Abstracts

14TH INTERNATIONAL WORKSHOP IN SET THEORY

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Omer Ben Neria: *Singular Stationarity*
Jörg Brendle: *Rearrangements and subseries*
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Arthur W. Apter

Tall, strong, and strongly compact cardinals

Abstract:

Relative to the appropriate hypotheses, I will discuss how to force and obtain models for the theory "ZFC + κ is strongly compact iff κ is strong" in which every strongly compact/strong cardinal is a limit of non-strong tall cardinals. This generalizes both earlier joint work with Cummings and earlier joint work with Gitik.

Omer Ben Neria

Singular Stationarity

Abstract:

In the 1990s, Foreman and Magidor introduced two generalized notions of stationarity for singular cardinals: Mutual stationarity and tight stationarity. We will discuss new consistency results related to these notions which involve methods of forcing with large cardinals, strong ideals, and extender forcing.

Jörg Brendle

Rearrangements and subseries

Abstract:

Let $\sum_n a_n$ be a conditionally convergent series of real numbers. The Riemann rearrangement theorem says that by choosing a permutation p of the natural numbers $\mathbb{N}$ appropriately, the rearranged series $\sum_n a_{p(n)}$ can be made to diverge or to converge to any prescribed real number. The *rearrangement number* $\mathfrak{rr}$ is the least size of a family $\mathcal{P}$ of permutations such that for every conditionally convergent $\sum_n a_n$ there is $p \in \mathcal{P}$ such that $\sum_n a_{p(n)}$ no longer converges to the same limit.

Furthermore, in a conditionally convergent series $\sum_n a_n$, the sum of positive terms diverges to $+\infty$ while the sum of negative terms diverges to $-\infty$. In particular there are always subsets $D \subseteq \omega$ such that the subseries $\sum_{n \in D} a_n$ diverges. The *subseries number* ß is the least size of a family $\mathcal{D}$ of subsets of ω such that for every conditionally convergent $\sum_n a_n$ there is $D \in \mathcal{D}$ such that $\sum_{n \in D} a_n$ diverges.

We compare $\mathfrak{rr}$ and ß to other cardinal invariants of the continuum and also discuss some of their relatives. This is joint work with A. Blass, W. Brian, J. Hamkins, M. Hardy, P. Larson, and J. Verner [1, 2].

References

[1] A. Blass, J. Brendle, W. Brian, J. Hamkins, M. Hardy, and Larson, *The rearrangement number*, preprint.

[2] J. Brendle, W. Brian, J. Hamkins, and J. Verner, *The subseries number*, in preparation.

Natasha Dobrinen

The universal homogeneous triangle-free graph has finite big Ramsey degrees

Abstract: We present the solution to a long-standing open problem regarding the big Ramsey degrees of the universal, homogeneous triangle-free graph. The development of several new techniques are involved in the proof. These techniques will likely be useful for solving a collection of open problems regarding big Ramsey degrees of universal structures.

Ramsey theory on relational structures can be studied from two vantage points. Classically, structural Ramsey theory extends Ramsey's theorem to certain classes of finite relational structures. A Fraïssé class of finite relational structures (such as finite ordered graphs) has the Ramsey property if for any structure A which embeds into a structure B, there is a structure C such that

for any coloring of all copies of A in C into finitely many colors, there is a copy of B in C in which all copies of A have the same color.

Of much recent interest is the study of colorings of copies of a finite structure inside an infinite homogenous, universal structure. For example, it is well-known that any finite coloring of the vertices of the Rado graph can be reduced to one color on a subgraph which is also a Rado graph. For edges and other structures with more than one vertex, Sauer has proved this to be impossible. However, Sauer also proved that given a finite graph A, there is a number $n(A)$ such that any coloring of all copies of A in the Rado graph into finitely many colors may be reduced to $n(A)$ colors on a copy of the Rado graph. Using the terminology of Kechris, Pestov and Todorcevic, we say that the Rado graph has finite *big Ramsey degrees*. Big Ramsey have been obtained for several other countable homogeneous structures, by Devlin, Laflamme, Laver, Nguyen Van Thé, and Sauer, though most are still open.

The problem of finite big Ramsey degrees for the universal, homogeneous triangle-free graph $\mathcal{H}_3$, constructed by Henson in 1971, has been open for some time, the problem being solved for vertex colorings by Komjath and Rodl in 1986, and for edge colorings by Sauer in 1998. The speaker has proved that for each finite triangle-free graph G, there is a number $n(G)$ such that for each coloring of all copies of G in $\mathcal{H}_3$ into finitely many colors, there is a subgraph $\mathcal{H}_3'$ of $\mathcal{H}_3$ which is again universal triangle-free, and in which all copies of G in $\mathcal{H}_3'$ take on no more than $n(G)$ many colors.

Our proof that the universal homogeneous triangle-free graph has finite big Ramsey degree in $\mathcal{H}_3$ hinges on the following developments: a new flexible method for constructing trees which code the universal triangle-free graph, called strong coding trees; a new notion of strict similarity type of finite subtrees of a strong coding tree; analogues of the Halpern-Läuchli and Milliken Theorems, obtaining a Ramsey theorem for strict similarity types of finite subtrees of a given strong coding tree; and a new notion of envelope. The proof of the Milliken-style theorem for strong coding trees uses forcing techniques and three new forcings, though the proof is in ZFC, building on ideas from Harrington's forcing proof of the Halpern-Läuchli Theorem.

Vera Fischer

Bounding, splitting and almost disjointness can be quite different

Abstract: The bounding, splitting and almost disjoint families are some of the well studied infinitary combinatorial objects on the real line. Their study has prompted the development of many interesting forcing techniques. Among those are the method of creature forcing, as well as Shelah's template iteration technique. In this talk, we will discuss some recent developments of Shelah's template iteration methods and in particular the construction of a model of $\aleph_1 < \mathfrak{s} < \mathfrak{b} = \mathfrak{d} < \mathfrak{a}$.

Laura Fontanella

From forcing models to realizability models

Abstract: We discuss classical realizability, a branch of mathematical logic that investigates the computational content of mathematical proofs by establishing a correspondence between proofs and programs. Research in this field has led to the development of highly technical constructions generalizing the method of forcing in set theory. In particular, models of realizability are models of ZF, and forcing models are special cases of realizability models.

Joel D. Hamkins

The hierarchy of second-order set theories between GBC and KM and beyond

Abstract: Recent work has clarified how various natural second-order set-theoretic principles, such as those concerned with class forcing or with proper class games, fit into a new robust hierarchy of second-order set theories between Gödel-Bernays GBC set theory and Kelley-Morse KM set theory and beyond. For example, the principle of clopen determinacy for proper class games is exactly equivalent to the principle of elementary transfinite recursion ETR, strictly between GBC and GBC+Π^1_1-comprehension; open determinacy for class games, in contrast, is strictly stronger; meanwhile, the class forcing theorem, asserting that every class forcing notion admits corresponding forcing relations, is strictly weaker, and is exactly equivalent to the fragment $\mathrm{ETR}_{\mathrm{Ord}}$ and to numerous other natural principles. What is emerging is a higher set-theoretic analogue of the familiar reverse mathematics of second-order number theory.

Piotr Koszmider

Noncommutative thin-tall algebras

Abstract: Thin-tall (superatomic) Boolean algebras are the algebras of the form $\mathcal{A} = \bigcup_{\alpha<\omega_1} \mathcal{I}_\alpha$, where

(1) the union is strictly increasing and continuous,
(2) $I_0 = \{0\}$,
(3) each $\mathcal{I}_\alpha$ is an ideal and
(4) $\mathcal{I}_{\alpha+1}/\mathcal{I}_\alpha$ is a dense ideal of $\mathcal{A}/\mathcal{I}_\alpha$ isomorphic to the ideal $\mathtt{Fin}(\mathbb{N})$ of all finite subsets of $\mathbb{N}$.

They carry interesting combinatorics which in the past motivated a considerable amount of research (Bagaria, Baumgartner, Bonnet, Dow, Juhasz, Just, Kunen, Roitman, Rubin, Shelah, Simon, Todorcevic, Weese, Weiss and others).

We propose investigating the combinatorics of noncommutative structures as above by allowing all finite matrices instead of all finite sets from the Boolean context as above (which correspond to the diagonal finite matrices). This results with a class of C*-algebras which we call *fully noncommutative* thin-tall (scattered) C*-algebras.

I will survey my recent results on this topic obtained with C. Hida and with S. Ghasemi. In particular we obtain the fully noncommutative version of the Kunen line (under $\diamondsuit$ or by forcing) or the noncommutative version of the algebra constructed by Simon and Weese (in ZFC). But the issue of the existence in ZFC of such an algebra with only trivial automorphisms remains unresolved.

No knowledge of noncommutative mathematics beyond multiplication of matrices is required to follow the talk.

John Krueger

Club isomorphisms on higher Aronszajn trees

Abstract: Solovay and Tennenbaum proved the consistency of Suslin's hypothesis, which states that there does not exist an ω_1-Suslin tree. Abraham and Shelah formulated a strengthening of Suslin's hypothesis, namely, the statement that any two normal ω_1-Aronszajn trees are club isomorphic, and proved its consistency. Laver and Shelah proved the consistency, assuming a weakly compact cardinal, of CH together with the ω_2-Suslin hypothesis, which asserts the nonexistence of an ω_2-Suslin tree. In this talk, we discuss our generalization of the Abraham-Shelah result to ω_2, which shows that it is consistent with CH, assuming an ineffable cardinal, that any two normal countably closed ω_2-Aronszajn trees are club isomorphic. This provides a natural strengthening of the ω_2-Suslin hypothesis which is analogous to the situation on ω_1.

Dominique Lecomte

Borel complexity of equivalence relations

Abstract: We recall and apply the Debs-Saint Raymond representation theorem for Borel sets to obtain progress concerning the Borel complexity of equivalence relations.

Benjamin Miller

On the existence of cocycle-invariant Borel probability measures

Abstract: We will discuss a generalization of Nadkarni's theorem characterizing the existence of a Borel probability measure that is invariant with respect to a given positive-real-valued Borel cocycle on a countable Borel equivalence relation.

Luca Motto Ros

Generalized descriptive set theory and classification

Abstract: We will discuss some intriguing applications of generalized descriptive set theory to the classification of uncountable structures and non-separable spaces, including a descriptive set theoretic version of Shelah's Main Gap theorem in terms of the complexity of the isomorphism relation between the uncountable models of the given countable first-order theory.

Assaf Rinot

Distributive Aronszajn trees

Abstract: It is well-known that that the statement "all $\aleph_1$-Aronszajn trees are special" is consistent with ZFC (Baumgartner, Malitz, and Reinhardt), and even with ZFC+GCH (Jensen). In contrast, Ben-David and Shelah proved that, assuming GCH, for every singular cardinal λ: if there exists a λ^+-Aronszajn tree, then there exists a non-special one. Furthermore:

Theorem (Ben-David and Shelah, 1986) Assume GCH and that λ is singular cardinal. If there exists a special λ^+-Aronszajn tree, then there exists a λ-distributive λ^+-Aronszajn tree.

This suggests that following stronger statement:

Conjecture. Assume GCH and that λ is singular cardinal. If there exists a λ^+-Aronszajn tree, then there exists a λ-distributive λ^+-Aronszajn tree.

The assumption that there exists a λ^+-Aronszajn tree is a very mild square-like hypothesis (that is, $\square(\lambda^+, \lambda)$). In order to bloom a λ-distributive tree from it, there is a need for a toolbox, each tool taking an abstract square-like sequence and producing a sequence which is slightly better than the original one. For this, we introduce the monoid of *postprocessing functions* and study how it acts on the class of abstract square sequences. We establish that, assuming GCH, the monoid contains some very powerful functions. We also prove that the monoid is closed under various mixing operations.

This allows us to prove a theorem which is just one step away from verifying the conjecture:

Theorem 1. Assume GCH and that λ is a singular cardinal.
If $\square(\lambda^+, <\lambda)$ holds, then there exists a λ-distributive λ^+-Aronszajn tree.

Another proof, involving a 5-steps chain of applications of postprocessing functions, is of the following theorem.

Theorem 2. Assume GCH. If λ is a singular cardinal and $\square(\lambda^+)$ holds, then there exists a λ^+-Souslin tree which is coherent mod finite.

This is joint work with Ari Brodsky. See: `http://assafrinot.com/paper/29`

Ralf Schindler
Varsovian models with more Woodin cardinals

Abstract: Inner model theoretic geology studies the collection of all grounds W of a given fine structural extender model $L[E]$, i.e., the collection of all inner models W of $L[E]$ such that $L[E]$ is a (set) generic extension of W. An exciting insight is that in many cases, there is a least such ground, called the mantle, and that the mantle can be verified to be a strategic extender model. We want to present examples produced by this line of research. This is joint work with Grigor Sargsyan and Stefan Miedzianowski.

Philipp Schlicht
The Hurewicz dichotomy for definable subsets of generalized Baire spaces

Abstract: By classical results of Hurewicz and Saint-Raymond, an analytic subset of a Polish space X is covered by a K_σ subset of X if and only if it does not contain a closed subset of X that is homeomorphic to the Baire space ${}^\omega\omega$. Moreover, Kechris proved that this result generalizes to the projective sets if projective determinacy is assumed. We consider the analogous statement, which is called the *Hurewicz dichotomy*, for subsets of the generalized Baire space ${}^\kappa\kappa$ for a given uncountable cardinal κ with $\kappa = \kappa^{<\kappa}$. We will sketch a proof of the consistency of the Hurewicz dichotomy for all subsets of the generalized Baire space ${}^\kappa\kappa$ that are definable from parameters in ${}^\kappa\kappa$. This is work in progress with Philipp Lücke and Luca Motto Ros.

Dima Sinapova
Prikry type forcing and combinatorial properties

Abstract: We will analyze consequences of various types of Prikry forcing on combinatorial properties at singular cardinals and their successors, focusing on weak square and simultaneous stationary reflection. The motivation is how much compactness type properties can be obtained at successors of singulars, and especially the combinatorics at $\aleph_{\omega+1}$.

Dániel T. Soukup
Monochromatic sumsets for colorings of $\mathbb{R}$

Abstract: N. Hindman, I. Leader and D. Strauss proved that if $2^{\aleph_0} < \aleph_\omega$ then there is a finite coloring of $\mathbb{R}$ so that no infinite sumset $X + X$ is monochromatic. Now, we prove a consistency result in the other direction: we show that consistently relative to a measurable cardinal for any $c : \mathbb{R} \to r$ with r finite there is an infinite $X \subseteq \mathbb{R}$ so that $c \restriction X + X$ is constant. The goal of this presentation is to discuss the motivation, ideas and difficulties involving this result, as well as the open problems around the topic. Joint work with P. Komjáth, I. Leader, P. Russell, S. Shelah and Z. Vidnyánszky .

Todor Tsankov

Universal minimal flows of homeomorphism groups of high-dimensional manifolds

Abstract: The first interesting case of a non-trivial, metrizable universal minimal flow (UMF) of a Polish group was computed by Pestov who proved that the UMF of the homeomorphism group of the circle is the circle itself. This naturally led to the question whether a similar result is true for homeomorphism groups of other manifolds (or more general topological spaces). A few years later, Uspenskij proved that the action of a group on its UMF is never 3-transitive, thus giving a negative answer to the question for a vast collection of topological spaces. Still, the question of metrizability of their UMFs remained open and he asked specifically whether the UMF of the homeomorphism group of the Hilbert cube is metrizable. I am going to report on a joint work with Yonatan Gutman that gives a negative answer to his question. Our proof works for manifolds of dimension at least 2, and also for more general continua, thus showing that metrizability of the UMF of a homeomorphism group is essentially a one-dimensional phenomenon.

Matteo Viale

An overview on category forcings

Abstract: I want to give the basic ideas and the simplest proofs of my work of the past years related to category forcings i.e. "forcing with forcings as conditions". I have already presented part of these results in previous occasions, but I have now a polished account of the general theory which I think it is worth presenting. The main outcomes of this theory can be summarized as follows:

Let Γ be a definable class of forcings closed under two steps iterations, let $\mathsf{B} \geqslant_\Gamma \mathsf{C}$ if there is $i : \mathsf{B} \to \mathsf{C}$ with a generic quotient in Γ.

Provided that Γ satisfies some natural properties (which by the way hold for the most interesting classes of forcings such as proper, SSP, semiproper, etc....) one has that:

- *$\Gamma \cap V_\delta \in \Gamma$ for most inaccessible cardinals δ. Moreover in such cases $\Gamma \cap V_\delta$ absorbs as a complete subforcing any element of $\Gamma \cap V_\delta$,*
- *for any G V-generic for some $\mathsf{B} \in \Gamma$, $\Gamma^{V[G]} = \Gamma^V/_G$,*
- *there is a cardinal κ_Γ attached to Γ such that $\Gamma \cap V_\delta$ forces a variety of forcing axioms (depending on the properties of the inaccessible cardinal δ, and on the choice of Γ) which yieds generic absoluteness for $H_{\kappa_\Gamma^+}$ and strenghten $\mathsf{FA}_{\kappa_\Gamma}(\mathsf{B})$ for all $\mathsf{B} \in \Gamma$ (remark that if Γ is the class of proper and semiproper forcings, $\kappa_\Gamma = \omega_1$),*
- .....

Asperó has shown that for uncountably many Γ with $\kappa_\Gamma = \omega_1$ the above machinery applies, yielding a zoo of pairwise incompatible Γ-generically invariant theories of H_{ω_2}, some of which implying CH.

Alessandro Vignati

Set theory and C^*-algebras: automorphisms of continuous quotients

Abstract: In the discrete setting, the work of Rudin, Shelah, Steprans, Velickovic and Farah among others showed that the structure of the automorphisms' group of discrete quotient structures depends on the axioms one assumes. We push this intuition to the ambient of quotients of C^*-algebras. In particular we show how the assumption of CH on one side, and of Forcing Axioms on the other, has an impact on the automorphisms group of corona C^*-algebras, non-commutative analogs of Stone-Čech remainders of topological spaces.

Lyubomyr Zdomskyy

Vitali-Hahn-Saks property of Boolean algebras in forcing extensions

Abstract: The talk is going to be devoted to the main ideas of the proof of the following

Theorem 1. *Let $\mathcal{A}$ be a σ-complete Boolean algebra, and $\mathbb{P}$ a proper poset which preserves ground model reals non-meager and has the Laver property. Then $\mathcal{A}$ has the Vitali-Hahn-Saks property in $V^{\mathbb{P}}$.*

Theorem 1 generalizes the main result of [1], which was the starting point of our investigations. The Vitali-Hahn-Saks property is known to be equivalent to the conjunction of the properties of Nikodym and Grothendieck. Schachermeier has proved that the Jordan algebra has the Nikodym property and fails to be Grothendieck. However, the only known examples of Boolean algebras with Grothendieck property and without the Nikodym one were constructed by Talagrand under CH. We shall discuss some other strategies to get such algebras in various models of ZFC suggested by our proof of Theorem 1.

The talk will be based on a joint work in progress with Damian Sobota

References

[1] Brech, C., *On the density of Banach spaces C(K) with the Grothendieck property*, Proceedings of the American Mathematical Society **134** (2006), 3653–3663.