p-ADIC ANALYTIC GEOMETRY AND DIFFERENTIAL EQUATIONS
GÉOMÉTRIE ANALYTIQUE ET ÉQUATIONS DIFFÉRENTIELLES p-ADIQUES

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Konstantin Ardakov: *Bounded linear endomorphisms of rigid analytic functions*

Francesco Baldassarri: *Convergence and spectral polygon of connections on Berkovich curves*

Vladimir Berkovich: *de Rham theorem in non-Archimedean analytic geometry*

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Tomoyuki Abe

Arithmetic $\mathcal{D}$-modules and existence of crystalline companion

Abstract: We will show that there exists a correspondence between smooth ℓ-adic sheaves and overconvergent F-isocrystals over a curve preserving the Frobenius eigenvalues. Moreover, we show the existence of ℓ-adic companions associated to overconvergent F-isocrystals for smooth varieties.
Some part of the work is done jointly with Esnault.

Konstantin Ardakov

Bounded linear endomorphisms of rigid analytic functions

Abstract: The classical Riemann-Hilbert correspondence relates regular holonomic $\mathcal{D}$-modules on a complex analytic manifold X with the constructible sheaves on X. In 2000, Prosmans and Schneiders generalised this correspondence to perfect complexes $C_\bullet$ of modules over the sheaf of infinite-order differential operators on X: it is possible to reconstruct $C_\bullet$ from its solution complex, provided one does not forget the topology on these solution spaces. I will talk about an analogue of this theory in the rigid analytic setting. This is joint work with Oren Ben-Bassat.

Francesco Baldassarri

Convergence and spectral polygon of connections on Berkovich curves

Abstract: I will discuss some foundational aspects of the theory of non-archimedean connections which seem to have been overlooked in previous work. One is the systematic use of "parallel transport" to refine Dwork's transfer principle. We clarify the difference between filtration according to the radius of convergence of solutions and according to the spectral radius: only the latter satisfies André's formalism of "tensor-bounded filtrations", and therefore splits canonically. Another point we will address is the extension of Temkin's notion of "Kaehler seminorm" to sheaves of jets and of differential operators. We indicate a simple way to compute these seminorms and their use in the definition of the spectral radius of convergence. Finally, we will report on joint work with Kedlaya aimed at a simplification of Poineau and Pulita's continuity results.

Vladimir Berkovich

de Rham theorem in non-Archimedean analytic geometry

Abstract: In my work in progress on complex analytic vanishing cycles for formal schemes, I have defined integral "etale" cohomology groups of a compact strictly analytic space over the field of Laurent power series with complex coefficients. These are finitely generated abelian groups provided with a quasi-unipotent action of the fundamental group of the punctured complex plane, and they give rise to all ℓ-adic etale cohomology groups of the space. After a short survey of this work, I will explain a theorem which, in the case when the space is rig-smooth, compares those groups and the de Rham cohomology groups of the space. The latter are provided with the Gauss-Manin connection and an additional structure which allow one to recover from them the "etale" cohomology groups with complex coefficients.

Velibor Bojković

Pushforwards of p-adic differential equations

Abstract: I will present a joint work done with Jérôme Poineau. Given a finite étale morphism $\varphi : Y \to X$ of quasi-smooth Berkovich curves over an algebraically closed, complete, nonarchimedean and nontrivially valued field of characteristic 0, a p-adic differential equation $(\mathcal{E}, \nabla)$ on Y and a point $x \in X$, we provide a relation between the radii of convergence of solutions of the pushforwarded equation $\varphi_*(\mathcal{E}, \nabla)$ at x and radii of convergence of solutions of $(\mathcal{E}, \nabla)$ at the

preimages of the point x in terms of the local invariants of the morphism at these points, the so-called profiles.

As a corollary we obtain the previously known cases of this problem, such as Kedlaya's Frobenius pushforward or the case where x has one preimage and φ is residually separable at it. We also provide formulas for the change of irregularity and Laplacian of $(\mathcal{E}, \nabla)$ under the pushforward by φ. We furthermore establish a relation between the radii of convergence of the pushforward of the constant connection on Y and the Herbrand function of the corresponding (presumed Galois) extension of completed residue field $\mathscr{H}(x)$.

Antoine Chambert–Loir

Formes et courants sur les espaces de Berkovich : retour aux sources

Abstract: J'expliquerai un travail en commun avec Antoine Ducros dans lequel nous définissons une notion de formes différentielles réelles et de courants (bigradués) sur des espaces de Berkovich non archimédiens. Durant la révision (toujours en cours) de ce travail, nous avons réalisé que ces constructions prennent naturellement place sur des squelettes. En particulier, toute partie polyédrale d'un espace de Berkovich est naturellement munie d'un calibrage canonique grâce auquel on peut intégrer les (n, n)-formes à support compact.

Bruno Chiarellotto

A non-abelian algebraic criterion for good reduction of curves I – Comparison of relative log de Rham fundamental groups

Abstract: Recently F. Andreatta, A. Iovita and M. Kim proved a criterion for good reduction of curves. They showed that for a proper curve $f\colon X \to \mathrm{Spec}K$ of genus $g \geqslant 2$ and semistable reduction defined over a discrete valuation field K of mixed characteristic $(0, p)$, the monodromy action of $\pi_1(\mathrm{Spec}K)$ on the p-adic unipotent fundamental group of the geometric generic fiber of f is crystalline if and only if f has good reduction. By deformation, they reduce themselves to the study of a family of proper complex curves $h\colon Y \to \Delta^*$ of genus $g \geqslant 2$ over a complex punctured disc Δ^*, having semistable reduction at the center. In this case, with trascendental methods, Oda was able to prove that the outer monodromy action of $\pi_1(\Delta^*) \cong \mathbb{Z}$ on the classical unipotent fundamental group of the generic fiber of h is trivial if and only if h has good reduction at the center. The goal of a joint project with V. Di Proietto and A. Shiho is to give a purely algebraic proof of this criterion (and this will amount to reprove Oda's result in a pure algebraic way). In this talk we introduce several definitions of the relative unipotent log de Rham fundamental group for log-schemes (not only for curves). We will prove their equivalence. This will be used later in our proof of the criterion.

Valentina Di Proietto

A non-abelian algebraic criterion for good reduction of curves II – Computation of the monodromy action

Abstract: In this second part we apply the comparison results of the first part to the algebraic analogue of a family of proper complex curves $h\colon Y \to \Delta^*$ of genus $g \geqslant 2$ over a puntured disc Δ^*. We show how we can explicitely calculate the monodromy action, and relate it to the degeneration of the family at the center.

Hélène Esnault

$\mathcal{D}$-modules and p-curvatures

Abstract: We show relations between rigidity of connections in characteristic 0 and nilpotency of their p-curvatures (a consequence of a conjecture by Simpson and of a generalization of Grothendieck?s p-curvature conjecture).
Work in progress with Michael Groechenig.

Charles Favre

Explosion of Lyapunov exponents using non-Archimedean geometry

Abstract: We consider a meromorphic family of endomorphisms of the complex projective space parameterized by the unit disk, and show that the blow-up of the Lyapunov exponent near the origin is controlled by a non-Archimedean quantity.

Walter Gubler

Differentiability of non-archimedean volumes and non-archimedean Monge-Ampère equations

Abstract: We study non-archimedean volumes, a tool which allows us to control the asymptotic growth of small sections of big powers of a metrized line bundle. We prove that the non-archimedean volume is differentiable at a continuous semipositive metric and that the derivative is given by integration with respect to a Monge-Ampère measure. Such a differentiability formula had been proposed by M. Kontsevich and Y. Tschinkel. In residue characteristic zero, it implies an orthogonality property for non-archimedean plurisubharmonic functions which allows us to drop an algebraicity assumption in a theorem of S. Boucksom, C. Favre and M. Jonsson about the solution to the non-archimedean Monge-Ampère equation.

Christine Huyghe

Coadmissible $\mathcal{D}$-modules over the generic fibre of a formal scheme

Abstract: This is joint work with D. Patel, T. Schmidt and M. Strauch. The motivation of this work comes from the localization theorem of Beilinson-Bernstein and Brylinski-Kashiwara. If G is reductive group over the field of complex numbers, the four previous authors proved that there is an equivalence of categories between the category of representations of $\mathrm{Lie}(G)$ with central character, and the category of $\mathcal{D}$-modules over the flag variety of G. Their result led to the proof of the Kazhdan-Lusztig conjectures. i will explain during this talk how one can generalize this localization result in the p-adic context for a split group scheme G over some discrete valuation ring, focusing on the $\mathcal{D}$-modules side. The category of $\mathcal{D}$-modules has then to be replaced by the category of coadmissible $\mathcal{D}$-modules over the generic fiber of a smooth formal scheme X. I will explain this construction that uses Raynaud's admissible blowing-ups of X and apply it to the p-adic completion of the flag variety of G. As in the classical case, one can prove some acyclicity results as well as Cartan's type theorems "A" and "B".

Kiran Kedlaya

Cyclic group actions and p-adic exponents

Abstract: The Christol-Mebkhout theory of p-adic exponents provides a (partial) structure theory for connections on a p-adic annulus satisfying the Robba condition. We explain how this theory can be refined by replacing the category of connections satisfying the Robba condition with a certain category of vector bundles with actions of the group $\mathbb{Q}_p/\mathbb{Z}_p$. Based on joint work with Atsushi Shiho.

Christopher David Lazda

A semistable Lefschetz $(1,1)$ ***theorem in equicharacteristic***

Abstract: By using some elementary properties of the (logarithmic) de Rham-Witt complex I will explain how to prove that a (rational) line bundle on the special fibre of a proper, semistable scheme over a power series ring $k[\![t]\!]$ in characteristic p lifts to the total space if and only if its first Chern class does. This generalises a result of Morrow in the smooth case, and provides an equicharactersitic analogue of a result of Yamashita. I will also explain a corollary concerning algebraicity of cohomology classes on varieties over global function fields. This is joint work with with Ambrus Pál.

François Loeser

Non-archimedean parametrizations, points of bounded height and applications

Abstract: Pila and Wilkie proved a general estimate for the number of rational points on the transcendental part of sets definable in an o-minimal structure; this has been used in a spectacular way by Pila to provide an unconditional proof of some cases of the André-Oort conjecture. They use in a fundamental way their o-minimal version of Yomdin-Gromov parametrizations.

Together with R. Cluckers and G. Comte we proved recently the existence of non-archimedean Yomdin-Gromov parametrizations. We used them to prove geometric versions of results of Bombieri and Pila over $\mathbb{C}((t))$ where counting points is replaced by counting dimensions. In the p-adic case we obtained a full version of the Pila-Wilkie result, which we recently used in joint work with A. Chambert-Loir to obtain a result of Ax-Lindemann type for products of Mumford curves.

Adriano Marmora

On the continuity of the finite Bloch-Kato cohomology

Abstract:

Bloch-Kato's finite cohomology groups give the local conditions, at places above a prime p, defining Selmer groups of a p-adic representation of the absolute Galois group of a number field. Precisely, let $K/\mathbb{Q}_p$ be a finite extension and let us denote by G_K the absolute Galois group of the local field K. To a crystalline $\mathbb{Z}_p$-representation T of G_K, Bloch and Kato associated a group $H^1_f(K,T)$, called finite cohomology, parametrizing crystalline extensions of the trivial representation $\mathbb{Z}_p$ by T. The behavior of finite cohomology under congruences between Galois representations is interesting and not well understood in general. In this lecture we present a functoriality result for these groups under congruences of representations, assuming some hypothesis on the ramification index of $K/\mathbb{Q}_p$ and on p-adic Hodge-Tate weights of the representations. This is a joint work with Adrian Iovita.

Shun Ohkubo

On the rationality of the logarithmic growth filtration of solutions of p***-adic differential equations***

Abstract: In his study of p-adic differential equations, B. Dwork proved that the power series solutions of certain p-adic differential equations such as the (p-adic) Gaussian hypergeometric one satisfy a mild growth condition. Recently, B. Chiarellotto and N. Tsuzuki reconsidered Dwork's work and conjectured a relationship between the order of growth and Frobenius slopes of the space of solutions. In this talk, we discuss some part of Chiarellotto-Tsuzuki conjecture.

Ambrus Pa

Rigid τ-crystals

Abstract: Rigid τ-crystals are equicharacteristic analogues of F-isocrystals. They arise naturally in the theory of motives in function field arithmetic, such as Drinfeld modules. I will report on recent work on the subject by Heuer, Ziegler and myself, including the analogues of some basic structural results, such as slope filtrations, the relation to stratified bundles, and full faithfulness results. It turns out that it is possible to prove suitable versions of some central results of the theory of F-isocrystals in this setting, including the analogue of a currently open conjecture, while other fundamental results are false. If time permits, I will also mention arithmetic applications.

Andrea Pulita

An Overview on Some Recent Results about p-Adic Differential Equations over Berkovich Curves

Abstract:

I will give an introductory talk on my recent results about p-adic differential equations on Berkovich curves, most of them in collaboration with J. Poineau. This includes the continuity of the radii of convergence of the equation, the finiteness of their controlling graphs, the global decomposition by the radii, a bound on the size of the controlling graph, and finally the finite dimensionality of their de Rham cohomology groups, together with some local and global index theorems relating the de Rham index to the behavior of the radii of the curve. If time permits I will say a word about some recent applications to the Riemann-Hurwitz formula.

Tobias Schmidt

Arithmetic $\mathcal{D}$-modules and p-adic representations

Abstract: In this talk I will explain some recent applications of the theory of arithmetic $\mathcal{D}$-modules to representations of p-adic reductive groups like $\mathrm{GL}_n(\mathbb{Q}_p)$. Such representations have gained much attention in the last years, mostly in connection with the p-adic local Langlands programme. A crucial input in our discussion is a certain arithmetic variant of the Beilinson-Bernstein localization theorem which allows to pass back and forth between $\mathcal{D}$-modules and representations. This is joint work with C. Huyghe, D. Patel and M. Strauch.

Michael Temkin

Logarithmic desingularization and semistable reduction

Abstract: I will tell about my work in progress with D. Abramovich and J. Wlodarczyk. We construct a canonical desingularization of log varieties of characteristic zero, which is functorial with respect to all log smooth morphisms (including Kummer coverings). The same algorithm should provide semistable reduction theorem for schemes and formal schemes over valuation rings of residue characteristic zero (work in progress), with the only technical difficulty coming from non-noetherianity. Our algorithm is a logarithmic adjustment, and even simplification, of the usual desingularization algorithm (we use the version of Wlodarczyk). Naturally, it runs by a canonical principalization of ideals on log smooth varieties. A surprising fact, though, is that in order to have the strongest functoriality we have to work with log smooth DM stacks and non-representable modifications that we call Kummer blow ups.

Simon Wadsley

Holonomic $\mathcal{D}$-modules on smooth rigid analytic spaces

Abstract: The speaker and Ardakov introduced a sheaf $\mathcal{D}$ of possibly infinite-order differential operators on any smooth rigid analytic space together with a notion of coadmissible $\mathcal{D}$-module that is supposed to be roughly analogous to the notion of a coherent $\mathcal{D}$-module on an algebraic variety. In this talk we will discuss a notion of a holonomic $\mathcal{D}$-module in this context (in characteristic 0) and some of its good properties and its limitations.