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Sisypheus. Franz von Stuck, 1920

## April 10–14, 2017

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Conference at the CIRM (Luminy)  
Date duration: April 10–14, 2017  
Topics: **Hodge Theory, Stokes Phenomenon and Applications**  
Organizers: Claus Hertling, Claude Sabbah & Christian Sevenheck  
Contacts: Claus Hertling, Claude Sabbah & Christian Sevenheck  
Registration closed on the CIRM web page

Invited speakers:  
Philip Boalch (CNRS, Université Paris Sud), Tom Bridgeland (University of Sheffield), Andrea D'Agnolo (Università degli Studi di Padova), Javier Fresán (ETH Zürich), Martin Guest (Waseda University, Tokyo), Frank Loray (CNRS, Université de Rennes), Todor Milanov (IPMU, University of Tokyo), Takuro Mochizuki (RIMS, Kyoto University), Thomas Reichelt (Universität Heidelberg), Masa-Hiko Saito (Kobe University), Carlos Simpson (CNRS, Université de Nice), Dmytro Shklyarov (Universität Chemnitz), Jean-Baptiste Teysier (KU Leuven), Kazushi Ueda (University of Tokyo), Jeng-Daw Yu (National Taiwan University).

Introductory lectures on Stokes structures by Claus Hertling (Universität Mannheim), Marco Hien (Universität Augsburg), Claude Sabbah (CNRS, École polytechnique).

### Program:

#### Monday, April 10

9h15–10h30 (1h15) **Claude Sabbah: Introduction to Stokes structures I (dimension one)**  
Abstract: Stokes phenomenon in dimension one, Stokes-filtered local systems, Deligne's Riemann–Hilbert correspondence, Stokes data, Dubrovin's conjecture. *Slides of talk 1, printable file of talk 1.*  
10h30–11h Coffee break  
11h–12h15 (1h15) **Philip Boalch: Wild character varieties, meromorphic Hitchin systems and Dynkin diagrams**  
Abstract: In 1987 Hitchin discovered a new family of algebraic integrable systems, solvable by spectral curve methods. One novelty was that the base curve was of arbitrary genus. Later on it was understood how to extend Hitchin's viewpoint, allowing poles in the Higgs fields, and thus incorporating many of the known classical integrable systems, which occur as meromorphic Hitchin systems when the base curve has genus zero. However, in a different 1987 paper, Hitchin also proved that the total space of his integrable system admits a hyperkähler metric and (combined with work of Donaldson, Corlette and Simpson) this shows that the differentiable manifold underlying the total space of the integrable system has a simple description as a character variety  $\text{Hom}(\pi_1(\Sigma), G)/G$  of representations of the fundamental group of the base curve  $\Sigma$  into the structure group  $G$ . This misses the main cases of interest classically, but it turns out there is an extension. In work with Biquard from 2001 Hitchin's hyperkähler story was extended to the meromorphic case, upgrading the speakers holomorphic symplectic quotient approach from 1999. Using the irregular Riemann–Hilbert correspondence, and Sabbah's 1999 extension of the result of Donaldson/Corlette, the total space of such integrable systems then has a simple explicit description in terms of monodromy and Stokes data, generalising the character varieties. The construction of such "wild character varieties", as algebraic symplectic varieties, was recently completed in work with D. Yamakawa, generalizing the author's construction in the untwisted case (2002–2014). For example, by hyperkähler rotation, the wild character varieties thus admit special Lagrangian fibrations. The main aim of this talk is to describe some simple examples of wild character varieties including some cases of complex dimension 2, familiar in the theory of Painlevé equations, although their structure as new examples of complete hyperkähler manifolds (gravitational instantons) is perhaps less well-known. The language of quasi-Hamiltonian geometry will be used and we will see how this leads to relations to quivers, Catalan numbers and triangulations, and in particular how simple examples of gluing wild boundary conditions for Stokes data leads to duplicit algebras in the sense of Loday. The new results to be discussed are joint work with R. Paluba and/or D. Yamakawa. *Slides of the talk, link to the preprint.*  
12h30 Lunch  
16h–17h (1h) **Frank Loray: A factorization theorem for rank-two irregular flat connections**  
Abstract: We will show that a rank-two meromorphic flat trace-free connection on a projective manifold which is irregular, with non trivial Stokes matrices, factors through a curve. This is a joint work with Jorge Pereira and Frederic Touzet. *Slides of the talk, link to the article.*  
17h–17h30 Coffee break  
17h30–18h30 (1h) **Thomas Reichelt: Hodge theory of GKZ systems**  
Abstract: GKZ hypergeometric systems were introduced by Gelfand, Kapranov and Zelevinsky as a generalization of Gauss hypergeometric differential equation. It can be shown that for certain parameters the GKZ-systems carry the structure of a mixed Hodge module in the sense of Morihiko Saito. We will discuss the Hodge and weight filtration of these  $D$ -modules.  
18h45–19h30: Discussion on the topics of the day  
19h30 Dinner

#### Tuesday, April 11

9h15–10h30 (1h15) **Claude Sabbah: Introduction to Stokes structures II (dimension  $\geq 2$ )**  
Abstract: Integrable meromorphic connections in dimension  $\geq 2$ , integrable deformations of meromorphic connections in dimension one, isomonodromic and iso-level deformations, normal form in dimension  $\geq 2$ , example of the partial Laplace transformation, Stokes-filtered local system on the real oriented blow-up space, Riemann–Hilbert correspondence in dimension  $\geq 2$ . *Slides of talk 2, printable file of talk 2.*  
10h30–11h Coffee break  
11h–12h15 (1h15) **Tom Bridgeland: Wall-crossing for Donaldson–Thomas invariants via quadratic differentials**  
Abstract: There is a very general story, due to Joyce and Kontsevich–Soibelman, which associates to a CY3 (three-dimensional Calabi–Yau) triangulated category equipped with a stability condition some rational numbers called Donaldson–Thomas (DT) invariants. The point I want to emphasise is that the wall-crossing formula, which describes how these numbers change as the stability condition is varied, takes the form of an iso-Stokes condition for a family of connections on the punctured disc, where the structure group is the infinite-dimensional group of symplectic automorphisms of an algebraic torus. I will not assume any knowledge of stability conditions, DT invariants etc but will instead focus on a family of examples where everything is quite concrete. In these examples stability conditions can be identified with meromorphic quadratic differentials on Riemann surfaces, and the DT invariants become numbers of finite-length trajectories.  
12h30 Lunch  
16h–17h (1h) **Masa-Hiko Saito: Geometry of moduli spaces of parabolic connections with irregular singularities on curves**  
Abstract: In this talk, I will explain about geometry of moduli spaces of parabolic connections with irregular singularities on curves based on our work of M. Inaba and M.-H. Saito (generic unramified case) and recent work on M. Inaba (generic ramified case). Fixing the formal types at each irregular singularity and introducing the stability conditions, moduli spaces can be constructed as smooth quasi-projective symplectic scheme by Geometric Invariant Theory. Some applications to geometric Painlevé property of isomonodromic equations will be discussed.  
17h–17h30 Coffee break  
17h30–18h30 (1h) **Jean-Baptiste Teysier: Skeletons and moduli of Stokes torsors**  
Abstract: In the local classification of differential equations of one complex variable, torsors under a certain sheaf of algebraic groups (the Stokes sheaf) play a central role. On the other hand, Deligne defined in positive characteristic a notion of skeletons for  $\ell$ -adic local systems on a smooth variety, constructed an algebraic variety parametrizing skeletons and raised the question whether every skeleton comes from an actual  $\ell$ -adic local system. We will explain how to use a variant of Deligne's skeleton conjecture in characteristic 0 to prove the existence of an algebraic variety parametrizing Stokes torsors. We will show how the geometry of this moduli can be used to prove new finiteness results on differential equations.  
18h45–19h30: Discussion on the topics of the day  
19h30 Dinner

#### Wednesday, April 12

9h–10h (1h) **Dmytro Shklyarov: Semi-infinite Hodge structures in noncommutative geometry**  
Abstract: Homological mirror symmetry asserts that the connection, discovered by physicists, between a count of rational curves in a Calabi–Yau manifold and period integrals of its mirror should follow from an equivalence between the derived Fukaya category of the first manifold and the derived category of coherent sheaves on the second one. Physicists' observation can be reformulated as, or rather upgraded to, a statement about an isomorphism of certain Hodge-like data attached to both manifolds, and a natural first step towards proving the above assertion would be to try to attach similar Hodge-like data to abstract derived categories. The aim of the talk is to report on some recent progress in this direction and illustrate the approach in the context of what physicists call Landau–Ginzburg  $B$ -models.  
10h–10h15 Coffee break  
10h15–11h15 (1h) **Jeng-Daw Yu: Recent developments in irregular Hodge filtrations**  
Abstract: The existence of a Hodge type filtration on certain connections with irregular singularities was suggested by the considerations of Landau–Ginzburg models in mirror symmetry and the analogies with wild ramifications in étale sheaves in positive characteristic. In this talk, we survey some recent results on the constructions of the irregular Hodge filtrations and focus on the structures of the resulting objects over a low dimensional base, e.g., the tensor product structure when the base is a point. Based on collaborations with Claude Sabbah.  
11h15–11h20 Break  
11h20–12h20 (1h) **Javier Fresán: Exponential motives**  
Abstract: I will sketch the construction—following ideas of Kontsevich and Nori—of a Tannakian category of exponential motives over a subfield of the complex numbers. It is a universal cohomology theory for pairs of varieties and regular functions, whose de Rham and Betti realizations are given by twisted de Rham and rapid decay cohomology respectively. The upshot is that one can attach to any such pair a motivic Galois group which conjecturally generalizes the Mumford–Tate group of a Hodge structure and, over number fields, governs all algebraic relations between exponential periods. This is a joint work with Peter Jossen (ETH).  
12h30 Lunch  
19h30 Dinner

#### Thursday, April 13

9h15–10h30 (1h15) **Claus Hertling: Introduction to Stokes structures III**  
Abstract: Riemann–Hilbert correspondence for holomorphic flat bundles  $(\mathcal{H}, \nabla)$  on  $(\Delta, 0)$  with extra assumptions. The case of holomorphic functions with isolated singularities, via this RH correspondence and a topological Fourier–Laplace transformation. *Slides of the talk.*  
10h30–11h Coffee break  
11h–12h15 (1h15) **Andrea d'Agnolo: On the Riemann–Hilbert correspondence for irregular holonomic  $D$ -modules**  
Abstract: The classical Riemann–Hilbert correspondence establishes an equivalence between the triangulated categories of regular holonomic  $D$ -modules and of constructible sheaves. In a joint work with Masaki Kashiwara, we proved a Riemann–Hilbert correspondence for holonomic  $D$ -modules which are not necessarily regular. The construction of our target category is based on the theory of ind-sheaves by Kashiwara–Schapira and is influenced by Tamarkin's work on symplectic topology. Among the main ingredients of our proof is the description of the structure of flat meromorphic connections due to Mochizuki and Kedlaya.  
12h30 Lunch  
16h–17h (1h) **Kazushi Ueda: Gamma conjecture for Brieskorn–Pham singularities**  
Abstract: Gamma conjecture of Galkin–Golyshev–Iritani relates the principal asymptotic classes (also known as the central connection matrices) of the quantum cohomologies of Fano manifolds with the combinations of the Gamma classes and the Chern characters of full exceptional collections. In the talk, we will discuss the analogue of gamma conjecture for Brieskorn–Pham singularities, and its relation with homological mirror symmetry.  
17h–17h30 Coffee break  
17h30–18h30 (1h) **Todor Milanov: Primitive forms and Frobenius structures on the Hurwitz spaces**  
Abstract: We develop a systematic way to construct semi-simple Frobenius structures on the Hurwitz spaces based on Saito's theory of primitive forms. The main motivation for our work is to provide a class of examples in which the classical theory of periods of Riemann surfaces can be used to understand various problems in the theory of semi-simple Frobenius manifolds. After explaining the key points in our construction I will explain how our results can be used to describe the relation between semi-simple Frobenius manifolds and the topological recursion of Eynard and Orantin. *Notes of the talk.*  
18h45–19h30: Discussion on the topics of the day  
19h30 Special Dinner (Bouillabaisse)

#### Friday, April 14

9h15–10h30 (1h15) **Marco Hien: Introduction to Stokes structures IV**  
Abstract: We will present several situations where we can determine the Stokes structure of an irregular singular meromorphic connection which arises as the Fourier–Laplace transform of some less complicated  $D$ -module. The results will be achieved in a purely topological way. We will use the approach via Stokes filtered local systems as well as the one via enhanced ind-sheaves. The principal goal is to show how to apply these ideas in order to obtain explicit computations. *Slides of the talk.*  
10h30–11h Coffee break  
11h–12h15 (1h15) **Takuro Mochizuki: Twistor  $D$ -modules and some examples**  
Abstract: In the study of mixed twistor  $D$ -modules, an important issue is to relate mixed twistor  $D$ -modules with concrete objects in various problems. Although we know that there exist many mixed twistor  $D$ -modules by an abstract existence theorem and by the functoriality, it is not easy to describe them explicitly, which is one of the blocks to find applications of the theory. In this talk, after giving a brief survey of the general theory, we shall describe some examples of mixed twistor  $D$ -modules for which the underlying  $\mathcal{R}$ -modules can be given explicitly. We shall also mention some examples for which  $V$ -filtrations can be computed, and we shall explain how they allow us to revisit some known results.  
12h30 Lunch  
13h45–14h45 (1h) **Martin Guest: On the monodromy data of the  $tt^*$ -Toda equations**  
Abstract: The  $tt^*$ -Toda equations are a rather special case of the  $tt^*$  equations, but they are historically important and nontrivial. We compute "the connection matrix" for all solutions which are smooth near 0. We discuss the structure of the monodromy data of such solutions, and its implications for the global (pure polarised) solutions found by Guest–Lin and Mochizuki.  
14h45–15h Coffee break  
15h–16h (1h) **Carlos Simpson: The Betti Hitchin fibration for  $\mathbb{P}^1$  minus 5 points**  
Abstract: A classical example of the character variety is the Fricke–Klein cubic curve, corresponding to rank 2 local systems on  $\mathbb{P}^1$  minus 4 points. Here, one can see rather easily the topological structure of the Hitchin fibration even on the Betti side. Our objective is to consider the next case of  $\text{SL}(2, \mathbb{C})$  local systems on  $\mathbb{P}^1$  minus 5 points with fixed conjugacy classes (we take  $\text{Tr} = 0$ ). We provide an explicit model with a hypersurface equation similar to Fricke–Klein. We then resolve singularities at infinity, with the help of computer algebra (SINGULAR). In the neighborhood of the divisor at infinity we can identify many of the features of the Hitchin fibration including the degenerate fibers over linked circles, and the monodromy around them. Link to the [preprint](#).  
19h30 Dinner