Monochromatic sumsets for colourings of $\mathbb{R}$

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Introduction

Consistently, modulo some large cardinal,

if $f : \mathbb{R} \to r$ with $r \in \omega$ then there is an **infinite** $X \subseteq \mathbb{R}$ so that

$$f \restriction X + X \text{ is constant.}$$

$X + X = \{x + y : x, y \in X\}$ i.e. **repetitions are allowed.**

- How does this fit into the theory of partition relations?
- What goes into the proof of this result?
- Joint result with P. Komjáth, I. Leader, P. Russell, S. Shelah, D. T. Soukup, and Z. Vidnyánszky.

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Evolving partition relations

... an incomplete overview ...

Evolving partition relations

If $f : \omega \to r$ then there is an infinite $X \subset \omega$ with $f \restriction X$ constant.

P. H. Principle

$$\omega \to (\omega)^1_r$$

Evolving partition relations

If $f : [\omega]^k \to r$ then there is an infinite $X \subset \omega$ with $f \restriction [X]^k$ constant.

P. H. Principle

$$\omega \to (\omega)^1_r$$

F. P. Ramsey, 1930

$$\omega \to (\omega)^k_r$$

Evolving partition relations

There is $f : [2^{\aleph_0}]^2 \to 2$ so that $f''[X]^2 = 2$ for any uncountable $X \subset 2^{\aleph_0}$.

P. H. Principle

$$\omega \to (\omega)^1_r$$

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$$\omega \to (\omega)^k_r$$

W. Sierpinski, 1933

$$2^{\aleph_0} \not\to (\aleph_1)^2_2$$

Evolving partition relations

If $f : [(2^{2^{\cdot^{\cdot^{\cdot^{2^{\aleph_0}}}}}})^+]^k \to r$ then $f \restriction [W]^k$ is constant for some uncountable W.

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$$\omega \to (\omega)^1_r$$

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Erdős, Rado 1956

$(2^{2^{\cdot^{\cdot^{\cdot^{2^{\aleph_0}}}}}})^+ \to (\omega_1)^k_r$ for all $r < \omega$.

Evolving partition relations

$FinSum(X) = \{x_0 + x_1 + \cdots + x_\ell : x_0 < \cdots < x_\ell \in X\}$ i.e. no repetitions.

P. H. Principle

$$\omega \to (\omega)^1_r$$

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$(2^{2^{\cdots^{2^{\aleph_0}}}})^+ \to (\omega_1)^k_r$ for all $r < \omega$.

N. Hindman, 1974

if $f : \mathbb{N} \to r$ then there is some infinite $X \subseteq \mathbb{N}$ so that $f \restriction FinSum(X)$ is constant.

Evolving partition relations

There is $f : [\aleph_1]^2 \to \aleph_1$ so that $f''[X]^2 = \aleph_1$ for any uncountable $X \subset \aleph_1$.

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$$\omega \to (\omega)^1_r$$

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$$\aleph_1 \not\to [\aleph_1]^2_{\aleph_1}$$

Evolving partition relations

If $f : [2^{\aleph_0}]^2 \to 3$ then there is an uncountable $X \subset 2^{\aleph_0}$ with $|f''[X]^2| \leq 2$.

P. H. Principle

$$\omega \to (\omega)^1_r$$

F. P. Ramsey, 1930

$$\omega \to (\omega)^k_r$$

W. Sierpinski, 1933

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$$\aleph_1 \not\to [\aleph_1]^2_{\aleph_1}$$

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$$\mathrm{Con}(2^{\aleph_0} \to [\aleph_1]^2_3)$$

Monochromatic sumsets in $\mathbb{N}$

Easy Ramsey consequence: if $f : \mathbb{N} \to r$ with $r \in \omega$ then there is an infinite $X \subseteq \mathbb{N}$ so that

$$f \restriction X \oplus X \text{ is constant.}$$

Here $X \oplus X = \{x + y : x \neq y \in X\}$ i.e. repetitions are not allowed.

Proof:

- if $f : \mathbb{N} \to r$ then let $g : [\mathbb{N}]^2 \to r$ defined by $g(x, y) = f(x + y)$,
- if $X \subset \mathbb{N}$ and $g \restriction [X]^2$ is constant then $f \restriction X \oplus X$ is constant too.

[Owings, Hindman 1970s] What happens if we allow repetition?

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Monochromatic sumsets in $\mathbb{N}$ - with repetitions?

$X + X = X \oplus X \cup \{2x : x \in X\}$.

There is $f : \mathbb{N} \to 4$ without infinite monochromatic sumsets:

$$f(x) = \lfloor \log_{\sqrt{2}}(x) \rfloor \bmod 4.$$

- Suppose that $X \subseteq \mathbb{N}$ is infinite and take $y << x \in X$.
- $|\log_{\sqrt{2}}(x) - \log_{\sqrt{2}}(x+y)| < 1$,
- $|f(x) - f(x+y)| \leq 1 \bmod 4$.
- $f(2x) = \lfloor \log_{\sqrt{2}}(x) + 2 \rfloor = f(x) + 2 \bmod 4$ so $f(2x) \neq f(x+y)$.

Can we do this with 2 colours???

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Monochromatic sumsets in $\mathbb{R}$

Started in **[Hindman, Leader, Strauss 2015]**

If $f : \mathbb{R} \to r$ is **Baire/Lebesgue measurable** then there is a **perfect** $\emptyset \neq X \subseteq \mathbb{R}$ so that

$$f \restriction X + X \text{ is constant.}$$

Without definability?

There is an $f : \mathbb{R} \to 2$ so that

$$f''X \oplus X = 2 \text{ for every uncountable } X \subset \mathbb{R}.$$

- **[HLS]** using CH, **[Komjáth, DTS, Weiss]** in ZFC, and consistently the number of colours is best possible.

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Continued by **[Fernandez-Breton, Rinot 2016]**:

- how to realize more colours on sets of the form $FinSum(X)$ (no repetitions),
- general theorems on **uncountable, commutative, cancellative semigroups** G.

Bottom line: without definabilty,

Strongest possible result on $\mathbb{R}$: infinite sumsets with repetition allowed.

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Monochromatic sumsets - with repetitions

Recall: $\exists\ f : \mathbb{N} \to 4$ so that $f \restriction X + X$ is **not constant** for an infinite $X \subset \mathbb{N}$.

Let $G(\kappa) = \bigoplus_\kappa \mathbb{Q}$ i.e. $x : \kappa \to \mathbb{Q}$ with $|supp(x)| < \omega$. E.g. $G(2^{\aleph_0}) \approx \mathbb{R}$.

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Monochromatic sumsets - with repetitions

If $f : G(\kappa) \to r$ then $f \restriction X + X$ is constant for some infinite $X \subset G(\kappa)$.

Let $G(\kappa) = \bigoplus_\kappa \mathbb{Q}$ i.e. $x : \kappa \to \mathbb{Q}$ with $|supp(x)| < \omega$. E.g. $G(2^{\aleph_0}) \approx \mathbb{R}$.
Notation:

$$G(\kappa) \xrightarrow{+} (\aleph_0)_r$$

Monochromatic sumsets - with repetitions

$\exists\ f : G(\kappa) \to r$ so that $f \restriction X + X$ is **not constant** for an infinite $X \subset G(\kappa)$.

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Notation:

$$G(\kappa) \overset{+}{\not\to} (\aleph_0)_r \text{ e.g. } \mathbb{N} \overset{+}{\not\to} (\aleph_0)_4$$

Monochromatic sumsets - with repetitions

$\exists\ f : G(\kappa) \to r$ so that $f \restriction X + X$ is **not constant** for an infinite $X \subset G(\kappa)$.

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[Hindman, Leader, Strauss]

Monochromatic sumsets - with repetitions

$\exists\ f : \mathbb{Q} \to 72$ so that $f \restriction X + X$ is **not constant** for an infinite $X \subset \mathbb{Q}$.

Let $G(\kappa) = \bigoplus_\kappa \mathbb{Q}$ i.e. $x : \kappa \to \mathbb{Q}$ with $|supp(x)| < \omega$. E.g. $G(2^{\aleph_0}) \approx \mathbb{R}$.

[Hindman, Leader, Strauss]

- $\mathbb{Q} \overset{+}{\not\to} (\aleph_0)_{72}$.

Monochromatic sumsets - with repetitions

$\exists\, f : G(m) \to 72$ so that $f \restriction X + X$ is **not constant** for an infinite $X \subset G(m)$.

Let $G(\kappa) = \bigoplus_{\kappa} \mathbb{Q}$ i.e. $x : \kappa \to \mathbb{Q}$ with $|supp(x)| < \omega$. E.g. $G(2^{\aleph_0}) \approx \mathbb{R}$.

[Hindman, Leader, Strauss]

- $\mathbb{Q} \overset{+}{\not\to} (\aleph_0)_{72}$.
- $G(m) \overset{+}{\not\to} (\aleph_0)_{72}$ **for** $m < \omega$.
-
-

Monochromatic sumsets - with repetitions

$\exists\ f : G(\aleph_0) \to 144$ so that $f \restriction X + X$ is **not constant** for an infinite $X \subset G(\aleph_0)$.

Let $G(\kappa) = \bigoplus_\kappa \mathbb{Q}$ i.e. $x : \kappa \to \mathbb{Q}$ with $|supp(x)| < \omega$. E.g. $G(2^{\aleph_0}) \approx \mathbb{R}$.

[Hindman, Leader, Strauss]

- $\mathbb{Q} \overset{+}{\not\to} (\aleph_0)_{72}$.
- $G(m) \overset{+}{\not\to} (\aleph_0)_{72}$ for $m < \omega$.
- $G(\aleph_0) \overset{+}{\not\to} (\aleph_0)_{144}$
-

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$\exists\ f : G(\aleph_m) \to 2^m \cdot 144$ so that $f \restriction X + X$ is **not constant** for an infinite X.

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Monochromatic sumsets - with repetitions

$\exists\, f : \mathbb{R} \to r$ so that $f \restriction X + X$ is **not constant** for an infinite X.

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- $G(\aleph_m) \overset{+}{\not\to} (\aleph_0)_{2^m \cdot 144}$ for $m < \omega$.

Corollary

If $2^{\aleph_0} < \aleph_\omega$ then

$$\mathbb{R} \overset{+}{\not\to} (\aleph_0)_r$$

for some $r < \omega$.

Positive relations through 'position invariance'

Given $s \in \mathbb{Q}^{<\omega}$ and $a \in [\kappa]^{|s|}$, let

$$x = s * a \in \bigoplus_{\kappa} \mathbb{Q}$$

by $supp(x) = a$ and $x(a(i)) = s(i)$.

Suppose that $c : \bigoplus_{\kappa} \mathbb{Q} \to 2$, and let $c_s : [\kappa]^{|s|} \to 2$ by

$$c_s(a) = c(s * a).$$

If κ is large enough then there is a large $W \subseteq \kappa$ so that c_{s_i} **are constant on** W for $i = 0, 1, 2$.

Assume that c_{s_1} and c_{s_2} are both constant 0.

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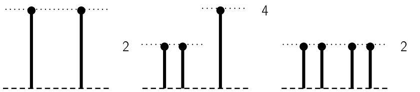

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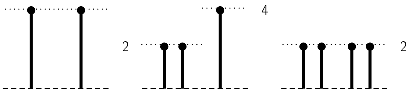

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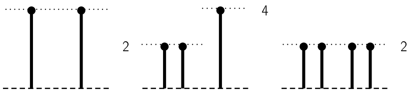

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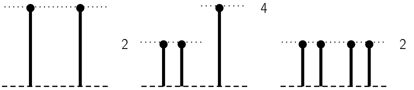

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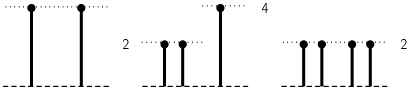

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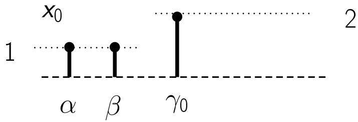

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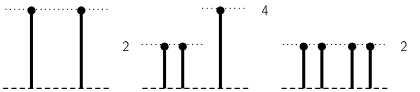

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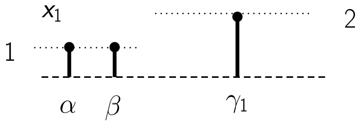

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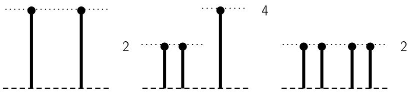

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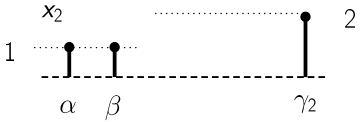

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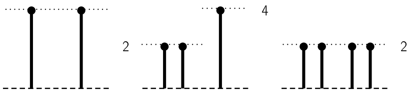

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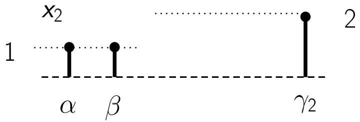

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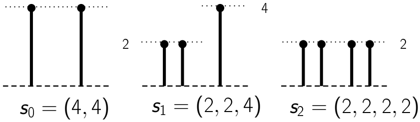

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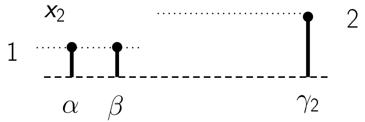

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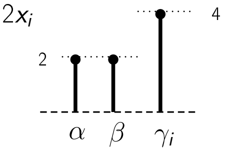

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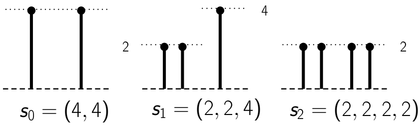

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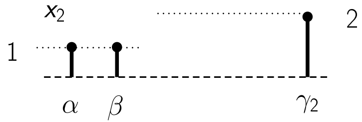

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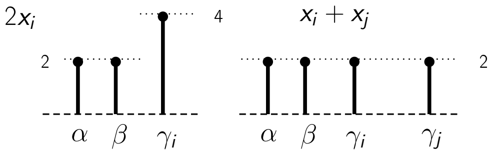

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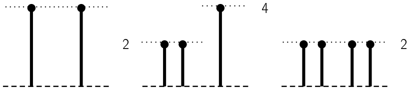

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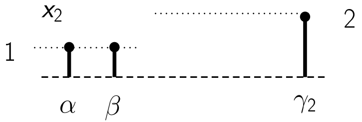

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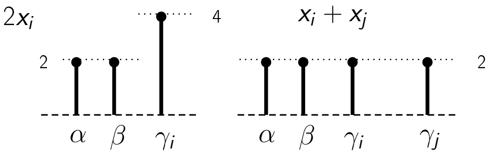

If c_{s_0}, c_{s_2} have the same constant then we need $tp(W) = \omega + \omega$.

Corollaries

[Komjáth] and **[Leader, Russell]** independently

$\Rightarrow G(\kappa) \xrightarrow{+} (\aleph_0)_r$ where $\kappa = \beth_{2r-1}(\aleph_0)$,

$\Rightarrow G(\aleph_\omega) \xrightarrow{+} (\aleph_0)_r$ for $r < \omega$ under GCH.

- using the **Erdős-Rado theorem.**

[DTS, Vidnyánszky]

$\Rightarrow G(\mathfrak{c}^+) \xrightarrow{+} (\aleph_0)_2$,

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Corollaries

[Komjáth] and [Leader, Russell] independently

$\Rightarrow G(\kappa) \xrightarrow{+} (\aleph_0)_r$ where $\kappa = \beth_{2r-1}(\aleph_0)$,

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Positive relations on $\mathbb{R}$ - the main result

Recall: if $2^{\aleph_0} < \aleph_\omega$ then $\mathbb{R} \overset{+}{\not\to} (\aleph_0)_r$ for some $r < \omega$.

Consistently, modulo an ω_1-Erdős cardinal,

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- **[S. Shelah, 2017]**
  "...you can suppose the coloring is continuous, right?"

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- **[S. Shelah, 1988]** Consistently, modulo an ω_1-Erdős cardinal, if $f : [2^{\aleph_0}]^{<\omega} \to r$ then there is an uncountable X and $F : X \hookrightarrow 2^\omega$ so that $f(\bar{x})$ **only depends on the type of the finite tree** $F[\bar{x}]$.

Various open problems

[Owings, 1974]

- $\mathbb{N} \overset{+}{\not\to} (\aleph_0)_2$???

Connected to our results:

- $\mathbb{R} \overset{+}{\to} (\aleph_0)_2$ in ZFC??
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- Is $2^{\aleph_0} = \aleph_m \to [\aleph_1]^2_3$ consistent for some $m < \omega$???
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What is the smallest r so that $G(\kappa) \overset{+}{\not\to} (\aleph_0)_r$ for a particular κ (finite, or $\aleph_m$)??

Monochromatic k-sumsets: $X + X + \cdots + X$?

[HLS] There is a finite colouring of $G(\aleph_n)$ with no infinite monochromatic k-sumsets ($n < \omega$),

[DS, Vidnyánszky] There is a finite coloring of $\mathbb{R}$ with no infinite monochromatic k-sumsets for $k \geq 3$.

We are far from a complete picture.

[Shelah, 1988]

- Is $2^{\aleph_0} = \aleph_m \to [\aleph_1]^2_3$ consistent for some $m < \omega$???
- Is $2^{\aleph_0} > \lambda \to [\aleph_1]^2_3$ consistent???

Various open problems – Thank you for your attention!

[Owings, 1974]

- $\mathbb{N} \overset{+}{\not\to} (\aleph_0)_2$???

Connected to our results:

- $\mathbb{R} \overset{+}{\to} (\aleph_0)_2$ in ZFC??
- $G(\aleph_\omega) \overset{+}{\to} (\aleph_0)_r$ **for** $r < \omega$ **in ZFC???**
- $\mathbb{R} \overset{+}{\to} (\aleph_0)_r$ if $2^{\aleph_0}$ is real-valued measurable?
- $\mathbb{R} \overset{+}{\to} (\aleph_0)_r$ without large cardinals?
- unbalanced sumsets $X + Y$ (or $X \oplus Y$)?

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