Research School: Random Structures in Statistical Mechanics and Mathematical Physics

March 6-10, 2017

Eric Cator: Last Passage Percolation in $\mathbb{R}^2$

We will introduce Last Passage Percolation (LPP) in $\mathbb{R}^2$, derive the corresponding shape function and the existence of semi-infinite geodesics. From this, we will define Busemann functions and show their relationship to stationary measures corresponding to LPP. In the last lecture we will look at consequences for second class particles and cube-root fluctuations.

Francis Comets: Brownian polymers in Poissonian medium

Keypoints: Weak versus Strong disorder. Diffusive aspects at weak disorder. Localization, complete localization in some parameter region. Qualitative and quantitiative estimates on the phase diagram. Quadratic shape and rate functions.

Kostya Khanin: Random Hamilton-Jacobi equation and KPZ problem

We shall discuss the random Hamilton-Jacobi equation in compact and non-compact settings in connection with the KPZ problem. We shall also present the main concepts of the weak-KAM theory, and discuss the hyperbolic properties of the minimisers for random Lagrangian systems.

Daniel Remenik: The KPZ fixed point

In these lectures I will present the recent construction of the KPZ fixed point, which is the scaling invariant Markov process conjectured to arise as the universal scaling limit of all models in the KPZ universality class, and which contains all the fluctuation behavior seen in the class.

In the first part of the minicourse I will describe this process and how it arises from a particular microscopic model, the totally asymmetric exclusion process (TASEP). Then I will present a Fredholm determinant formula for its distribution (at a fixed time) and show how all the main properties of the fixed point (including the Markov property, space and time regularity, symmetries and scaling invariance, and variational formulas) can be derived from the formula and the construction, and also how the formula reproduces known self-similar solutions such as the $Airy_1 and Airy_2$ processes.

The second part of the course will be devoted to explaining how the KPZ fixed point can be computed starting from TASEP. The method is based on solving, for any initial condition, the biorthogonal ensemble representation for TASEP found by Sasamoto '05 and Borodin-Ferrari-Prähofer-Sasamoto '07. The resulting kernel involves transition probabilities of a random walk forced to hit a curve defined by the initial data, and in the KPZ 1:2:3 scaling limit the formula leads in a transparent way to a Fredholm determinant formula given in terms of analogous kernels based on Brownian motion.
Based on joint work with K. Matetski and J. Quastel.

Timo Seppalainen: Variational formulas, Busemann functions, and fluctuation exponents for the corner growth model with exponential weights

Lecture 1. Variational formulas for limit shapes of directed last-passage percolation models. Connections of minimizing cocycles of the variational formulas to geodesics, Busemann functions, and stationary percolation.

Lecture 2. Busemann functions for the two-dimensional corner growth model with exponential weights. Derivation of the stationary corner growth model and its use for calculating the limit shape and proving existence of Busemann functions.

Lecture 3. Kardar-Parisi-Zhang fluctuation exponent for the last-passage value of the two-dimensional corner growth model with exponential weights. We sketch the proof of the fluctuation exponent for the stationary corner growth process, and if time permits indicate how the exponent is derived for the percolation process with i.i.d. weights.

Senya Shlosman: KPZ exponents in classical statistical mechanics

I will describe recent results concerning the behavior of the 2D and 3D Ising model. More specifically, I will talk about the layering transitions. I will exhibit certain observables, which are scaled as $N^1/3$ in the system of linear size N. Based on joint works with Dima Ioffe and Yvan Velenik.