Winter School on *Combinatorics and interactions*
Abstracts and timetable

Organizers: Jérémie Bouttier, Guillaume Chapuy and Enrica Duchi

CIRM, 9-13 January 2017

1 Mini-courses

- **Nicolas Curien** (Université Paris-Sud Orsay)
  Peeling random planar maps

  The spatial Markov property of random planar maps is one of the most important properties of these random lattices. Roughly speaking, this property says that, after a region of the map has been explored, the law of the remaining part only depends on the perimeter of the discovered region. The spatial Markov property was first used in the physics literature, without a precise justification: Watabiki introduced the so-called "peeling process", which is a growth process discovering the random lattice step by step and used it to derived the so-called "two-point" function of 2D quantum gravity. A rigorous version of the peeling process and its Markovian properties was given by Angel in the case of the Uniform Infinite Planar Triangulation (UIPT), which had been defined by Angel and Schramm as the local limit of uniformly distributed plane triangulations with a fixed size. The peeling process has been used since to derive information about the metric properties of the UIPT, about percolation and simple random walk on the UIPT and its generalizations, and more recently about the conformal structure of random planar maps. It also plays a crucial role in the construction of "hyperbolic" random triangulations. In this course we review and extend these results via the new and more universal peeling process recently introduced by Budd which enables us to treat all the Boltzmann map models at once.

- **Piotr Śniady** (Uniwersytet im. Adama Mickiewicza w Poznaniu)
  Characters, maps, free cumulants

  - Normalized characters of the symmetric groups,
  - Kerov polynomials and Kerov positivity conjecture,
  - Stanley character polynomials and multirectangular coordinates of Young diagrams,
  - Stanley character formula and maps,
  - Jack characters - characterization, partial results.

- **Béatrice de Tilière** (Université Paris-Est Créteil)
  Dimers and related models in statistical mechanics

  The dimer model represents the adsorption of diatomic molecules on the surface of a crystal. Our first goal for these lectures is to present the founding result of Kasteleyn, Temperley & Fisher proving an exact formula for the partition function, the local probability formula due to Kenyon, and the discrete surface interpretation due to Thurston. Then, we will give a brief overview of the phase diagram obtained by Kenyon, Okounkov and Sheffield in the bipartite case.

The 2-dimensional Ising model is a well known model for ferromagnetism. Our second goal is to explain the notion of Z-invariance for this model, a concept extensively developed by Baxter. To this purpose we will introduce isoradial graphs and elliptic functions.

One way of studying the Ising model is to use Fisher's correspondence relating it to the dimer model on a decorated graph. Using this approach, our third goal is to present results obtained with C. Boutillier and K. Raschel, proving local formulas for the free energy and probabilities in the Z-invariant case.

- **Dimitri Zvonkine** (CNRS, Institut de Mathématiques de Jussieu-Paris Rive Gauche)
  Hurwitz numbers

  – Lecture 1: Hurwitz numbers and integrable hierarchies

    In this lecture we define Hurwitz numbers, write out its generating series using the irreducible representation of the symmetric group, give an introduction to the classical integrable hierarchies (specifically, the Korteweg-de Vries, Kadomtsev-Petviashvili, Gelfand-Dickey, and Hirota hierarchies), and prove that the generating series of Hurwitz numbers is a solution of the Kadomtsev-Petviashvili hierarchy.

  – Lecture 2: Hurwitz numbers and the intersection theory on moduli spaces of curves

    In this lecture we will introduce moduli spaces of curves, the Deligne-Mumford compactification, and some natural cohomology classes on these spaces. We will present the ELSV formula that expresses Hurwitz numbers as an integral of cohomology classes over the moduli space of curves.

  – Lecture 3: Hurwitz numbers and the topological recursion

    The topological recursion is a way to construct invariants starting from a plane curve. The construction originates from the study of matrix models, whose free energy satisfies the so-called "loop equation" that can be written out exclusively in terms of the spectral curve of the matrix model. This allows one to generalize the loop equation to any plane curve and construct its solution. We will present a proof of the Bouchard-Mariño conjecture stating that Hurwitz numbers appear as a solution of the loop equation for a particular plane curve.

2 Research talks

- **Mathilde Bouvel** (Universität Zürich)
  Studying permutation classes using the substitution decomposition

  The notion of "pattern" in a permutation provides a natural notion of substructure for permutations. Permutation classes are downsets for the corresponding partial order. Permutation patterns and permutation classes were first defined in the seventies, in connection with sorting devices. But since then, most of the work done in the area is in enumerative combinatorics (although probabilists recently started to be interested in the topic). In my talk, I will present the substitution decomposition of permutations, which allows to encode permutations as trees. I will show several examples of results on permutation classes that can be derived using substitution decomposition. These will include exact enumerative results for specific permutation classes, general enumerative results for families of permutation classes, and a probabilistic result describing the "limit shape" of permutations taken in the class of so-called separable permutations.

- **Élise Goujard** (Université Paris-Sud Orsay)
  Flat surfaces and combinatorics

  Billiards in polygons are related to dynamics of the linear flow on flat surfaces. Through some examples of counting problems on flat surfaces and on moduli spaces of flat surfaces, we will see how combinatorics can lead to interesting dynamical results in this setting.

- **Danilo Lewanski** (Universiteit van Amsterdam)
  Orbifold Hurwitz numbers, topological recursion and ELSV-type formulae

  ELSV-type formulae express Hurwitz numbers of certain type in terms of the intersection theory on moduli spaces of curves. ELSV-type formulae and Topological Recursion for the appropriate spectral curve are in many examples proved to be equivalent results. In particular, a specialisation of the ELSV- type formula due to Johnson, Pandharipande and Tseng (JPT) can be shown to be equivalent to the topological recursion for the r-orbifold Hurwitz numbers, using Chiodo classes. We will discuss this equivalence and how to prove the topological recursion, obtaining a new proof of the specialised JPT formula.

  Based on two joined works with (subsets of) P. Dunin-Barkowski, A. Popolitov, S. Shadrin, D. Zvonkine.

- **Alexander Moll** (Institut des Hautes Études Scientifiques)
  A new spectral theory for Schur polynomials and applications

  After Fourier series, the quantum Hopf-Burgers equation $v_t + vv_x = 0$ with periodic boundary conditions is equivalent to a system of coupled quantum harmonic oscillators, which may be prepared in Glauber's coherent states as initial conditions. Sending the displacement of each oscillator to infinity at the same rate, we (1) confirm and (2) determine corrections to the quantum-classical *correspondence principle*. After diagonalizing the Hamiltonian with Schur polynomials, this is equivalent to proving (1) the concentration of profiles of Young diagrams around a limit shape and (2) their global Gaussian fluctuations for Schur measures with symbol $v : T \to R$ on the unit circle T. We identify the emergent objects with the *push-forward* along v of (1) the uniform measure on T and (2) $H^{1/2}$ noise on T. Our proofs exploit the integrability of the model as described by Nazarov-Sklyanin (2013). As time permits, we discuss structural connections to the theory of the topological recursion.

- **Jonathan Novak** (University of California, San Diego)
  Monotone Hurwitz numbers and the HCIZ integral

  The Harish-Chandra/Itzykson-Zuber integral is a basic special function in representation theory and random matrix theory. In representation theory, it appears in the Harish-Chandra/Kirillov character formula for the general linear groups; in random matrix theory, it describes the spectrum of coupled random matrices with the so-called "AB interaction." I will describe joint work with I. Goulden and M. Guay-Paquet which reveals that the HCIZ integral is a generating function for a desymmetrized version of the double Hurwitz numbers, known as monotone double Hurwitz numbers. This combinatorial model for the HCIZ integral is especially useful for analyzing its asymptotic behaviour in various scaling limits.

- **Gourab Ray** (University of Cambridge)
  Universality of fluctuations of the dimer model

  We introduce a new robust technique of tackling the problem of universality of fluctuations of the height function of dimer models on general graphs (on the hexagonal lattice, this model is also known as lozenge tilings). This will use the exact solvability of the model through various forms of bijections and not through the popular method of analysing Kasteleyn matrices. For this, we exploit the new GFF/SLE coupling results of imaginary geometry due to Dubedat, Miller and Sheffield. As an application, I will try to convince you of the universal behaviour of the fluctuations under only a CLT assumption on a related graph (called T graphs) for the Gibbs measure on lozenge tiling with any mean slope. Time permitting, I will also discuss an ongoing work on the universality of height function fluctuation on a surface with more general topology (e.g. a torus).

  This is joint work with Nathanael Berestycki and Benoit Laslier.

3 Posters

- **Florian Aigner** (Universität Wien)
  Polynomiality phenomena for FPLs and ASTs

- **Tova Brown** (University of Arizona)
  Integrable dynamics used for solving recursions in map enumeration problems

  Families of map enumeration problems whose generating functions are given by recurrence relations in the form of autonomous discrete Painlevé-I equations were discovered to have nice, simple closed forms for their recurrence relations (Bouttier, Di Francesco, Guitter 2003; Bousquet-Mélou 2006). By viewing the recurrence relations as discrete dynamical systems, the integrability of these equations can be exploited to explain how the closed forms for these combinatorial problems are a beautiful consequence of the dynamics and special initial conditions.

- **Linxiao Chen** (Université Paris-Sud et CEA Saclay) and **Joonas Turunen** (University of Helsinki)
  Critical Boltzmann Ising triangulations of the half plane

  We consider a random triangulation of the $(p+q)$-gon coupled to an Ising model on its faces, with a Dobrushin boundary condition $+^p-^q$. A Botlzmann weight is assigned to every face of the triangulation. This type of boundary condition is preserved by the simple peeling process that explores the Ising interface imposed by the boundary condition. We exploit this fact to compute explicitly the partition function of these random triangulations by solving the associated Tutte's equation. This partition function gives rise to a perimeter exponent different from that of a uniform triangulation for one unique value of the Ising coupling constant. We concentrate on this critical phase. Using exact asymptotics of the partition function, we show that when p and q tend to infinity one after the other, the above random triangulation converges locally in distribution to an infinite triangulation of the half plane. Moreover, on this infinite triangulation there is essentially one unique infinite Ising interface, which touches the boundary only finitely often. A scaling limit result of cluster perimeter is also obtained.

- **Dario De Stavola** (Universität Zürich)
  A Plancherel measure associated to set partitions

  In recent years increasing attention has been paid on the area of supercharacter theories, especially to those of the upper unitriangular group. A particular supercharacter theory, in which supercharacters are indexed by set partitions, has several interesting properties, which make it object of further study. We define a natural generalization of the Plancherel measure, called superplancherel measure, and prove a limit shape result for a random set partition according to this distribution. We also give a description of the asymptotical behavior of two set partition statistics related to the supercharacters. The study of these statistics when the set partitions are uniformly distributed has been done by Chern, Diaconis, Kane and Rhoades.

- **Bérénice Delcroix-Oger** (Université Toulouse 3)
  Rigidity theorem for generalised bialgebras

  Most of studied bialgebras in combinatorics are Hopf algebras. However, looking at the algebra of words endowed with concatenation and deconcatenation, the relation linking product and coproduct is clearly of another type. Markl, Fox and Loday have introduced the notion of generalised bialgebra to keep this kind of relations into account. It then appears that some of these generalised bialgebras satisfy a strong algebraic property, called rigidity theorem.

We will first recall the required notions and then present the rigidity theorem and the intertwinement between algebra and combinatorics appearing in this frame.

- **Gustina Elfiyanti** (Institut Teknologi Bandung)
  Construction of derived category of U-complexes

  Davvaz and Shabani-Solt introduced a notion of U-complexes as a generalization of complexes by replacing the kernels with submodules U. They used the concept to make some generalizations in homological algebra. Here, we continue their research in category theory. We propose to define the derived category of U-complexes. First we define the category of U-complexes and homotopy category of U-complexes. We proved that the categories are abelian and triangulated respectively. Then we show that the set of all U-exact sequences is a null system and the set of all quasi-isomorphism in the category of U-complexes is a multiplicative system. Using those results we construct the derived category of U-complexes as a localization of homotopy category of U-complexes with respects to quasi isomorphism of U-complexes.

  This research is part of my dissertation under supervision of Dr. Intan Muchtadi-Alamsyah and Dr. Dellavitha Nasution.

- **Mathias Lepoutre** (École polytechnique)
  Open diagrams for walks in the plane

- **Philippe Marchal** (CNRS et Université Paris-Nord)
  Rectangular Young tableaux

  It has been shown by Pittel and Romik that the random surface associated with a large rectangular Young tableau converges to a deterministic limit. We study the fluctuations from this limit along the edges of the rectangle. We show that in the corner, these fluctuations are gaussian wheras, away from the corner and when the rectangle is a square, the fluctuations are given by the Tracy-Widom distribution. Our method is based on a connection with the Jacobi ensemble.

- **Paul Melotti** (Université Pierre et Marie Curie)
  Combinatorial interpretation of spatial recurrences

  Some important relations appearing in cluster algebras and in statistical mechanics can be expressed as spatial recurrences, e.g. the octahedron recurrence, cube recurrence, etc. In some cases, the solutions to these recurrences can be expressed as combinatorial objects : perfect matchings (Speyer), cube groves (Carroll, Speyer), double dimers (Kenyon, Pemantle). We give a combinatorial interpretation of Kashaev's recurrence, which appeared in the study of the star-triangle relation for the Ising model, in terms of bicolor loops. Using a standard procedure we also provide limit shapes for this loop model.

- **Arthur Nunge** (Université Paris-Est Marne-la-Vallée)
  2-species exclusion processes and combinatorial algebras

- **Drazen Petrovic** (Indiana University and Purdue University Indianapolis)
  Pfaffian sign theorem for the dimer model on a triangular lattice on the torus

- **Anna Weigandt** (University of Illinois at Urbana-Champaign)
  Partition identities and quiver representations

  We present a particular connection between classical partition combinatorics and the theory of quiver representations. Specifically, we give a bijective proof of an analogue of A. L. Cauchy's Durfee square identity to multipartitions. We then use this result to give a new proof of M. Reineke's identity in the case of quivers Q of Dynkin type A of arbitrary orientation. Our identity is stated in terms of the lacing diagrams of S. Abeasis - A. Del Fra, which parameterize orbits of the representation space of Q for a fixed dimension vector.

  This poster is based on joint work with Richard Rimanyi and Alexander Yong.

CIRM Winter School on Combinatorics and Interactions — Mon 2017-01-09 – Fri 2017-01-13 (Paris)

	Mon 1/9	Tue 1/10	Wed 1/11	Thu 1/12	Fri 1/13
09:00	Śniady 1 09:00 - 10:30	**Curien 2** 09:00 - 10:30	**de Tilière 1** 09:00 - 10:30	**de Tilière 2** 09:00 - 10:30	**Zvonkine 3** 09:00 - 10:30
10:50					**Ray** 10:50 - 11:40
11:00	**Curien 1** 11:00 - 12:30	Śniady 3 11:00 - 12:30	**Zvonkine 1** 11:00 - 12:30	**Zvonkine 2** 11:00 - 12:30	
11:40					**Lewanski** 11:40 - 12:30
12:30	**[lunch]** 12:30 - 13:30	**[lunch]** 12:30 - 13:30	**[lunch]** 12:30 - 13:30	**[lunch]** 12:30 - 13:30	**[lunch]** 12:30 - 13:30
14:00					**Novak** 14:00 - 14:50
15:00	Śniady 2 15:00 - 16:30	**Curien 3** 15:00 - 16:30		**de Tilière 3** 15:00 - 16:30	**Zvonkine exercises** 15:00 - 16:00
17:00	**Bouvel** 17:00 - 17:50	**Moll** 17:00 - 17:50		**Goujard** 17:00 - 17:50	
18:00	Śniady exercises 18:00 - 19:00	**Curien exercises** 18:00 - 19:00		**de Tilière exercises** 18:00 - 19:00	
19:30	**[dinner]** 19:30 - 20:30	**[dinner]** 19:30 - 20:30	**[dinner]** 19:30 - 20:30	**[dinner]** 19:30 - 20:30	
20:30		**[poster session]** 20:30 - 22:00 [poster session]			