Geometry of the moduli of parabolic bundles on elliptic curves

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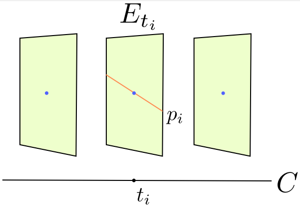
Parabolic vector bundles

Let X be a smooth projective curve over $\mathbb{C}$.
$T = t_1 + \cdots + t_n$ divisor on X.

Definition

A *rank 2 parabolic bundle* $(E, \boldsymbol{p} = (p_1, \ldots, p_n))$ over (X, T) is a rank 2 vector bundle E over X together with 1-dimensional linear subspaces p_i of the fiber E_{t_i} of t_i for $i = 1, \ldots, n$.



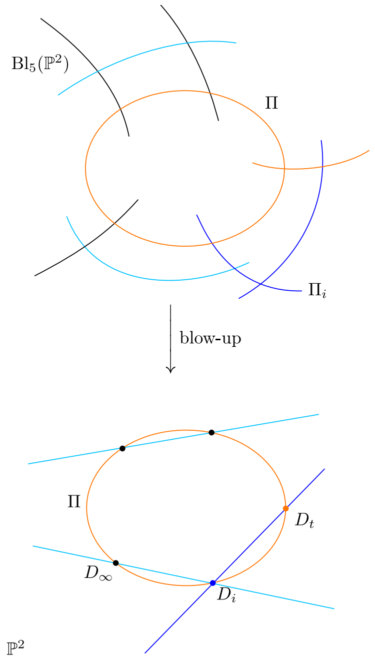
Objective: to study the GIT moduli space $\mathrm{Bun}_L(X, T)$ of rank 2 parabolic bundles with fixed determinant L over (X, T).

$(X = \mathbb{P}^1, D = 0 + 1 + \infty + \lambda + t)$

Theorem (F. Loray, M.-H. Saito (2013))

The moduli space $\mathrm{Bun}(\mathbb{P}^1, D)$ *is isomorphic to a 5-point blow-up of* $\mathbb{P}^2$*.*

$\mathrm{Bun}(\mathbb{P}^1, D) \cong \mathrm{Bl}_5(\mathbb{P}^2)$ is a del Pezzo surface of degree 4.


$\mathrm{Bl}_5(\mathbb{P}^2)$
Π
Π_i
blow-up
Π
D_t
D_∞
D_i
$\mathbb{P}^2$

The case X = elliptic curve

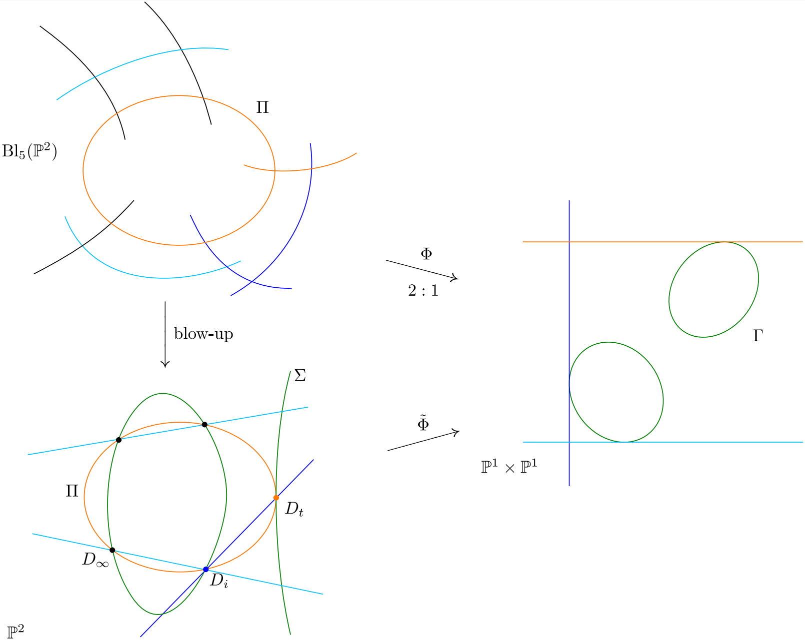
(X = elliptic curve, $T = t_1 + t_2$)

Theorem (N. F., 2015)

- *The moduli space* $\mathrm{Bun}(X, T)$ *is isomorphic to* $\mathbb{P}^1 \times \mathbb{P}^1$.
- *The semistable locus is a* $(2, 2)$ *curve* $\Gamma \subset \mathbb{P}^1 \times \mathbb{P}^1$.
  $\Gamma \cong X$, T *is determined by the embedding (Torelli).*
- *There is a 2:1 modular covering*

$$\Phi : Bl_5(\mathbb{P}^2) \cong \mathrm{Bun}(\mathbb{P}^1, D) \to \mathrm{Bun}(X, T) \cong \mathbb{P}^1 \times \mathbb{P}^1$$

ramified over Γ.


Π
$Bl_5(\mathbb{P}^2)$
Φ
2 : 1
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blow-up
Σ
$\tilde{\Phi}$
$\mathbb{P}^1 \times \mathbb{P}^1$
Π
D_t
D_∞
D_i
$\mathbb{P}^2$