Normal approximation for stabilizing functionals

Matthias Schulte

University of Bern

May 18, 2017

joint work with Raphael Lachieze-Rey and Joe Yukich

Stabilization

- Let $\mathcal{X}$ be a Poisson point process $\mathcal{P}_s$ or a binomial point process $\mathcal{X}_n$.

Stabilization

- Let $\mathcal{X}$ be a Poisson point process $\mathcal{P}_s$ or a binomial point process $\mathcal{X}_n$.
- Many functionals of interest in stochastic geometry are of the form

$$h(\mathcal{X}) = \sum_{x \in \mathcal{X}} \xi(x, \mathcal{X}).$$

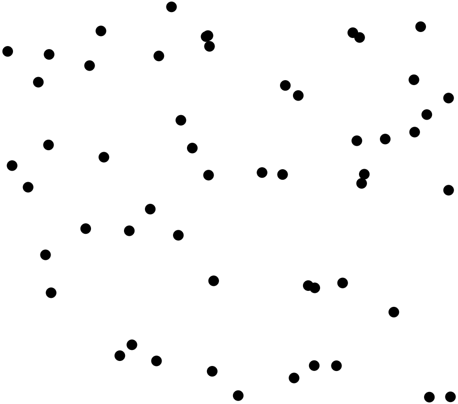
Stabilization

- Let $\mathcal{X}$ be a Poisson point process $\mathcal{P}_s$ or a binomial point process $\mathcal{X}_n$.
- Many functionals of interest in stochastic geometry are of the form

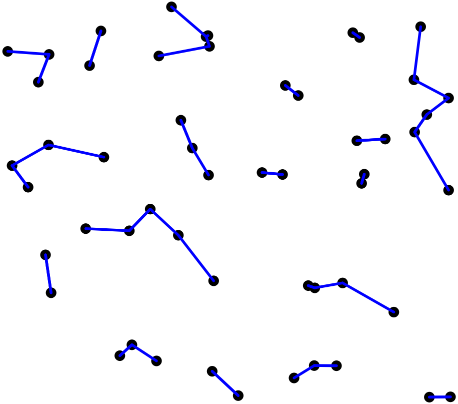
$$h(\mathcal{X}) = \sum_{x \in \mathcal{X}} \xi(x, \mathcal{X}).$$

- We have stabilization if $\xi(x, \mathcal{X})$ is locally defined in the sense that $\xi(x, \mathcal{X})$ only depends on x and the points of $\mathcal{X}$ in a random neighbourhood around x.

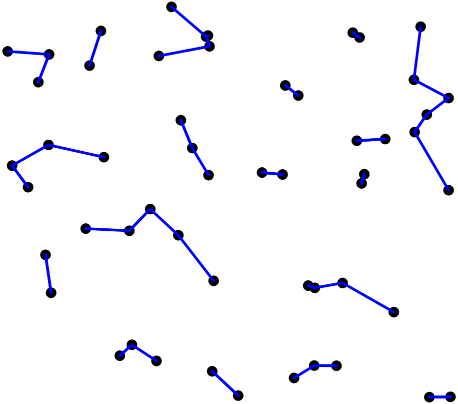
Nearest neighbour graph



Nearest neighbour graph

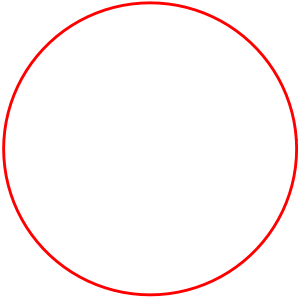


Nearest neighbour graph

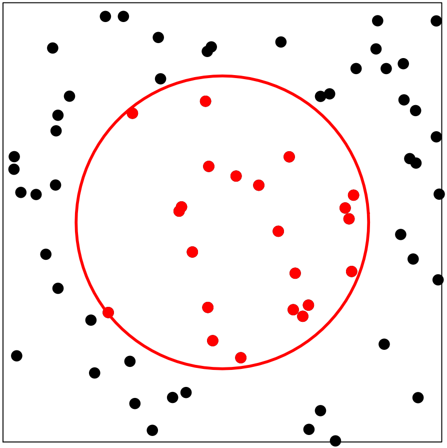


$$L(\mathcal{X}) = \sum_{x \in \mathcal{X}} \xi(x, \mathcal{X}), \quad \xi(x, \mathcal{X}) = \begin{cases} \|x - x_{NN}\|/2, & x = (x_{NN})_{NN} \\ \|x - x_{NN}\|, & x \neq (x_{NN})_{NN} \end{cases}$$

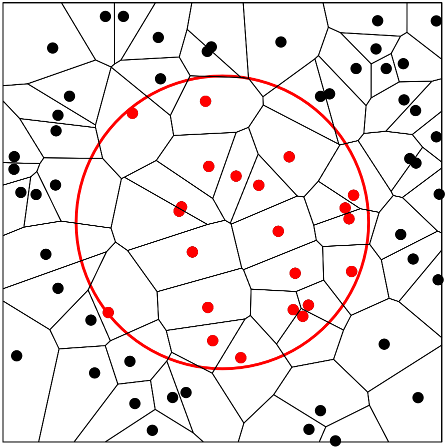
Voronoi approximation



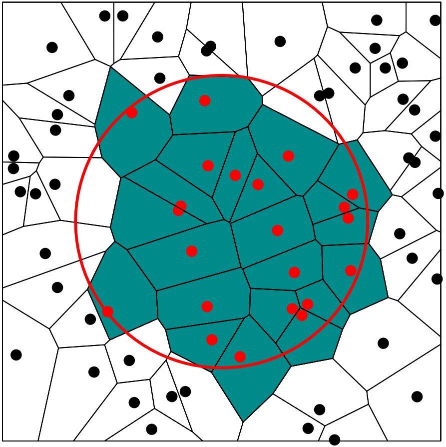
Voronoi approximation



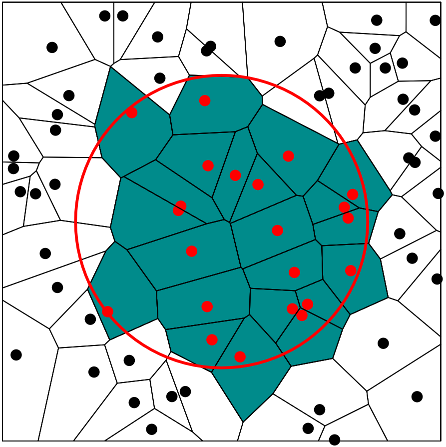
Voronoi approximation



Voronoi approximation



Voronoi approximation



$$A_W(\mathcal{X}) - V_d(W) = \sum_{x \in \mathcal{X}} \xi(x, \mathcal{X}), \quad \xi(x, \mathcal{X}) = \begin{cases} V_d(C_x(\mathcal{X}) \cap W^C), & x \in W \\ -V_d(C_x(\mathcal{X}) \cap W), & x \in W^C \end{cases}$$

CLTs for stabilizing functionals

- Many results on CLTs for stabilizing functionals by Barbour, Baryshnikov, Penrose, Schreiber, Xia, Yukich, ...

CLTs for stabilizing functionals

- Many results on CLTs for stabilizing functionals by Barbour, Baryshnikov, Penrose, Schreiber, Xia, Yukich, . . .
- Usually there are no or presumably suboptimal rates of convergence.

CLTs for stabilizing functionals

- Many results on CLTs for stabilizing functionals by Barbour, Baryshnikov, Penrose, Schreiber, Xia, Yukich, ...
- Usually there are no or presumably suboptimal rates of convergence.
- Sharp bounds derived by the Malliavin-Stein method in Last/Peccati/S. (2016) and Lachieze-Rey/Peccati (2015+)

CLTs for stabilizing functionals

- Many results on CLTs for stabilizing functionals by Barbour, Baryshnikov, Penrose, Schreiber, Xia, Yukich, ...
- Usually there are no or presumably suboptimal rates of convergence.
- Sharp bounds derived by the Malliavin-Stein method in Last/Peccati/S. (2016) and Lachieze-Rey/Peccati (2015+)
- Apply these recent finding to general stabilizing functionals

Framework

- $(\mathbb{X}, \mathcal{F})$ measurable space generated by a semi-metric d and equipped with a σ-finite non-atomic measure $\mathbb{Q}$
- Assume that there are constants $\gamma, \kappa > 0$ such that

$$(M) \quad \limsup_{\varepsilon \to 0} \frac{\mathbb{Q}(B(x, r+\varepsilon)) - \mathbb{Q}(B(x, r))}{\varepsilon} \leq \kappa \gamma r^{\gamma - 1}, \quad r \geq 0, x \in \mathbb{X}.$$

Framework

- $(\mathbb{X}, \mathcal{F})$ measurable space generated by a semi-metric d and equipped with a σ-finite non-atomic measure $\mathbb{Q}$
- Assume that there are constants $\gamma, \kappa > 0$ such that

$$(M) \quad \limsup_{\varepsilon \to 0} \frac{\mathbb{Q}(B(x, r+\varepsilon)) - \mathbb{Q}(B(x,r))}{\varepsilon} \leq \kappa \gamma r^{\gamma-1}, \quad r \geq 0, x \in \mathbb{X}.$$

Examples:

- $\mathbb{X} = W \subseteq \mathbb{R}^d$, d Euclidean norm, $\mathbb{Q}$ restriction of the Lebesgue measure to W
- $\mathbb{X}$ m-dimensional manifold in $\mathbb{R}^d$ (with some additional assumptions), d geodesic distance and $\mathbb{Q}$ Hausdorff measure, for example the sphere

Framework

- $\mathcal{P}_s$ Poisson point process with intensity measure $s\mathbb{Q}$, $s \geq 1$
- $\mathcal{X}_n$ binomial point process of n independent points distributed according to $\mathbb{Q}$ if $\mathbb{Q}(\mathbb{X}) = 1$

Framework

- $\mathcal{P}_s$ Poisson point process with intensity measure $s\mathbb{Q}$, $s \geq 1$
- $\mathcal{X}_n$ binomial point process of n independent points distributed according to $\mathbb{Q}$ if $\mathbb{Q}(\mathbb{X}) = 1$
- For a family of measurable scores $\xi_s : \mathbb{X} \times \mathbf{N} \to \mathbb{R}$, $s \geq 1$, let

$$h(\mathcal{P}_s) = \sum_{x \in \mathcal{P}_s} \xi_s(x, \mathcal{P}_s) \quad \text{and} \quad h(\mathcal{X}_n) = \sum_{x \in \mathcal{X}_n} \xi_n(x, \mathcal{X}_n).$$

Framework

- $\mathcal{P}_s$ Poisson point process with intensity measure $s\mathbb{Q}$, $s \geq 1$
- $\mathcal{X}_n$ binomial point process of n independent points distributed according to $\mathbb{Q}$ if $\mathbb{Q}(\mathbb{X}) = 1$
- For a family of measurable scores $\xi_s : \mathbb{X} \times \mathbf{N} \to \mathbb{R}$, $s \geq 1$, let

$$h(\mathcal{P}_s) = \sum_{x \in \mathcal{P}_s} \xi_s(x, \mathcal{P}_s) \quad \text{and} \quad h(\mathcal{X}_n) = \sum_{x \in \mathcal{X}_n} \xi_n(x, \mathcal{X}_n).$$

- We call $R_s : \mathbb{X} \times \mathbf{N} \to \mathbb{R}$ a radius of stabilization if

$$\xi_s(x, \mathcal{M} \cup \{x\} \cup \mathcal{A}) = \xi_s(x, (\mathcal{M} \cup \{x\} \cup \mathcal{A}) \cap B(x, R_s(x, \mathcal{M} \cup \{x\})))$$

for all $x \in \mathbb{X}$, $\mathcal{M} \in \mathbf{N}$ and $\mathcal{A} \subset \mathbb{X}$ with $|\mathcal{A}| \leq 7$.

Assumptions for the Poisson case

- (A1) There are radii $(R_s)_{s\geq 1}$ of stabilization and $C_{stab}, c_{stab}, \alpha_{stab} > 0$ such that

$$\mathbb{P}(R_s(x, \mathcal{P}_s \cup \{x\}) \geq r) \leq C_{stab} \exp(-c_{stab}(s^{1/\gamma} r)^{\alpha_{stab}})$$

for $x \in \mathbb{X}, r \geq 0, s \geq 1$.

Assumptions for the Poisson case

- (A1) There are radii $(R_s)_{s\geq 1}$ of stabilization and $C_{stab}, c_{stab}, \alpha_{stab} > 0$ such that

$$\mathbb{P}(R_s(x, \mathcal{P}_s \cup \{x\}) \geq r) \leq C_{stab} \exp(-c_{stab}(s^{1/\gamma} r)^{\alpha_{stab}})$$

  for $x \in \mathbb{X}, r \geq 0, s \geq 1$.

- (A2) For some $p \in (0,1]$ and $C_p > 0$,

$$\sup_{s\in[1,\infty), x\in\mathbb{X}, \mathcal{A}\subset\mathbb{X}, |\mathcal{A}|\leq 7} \mathbb{E}|\xi_s(x, \mathcal{P}_s \cup \{x\} \cup \mathcal{A})|^{4+p} \leq C_p.$$

Assumptions for the Poisson case

- (A1) There are radii $(R_s)_{s\geq 1}$ of stabilization and $C_{stab}, c_{stab}, \alpha_{stab} > 0$ such that

$$\mathbb{P}(R_s(x, \mathcal{P}_s \cup \{x\}) \geq r) \leq C_{stab} \exp(-c_{stab}(s^{1/\gamma} r)^{\alpha_{stab}})$$

  for $x \in \mathbb{X}, r \geq 0, s \geq 1$.

- (A2) For some $p \in (0,1]$ and $C_p > 0$,

$$\sup_{s\in[1,\infty), x\in\mathbb{X}, \mathcal{A}\subset\mathbb{X}, |\mathcal{A}|\leq 7} \mathbb{E}|\xi_s(x, \mathcal{P}_s \cup \{x\} \cup \mathcal{A})|^{4+p} \leq C_p.$$

- (A3) There are a set $K \in \mathcal{F}$ and $C_K, c_K, \alpha_K > 0$ such that

$$\mathbb{P}(\xi_s(x, \mathcal{P}_s \cup \{x\} \cup \mathcal{A}) \neq 0) \leq C_K \exp(-c_K s^{\alpha_K/\gamma} \mathrm{d}(x, K)^{\alpha_K}).$$

  for all $x \in \mathbb{X}$, $s \geq 1$ and $\mathcal{A} \subset \mathbb{X}$ with $|\mathcal{A}| \leq 7$.

Main result for the Poisson case

For two random variables Y, Z let

$$d_K(Y, Z) := \sup_{u \in \mathbb{R}} |\mathbb{P}(Y \leq u) - \mathbb{P}(Z \leq u)|.$$

Main result for the Poisson case

For two random variables Y, Z let

$$d_K(Y,Z) := \sup_{u\in\mathbb{R}} |\mathbb{P}(Y \le u) - \mathbb{P}(Z \le u)|.$$

Theorem: Lachieze-Rey/S./Yukich (2017)

Assume (A1), (A2), (A3) with $\alpha = \min\{\alpha_{stab}, \alpha_K\}$, $c = \min\{c_{stab}, c_K\}$ and

$$s \int_{\mathbb{X}} \exp\left(- \frac{c\, p\, s^{\alpha/\gamma} \mathrm{d}(x,K)^{\alpha}}{36 \cdot 4^{\alpha+1}} \right) \mathbb{Q}(\mathrm{d}x) \le C_{\mathrm{Var}} \operatorname{Var} h(\mathcal{P}_s), \quad s \ge 1,$$

with $C_{\mathrm{Var}} > 0$. Let N be a standard Gaussian random variable. Then there is a $C > 0$ such that

$$d_K\left(\frac{h(\mathcal{P}_s) - \mathbb{E} h(\mathcal{P}_s)}{\sqrt{\operatorname{Var} h(\mathcal{P}_s)}}, N \right) \le \frac{C}{\sqrt{\operatorname{Var} h(\mathcal{P}_s)}}, \quad s \ge 1.$$

Main result for the Poisson case

For two random variables Y, Z let

$$d_K(Y,Z) := \sup_{u\in\mathbb{R}} |\mathbb{P}(Y \le u) - \mathbb{P}(Z \le u)|.$$

Theorem: Lachieze-Rey/S./Yukich (2017)

Assume (A1), (A2), (A3) with $\alpha = \min\{\alpha_{stab}, \alpha_K\}$, $c = \min\{c_{stab}, c_K\}$ and

$$s \int_{\mathbb{X}} \exp\left(- \frac{c\, p\, s^{\alpha/\gamma} \mathrm{d}(x,K)^\alpha}{36 \cdot 4^{\alpha+1}} \right) \mathbb{Q}(\mathrm{d}x) \le C_{\mathrm{Var}} \operatorname{Var} h(\mathcal{P}_s), \quad s \ge 1,$$

with $C_{\mathrm{Var}} > 0$. Let N be a standard Gaussian random variable. Then there is a $C > 0$ such that

$$d_K\left(\frac{h(\mathcal{P}_s) - \mathbb{E} h(\mathcal{P}_s)}{\sqrt{\operatorname{Var} h(\mathcal{P}_s)}}, N \right) \le \frac{C}{\sqrt{\operatorname{Var} h(\mathcal{P}_s)}}, \quad s \ge 1.$$

For $K = \mathbb{X}$ the variance conditions becomes $\operatorname{Var} h_s(\mathcal{P}_s) \ge s\mathbb{Q}(\mathbb{X})/C_{\mathrm{Var}}$.

Assumptions for the binomial case

- (A1′) There are radii of stabilization $(R_n)_{n\in\mathbb{N}}$ and $C_{stab}, c_{stab}, \alpha_{stab} > 0$ such that

$$\mathbb{P}(R_n(x, \mathcal{X}_{n-8} \cup \{x\}) \geq r) \leq C_{stab} \exp(-c_{stab}(n^{1/\gamma} r)^{\alpha_{stab}})$$

for $x \in \mathbb{X}, r \geq 0, n \geq 9$.

Assumptions for the binomial case

- (A1′) There are radii of stabilization $(R_n)_{n\in\mathbb{N}}$ and $C_{stab}, c_{stab}, \alpha_{stab} > 0$ such that

$$\mathbb{P}(R_n(x, \mathcal{X}_{n-8} \cup \{x\}) \geq r) \leq C_{stab} \exp(-c_{stab}(n^{1/\gamma} r)^{\alpha_{stab}})$$

  for $x \in \mathbb{X}, r \geq 0, n \geq 9$.

- (A2′) For some $p \in (0, 1]$ and $C_p > 0$,

$$\sup_{n\geq 9, x\in\mathbb{X}, \mathcal{A}\subset\mathbb{X}, |\mathcal{A}|\leq 7} \mathbb{E}|\xi_n(x, \mathcal{X}_{n-8} \cup \{x\} \cup \mathcal{A})|^{4+p} \leq C_p.$$

Assumptions for the binomial case

- (A1′) There are radii of stabilization $(R_n)_{n\in\mathbb{N}}$ and $C_{stab}, c_{stab}, \alpha_{stab} > 0$ such that

$$\mathbb{P}(R_n(x, \mathcal{X}_{n-8} \cup \{x\}) \geq r) \leq C_{stab} \exp(-c_{stab}(n^{1/\gamma} r)^{\alpha_{stab}})$$

  for $x \in \mathbb{X}, r \geq 0, n \geq 9$.

- (A2′) For some $p \in (0, 1]$ and $C_p > 0$,

$$\sup_{n\geq 9, x\in\mathbb{X}, \mathcal{A}\subset\mathbb{X}, |\mathcal{A}|\leq 7} \mathbb{E}|\xi_n(x, \mathcal{X}_{n-8} \cup \{x\} \cup \mathcal{A})|^{4+p} \leq C_p.$$

- (A3′) There are a set $K \in \mathcal{F}$ and $C_K, c_K, \alpha_K > 0$ such that

$$\mathbb{P}(\xi_n(x, \mathcal{X}_{n-8} \cup \{x\} \cup \mathcal{A}) \neq 0) \leq C_K \exp(-c_K(n^{1/\gamma} \mathrm{d}(x, K))^{\alpha_K}).$$

  for all $x \in \mathbb{X}$, $n \geq 9$ and $\mathcal{A} \subset \mathbb{X}$ with $|\mathcal{A}| \leq 7$.

Main result for the binomial case

Theorem: Lachieze-Rey/S./Yukich (2017)

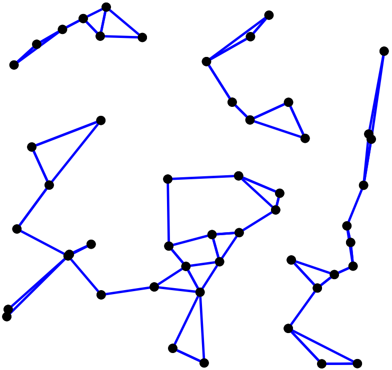
Assume (A1′), (A2′), (A3′) with $\alpha = \min\{\alpha_{stab}, \alpha_K\}$, $c = \min\{c_{stab}, c_K\}$ and

$$n \int_{\mathbb{X}} \exp\left(- \frac{c\, p\, n^{\alpha/\gamma} \mathrm{d}(x, K)^{\alpha}}{36 \cdot 4^{\alpha+1}} \right) \mathbb{Q}(\mathrm{d}x) \leq C_{\mathrm{Var}} \operatorname{Var} h(\mathcal{X}_n), \quad n \geq 9,$$

with $C_{\mathrm{Var}} > 0$. Let N be a standard Gaussian random variable. Then there is a $C > 0$ such that

$$d_K\left(\frac{h(\mathcal{X}_n) - \mathbb{E} h(\mathcal{X}_n)}{\sqrt{\operatorname{Var} h(\mathcal{X}_n)}}, N \right) \leq \frac{C}{\sqrt{\operatorname{Var} h(\mathcal{X}_n)}}, \quad n \geq 9.$$

k-nearest neighbour graph



- $W \subset \mathbb{R}^d$ compact convex with $V_d(W) > 0$ and $\mathbb{Q}(\cdot) = V_d(\cdot)/V_d(W)$
- For $\alpha \geq 0$ let

$$L^{(\alpha)}(\mathcal{M}) = \frac{1}{2} \sum_{(x,y) \in \mathcal{M}^2_{\neq}} \mathbf{1}\{(x,y) \in NNG_k(\mathcal{M})\} \|x-y\|^{\alpha}, \quad \mathcal{M} \in \mathbf{N}.$$

k-nearest neighbour graph

Theorem: Last/Peccati/S. 2016, Lachieze-Rey/S./Yukich 2017

There is a constant $C_\alpha > 0$ for any $\alpha \geq 0$ such that

$$d_K\left(\frac{L^{(\alpha)}(\mathcal{P}_s) - \mathbb{E}L^{(\alpha)}(\mathcal{P}_s)}{\sqrt{\operatorname{Var} L^{(\alpha)}(\mathcal{P}_s)}}, N\right) \leq \frac{C_\alpha}{\sqrt{s}}, \quad s \geq 1,$$

and

$$d_K\left(\frac{L^{(\alpha)}(\mathcal{X}_n) - \mathbb{E}L^{(\alpha)}(\mathcal{X}_n)}{\sqrt{\operatorname{Var} L^{(\alpha)}(\mathcal{X}_n)}}, N\right) \leq \frac{C_\alpha}{\sqrt{n}}, \quad n \geq 9.$$

k-nearest neighbour graph

Theorem: Last/Peccati/S. 2016, Lachieze-Rey/S./Yukich 2017

There is a constant $C_\alpha > 0$ for any $\alpha \geq 0$ such that

$$d_K\left(\frac{L^{(\alpha)}(\mathcal{P}_s) - \mathbb{E}L^{(\alpha)}(\mathcal{P}_s)}{\sqrt{\operatorname{Var} L^{(\alpha)}(\mathcal{P}_s)}}, N\right) \leq \frac{C_\alpha}{\sqrt{s}}, \quad s \geq 1,$$

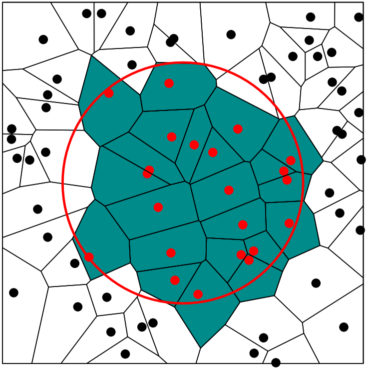
and

$$d_K\left(\frac{L^{(\alpha)}(\mathcal{X}_n) - \mathbb{E}L^{(\alpha)}(\mathcal{X}_n)}{\sqrt{\operatorname{Var} L^{(\alpha)}(\mathcal{X}_n)}}, N\right) \leq \frac{C_\alpha}{\sqrt{n}}, \quad n \geq 9.$$

Previous results with (weaker) rates of convergence:

- Poisson case: Avram/Bertsimas 1993, Penrose/Yukich 2005
- binomial case: Bickel/Breimann 1983, Chatterjee 2008, Lachieze-Rey/Peccati 2015+

Voronoi approximation



- $W \subset (0,1)^d$ compact convex with $V_d(W) > 0$ and $\mathbb{Q}(\cdot) = V_d(\cdot \cap [0,1]^d)$
- For $\mathcal{M} \in \mathbf{N}$,

$$A_W(\mathcal{M}) = V_d\Big(\bigcup_{x \in \mathcal{M} \cap A} C(x,\mathcal{M}) \cap [0,1]^d \Big).$$

Voronoi approximation

Theorem: Lachieze-Rey/S./Yukich 2017+

There is a constant $C_W > 0$ such that

$$d_K\left(\frac{A_W(\mathcal{P}_s) - \mathbb{E}A_W(\mathcal{P}_s)}{\sqrt{\operatorname{Var} A_W(\mathcal{P}_s)}}, N\right) \leq \frac{C_W}{s^{(d-1)/(2d)}}, \quad s \geq 1,$$

and

$$d_K\left(\frac{A_W(\mathcal{X}_n) - \mathbb{E}A_W(\mathcal{X}_n)}{\sqrt{\operatorname{Var} A_W(\mathcal{X}_n)}}, N\right) \leq \frac{C_W}{n^{(d-1)/(2d)}}, \quad n \geq 9.$$

Voronoi approximation

Theorem: Lachieze-Rey/S./Yukich 2017+

There is a constant $C_W > 0$ such that

$$d_K\left(\frac{A_W(\mathcal{P}_s) - \mathbb{E}A_W(\mathcal{P}_s)}{\sqrt{\operatorname{Var} A_W(\mathcal{P}_s)}}, N\right) \leq \frac{C_W}{s^{(d-1)/(2d)}}, \quad s \geq 1,$$

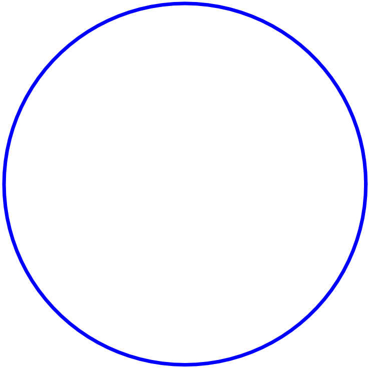
and

$$d_K\left(\frac{A_W(\mathcal{X}_n) - \mathbb{E}A_W(\mathcal{X}_n)}{\sqrt{\operatorname{Var} A_W(\mathcal{X}_n)}}, N\right) \leq \frac{C_W}{n^{(d-1)/(2d)}}, \quad n \geq 9.$$

Previous results with (weaker) rates of convergence:

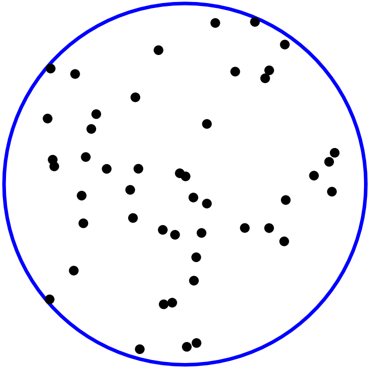
- Poisson case: Schulte 2012, Yukich 2015
- binomial case: Lachieze-Rey/Peccati 2015+, Lachieze-Rey/Vega 2015+

Convex hull of random points



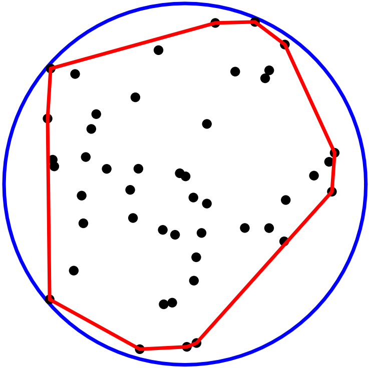
- $A \subset \mathbb{R}^d$ compact convex with C^2-boundary and positive Gaussian curvature and $\mathbb{Q}(\cdot) = V_d(\cdot \cap A)/V_d(A)$

Convex hull of random points



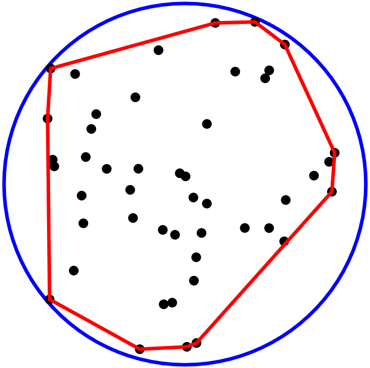
- $A \subset \mathbb{R}^d$ compact convex with C^2-boundary and positive Gaussian curvature and $\mathbb{Q}(\cdot) = V_d(\cdot \cap A)/V_d(A)$

Convex hull of random points



- $A \subset \mathbb{R}^d$ compact convex with C^2-boundary and positive Gaussian curvature and $\mathbb{Q}(\cdot) = V_d(\cdot \cap A)/V_d(A)$
- $\mathrm{Conv}(\mathcal{M})$ convex hull of $\mathcal{M} \subset A$

Convex hull of random points



- $A \subset \mathbb{R}^d$ compact convex with C^2-boundary and positive Gaussian curvature and $\mathbb{Q}(\cdot) = V_d(\cdot \cap A)/V_d(A)$
- $\mathrm{Conv}(\mathcal{M})$ convex hull of $\mathcal{M} \subset A$
- $f_k(\mathrm{Conv}(\mathcal{M}))$ number of k-faces of $\mathrm{Conv}(\mathcal{M})$
- $V_i(\mathrm{Conv}(\mathcal{M}))$ i-th intrinsic volume of $\mathrm{Conv}(\mathcal{M})$

Convex hull of random points

Theorem: Lachieze-Rey/S./Yukich 2017

For any $h \in \{f_0, \dots, f_{d-1}, V_1, \dots, V_d\}$ there is a constant C_h also depending on A such that

$$d_K\left(\frac{h(\mathrm{Conv}(\mathcal{P}_s)) - \mathbb{E}h(\mathrm{Conv}(\mathcal{P}_s))}{\sqrt{\mathrm{Var}\, h(\mathrm{Conv}(\mathcal{P}_s))}}, N\right) \leq C_h s^{-\frac{d-1}{2(d+1)}}, \quad s \geq 1,$$

and

$$d_K\left(\frac{h(\mathrm{Conv}(\mathcal{X}_n)) - \mathbb{E}h(\mathrm{Conv}(\mathcal{X}_n))}{\sqrt{\mathrm{Var}\, h(\mathrm{Conv}(\mathcal{X}_n))}}, N\right) \leq C_h n^{-\frac{d-1}{2(d+1)}}, \quad n \geq \max\{9, d+2\}.$$

Convex hull of random points

Theorem: Lachieze-Rey/S./Yukich 2017

For any $h \in \{f_0, \dots, f_{d-1}, V_1, \dots, V_d\}$ there is a constant C_h also depending on A such that

$$d_K\left(\frac{h(\mathrm{Conv}(\mathcal{P}_s)) - \mathbb{E}h(\mathrm{Conv}(\mathcal{P}_s))}{\sqrt{\mathrm{Var}\, h(\mathrm{Conv}(\mathcal{P}_s))}}, N\right) \leq C_h s^{-\frac{d-1}{2(d+1)}}, \quad s \geq 1,$$

and

$$d_K\left(\frac{h(\mathrm{Conv}(\mathcal{X}_n)) - \mathbb{E}h(\mathrm{Conv}(\mathcal{X}_n))}{\sqrt{\mathrm{Var}\, h(\mathrm{Conv}(\mathcal{X}_n))}}, N\right) \leq C_h n^{-\frac{d-1}{2(d+1)}}, \quad n \geq \max\{9, d+2\}.$$

Previous results by Renyi/Sulanke 1963/1964, Reitzner 2005, Vu 2006, Calka/Schreiber/Yukich 2013, Thäle/Turchi/Wespi 2017

Malliavin-Stein bounds

Theorem: Last/Peccati/S. 2016

Let $f(\mathcal{P}_s)$ be square integrable with $\sigma^2 = \operatorname{Var} f(\mathcal{P}_s)$. Assume

$$\mathbb{E}|D_x f(\mathcal{P}_s \cup \mathcal{A})|^{4+p} \leq c, \quad \mathbb{Q}\text{-a.e. } x \in \mathbb{X}, \mathcal{A} \subset \mathbb{X}, |\mathcal{A}| \leq 1.$$

with $c, p > 0$. Then there is a constant $C > 0$ such that

$$\begin{aligned} d_K\left(\frac{f(\mathcal{P}_s) - \mathbb{E} f(\mathcal{P}_s)}{\sigma}, N\right) \leq \frac{C}{\sigma^2}\Bigg(&\Gamma_s^{1/2} + \frac{2\Gamma_s}{\sigma} + \frac{\Gamma_s^{5/4} + 2\Gamma_s^{3/2}}{\sigma^2} \\ &+ \left[s^3 \int_{\mathbb{X}} \left(\int_{\mathbb{X}} \mathbb{P}(D^2_{x,y} f(\mathcal{P}_s) \neq 0)^{p/(16+4p)}\, \mathbb{Q}(\mathrm{d}y)\right)^2 \mathbb{Q}(\mathrm{d}x)\right]^{1/2} \\ &+ \left[s^2 \int_{\mathbb{X}^2} \mathbb{P}(D^2_{x,y} f(\mathcal{P}_s) \neq 0)^{p/(8+2p)}\, \mathbb{Q}^2(\mathrm{d}(x,y))\right]^{1/2}\Bigg) \end{aligned}$$

with $$\Gamma_s := s \int_{\mathbb{X}} \mathbb{P}(D_x f(\mathcal{P}_s) \neq 0)^{p/(8+2p)}\, \mathbb{Q}(\mathrm{d}x).$$

Malliavin-Stein bounds

Theorem: Lachieze-Rey/S./Yukich 2017, Lachieze-Rey/Peccati 2015+

Let $f(\mathcal{X}_n)$ be square integrable with $\sigma^2 = \operatorname{Var} f(\mathcal{X}_n)$ and assume that

$$\mathbb{E}|D_x f(\mathcal{X}_{n-1-|\mathcal{A}|} \cup \mathcal{A})|^{4+p} \leq c, \quad \mathbb{Q}\text{-a.e. } x \in \mathbb{X}, \mathcal{A} \subset \mathbb{X}, |\mathcal{A}| \leq 2.$$

Define

$$\Gamma_n = n \int_{\mathbb{X}} \mathbb{P}(D_x f(\mathcal{X}_{n-1}) \neq 0)^{\frac{p}{8+2p}} \, \mathbb{Q}(\mathrm{d}x),$$

$$\psi_n(x,y) = \sup_{\mathcal{A} \subset \mathbb{X}: |\mathcal{A}| \leq 1} \mathbb{P}(D^2_{x,y} f(\mathcal{X}_{n-2-|\mathcal{A}|} \cup \mathcal{A}) \neq 0)^{p/(8+2p)}, \quad x, y \in \mathbb{X}.$$

Then there is a constant $C > 0$ such that

$$d_K\left(\frac{f(\mathcal{X}_n) - \mathbb{E} f(\mathcal{X}_n)}{\sigma}, N\right) \leq \frac{C}{\sigma^2}\Bigg[\frac{\Gamma_n}{\sigma} + \frac{\sqrt{\Gamma_n}^3 + \Gamma_n}{\sigma^2} + \sqrt{\Gamma_n}$$
$$+ n\sqrt{\int_{\mathbb{X}^2} \psi_n(x,y) \, \mathbb{Q}^2(\mathrm{d}(x,y))} + n^{3/2}\sqrt{\int_{\mathbb{X}} \left(\int_{\mathbb{X}} \psi_n(x,y) \, \mathbb{Q}(\mathrm{d}y)\right)^2 \mathbb{Q}(\mathrm{d}x)}\Bigg].$$

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence
- Main result applicable to

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence
- Main result applicable to
  - Poisson or binomial input

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence
- Main result applicable to
  - Poisson or binomial input
  - a large class of underlying spaces

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence
- Main result applicable to
  - Poisson or binomial input
  - a large class of underlying spaces
  - different kinds of variance rescaling

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence
- Main result applicable to
  - Poisson or binomial input
  - a large class of underlying spaces
  - different kinds of variance rescaling
- Version for marked Poisson or binomial point processes

Conclusion

- Central limit theorems for stabilizing functionals with presumably optimal rates of convergence
- Main result applicable to
  - Poisson or binomial input
  - a large class of underlying spaces
  - different kinds of variance rescaling
- Version for marked Poisson or binomial point processes
- Many further examples of stabilizing functionals

Thank you!