Photonic band control in a quantum metamaterial

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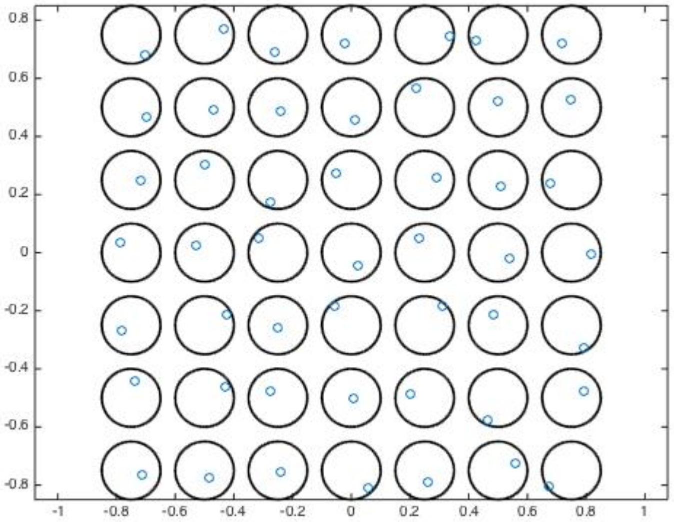
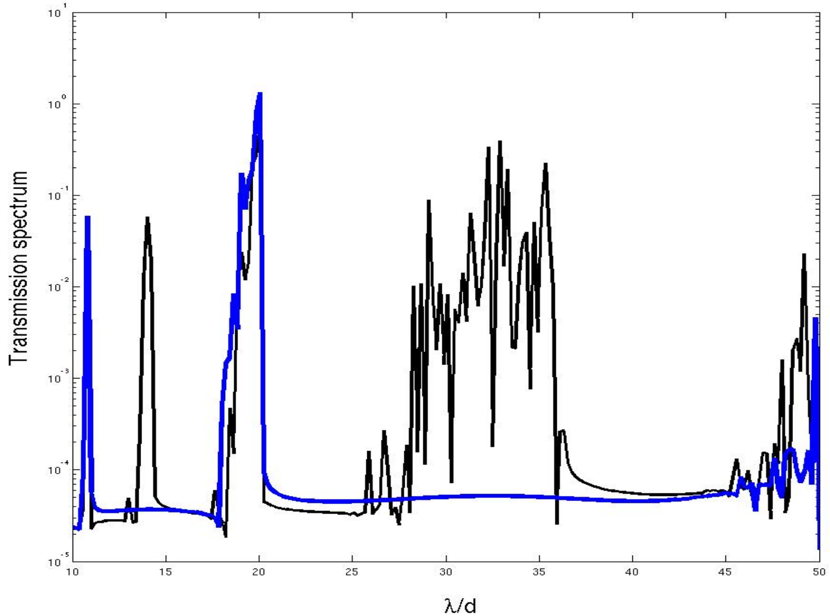
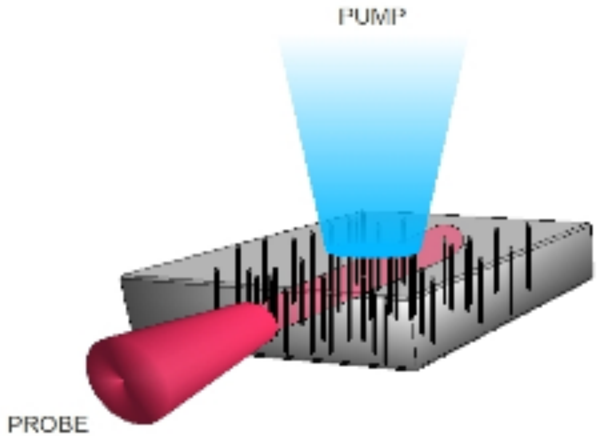
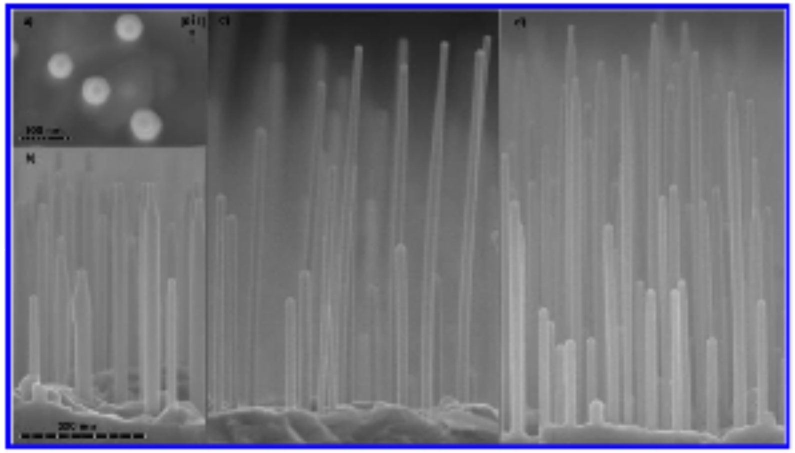
Introducing quantum dots in a photonic structure



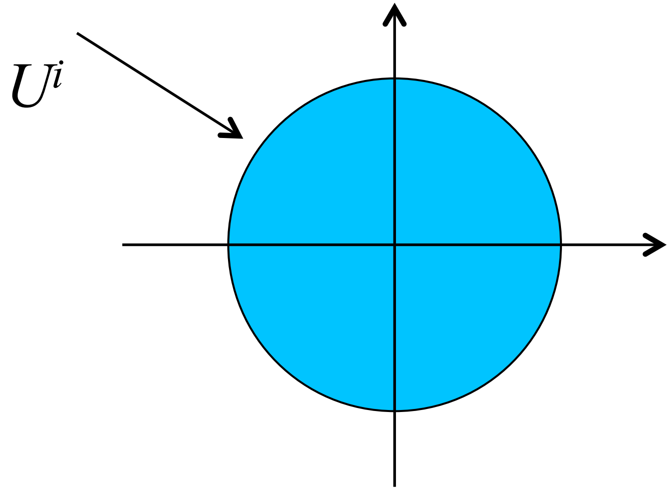






Scattering by one scatterer: resonances

$$U(r,\theta)\, e^{-i\omega t}$$



$$\Delta U + k_0^2\, \varepsilon\, U = 0$$
$$U^s = U - U^i$$

$$R_0 = (-\Delta - k_0^2)^{-1}$$

$$R_\varepsilon(k_0) = R_0(k_0)\left(I_d - (\varepsilon - 1)k_0^2 \circ R_0(k_0)\right)^{-1}$$

$$\Delta U^s + k_0^2\, \varepsilon\, U^s = (1-\varepsilon)k_0^2 U^i$$

$$U^s = R_\varepsilon(k_0)(1-\varepsilon)k_0^2 U^i$$

Scattering amplitude

Usual approach:

$$U = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{e^{ik_0 r}}{\sqrt{k_0 r}} \Phi_\varepsilon(k_0, \mathbf{r})$$

Awkward to use plane wave for a circular scatterer

$$U_n^i(r,\theta) = J_n(k_0 r)\, e^{in\theta}$$

$$e^{ir\sin\theta} = \sum_p J_p(r) e^{ip\theta}$$

Field "at infinity"

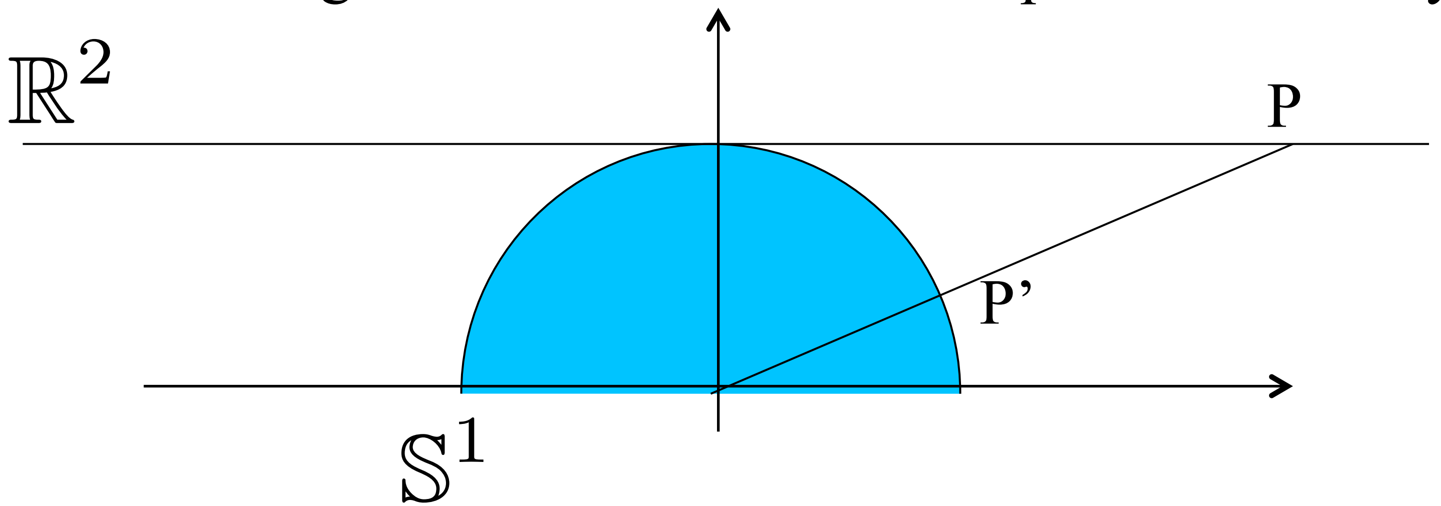
$$U_n^s \sim \frac{e^{ik_0 r}}{\sqrt{k_0 r}} s_n\, e^{in\theta}$$

Scattering amplitude

$$U^i(r,\theta) = \sum_n i_n J_n(k_0 r) e^{in\theta}$$

$$U^s(r,\theta) \sim \frac{e^{ik_0 r}}{\sqrt{k_0 r}} \sum_n s_n e^{in\theta}$$

Compactification: field given as a function on the sphere "at infinity"



$$T(k_0)[(i_n)] = (s_n)$$

Low frequency

$$U^i(r,\theta) = \sum_n i_n J_n(k_0 r) e^{in\theta}$$

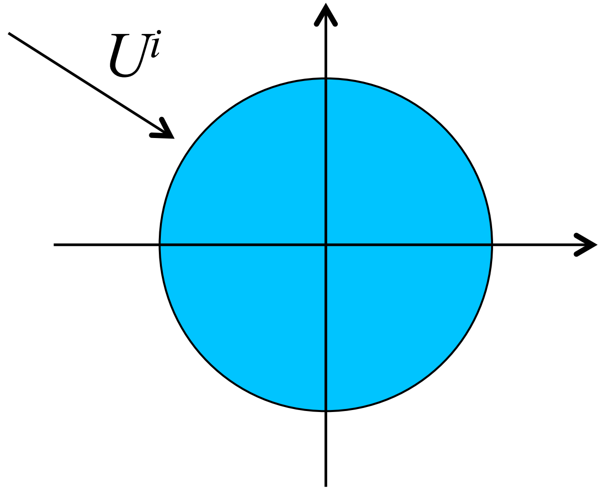
$$U^s(r,\theta) = \sum_n s_n H_n^{(1)}(k_0 r) e^{in\theta}$$

Low frequency (λ>>a): obstacle equivalent to two dipoles:

$$\mathbf{P} = \frac{4\varepsilon_0}{ik_0^2} s_0 \, \mathbf{e}_z \qquad \mathbf{M} = \frac{4}{ik_0^2 Z_0} \begin{pmatrix} s_1 + s_{-1} \\ s_1 - s_{-1} \end{pmatrix}$$

Scattering by one scatterer: resonances

$$U^s(r,\theta) = \sum_n s_n H_n^{(1)}(k_0 r) e^{in\theta}$$

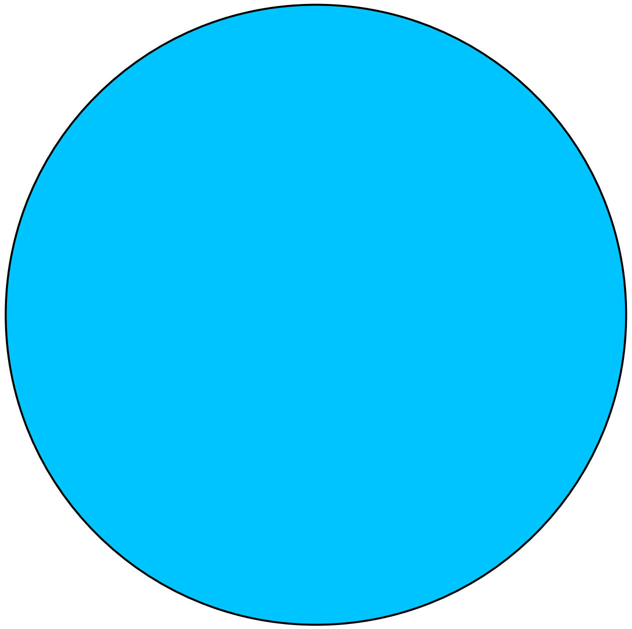


$$s_n = \frac{-1}{1 + iR_n}$$

$$R_n = \left(\frac{Y_n(ka)}{J_n(ka)}\right) \frac{F(k\sqrt{\varepsilon}a) - \frac{Y'(ka)}{kaY(ka)}}{F(k\sqrt{\varepsilon}a) - \frac{J'(ka)}{kaJ(ka)}}$$

$$F(ka\sqrt{\varepsilon}) = \frac{J_n'(ka\sqrt{\varepsilon})}{ka\sqrt{\varepsilon}J_n(ka\sqrt{\varepsilon})}$$

Origin of the poles of one scatterer



Solve a cavity spectral problem

$$\Delta U + k_0^2 \varepsilon U = 0$$

$$U = 0 \text{ on } \partial\Omega$$

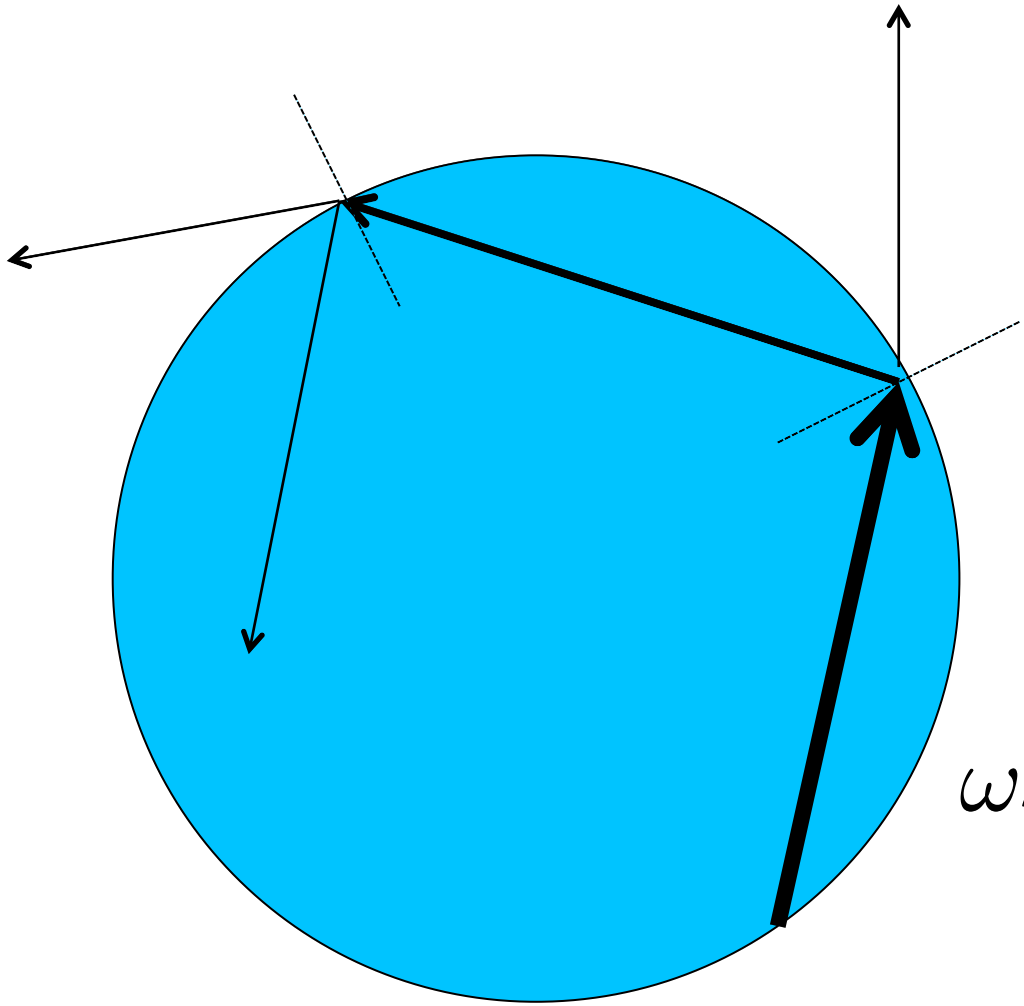
self-adjoint in $L^2(\Omega)$ with compact resolvent

Real discrete spectrum

$$k_1^2 < k_2^2 < \ldots$$

Relax the Dirichlet condition to radiating conditions

Complex poles of the scattering amplitude=Finite life-time of inner modes due to leakage



Poles of the scattering amplitude

Finite life-time of inner modes due to leakage

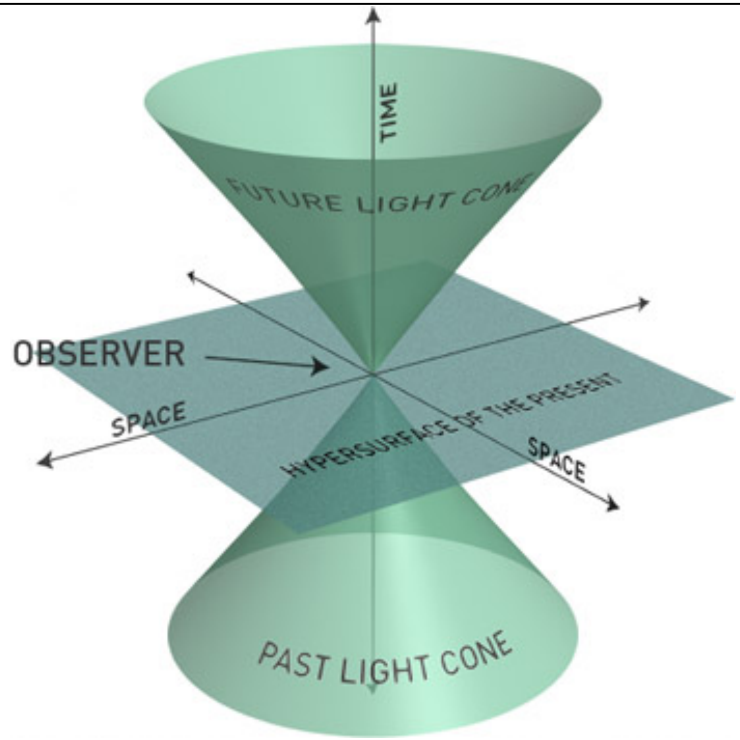
Resonances at

$$\omega_r = \omega - i\Gamma, \Gamma > 0$$

$$\omega_r = \omega - i\Gamma, \Gamma > 0$$

$$e^{-i\omega_r t} = e^{-i\omega t}\, e^{-\Gamma t}$$

$$e^{ik_r r} = e^{i\omega r/c}\, e^{\Gamma r/c}$$

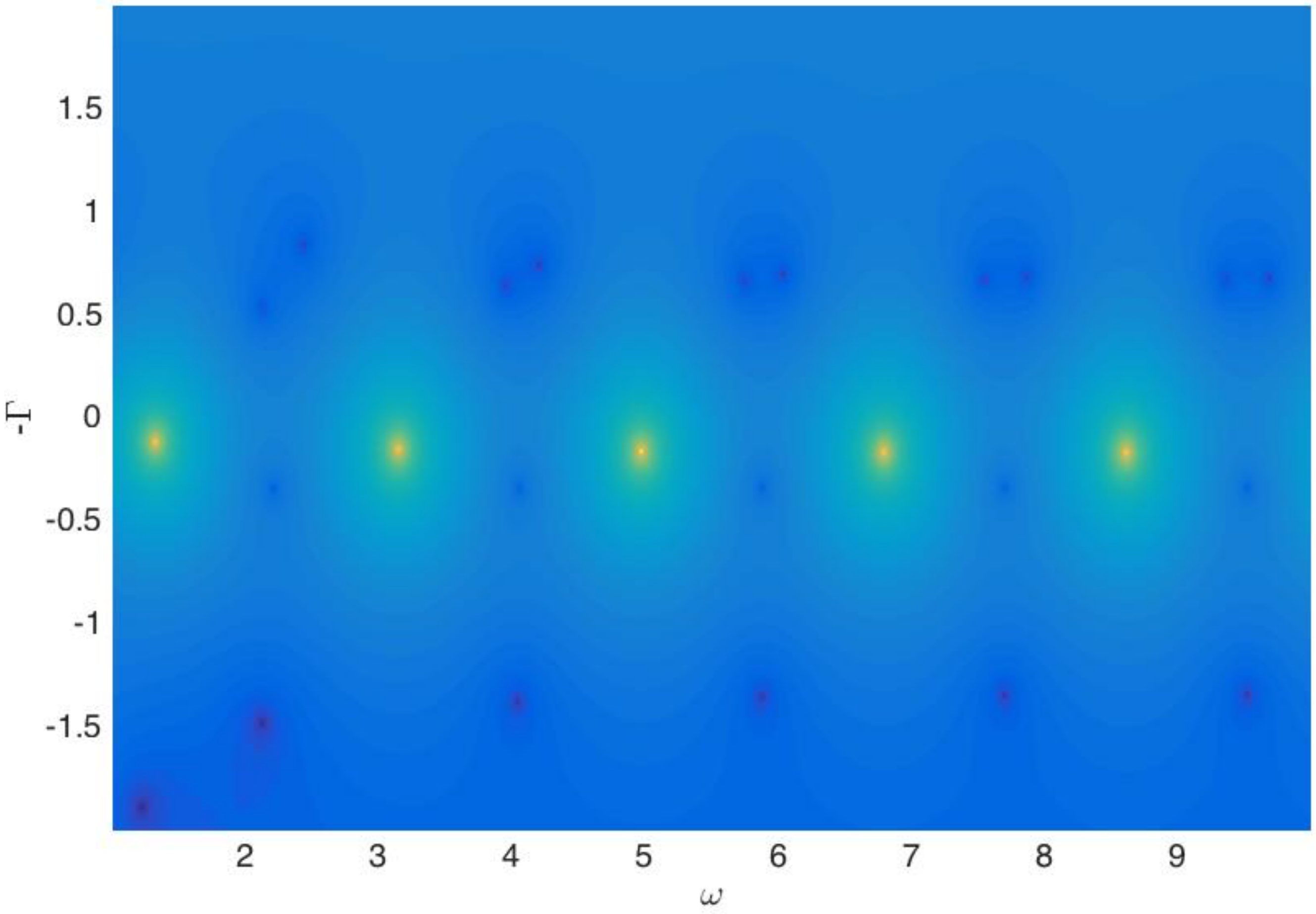


Do not forget (special) relativity:

$$e^{i(k_r r - \omega_r t)} = e^{i\frac{\omega}{c}(r-ct)}\, e^{\frac{\Gamma}{c}(r-ct)}$$

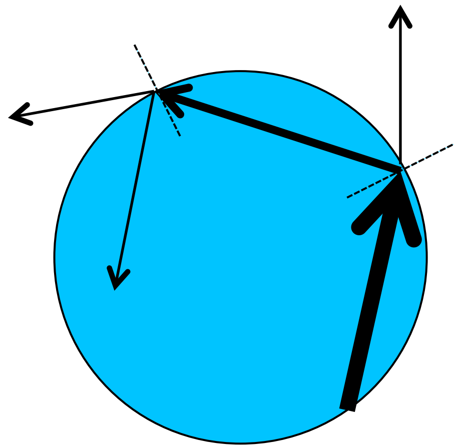
Exponentially decreasing in the light cone *(r-ct)<0*

Modulus of the determinant of the scattering amplitude

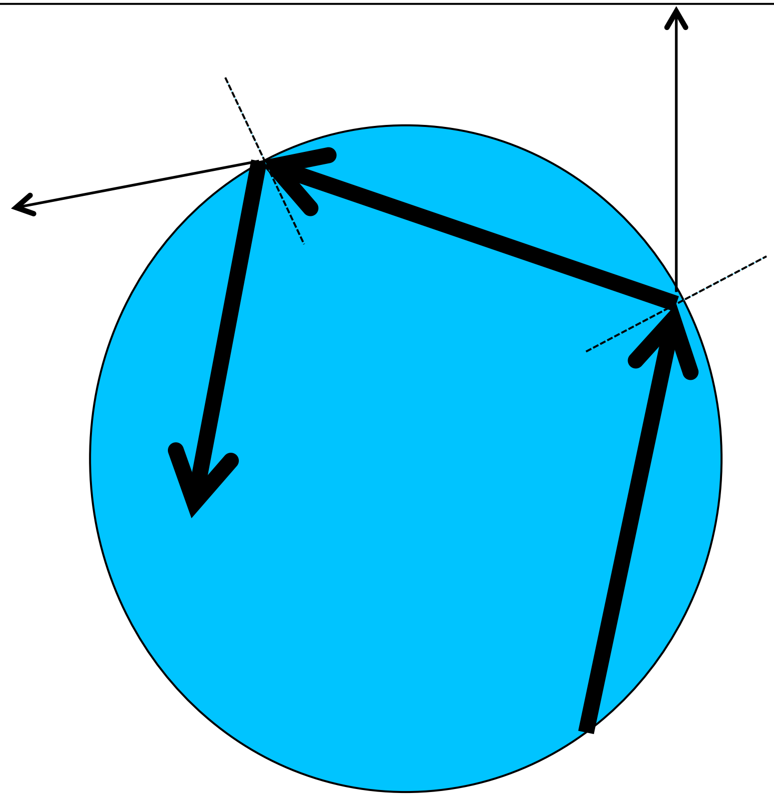


Poles of the scattering amplitude

Finite life-time of inner modes due to leakage



$$\omega_r = \omega - i\Gamma, \Gamma > 0$$



$$e^{ik_0\sqrt{\varepsilon}r}$$

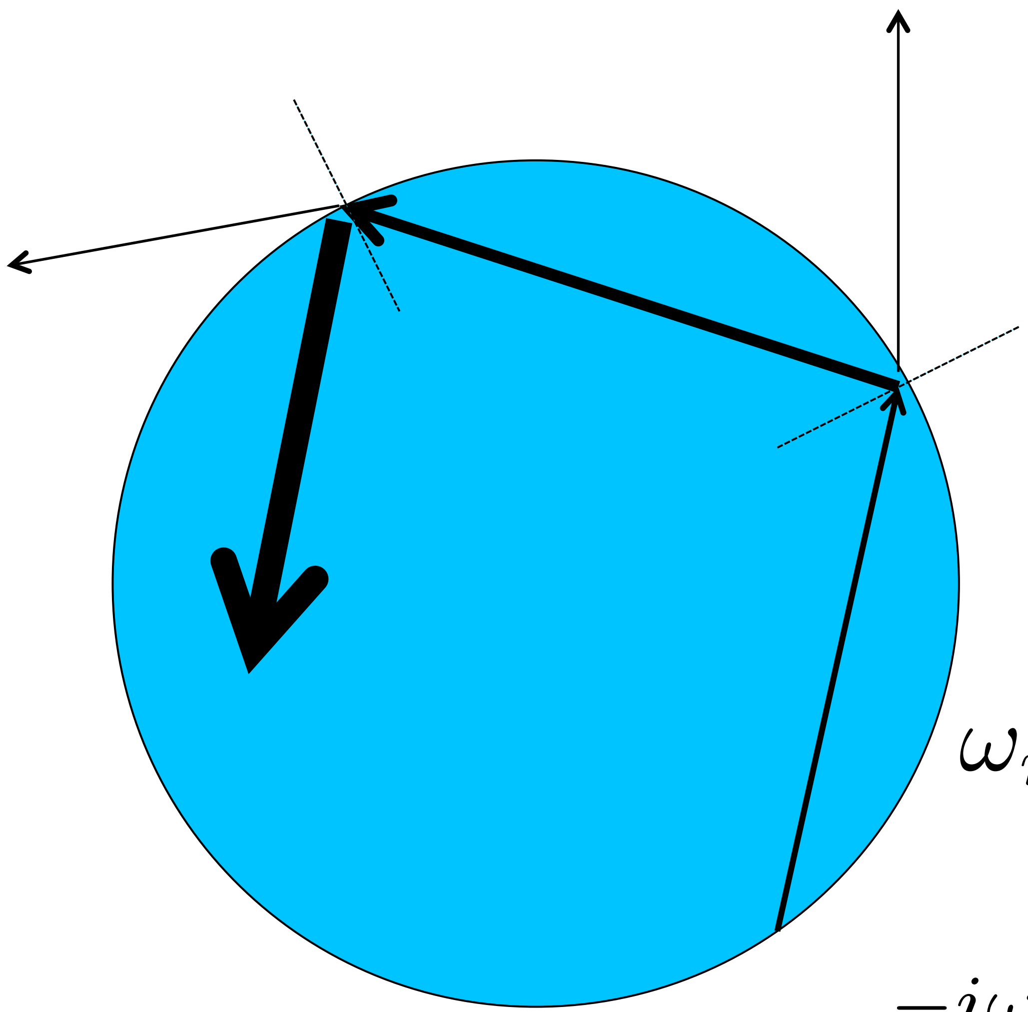
Poles of the scattering amplitude

Adding gain through the permittivity

$$\varepsilon = \varepsilon' - i\varepsilon''$$

make poles shift towards the upper half of the complex plane (perturbation theory)

Instability when crossing the real line: infinite life-time=Embedded eigenvalue

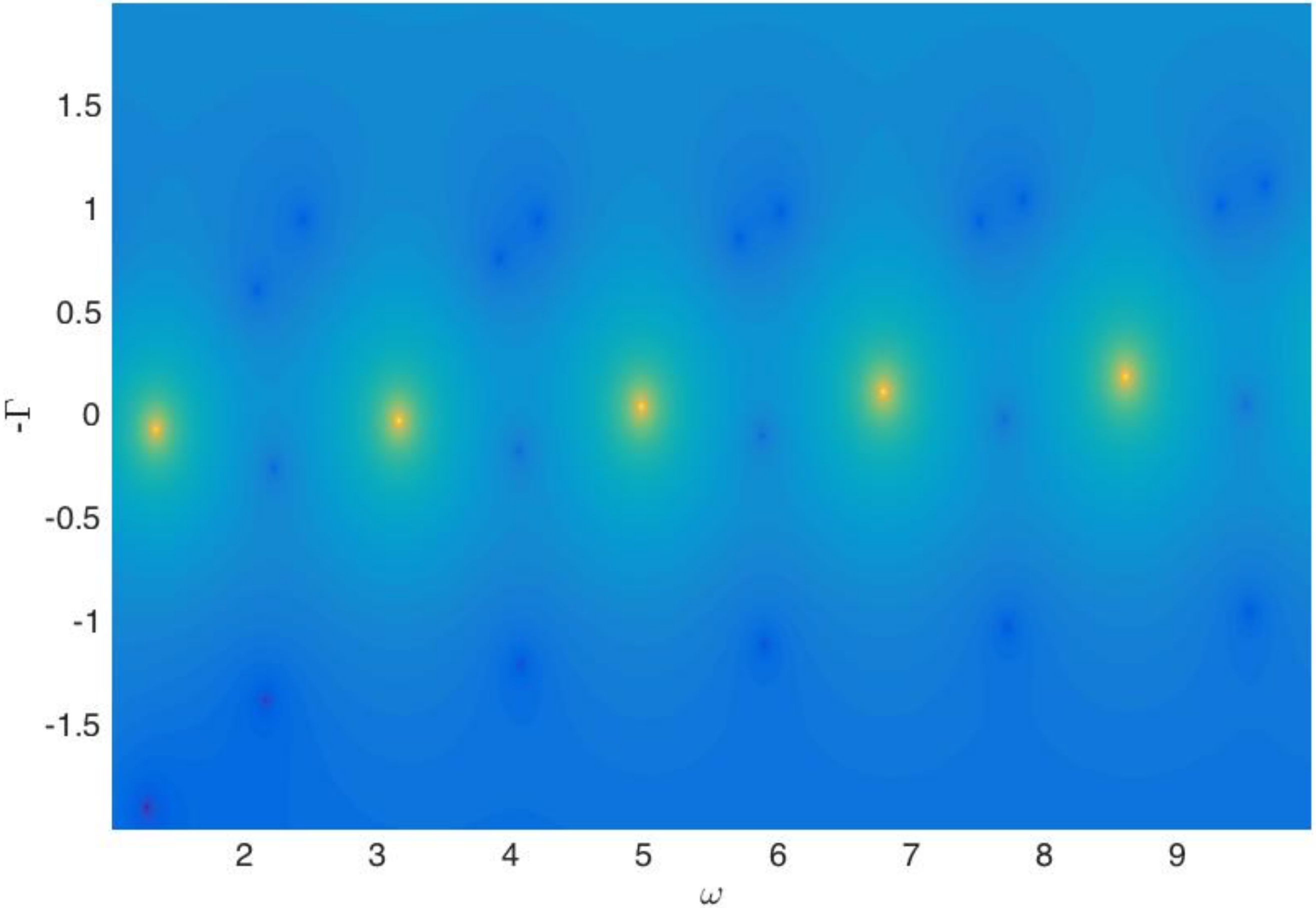


Poles of the scattering amplitude

Light Amplification in the upper sheet

$$\omega_r = \omega + i\Gamma, \Gamma > 0$$

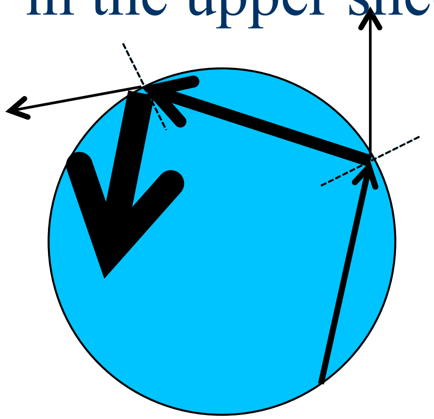
$$e^{-i\omega_r t} = e^{-i\omega t}\, e^{\Gamma t}$$



Poles of the scattering matrix

Adding gain make poles shift towards the upper half of the complex plane

Light Amplification in the upper sheet



Rods behave as (open) electromagnetic cavities

For a closed cavity filled with a dielectric:
if the radius is divided by η and the index is multiplied by η, the cavity is unchanged
The open cavity behave in the same way if the permittivity is high enough

In other words: the resonances are (asymptotically) invariant under the transformation:

$$a \longrightarrow \eta a$$

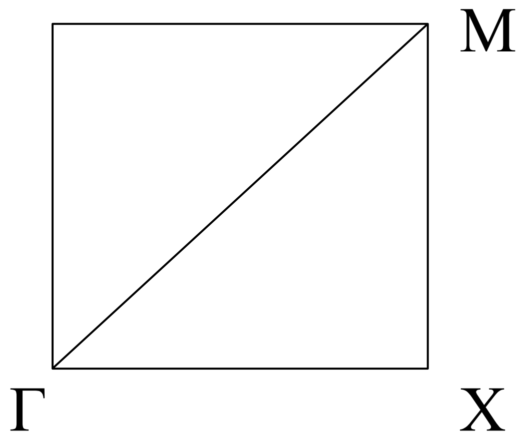
$$\varepsilon \longrightarrow \frac{\varepsilon}{\eta^2}$$

The infinite structure

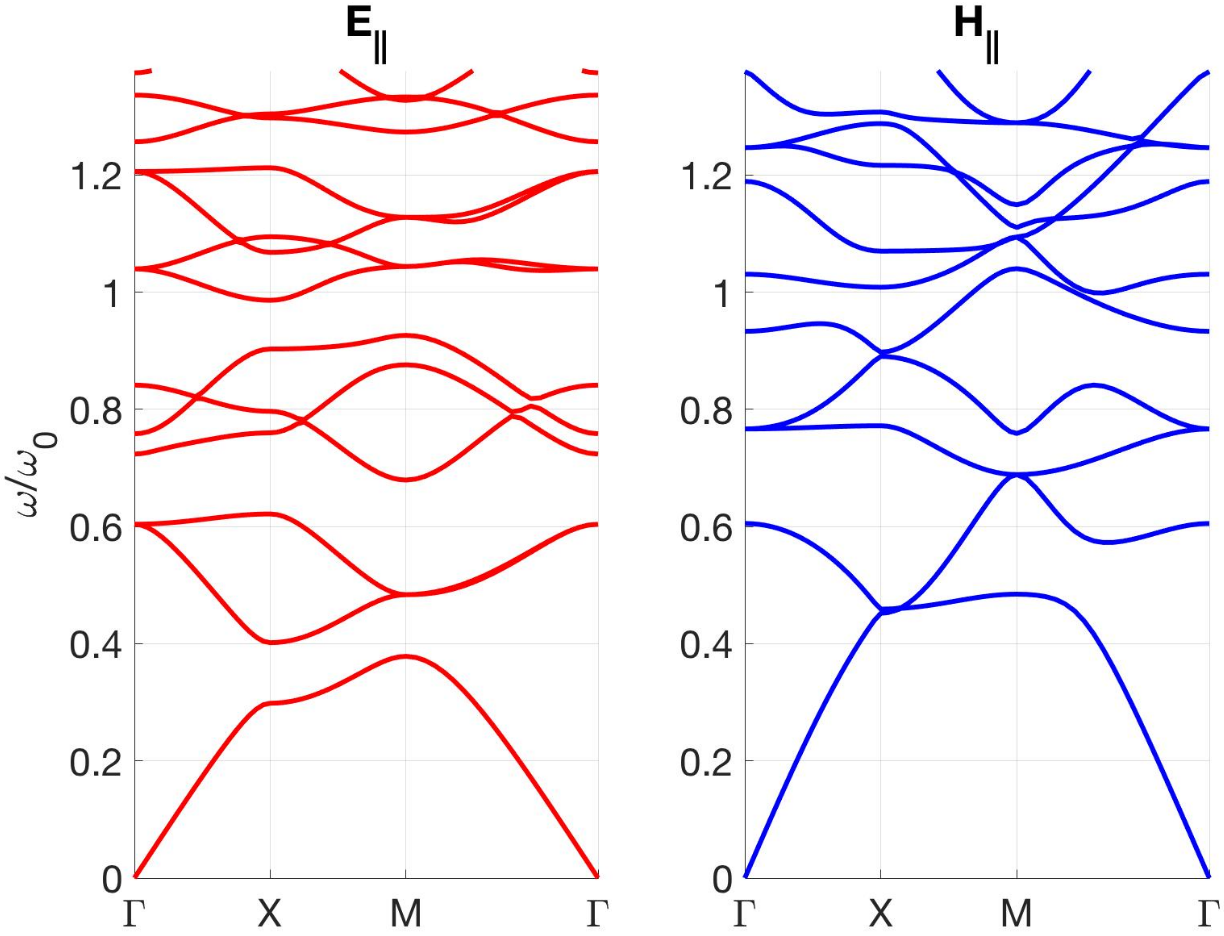
Bloch wave analysis

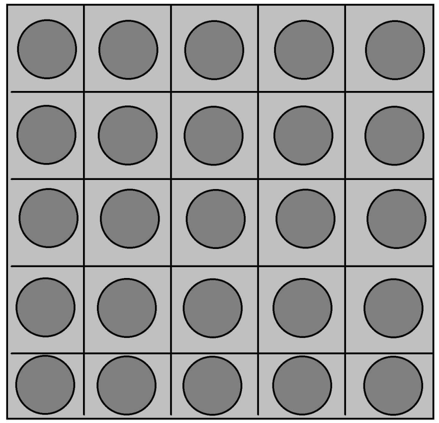
$$U(\mathbf{x};\mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{x}} V_{\mathbf{k}}(\mathbf{x})$$

$$\mathbf{k} \in (\mathbb{R}/2\pi\mathbb{Z})^2$$

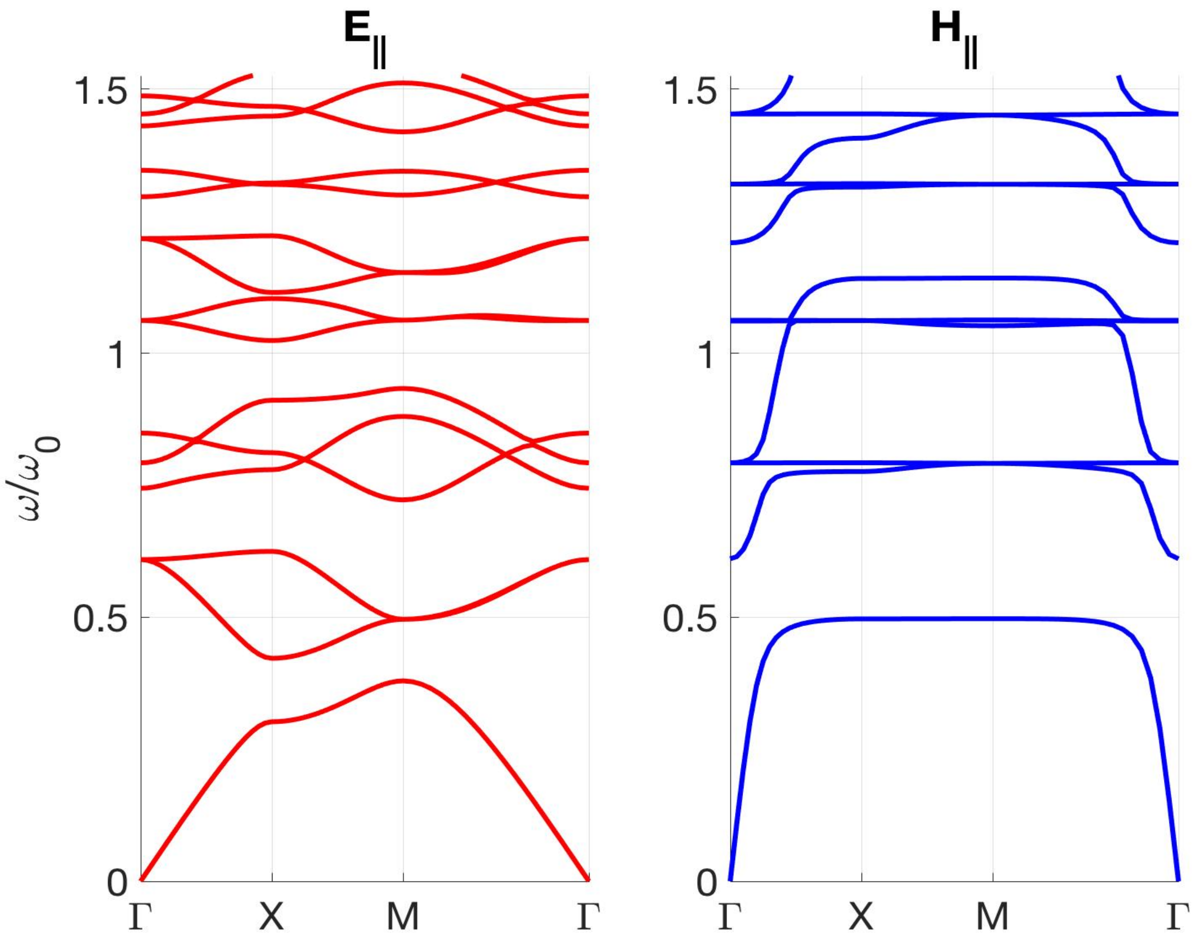


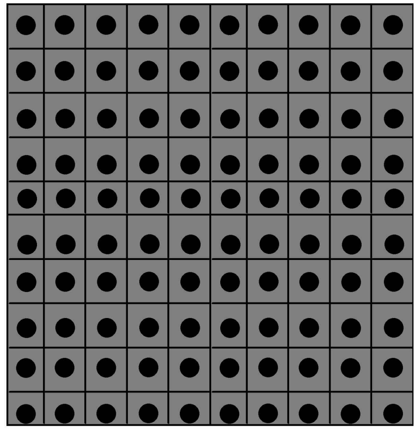
Bands induced by Mie resonances





Rescaling $\eta=1/10$ $(a\eta,\ \varepsilon/\eta^2)$





Quite insensitive to scaling for E|| waves

Bands induced by Mie resonances: homogenization

Band structure can be described using homogenized parameters (ε, μ)

D. Felbacq, G. Bouchitté, Phys. Rev. Lett. **94**, 183902 (2005) and C. R. Acad. Sci. Paris, Ser. I **339**, 377-382 (2004)

G. Bouchitté, C. Bourel, D. Felbacq, C. R. Acad. Sci. Paris, Ser. I **347** (2009) 571–576

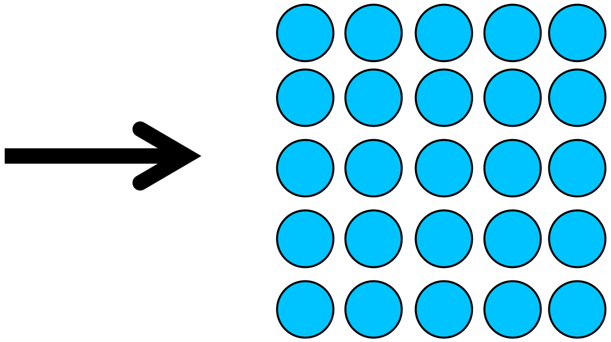
G. Bouchitté, C. Bourel, D. Felbacq, *Homogenization of the 3D Maxwell system near resonances and artificial magnetism,* C. R. Acad. Sci. Paris, Ser. I **347** (2009) 571–576

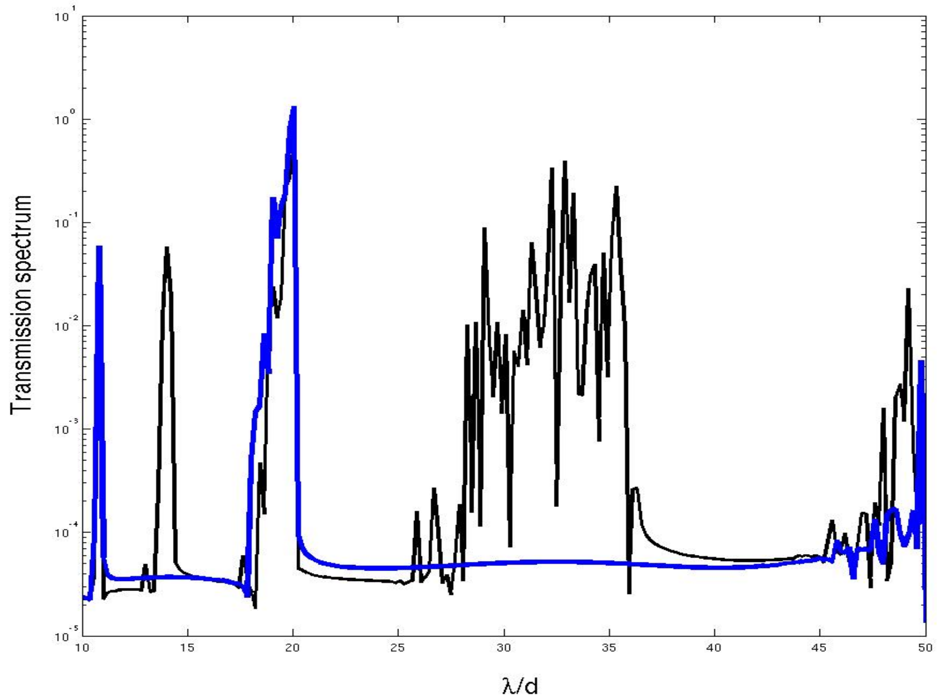
K.Vynck, D. Felbacq, Phys. Rev. Lett. **102**, 133901 (2009)

G. Bouchitté, C. Bourel, D. Felbacq, *Homogenization near resonances and artificial magnetism in 3D dielectric metamaterials,* Arch. Rat. Mech. Anal. 2017

Bands induced by Mie resonances: finite structure

Conduction band depends on both the electric and magnetic dipoles

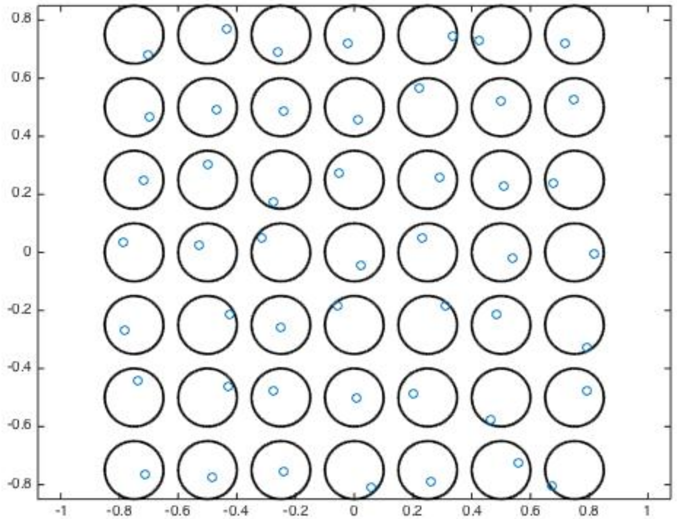




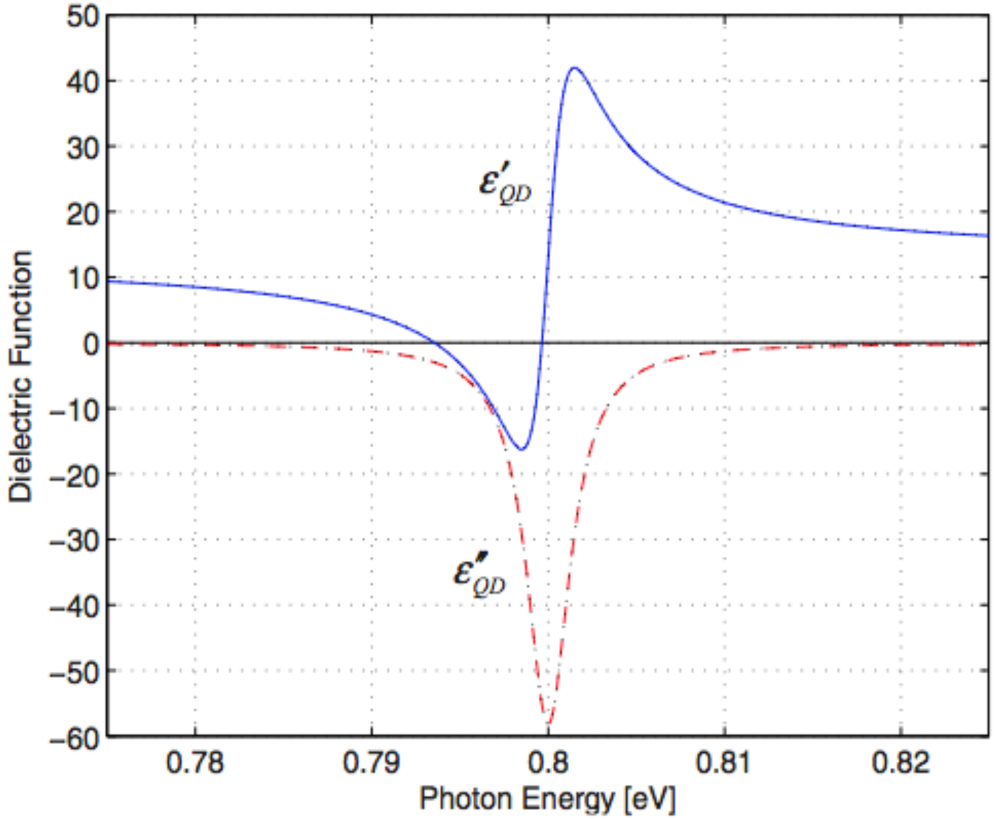
$$\hat{s}_k = T_k \hat{\imath}_k + T_k \sum_{n \neq k} \tau_k^n \, \hat{s}_n$$

$$\mathcal{S} = (1 - \mathcal{T}\Sigma)^{-1} \mathcal{T}\mathcal{I}$$

Introducing quantum dots in the photonic structure

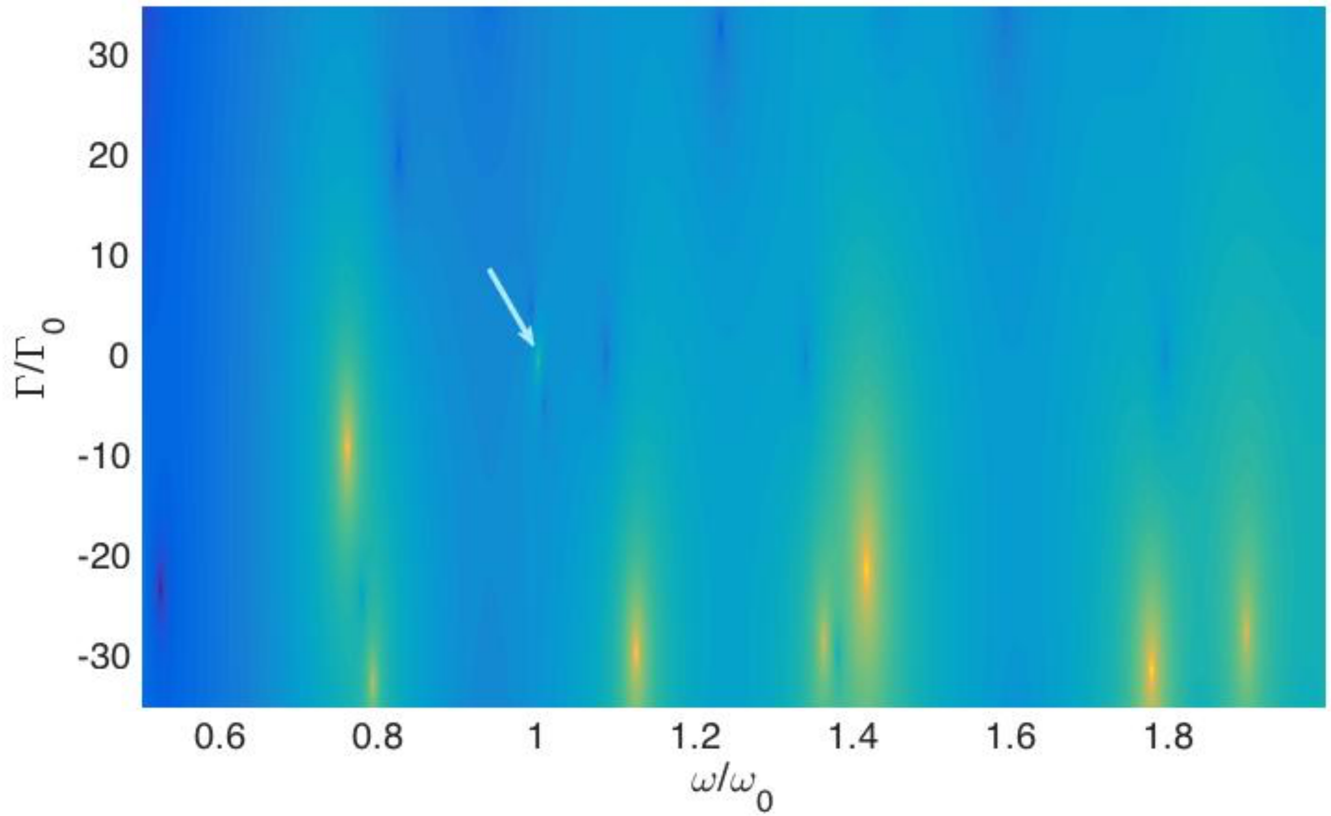


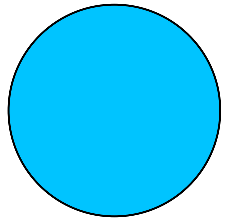
$$\varepsilon_{QD}(\omega) = \varepsilon_b \pm \frac{f}{\omega^2 - \omega_0^2 + i2\omega\gamma}$$



The minus sign corresponds to the inversion regime (gain-pumped QDs)

One scatterer with a QD: poles of the scattering amplitude

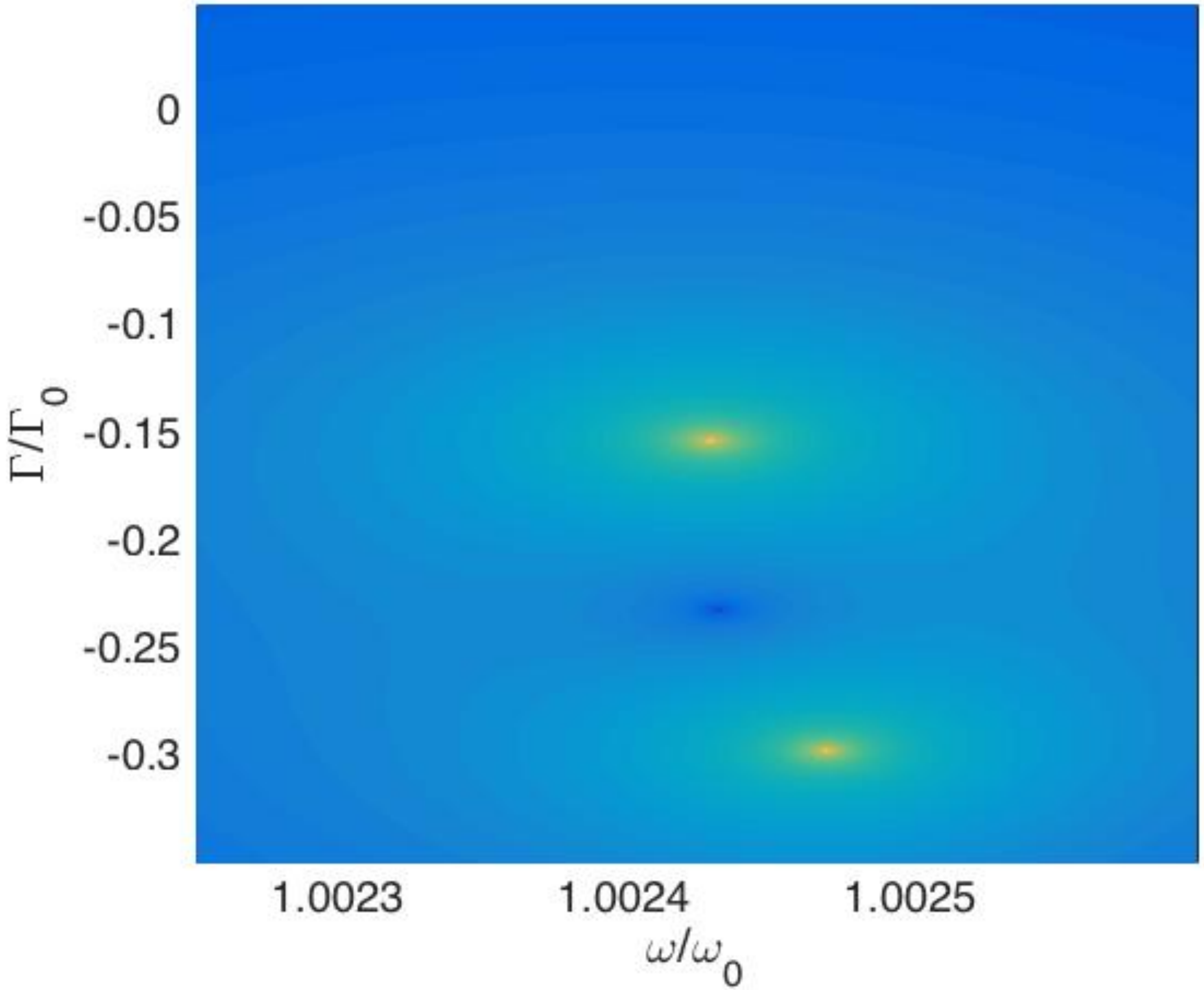


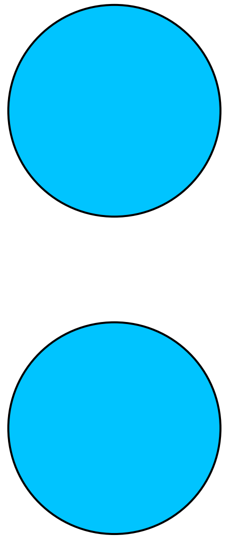


$$\omega_r = \omega + i\Gamma$$

$$\varepsilon_{QD}(\omega) = \varepsilon_b \pm \frac{f}{\omega^2 - \omega_0^2 + i2\omega\gamma}$$

2 scatterers with a QD: poles of the scattering amplitude



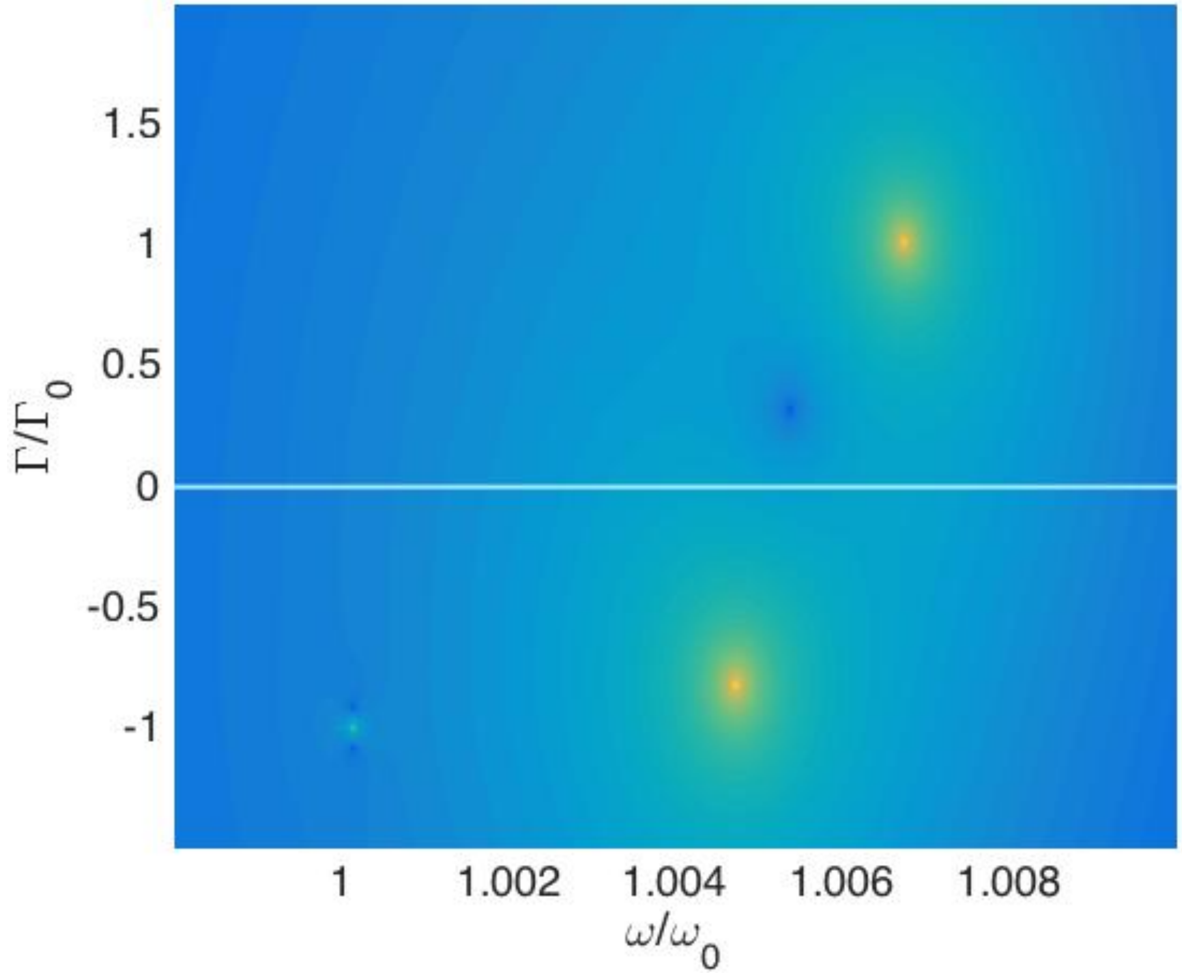


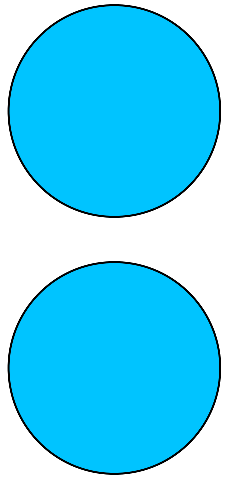
$$S(\omega + i\Gamma)$$

$$\omega_r = \omega - i\Gamma$$

$$\varepsilon_{QD}(\omega) = \varepsilon_b \pm \frac{f}{\omega^2 - \omega_0^2 + i2\omega\gamma}$$

2 scatterers with a QD: poles of the scattering amplitude

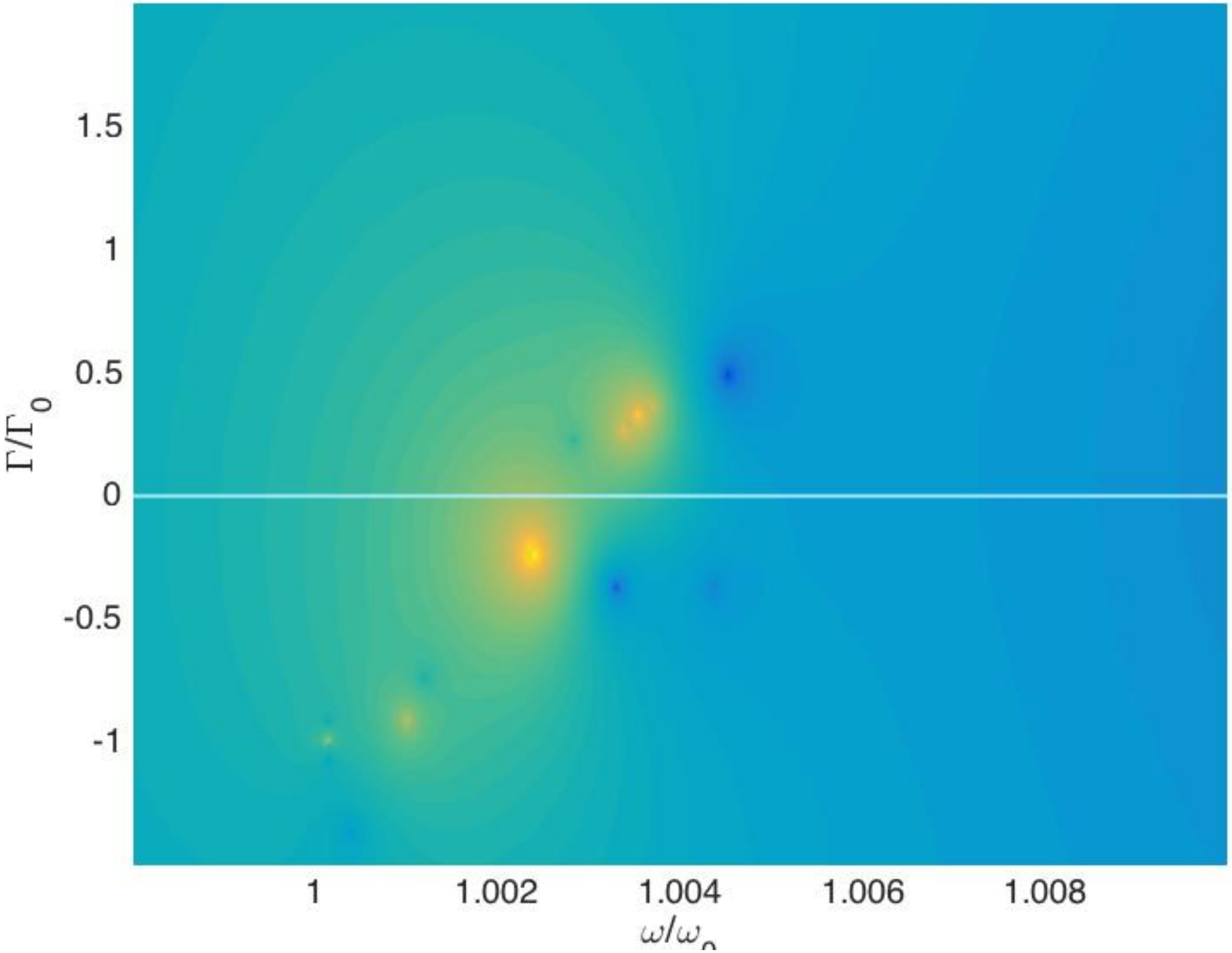


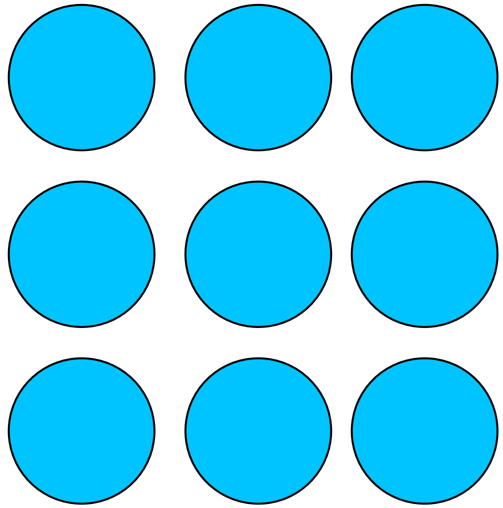


$$S(\omega + i\Gamma)$$

$$\omega_r = \omega + i\Gamma \qquad \varepsilon_{QD}(\omega) = \varepsilon_b \pm \frac{f}{\omega^2 - \omega_0^2 + i2\omega\gamma}$$

3x3 scatterers with a QD: poles of the scattering matrix

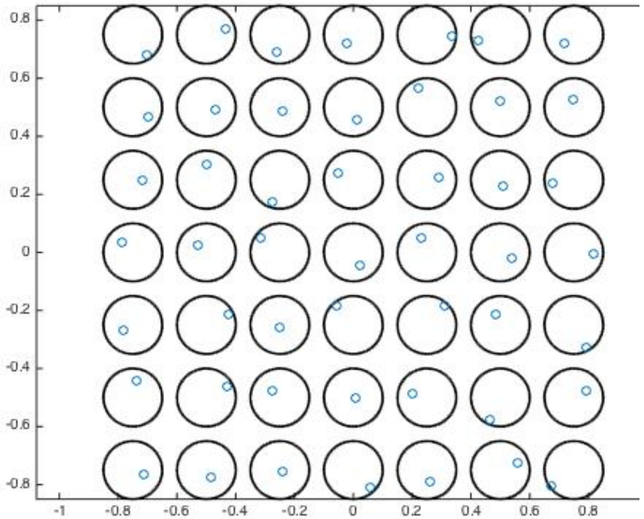


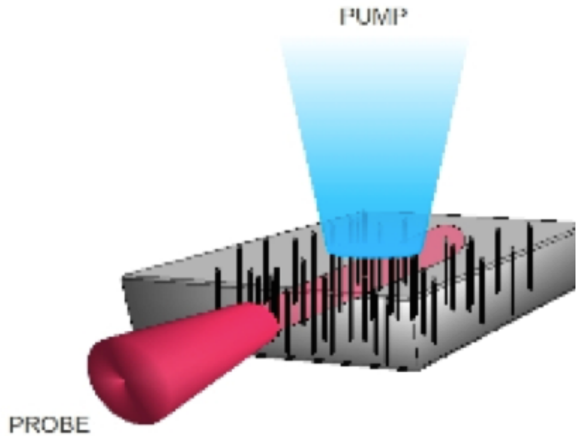


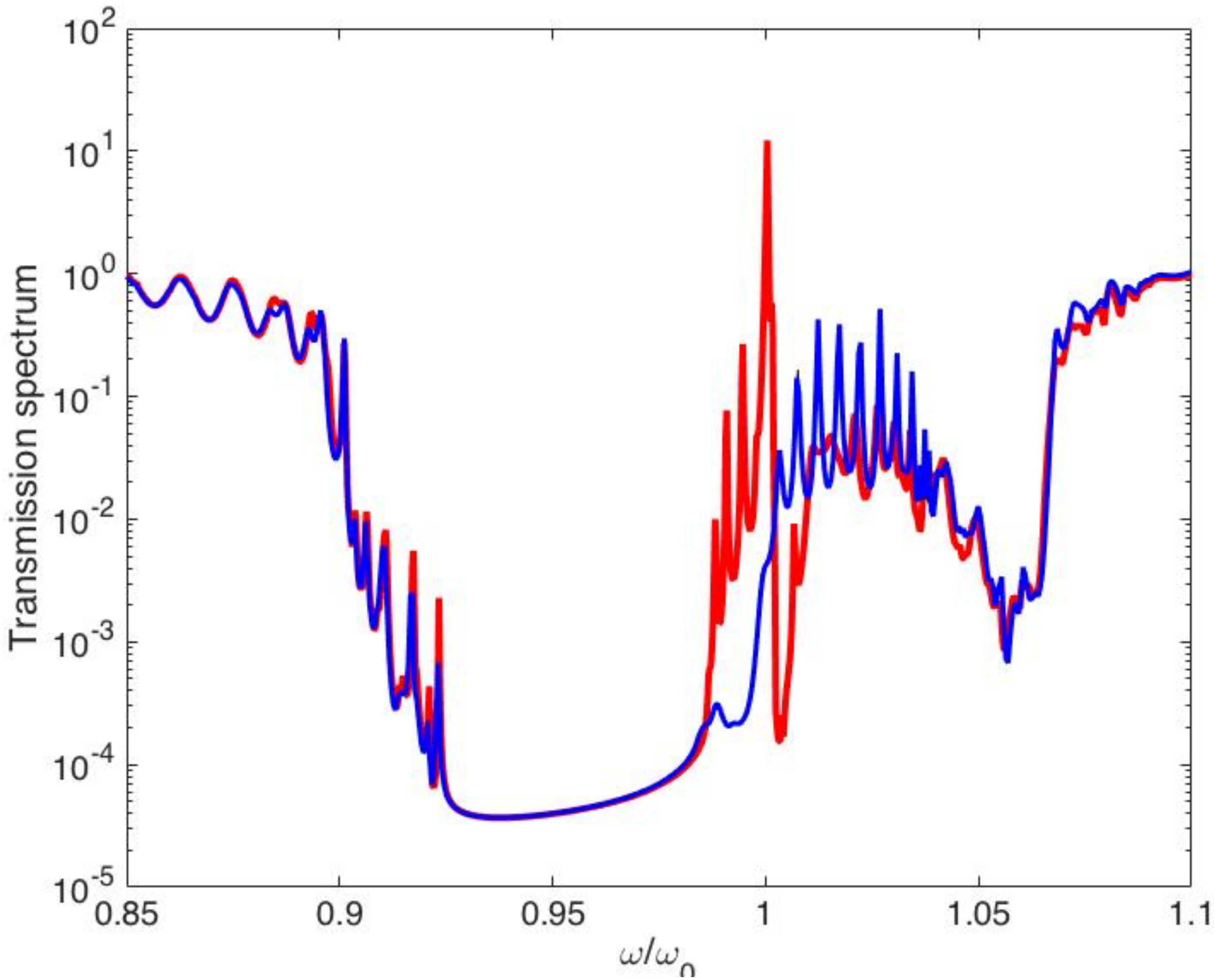
$$\omega_r = \omega + i\Gamma$$

$$\varepsilon_{QD}(\omega) = \varepsilon_b \pm \frac{f}{\omega^2 - \omega_0^2 + i2\omega\gamma}$$

Transmission through the structure



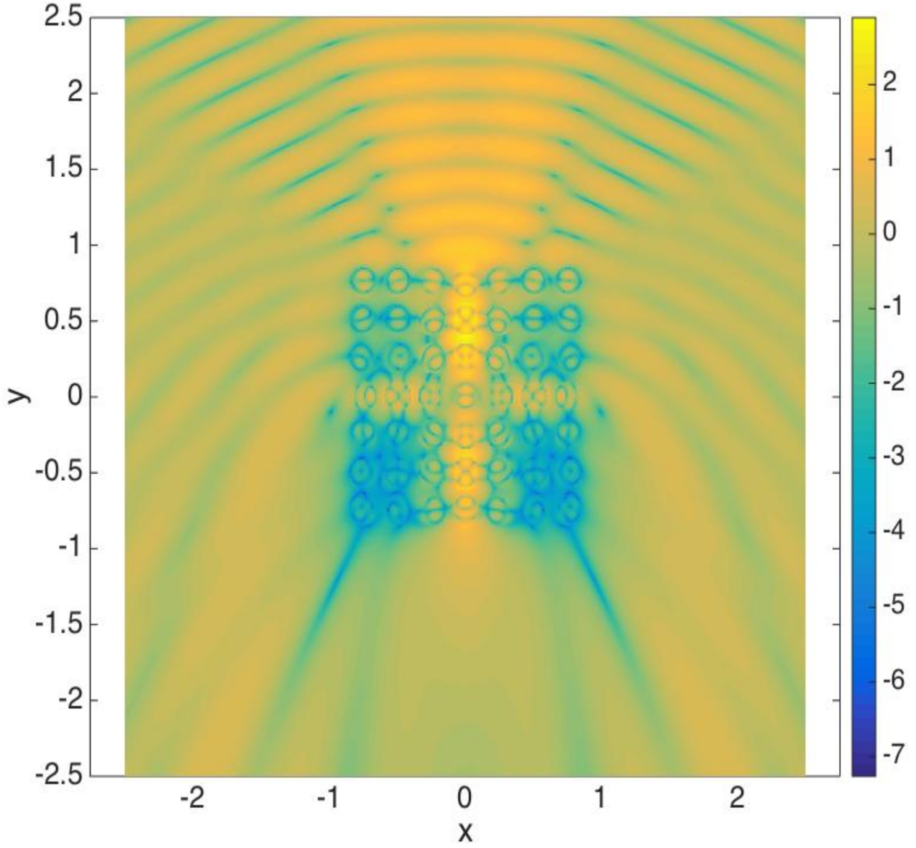




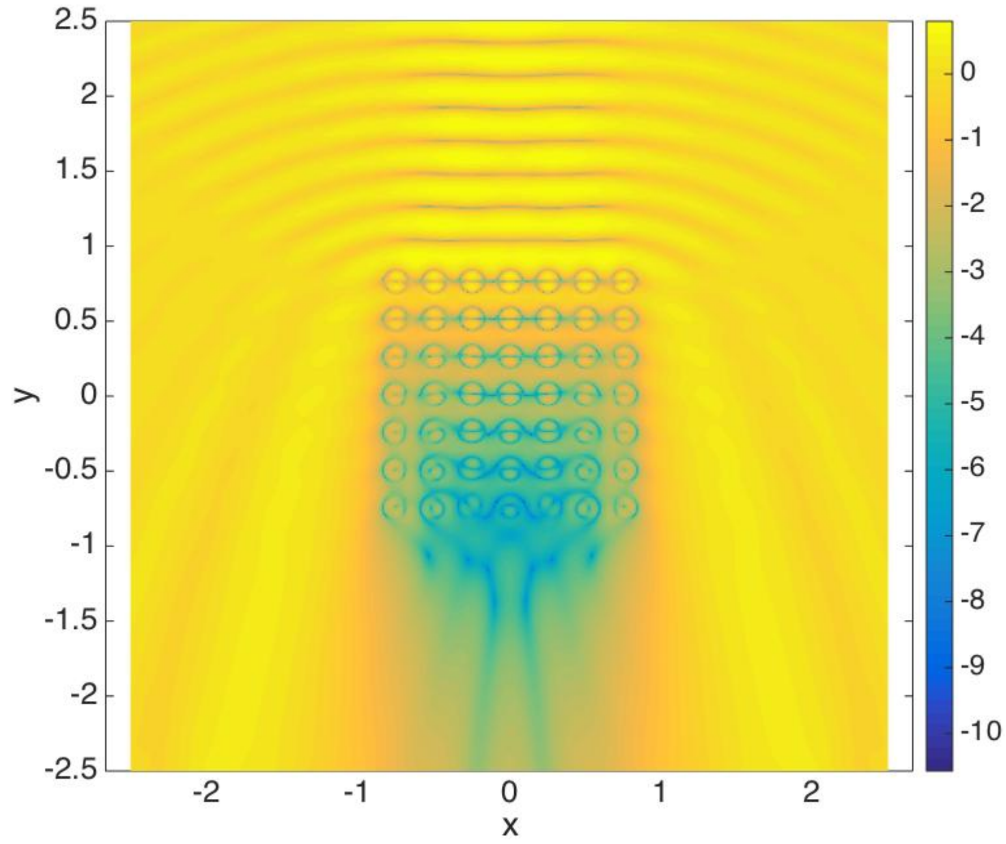
D. Felbacq, *all-optical photonic band control in a quantum metamaterial*, Annalen der Physik. 2017

Map of the field

On resonance

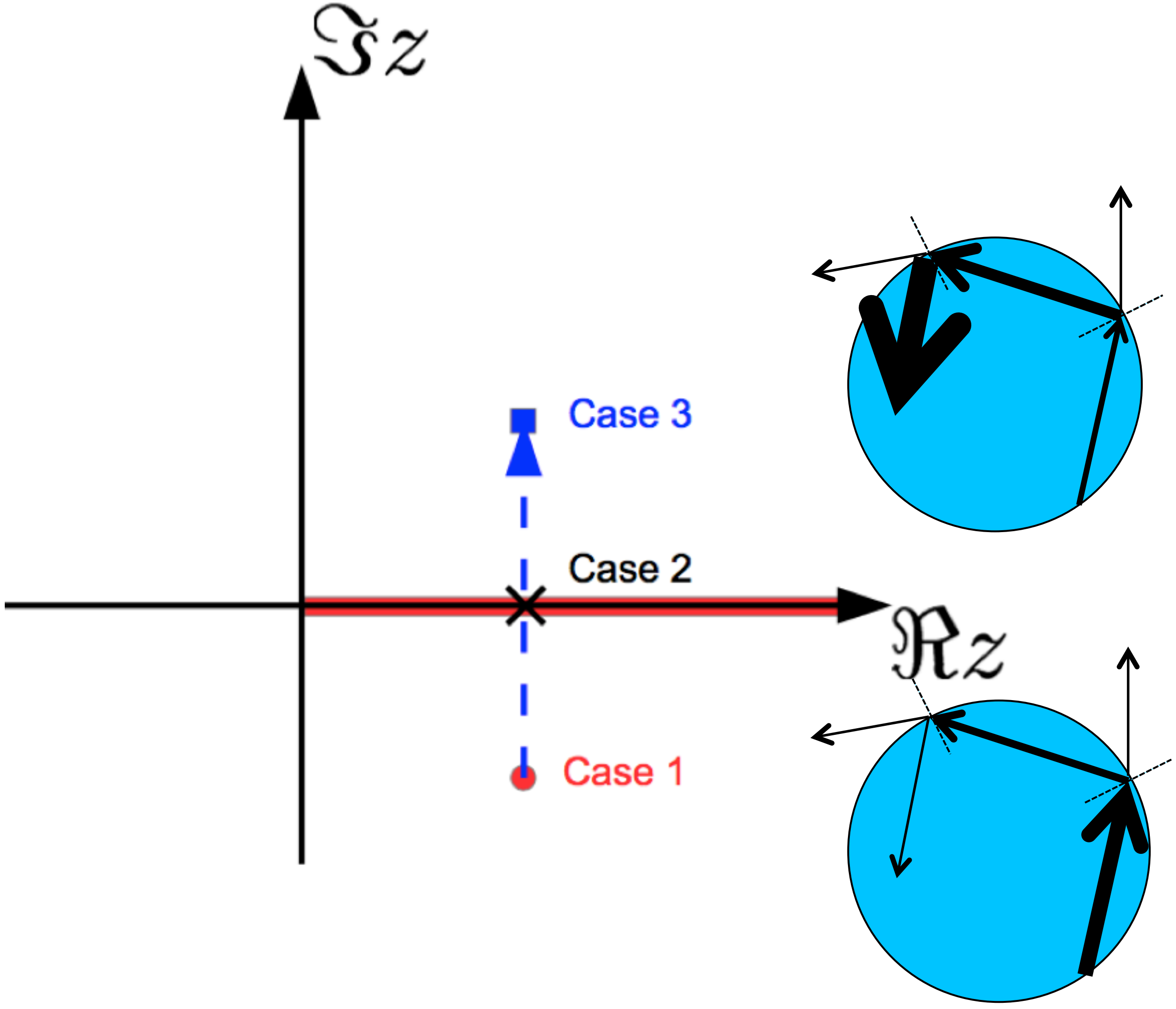


Off resonance



Validity of the model

What is the meaning of a pole in the upper sheet?



Validity of the model

Consider the temporal behavior of the field:

$$U(x,t) = \int \frac{r(x,\omega)}{\omega - \omega_0 + i\Gamma} e^{-i\omega t} d\omega$$

Validity of the model

When the pole is in the lower sheet ($\Gamma > 0$), this corresponds to an exponentially decreasing field in time.

$$\int \frac{e^{-i\omega t}}{\omega - \omega_0 + i\Gamma} d\omega = \theta(x) e^{-i\omega_0 t} e^{-\Gamma t}$$

When the pole is in the upper sheet ($\Gamma < 0$), this corresponds to a non-causal field

$$\int \frac{e^{-i\omega t}}{\omega - \omega_0 + i\Gamma} d\omega = \theta(-x) e^{-i\omega_0 t} e^{-\Gamma t}$$

The true behavior should be obtained by means of analytical continuation

Validity of the model

$$\mathcal{U}_{(x,t)}(z) = \int_{\mathbb{R}+} \frac{r(x,\omega)}{\omega - z} e^{-i\omega t} d\omega, \Im z < 0$$

This expression also makes sense for Im(z) > 0.

Denote $\widetilde{\mathcal{U}}$ the corresponding function.

There is a cut line on $\mathbb{R}^+$, from Plemelj- Sokhotski theorem, we get:

$$\mathcal{U}(\omega_0 - i0) = \fint_{\mathbb{R}+} \frac{r(x,\omega)}{\omega - \omega_0} e^{-i\omega t} d\omega - i\pi r(x,\omega_0) e^{-i\omega_0 t}$$

Implying:

$$\mathcal{U}(\omega_0 - i0) = \widetilde{\mathcal{U}}(\omega_0 + i0) - 2i\pi r(x,\omega_0) e^{-i\omega_0 t}$$

Validity of the model

The jump is an entire function of ω_0, showing that the field can be analytically continued by posing:

$$\text{For } z \in \mathbb{C}^+, \mathcal{U}(z) = \widetilde{\mathcal{U}}(z) - 2i\pi r(x,z)e^{-izt}$$

Therefore the field is exponentially growing when the pole enters the upper sheet.
Physically, this means that this approach can only account for the early times, afterwards, saturation and nonlinearity cannot be neglected.

Towards quantum metamaterials

A quantum formalism :

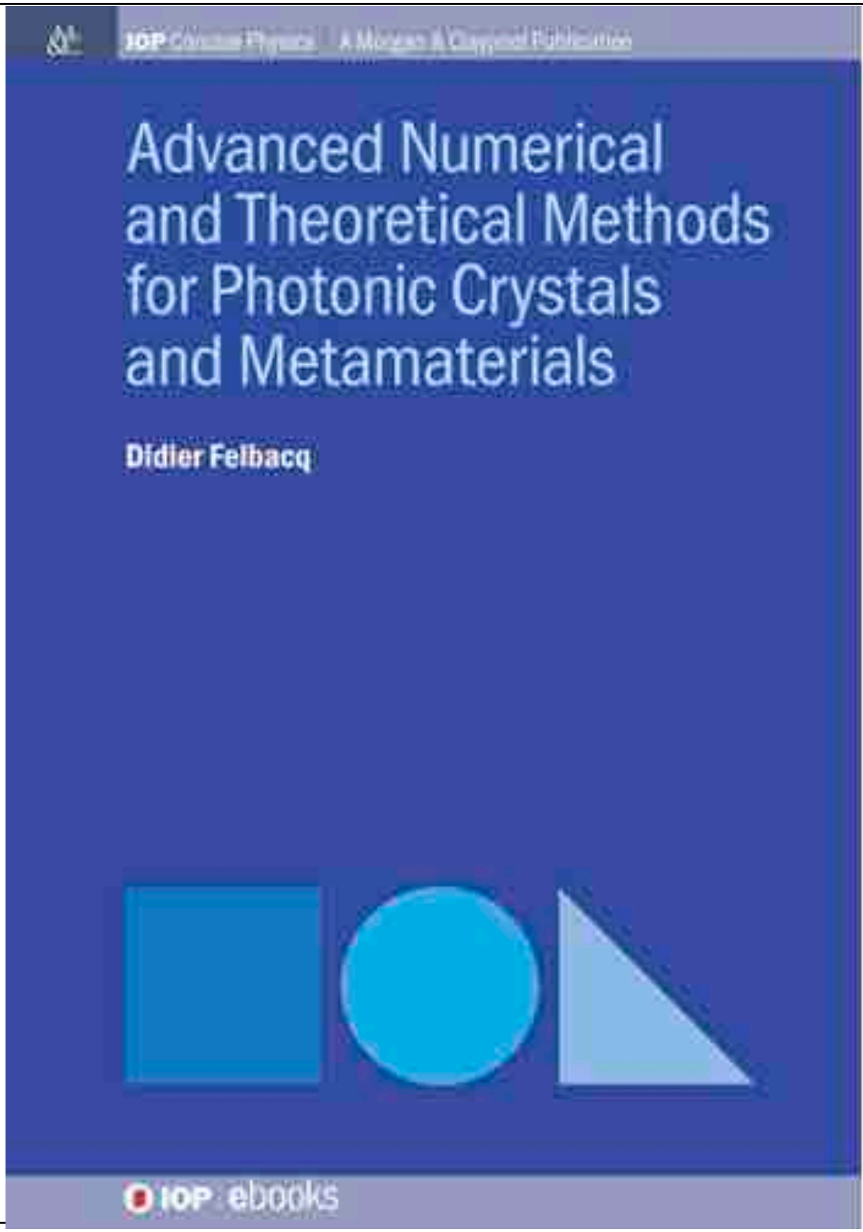
- Set of two-level systems
- Dipole coupling
- Not necessarily RWA

$$H_{QD} = \sum_n \frac{\hbar\omega_0}{2}\sigma_z \otimes \delta(r - r_n)$$

$$P = d\sum_n (\sigma^+ + \sigma^-) \otimes \delta(r - r_n)$$

Similar to the Dicke model for super-radiance
Except for the spatial variation of the field

Polaritons are expected to exist as collective modes of QDs mediated by Bloch waves, inducing non-local effects (k is a good quantum number here!)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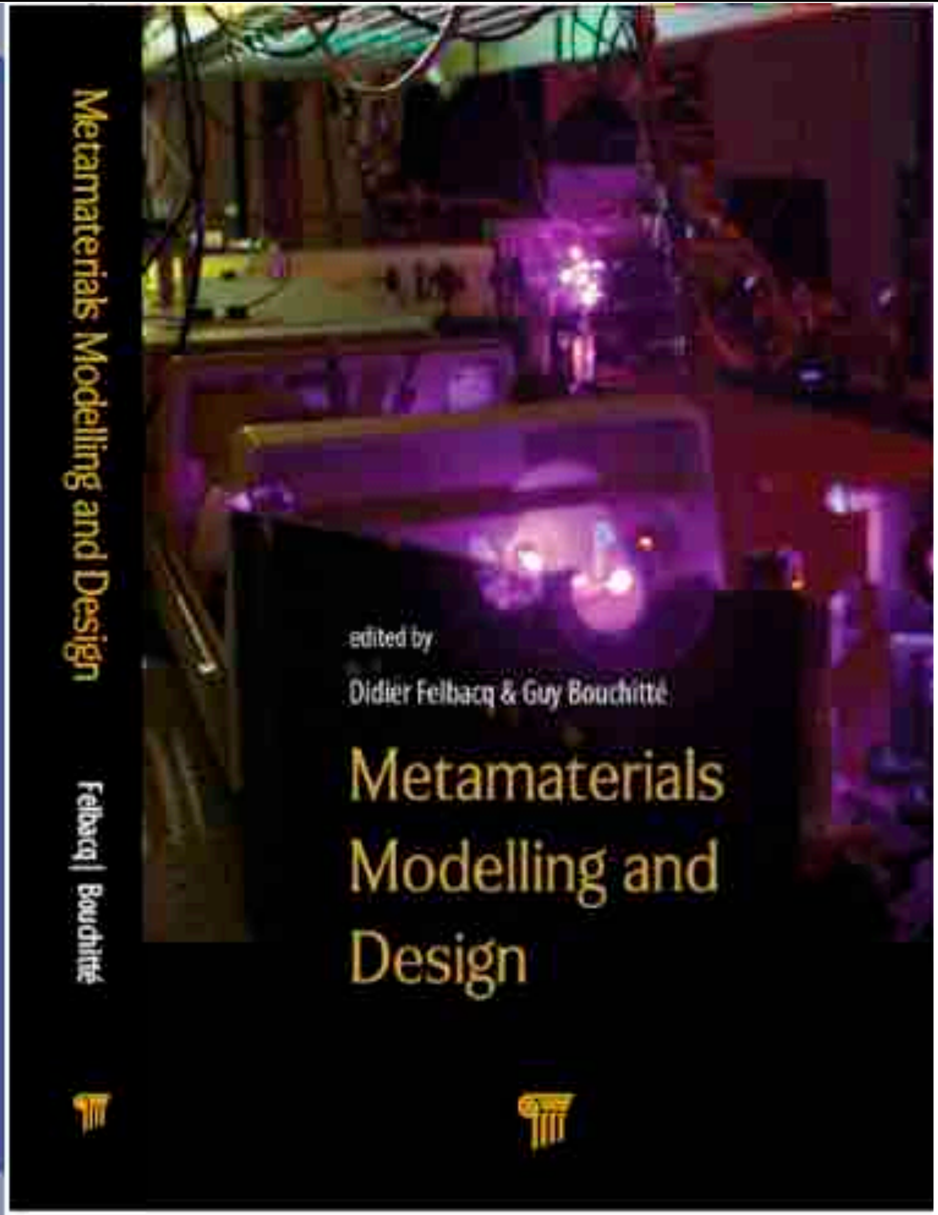
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