Analyse asymptotique des équations d'évolution

Centre International de Rencontres Mathématiques – Luminy
3 au 7 Juillet 2017

Liste des conférences

Thimoty Candy (University of Bielefeld): **Scattering for the Cubic Dirac equation**

Abstract: The asymptotic behavior of the cubic Dirac equation is particularly delicate in small dimensions due to the lack of available Strichartz estimates. However, recently a number of small data scattering results have been obtained, which in particular show the existence of a modified asymptotic profile in one dimension. We summarize these developments and related results.

Charles Collot (Université de Nice): **On dynamics near the ground state for the energy critical semi-linear heat equation**

Abstract: This talk is about solutions to

$$\partial_t u = \Delta u + |u|^{p-1}u, \quad p > 1, \; x \in \mathbb{R}^d$$

in the energy critical case $p = 1 + 4/(d-2)$ in the vicinity of the family of radial stationary states $\left((\lambda^{-2/(p-1)} Q((x-x_0)/\lambda)\right)_{\lambda>0}$ obtained by rescaling and translations in space of a given radial solution to $\Delta Q + |Q|^{p-1}Q = 0$. These special solutions can appear in the asymptotic description in time of arbitrary solutions, either in the case of global dynamics or when finite time blow-up happens.
For a solution starting in their neighbourhood $u(0,x) = Q(|x|) + \varepsilon(0,x)$, as long as it stays there, the stationary state will be modulated, that will in turn affect its interaction with the perturbation, $u(t,x) = \lambda(t)^{-2/(p-1)} Q\left(|x - x(t)|/\lambda(t)\right) + \varepsilon(t,x)$. We will first review some known cases when the scale can shrink to 0 in finite time, forming a singularity. The solution can also escape from this neighbourhood, and display another behaviour in large times. We will then mainly focus on the large dimension case $d \geq 7$ where a classification of dynamics was achieved in collaboration with F. Merle and P. Raphaël.

Thibault de Poyferré (École normale supérieure, Paris): **Gravity water waves and emerging bottom**

Abstract: To understand the behavior of waves at a fluid surface in configurations where the surface and the bottom meet (islands, beaches...), one encounters a difficulty : the presence in the bulk of the fluid of an edge, at the triple line. To solve the Cauchy problem, we need to study elliptic regularity in such domains, understand the linearized operator around an arbitrary solution, and construct an appropriate procedure to quasi-linearize the equations. Using those tools, I will present some a priori estimates, a first step towards a local existence result.

Erwan Faou (INRIA et Université de Rennes 1): **On the long time stability of travelling wave for discrete nonlinear Schrödinger equations**

Abstract: I will discuss the possible existence of travelling wave solutions for discrete nonlinear Schrödinger equations on a grid. I will show the influence of the nonlinearity in this problem and give some partial results for the long time stability.
This is a joint work with Dario Bambusi, Joackim Bernier, Benoît Grébert and Alberto Maspero.

Sandrine Grellier (Université d'Orléans): **Various aspects of the dynamics of the cubic Szegő solutions**

Abstract: The cubic Szegő equation has been introduced as a toy model for totally non dispersive evolution equations. It turned out that it is a complete integrable Hamiltonian system for which we built a non linear Fourier transform giving an explicit expression of the solutions.
This explicit formula allows to study the dynamics of the solutions. We will explain different aspects of it: almost-periodicity of the solutions in the energy space, uniform analyticity for a large set of initial data, turbulence phenomenon for a dense set of smooth initial data in large Sobolev spaces.
From joint works with Patrick Gérard.

Stephen Gustafson (University of British Columbia, Vancouver): **Asymptotic behaviour for Landau-Lifshitz and nonlinear heat equations**

Abstract: We describe some results on long-time asymptotics for energy-critical dissipative problems – including the (above-threshold) equivariant Landau-Lifshitz system and (below-threshold) nonlinear heat equation – with methods inspired by dispersive PDE theory.
Joint work with D. Roxanas.

Zaher Hani (Georgia Tech University): **Effective dynamics of nonlinear Schrödinger equations on large domains**

Abstract: In this talk, we will be mainly concerned with the following question: Suppose we consider a nonlinear dispersive or wave equation on a large domain of characteristic size L: What is the effective dynamics when L is very large? This question is relevant for equations that are naturally posed on large domains (like water waves on an ocean), and in turbulence theories for dispersive equations. It's not hard to see that the answer is intimately related to the particular time scales at which we study the equation, and one often obtains different effective dynamics on different timescales. After discussing some relatively "trivial" time scales (and their corresponding effective dynamics), we shall attempt to go to longer times scales and try to describe the effective equations that govern the dynamics there. The ultimate goal is to reach the so-called "Kinetic time scale" over which the dynamics are effectively described by a kinetic equation called the "wave kinetic equation". This is the main claim of wave turbulence theory. We will discuss several recent results, obtained in collaboration with Tristan Buckmaster, Pierre Germain, and Jalal Shatah (all at Courant Institute, NYU), that are aimed at addressing the above problematic for the nonlinear Schrödinger equation.

Sebastian Herr (University of Bielefeld): **Nonlinear Dirac equations and systems**

Abstract: Results concerning the longtime behaviour of solutions of cubic Dirac equations and of the Dirac-Klein-Gordon system will be presented. In particular, small perturbations of a certain class of large solutions will be considered.

Cécile Huneau (Université de Grenoble-Alpes): **High frequency back reaction for the Einstein equations**

Abstract: It has been observed by physicists (Isaacson, Burnett, Green-Wald) that metric perturbations of a background solution, which are small amplitude but with high frequency, yield at the limit to a non trivial contribution which corresponds to the presence of an energy impulsion tensor in the equation for the background metric. This non trivial contribution is of due to the nonlinearities in Einstein equations, which involve products of derivatives of the metric. It has been conjectured by Burnett that the only tensors which can be obtained this way are massless Vlasov, and it has been proved by Green and Wald that the limit tensor must be traceless and satisfy the dominant energy condition. The known exemples of this phenomena are constructed under symmetry reductions which involve two Killing fields and lead to an energy impulsion tensor which consists in at most two dust fields propagating in null directions. In this talk, I will explain our construction, under a symmetry reduction involving one Killing field, which leads to an energy impulsion tensor consisting in N dust fields propagating in arbitrary null directions.
This is a joint work with Jonathan Luk (Stanford).

Mihaela Ifrim (University of California, Berkeley): **Normal form methods in water waves models**

Abstract: The aim of the talk is toward better understanding the long time dynamics of solutions to water waves models by means of implementing well chosen normal form transformations.

Jacek Jendrej (University of Chicago): **Two-bubble dynamics for threshold solutions to the wave maps equation**

Abstract: We consider the energy-critical wave maps equation $\mathbb{R}^{1+2} \to \mathbb{S}^2$ in the equivariant case, with equivariance degree $k \geq 2$. It is known that initial data of energy $< 8\pi k$ and topological degree 0 lead to global solutions that scatter in both time directions. We consider the threshold case of energy $8\pi k$. We prove that the solution is defined for all time and either scatters in both time directions, or converges to a superposition of two harmonic maps in one time direction and scatters in the other time direction. In the latter case, we describe the asymptotic behavior of the scales of the two harmonic maps.
The proof combines the classical concentration-compactness techniques of Kenig-Merle with a modulation analysis of interactions of two harmonic maps in the absence of excess radiation.
Joint work with Andrew Lawrie.

Joachim Krieger (École polytecnique fédérale de Lausanne): **On stability of type II blow up solutions for the critical nonlinear wave equation**

Abstract: The talk will discuss a recent result showing that certain type II blow up solutions constructed by Krieger-Schlag-Tataru are actually stable under small perturbations along a co-dimension one Lipschitz hypersurface in a suitable topology. This result is qualitatively optimal.
Joint work with Stefano Burzio (EPFL).

Fabricio Macià (Universidad Politécnica de Madrid): **Dynamics, dispersion and control of Schrödinger equations**

Abstract: We study the dynamics of linear Schrödinger equations:

$$i\partial_t u(t,x) + \Delta_x u(t,x) - V(t,x)u(t,x) = 0, \quad (t,x) \in \mathbb{R} \times M,$$

in bounded geometries; M will be typically a compact manifold, equipped with a Riemannian metric, or a bounded domain in Euclidean space.
We are particularly interested in understanding the structure of those subsets on which high-frequency solutions can concentrate (in the sense of the L^2 norm); that is, regions on which the position probability densities $|u_n(t,x)|^2$ of a normalized sequence of solutions can accumulate.
This is problem related to quantifying dispersion and understanding controllability properties for Schrödinger equations.
We give a detailed answer to this question for systems whose underlying classical dynamics (the geodesic flow or the billiard flow) is completely integrable (as flat tori, spheres or the planar disk). Our analysis is based on understanding the structure of microlocal defect measures associated to sequences of solutions. We accomplish that by means of second microlocalizations with respect to a partition of phase space adapted to the classical dynamical system.
This talk is based on joint works with Nalini Anantharaman, Clotilde Fermanian-Kammerer, Matthieu Léautaud and Gabriel Rivière.

Tadahiro Oh (University of Edinburgh): **On the transport property of Gaussian measures under Hamiltonian PDE dynamics**

Abstract: In probability theory, the transport property of Gaussian measures have attracted wide attention since the seminal work of Cameron and Martin '44. In this talk, we discuss recent development on the study of the transport property of Gaussian measures on spaces of functions under nonlinear Hamiltonian PDE dynamics. As an example, we will discuss the case for the 2-d cubic nonlinear wave equation, for which we introduce a simultaneous renormalization of the energy functional and its time derivative to study the transport property of Gaussian measures on Sobolev spaces.

Benoît Pausader (Brown University): **On the Einstein equation with a massive scalar field**

Abstract: A toy model to incorporate matter in General relativity is to consider the stress energy tensor of a massive scalar field. This leads to couple the Einstein equation on the metric (wave equations) with a Quasilinear Klein-Gordon equation on the space. We show that the Minkowski space is asymptotically stable for this model. For restricted initial data, this was previously studied by Wang and LeFloch-Ma. We extend this to general initial data (under suitable decay hypothesis).
This is a joint work with A. Ionescu.

Annalaura Stingo (Université Paris 13): **Global existence and asymptotics for quasi-linear one-dimensional Klein-Gordon equation with small mildly decaying Cauchy data**

Abstract: Let u be a solution to a quasi-linear Klein-Gordon equation in one-space dimension, $\Box u + u = P(u, \partial_t u, \partial_x u; \partial_t \partial_x u, \partial_x^2 u)$, where P is a homogeneous polynomial of degree three, and with smooth Cauchy data of size $\varepsilon \to 0$. It is known that, under a suitable condition on the nonlinearity, the solution is global-in-time for compactly supported Cauchy data. We prove that the result holds even when data are not compactly supported but just decaying as $\langle x \rangle^{-1}$ at infinity, combining the method of Klainerman vector fields with a semiclassical normal forms method introduced by Delort. Moreover, we get a one term asymptotic expansion for u when $t \to +\infty$, showing that there is modified scattering.

Daniel Tataru (University of California, Berkeley): **Geometric heat flows and caloric gauges**

Abstract: Choosing favourable gauges is a crucial step in the study of nonlinear geometric dispersive equations. A very successful tool, that has emerged originally in work of Tao on wave maps, is the use of caloric gauges, defined via the corresponding geometric heat flows. The aim of this talk is to describe two such flows and their associated gauges, namely the harmonic heat flow and the Yang-Mills heat flow.

Joseph Thirouin (École normale supérieure, Paris): **Optimal estimates on the growth of Sobolev norms for a quadratic Szegő equation**

Abstract: In this talk, I would like to introduce a Hamiltonian equation which resembles the cubic Szegő equation studied by Gérard and Grellier, but which only involves quadratic nonlinearities. I will review its integrability properties, and show how explicit computations can prove the existence of weakly turbulent orbits, *i.e.* smooth solutions which blow-up in infinite time in the sense of certain Sobolev norms. Starting from this exemple, I will discuss the optimality of a-priori estimates on the growth of both smooth and rough solutions (the latter belonging to the BMO space of John and Nirenberg).

Luis Vega (BCAM et Universidad del País Vasco, Bilbao): **Critical perturbations of Dirac Hamiltonians: selfadjointness and spectrum**

Abstract: I shall present some recent results about some singular perturbations of Dirac operator and its connection with the boundedness of the Cauchy operator and Calderon's projector operator. Also I will sketch the proof of an isoperimetric type inequality.

Monica Visan (University of California, Los Angeles): **Almost sure scattering for the energy-critical Schrödinger equation in 4D with radial data**

Abstract: Inspired by a recent result of Dodson-Luhrmann-Mendelson, who proved almost sure scattering for the energy-critical wave equation with radial data in four dimensions, we establish the analogous result for the Schrödinger equation.
This is joint work with R. Killip and J. Murphy.

Sijue Wu (University of Michigan, Ann Arbor): **On two dimensional water waves with angled crests**

Abstract: I will discuss recent progress on the local existence and uniqueness of the two dimensional water wave equation in a class that include water waves with non C^1 interfaces.

Christian Zillinger (University of Southern California): **On mixing for circular flows**

Abstract: In recent years, following the seminal works of Villani and Mouhot on Landau damping, phase-mixing as a damping mechanism and inviscid damping in fluid dynamics have attracted much interest. Here, mixing in physical space and weak convergence interact with the Biot-Savart law and results in damping of the velocity field. In this talk, I will discuss linear stability and damping with optimal decay rates around 2D Taylor-Couette flow between two concentric cylinders and similar circular flows. A particular focus will be on asymptotic singularity formation at the boundary and stability in weighted spaces.