi_{\vec{t}} := \varphi_{|A_{\vec{t}}}$ (it will be a connection on a small open annulus $A_{\vec{v}} = A(0; q^d, 1)$ attached to $x = \varphi(y)$).
In some coordinates we have $S = \varphi_{\vec{t}}(T) = T^d(1 + h(T))$ and $\frac{dS}{dT} = aT^{\sigma}(1 + g(T))$. We put $\nu := \sigma - d + 1$.

Theorem (B-P)

For each $\rho < 1$ close enough to 1, we have

$$h(\eta_{\rho^d}, (\mathcal{F}, \nabla)_{\vec{t}}) = d\, h(\eta_{\rho}, (\mathcal{E}, \nabla)_{\vec{t}}) - r\nu \log \rho^d - rd \log |a|.$$

In particular, we have

$$Irr_{\vec{v}}(x, (\mathcal{F}, \nabla)_{\vec{t}}) = Irr_{\vec{t}}(y, (\mathcal{E}, \nabla)_{\vec{t}}) + r\nu.$$

Laplacians

Theorem (B-P)

Suppose that $\varphi^{-1}(x) = \{y\}$ and let $(\mathcal{F}, \nabla) := \varphi_*(\mathcal{E}, \nabla)$. Let Γ_x be a finite subset of $T_x X$ and let $\Gamma_y := \vec{\varphi}_y^{-1}(\Gamma_x) \subset T_y Y$. Then

$$\Delta_y(\Gamma_y, (\mathcal{E}, \nabla)) = \Delta_x(\Gamma_x, (\mathcal{F}, \nabla)) + r \sum_{\vec{t} \in \Gamma_y} \nu_{\vec{t}}.$$

Theorem (B-P)

Let $x \in X$ be an inner point of type 2. Let $i \in \{1, \dots, r\}$. Then, for each finite subset Γ_x of $T_x X$ we have

$$\Delta_x^i(\Gamma_x, (\mathcal{E}, \nabla)) \leq (2g(x) - 2 + \#\Gamma_x)\, i$$

with equality in the following case: $i = r$ and $\mathcal{R}_r(x, (\mathcal{E}, \nabla)) < 1$.

Thank you!

Merci!

Хвала!