Carlos Matheus Silva Santos (IMPA Brasil)
Mini-course *The Lagrange and Markov spectra from the dynamical point of view*

Lecture 1:

- Diophantine approximations, Dirichlet theorem, Hurwitz theorem, Lagrange spectrum
- Indefinite binary quadratic forms, Markov spectrum, Markov theorem, Markov tree and Vieta involutions
- Continued fraction algorithm, best rational approximations, Perron's characterization of Lagrange and Markov spectra
- Basic properties of the Lagrange and Markov spectra

Lecture 2:

- Digression: geometrical version of Perron characterization of Lagrangre spectrum (in terms of cusp excursions on the modular surface)
- Hall ray and Freiman's constant
- Moreira's theorem (and its dynamical generalizations)
- Global view on the structure of the Lagrange and Markov spectra
- Introduction to Hausdorff and box-counting dimensions

Lecture 3:

- General strategy of proof of Moreira's theorem
- Dynamical Cantor sets
- Examples of dynamical Cantor sets: affine Cantor sets and Gauss-Cantor sets
- Non-essentially affine Cantor sets
- Moreira's dimension formula
- Euler's remark
- 1st step of proof of Moreira's theorem: projections of products of Gauss-Cantor sets

Lecture 4:

- 2nd step of proof of Moreira's theorem: approximation of Lagrange and Markov spectra by projections of Gauss-Cantor sets
- 3rd step of proof of Moreira's theorem: lower semicontinuity of Hausdorff dimension across Lagrange and Markov spectra
- 4th step of proof of Moreira's theorem: upper semicontinuity of Hausdorff dimension across Lagrange and Markov spectra via an elementary com-

pactness argument

- End of proof of Moreira's theorem: behavior of the Hausdorff dimension near 3 and near $\sqrt{12}$.