

Invited speakers

THE POISSON–SADDLEPOINT APPROXIMATION

Adrian Baddeley (Curtin University, Perth, Australia),
Adrian.Baddeley@curtin.edu.au

Abstract: Gibbs spatial point processes are important models in theoretical physics and in spatial statistics. After a brief survey of Gibbs point processes, we will present a method for approximating their most important characteristic, the intensity of the process. The method has some affinity with the classical saddlepoint approximations of probability densities. For pairwise-interaction processes the approximation can be computed directly : it performs very well in many cases, but not in all cases. For higher-order interactions, we invoke limit results from stochastic geometry due to Roger Miles and the late Peter Hall, in order to compute the approximation.

Joint work with Gopalan Nair

A STATISTICAL METHODOLOGY TO DETECT LOW-DIMENSIONALITY

Antonio Cuevas (Departamento de Matemáticas, Universidad Autónoma de Madrid),
antonio.cuevas@uam.es

Abstract: Our object of interest is a, possibly lower-dimensional, compact set $\mathcal{M} \subset \mathbb{R}^d$. The general aim is to identify (via stochastic procedures) some qualitative or quantitative features of \mathcal{M} , of geometric or topological character. The available information is just a random sample of points drawn either on \mathcal{M} (noiseless case) or on a parallel set around \mathcal{M} (noisy case). The term “to identify” means here to achieve a correct answer almost surely (a.s.) when the sample size tends to infinity. All the proposed methods are based on explicit, feasible algorithms. Our proposal includes the following results :

(a) A method to identify (eventually a.s.) whether or not the interior of \mathcal{M} is empty. Under some regularity conditions this amounts to decide whether or not \mathcal{M} has a dimension smaller than that of the ambient space. Both sample models (noisy and noiseless) are considered.

(b) A method to partially “denoise” a sample.

(c) A method to estimate the measure of the boundary of \mathcal{M} , as given by the Minkowski content. Again both sample models are studied in this problem.

(d) Some simulations and graphical illustrations.

Joint work with Catherine Aaron and Alejandro Cholaquidis

ITERATION OF BOOLEAN RANDOM VARIETIES. APPLICATION TO FRACTURE STATISTICS

Dominique Jeulin (MINES ParisTech, PSL Research University, Centre de Morphologie Mathématique, Fontainebleau),
`dominique.jeulin@mines-paristech.fr`

Abstract: Models of random sets and of point processes are introduced to simulate some specific large scale clustering. A particular case is given by points on random lines in \mathbb{R}^2 and \mathbb{R}^3 and on random planes in \mathbb{R}^3 . The corresponding point processes are special cases of Cox processes. The generating distribution function of the probability distribution of the number of points in a convex set K and the Choquet capacity $T(K)$ are given. An instructive range of applications is to model point defects in materials showing some degree of alignment, like point defects on random fibers or on the grain boundaries of polycrystals. Theoretical results on the probability of fracture of convex specimens in the framework of the weakest link assumption are derived, and are used to compare some potential geometrical effects on the sensitivity of materials to fracture.

[1] Jeulin, D. (2016). Iterated Boolean random varieties and application to fracture statistics models. *Applications of Mathematics*, 61(4), 363-386.

INDEPENDENT EDGE MARKING OF POISSON PROCESSES

Günter Last (Karlsruhe Institute of Technology),
`guenter.last@kit.edu`

Abstract: We consider the complete graph supported by the points of a Poisson process on a general phase space with a diffuse intensity measure. Given a distribution on another Borel (mark) space, each edge is marked independently according to this distribution. The resulting random graph is a generic model of stochastic geometry. For instance it can be used to define the Gilbert graph with random radii or the random connection model. We shall discuss second order properties of functionals of this random graph. In particular we derive variance inequalities and Berry–Esseen type bounds for normal approximation.

Joint work with Franz Nestmann and Matthias Schulte

DETECTION THEORY WITHOUT LEARNING, WITH SOME ILLUSTRATIVE EXAMPLES

Jean-Michel Morel (Ecole Normale Supérieure Paris-Saclay),
moreljeanmichel@gmail.com

Abstract: In this presentation based on on-line demonstrations of algorithms and on the examination of several practical examples, I will reflect on the problem of modeling a detection task in images. I will place myself in the (very frequent) case where the detection task can not be formulated in a Bayesian framework or, rather equivalently that can not be solved by simultaneous learning of the model of the object and that of the background. (In the case where there are plenty of examples of the background and of the object to be detected, the neural networks provide a practical answer, but without explanatory power). Nevertheless for the detection without "learning", I will show that we can not avoid building a background model, or possibly learn it. But this will not require many examples.

Joint works with Axel Davy, Tristan Dagobert, Agnes Desolneux, Thibaud Ehret.

CANCELLATIONS IN RANDOM NODAL SETS

Giovanni Peccati (RMATH, Luxembourg University),
giovanni.peccati@gmail.com

Abstract: I will discuss second order results for the length of nodal sets and the number of phase singularities associated with Gaussian random Laplace eigenfunctions, both on compact manifolds (the flat torus) and on subset of the plane. I will mainly focus on 'cancellation phenomena' for nodal variances in the high-frequency limit, with specific emphasis on central and non-central second order results.

Based on joint works with F. Dalmao, D. Marinucci, I. Nourdin, M. Rossi and I. Wigman

HYPERPLANE TESSELLATIONS IN EUCLIDEAN AND SPHERICAL SPACES

Rolf Schneider (University of Freiburg, Germany),
rolf.schneider@math.uni-freiburg.de

Abstract: Random mosaics generated by stationary Poisson hyperplane processes in Euclidean space are a much studied object of Stochastic Geometry, and their typical cells or zero cells belong to the most prominent models of random polytopes. After a brief review, we turn to analogues in spherical space or, roughly equivalently, in a conic setting. A given number of i.i.d. random hyperplanes through the origin in \mathbb{R}^d generate a tessellation of \mathbb{R}^d into polyhedral cones. The typical cone of this tessellation, called a 'random Schläfli cone', is the object of our study. We provide first moments and mixed second moments of some geometric functionals, and compute probabilities of non-trivial intersection of a random Schläfli cone with a fixed polyhedral cone, or of two independent random Schläfli cones.

Parts are joint work with Matthias Reitzner, others with Daniel Hug

☛ RANDOM WALKS ON DYNAMICAL PERCOLATION

Perla Sousi (University of Cambridge),
`p.sousio@statslab.cam.uk`

Abstract: We study the behaviour of random walk on dynamical percolation. In this model, the edges of a graph are either open or closed and refresh their status at rate μ , while at the same time a random walker moves on G at rate 1, but only along edges which are open. On the d -dimensional torus with side length n , when the bond parameter is subcritical, the mixing times for both the full system and the random walker were determined by Peres, Stauffer and Steif. I will talk about the supercritical case, which was left open, but can be analysed using evolving sets.

Joint work with Y. Peres and J. Steif

☛ CURVATURE MEASURES OF RANDOM SETS

Martina Zähle (Friedrich Schiller University Jena, Germany),
`martina.zaehle@uni-jena.de`

Abstract: A survey on some developments in curvature theory for random sets will be given. We first consider previous models with classical singularities like polyconvex sets or unions of sets with positive reach. The main part of the talk concerns extensions to certain classes of random fractals which have been investigated in the last years. In these cases limits of rescaled versions for suitable approximations are used.

☛ ESTIMATING PARTICLE SHAPE AND ORIENTATION USING VOLUME TENSORS

Johanna F. Ziegel (University of Berne),
`johanna.ziegel@stat.unibe.ch`

Abstract: We present procedures for estimating shape and orientation of arbitrary three-dimensional particles such as cells in biological applications. We focus on the case where particles cannot be observed directly, but only via sections as it occurs in microscopy. Volume tensors are used for describing particle shape and orientation, and we derive stereological estimators of the tensors. These estimators are combined to provide consistent estimators of the moments of the so-called particle cover density. The covariance structure associated with the particle cover density depends on the orientation and shape of the particles. For instance, if the distribution of the typical particle is invariant under rotations, then the covariance matrix is proportional to the identity matrix. We develop a non-parametric test for such isotropy. The developed methods are used to study the cell organization in the human brain cortex.

This is joint work with Eva B. Vedel Jensen, Markus Kiderlen, Astrid Kousholt, Jens R. Nyengaard and Ali H. Rafati.

Contributed talks

✦ PLANAR SEGMENT PROCESSES WITH REFERENCE MARK DISTRIBUTIONS, MODELING AND ESTIMATION

Viktor Beneš (Charles University, Faculty of Mathematics and Physics, Czech Republic),
`benesv@karlin.mff.cuni.cz`

Abstract: The talk deals with planar segment processes given by a density with respect to the Poisson process. Parametric models involve reference distributions of directions and/or lengths of segments. These distributions generally do not coincide with the corresponding observed distributions. Statistical methods are presented which first estimate scalar parameters by known approaches and then the reference distribution is estimated non-parametrically. Besides a general theory we offer two models, first a Gibbs type segment process with reference directional distribution and secondly an inhomogeneous process with length reference distribution. The estimation is demonstrated in simulation studies where the variability of estimators is presented graphically.

Joint work with Jakub Večeřa and Milan Pultar

✦ MEAN GEOMETRY OF EXCURSION SETS FOR 2D RANDOM FIELDS

Hermine Biermé (LMA Université de Poitiers),
`hermine.bierme@math.univ-poitiers.fr`

Abstract: We consider mean geometry of excursion sets for 2D stationary random fields. We adopt a weak functional framework, allowing to get explicit formulas for almost all level of excursion. We introduce the level perimeter and total curvature integrals associated with a real valued function defined on the plane. Using a co-area formula, this permits to compute length and total (signed) curvature of the boundary of excursion sets above almost all level. Thanks to Gauss-Bonnet Theorem, the total curvature is directly related to the Euler Characteristic of the excursion set. This setting allows explicit computations for the mean geometry of excursion sets of some stationary 2D random fields, beyond the Gaussian smooth framework. In particular, considering shot noise random fields, this generalizes results of the literature about the Boolean model or about random configurations.

Joint work with Agnès Desolneux (CNRS, CMLA, ENS Cachan)

THE ACCUMULATIVE PERSISTENCE FUNCTION, A USEFUL FUNCTIONAL SUMMARY STATISTIC FOR TOPOLOGICAL DATA ANALYSIS

Christophe Biscio (Aalborg University, Denmark),
christophe@math.aau.dk

Abstract: A persistent diagram is a multiset of points in the plane describing the persistence of topological features of a compact set when a scale parameter varies. Since statistical methods are difficult to apply directly on persistence diagrams, various alternative functional summary statistics have been suggested, but either they do not contain the full information of the persistence diagram or they are two-dimensional functions. This talk suggests a new functional summary statistic that is one-dimensional and hence easier to handle, and which under mild conditions contains the full information of the persistence diagram. Its usefulness is illustrated in various statistical settings concerned with point clouds and brain artery trees.

Joint work with Jesper Møller.

MAXIMUM DEGREE IN A POISSON-DELAUNAY TRIANGULATION

Gilles Bonnet (Ruhr Universität Bochum, Germany),
gilles.bonnet@rub.de

Abstract: For a given locally finite vertex set $\chi \subset \mathbb{R}^2$, the Delaunay graph $Del(\chi)$ is one of the most investigated graph. It is the key ingredient of the first algorithm for computing the minimum spanning tree. It is extensively used in medical image segmentation, in finite element method to build meshes and is a powerful tool for reconstructing a 3D set from a discrete point set.

In this paper, we consider the case where $\chi = \mathbb{X}$ is a homogeneous Poisson point process of intensity 1. In order to describe the regularity of the Delaunay graph $Del(\mathbb{X})$ we investigate the maximal degree of the graph when observed in a window. More precisely, let $\mathbb{W}_n = n^{1/2}[0, 1]^2$ with $n > 0$ and let

$$\Delta_n := \max_{x \in \mathbb{X} \cap \mathbb{W}_n} d_{\mathbb{X}}(x),$$

where $d_{\mathbb{X}}(x)$ denotes the degree of any point x , i.e. the number of edges passing through x . Bern *et al.* showed that $\mathbb{E}\Delta_n = \Theta\left(\frac{\log n}{\log \log n}\right)$, see [The expected extremes in a Delaunay triangulation. *ICALP*, pages 674-685, 1991]. We strengthen this result by showing a concentration of Δ_n on two consecutive integers, for which we provide the exact order.

Theorem. Let Δ_n be the maximal degree in a Poisson-Delaunay graph. Then there exists a deterministic sequence $(I_n)_{n \geq 1}$ such that

1. $\mathbb{P}(\Delta_n \in \{I_n, I_n + 1\}) \rightarrow 1$, as $n \rightarrow \infty$;
2. $I_n \sim \frac{1}{2} \cdot \frac{\log n}{\log \log n}$, as $n \rightarrow \infty$.

Joint work with Nicolas Chenavier

THE MOMENTS OF THE NUMBER OF VERTICES OF A RANDOM POLYTOPE

Christian Buchta (Salzburg University, Austria),
christian.buchta@sbg.ac.at

Abstract: Consider n random points distributed independently and uniformly in a convex body K of volume $\text{vol } K$. Denote by N_n the number of vertices and by V_n the volume of their convex hull. We prove that the k -th moment of N_n , i.e. $\mathbb{E}N_n^k$, can be expressed by the moments $\mathbb{E}V_{n-j}^j$, $j = 0, \dots, k$. More precisely, there are polynomials $p_j^{(k)}(n)$ of degree k in n such that

$$\mathbb{E}N_n^k = \sum_{j=0}^k p_j^{(k)}(n) \frac{\mathbb{E}V_{n-j}^j}{(\text{vol } K)^j}. \quad (1)$$

Denoting the Stirling numbers of the second kind by $\left\{ \begin{smallmatrix} k \\ i \end{smallmatrix} \right\}$, these polynomials are given by $p_j^{(k)}(n) = (-1)^j \sum_{i=j}^k \left\{ \begin{smallmatrix} k \\ i \end{smallmatrix} \right\} n_{(i)}$, with $n_{(i)} = \frac{n!}{(n-i)!}$. Alternatively, there are polynomials $q_j^{(k)}(n)$ of degree $k-j$ in n , such that

$$\mathbb{E}N_n^k = \sum_{j=0}^k q_j^{(k)}(n) n_{(j)} \frac{\mathbb{E}V_{n-j}^j}{(\text{vol } K)^j} \quad \text{with} \quad q_j^{(k)}(n) = \sum_{i=j}^k (-1)^i \left\{ \begin{smallmatrix} k \\ i \end{smallmatrix} \right\} \left\{ \begin{smallmatrix} i \\ j \end{smallmatrix} \right\} n^{k-i}. \quad (2)$$

As a by-product, the equivalence of formulae (1) and (2) implies an identity for the Stirling numbers of the second kind which is not obvious. If the equivalence arising from the geometric background is not made use of, the verification of this identity is non-trivial. In the special case $k = 1$, both formulae reduce to Efron's classical result (1965) that $\mathbb{E}N_n = n(1 - \mathbb{E}V_{n-1})$. Note that all these results do not depend on the dimension.

COST ESTIMATION OF A FIXED NETWORK DEPLOYMENT OVER AN URBAN TERRITORY

Elie Cali, Catherine Gloaguen (Orange Labs, France),
catherine.gloaguen@orange.com, elie.cali@orange.com

Abstract: Using theoretical results presented in former papers, two prototypes were developed, aiming at estimating within minutes the cost of a fibre network deployment on a given territory. The first one helps defining the limit of an urban territory and computes mathematical parameters representing its street system. The second one allows a final user to easily design various network architectures and specify his own engineering rules; it then gives global information on a fixed network deployment on this territory, namely the probability distributions of distances and attenuation from a node of the network to the final customer, and an evaluation of the deployment cost. This makes it possible to compare different deployment scenarios, and to optimize the budget and the efficiency of the network in a few minutes. The results were benchmarked on two real French urban territories (in Tours and Rouen) against those given by an optimization tool currently used by Orange.

✦ VORONOI DIAGRAM ON RIEMANNIAN MANIFOLDS.

Aur lie Chapron (Modal'X, Paris Nanterre and LMRS, Rouen),
`aurelie.chapron1@inv.univ-rouen.fr`

Abstract: We examine the Voronoi diagram generated by a homogeneous Poisson point process on a Riemannian manifold. We aim to recover the local geometry of the manifold from the statistics of the Voronoi cells. We first obtain the asymptotics of the mean number of vertices of a particular Voronoi cell. We then prove limit theorems for the total number of vertices of the Voronoi tessellation and we derive from them an estimator of the scalar curvature of the manifold. Finally, we will discuss the possibility to recover the global geometry of the manifold, providing a probabilistic proof of the Gauss-Bonnet theorem in the two dimensional case.

✦ CLUSTER SIZE DISTRIBUTIONS OF EXTREME VALUES FOR THE POISSON-VORONOI TESSELLATION

Nicolas Chenavier (Universit  du Littoral C te d'Opale),
`nicolas.chenavier@univ-littoral.fr`

Abstract: We consider the Voronoi tessellation based on a homogeneous Poisson point process in \mathbf{R}^d . For a geometric characteristic of the cells (e.g. the inradius, the circumradius, the volume), we investigate the point process of the nuclei of the cells with large values. Conditions are obtained for the convergence in distribution of this point process of exceedances to a homogeneous compound Poisson point process. We provide a characterization of the asymptotic cluster size distribution which is based on the Palm version of the point process of exceedances. This characterization allows us to compute efficiently the values of the extremal index and the cluster size probabilities by simulation for various geometric characteristics. The extension to the Poisson-Delaunay tessellation is also discussed.

Joint work with Christian Y. Robert (ISFA, Lyon 1)

✦ REGULARIZED POISSON AND LOGISTIC REGRESSION METHODS FOR SPATIAL POINT PROCESSES INTENSITY ESTIMATION WITH A DIVERGING NUMBER OF COVARIATES

Achmad Choiruddin (Laboratory Jean Kuntzmann, University Grenoble Alpes, France),
`achmad.choiruddin@univ-grenoble-alpes.fr`

Abstract: Many methods for estimating parametrically the intensity function for inhomogeneous spatial point processes are available in the literature. Most of them consider the number of covariates moderate. Our study considers feature selection procedures based on convex and non-convex regularization techniques to deal with such data. We propose regularized versions of estimating equations based on Campbell theorem derived from Poisson and logistic regression likelihoods. We investigate theoretical and computational aspects. In particular, we consider asymptotic properties which make our results available for several penalty functions and large classes of spatial point processes.

Joint work with Jean-Fran ois Coeurjolly and Fr d rique Letu 

DESIGN UNBIASED POPULATION SIZE ESTIMATION ON GIGAPIXEL IMAGES

Marcos Cruz (Univ. Cantabria, Spain),
`marcos.cruz@unican.es`

Abstract: We propose a design unbiased method for population size estimation. It is based on geometric sampling and typically achieves relative standard errors of 5%-10%. However the large perspective effect introduced by gigapixel images may boost these errors to 30%-40%. We show that projecting the sampling grid from a map onto the gigapixel image using the camera projection, neutralizes the variance due to perspective effects and restores the relative standard errors back into the 5%-10% range.

Joint work with Javier González Villa

ON ASYMPTOTIC DISTRIBUTION OF RST INFINITE BRANCHES

Youri Davydov (Lille1, France),
`youri.davydov@math.univ-lille1.fr`

Abstract: The Radial Spanning Tree (RST) is a geometric random tree defined on an homogeneous Poisson point process by geometric local rules. It has been introduced in 2007 by F. Baccelli and C. Bordenave to modelize communication networks. In dimension 2, it is known that the RST admits semi-infinite branches in any direction with probability 1. We state that the random probability measure μ_n given by the angles of the semi-infinite branches at level n converges in probability, as n tends to infinity, to the uniform distribution on the disc.

Joint work with David Coupier

IDENTIFICATION AND ISOTROPY CHARACTERIZATION OF DEFORMED RANDOM FIELDS THROUGH EXCURSION SETS

Julie Fournier (MAP5, universit  Paris-Descartes),
`julie.fournier@parisdescartes.fr`

Abstract: A deterministic application $\theta : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ deforms bijectively and regularly the plane and allows to build a deformed random field $X \circ \theta : \mathbb{R}^2 \rightarrow \mathbb{R}$ from a regular, stationary and isotropic random field $X : \mathbb{R}^2 \rightarrow \mathbb{R}$. The deformed field $X \circ \theta$ is in general not isotropic, however we give an explicit characterization of the deformations θ that preserve the isotropy. Further assuming that X is Gaussian, we introduce a weak form of isotropy of the field $X \circ \theta$, defined by an invariance property of the mean Euler characteristic of some of its excursion sets. Deformed fields satisfying this property are proved to be strictly isotropic. Besides, assuming that the mean Euler characteristic of excursions sets of $X \circ \theta$ over some basic domains is known, we are able to identify θ .

CLUSTER MARKED CLUSTER PROCESSES

Ute Hahn (CSGB, Aarhus University, Denmark),
ute@math.au.dk

Abstract: Consider marked cluster processes where all points belonging to the same cluster bear the same mark, but the marks from different clusters are independent. For this model, I will present a relation between the pair correlation function and the mark correlation function that can be used for checking the model assumptions and gives information on the parent process. Research was motivated by a data set from ultra microscopy, where points represent localizations of single molecules. The points are obtained from fluorescence photons that are registered in a video, and thus they are equipped with time marks. Usually, these data sets are only analysed as point processes in space, but the extra information given by the time marks gives interesting insight into the process that generates the data.

CLTs FOR NON-POISSONIAN HYPERPLANE PROCESSES IN EXPANDING CONVEX BODIES

Lothar Heinrich (University of Augsburg, Germany),
heinrich@math.uni-augsburg.de

Abstract: We consider hyperplane processes $\{H_i, i \in \mathbb{Z}\}$ in an expanding convex body ϱK (as $\varrho \rightarrow \infty$) generated by stationary independently marked point processes $\Psi = \{[p_i, v_i], i \in \mathbb{Z}\}$ on the real axis, where p_i gives the signed perpendicular distance between H_i and the origin o and the i.i.d. marks $v_i \in \mathbb{S}^{d-1}$ are unit vectors orthogonal to H_i . K is a convex body in \mathbb{R}^d containing o as inner point. Under a suitable mixing condition (e.g. Brillinger-mixing) put on the unmarked stationary (non-Poisson) point process $\{p_i, i \in \mathbb{Z}\}$ we prove CLTs for the total number as well as for the total $(d-k)$ -volume of the (in general non-stationary) intersection $(d-k)$ -flat process ($k = 1, \dots, d$) generated by the H_i 's in ϱK . The obtained results generalize CLTs derived in [1], [2], [3] for Poisson hyperplane processes partly in special cases (e.g. under isotropy assumption or only for the unit ball $K = B_d$) by applying quite different proving techniques.

[1] Paroux, K. (1998) Adv. Appl. Probab. **30**, 640 - 656.

[2] Heinrich, L., Schmidt, H., Schmidt, V. (2006) Ann. Appl. Probab. **16**, 919 - 950.

[3] Heinrich, L. (2009) Rend. Circ. Mat. Palermo, Serie II, Suppl. **81**, 187-212.

ON MAXIMAL HARD-CORE THINNINGS OF STATIONARY PARTICLE PROCESSES

Christian Hirsch (LMU Munich, Germany),
hirsch@math.lmu.de

Abstract: We consider existence and uniqueness of subclasses of stationary hard-core particle systems arising as thinnings of stationary particle processes. These subclasses are defined by natural maximality criteria. We investigate two specific criteria, one related to the intensity of the hard-core particle process, the other one being a local optimality criterion. In fact, the criteria are equivalent under suitable moment conditions. We show that stationary hard-core thinnings satisfying such criteria exist and are frequently unique. More precisely, uniqueness holds in subcritical and barely supercritical regimes of continuum percolation. Additionally, based on the analysis of a specific example, we argue that fluctuations in grain sizes can play an important role for establishing uniqueness at high intensities.

Joint work with Günter Last (KIT)

DISAGREEMENT PERCOLATION FOR MARKED GIBBS POINT PROCESSES

Christoph Hofer-Temmel (NLDA, Den Helder and CWI, Amsterdam),
`math@temmel.me`

Abstract: Disagreement percolation is a technique to control the differing boundary conditions in a Gibbs specification by a simpler percolation model. In the high temperature regime, the percolation model does not percolate and implies the uniqueness of the Gibbs measure. If the percolation has exponentially decaying connection probabilities, then exponential decay of correlations for the Gibbs measure follows, too. We extend this technique from the discrete case and bounded range interaction simple Gibbs point processes to finite range interaction marked Gibbs point process and general Boolean models. A core building block is a dependent thinning from a Poisson point process to a dominated Gibbs point process within a finite volume, where the thinning probability is related to the derivative of the free energy of the Gibbs point process.

Joint work with Pierre Houdebert.

ON KENDALL'S PROBLEM IN SPHERICAL SPACE

Daniel Hug (Karlsruhe Institute of Technology),
`daniel.hug@kit.edu`

Abstract: Stochastic geometry in Euclidean space \mathbb{R}^d has been studied extensively and much progress has been made within the last two decades. In many situations, the Euclidean setting is particularly convenient and fruitful, since the geometry is reasonably easy to visualize and Euclidean functionals are thoroughly understood.

One of the inspiring problems of stochastic geometry in Euclidean space goes back to David Kendall. It is concerned with the problem of determining the asymptotic or limit shape (if it exists) of large random cells in Poisson driven random tessellations. The topic has been successfully explored in a sequence of contributions since 2004, jointly with Rolf Schneider and partly with Matthias Reitzner (with related contributions also by Pierre Calka).

Recently, stochastic geometry in *spherical space* has come into focus. Here we consider a spherical version of Kendall's problem. In the spherical setting, the problem has to be modified since "large cells" cannot occur. We indicate several results concerning this problem, which is usually connected with interesting results of isoperimetric type.

Joint work with Andreas Reichenbacher

🍃 LIMIT THEOREMS FOR MULTI-DIMENSIONAL RENEWAL SETS

Andrii Ilienکو (National Technical University of Ukraine),
ilienکو@matan.kpi.ua

Abstract: We are interested in the geometry of renewal sets constructed by partial sums of i.i.d.r.v. with multi-dimensional indices. Given an infinite multi-indexed array of i.i.d.r.v. with a finite mean $\mu > 0$, consider the family of super-level sets constructed by their partial sums over multi-dimensional rectangles. This family may be regarded as a set-valued random field, which is a multi-dimensional counterpart of the classic renewal process. The talk provides a short overview of some known limit theorems concerning the asymptotic behaviour of multi-dimensional renewal processes as well as new asymptotic results about the location and the shape of renewal sets. In more detail, we show that appropriately rescaled renewal sets converge to the (non-random) set $\mathcal{H} = \{x \in \mathbb{R}_+^d : x_1 \cdot \dots \cdot x_d \geq \mu^{-1}\}$. The rate of convergence (in the form of the Marcinkiewicz-Zygmund strong law of large numbers) and the law of the iterated logarithm are studied as well.

The talk is based on joint work with Ilya Molchanov (University of Bern).

🍃 ANISOTROPY IN FINITE CONTINUUM PERCOLATION : THRESHOLD ESTIMATION BY MINKOWSKI FUNCTIONALS

Michael A. Klatt (KIT, Germany),
michael.klatt@kit.edu

Abstract: What is the interplay between anisotropy and percolation, that is, the spontaneous formation of a system spanning cluster in an anisotropic model? We address this question in simulations of a benchmark model of continuum percolation, the Boolean model. Only in finite systems, distinct differences between effective percolation thresholds for different directions appear. If extrapolated to infinite system sizes, these differences vanish independently of the details of the model, because of the uniqueness of the percolating cluster. While percolation is isotropic even for anisotropic processes, the value of the percolation threshold depends on the model parameters. We discuss analytic formulas for approximations based on the excluded area or the Euler characteristic and compare them to our simulation results. Empirical parameters from similar systems allow for accurate predictions of the percolation thresholds, but even without any empirical parameters, the explicit approximations from integral geometry capture well the qualitative behavior. As an outlook, further candidates for explicit approximations based on second moments of Minkowski functionals are presented.

Joint work with Gerd E. Schröder-Turk and Klaus Mecke; see M. A. Klatt, G. E. Schröder-Turk, K. Mecke. J. Stat. Mech. 2017 :023302 (2017).

COVARIOGRAMS AND GEOMETRIC FUNCTIONALS OF RANDOM EXCURSIONS

Raphaël Lachieze-Rey (Université Paris Descartes),
`raphael.lachieze-rey@parisdescartes.fr`

Abstract: The covariogram of a measurable set A in \mathbb{R}^d is the function that associates to a vector u the Lebesgue measure of the intersection of A with $A + u$. We introduce similarly the bicovariogram as the measure of the intersection between A , $A + u$, and $A + v$, for u, v in \mathbb{R}^2 . It turns out that, if A is a compact C^1 manifold of \mathbb{R}^2 with Lipschitz boundary, its Euler characteristic can be expressed directly as a linear combination of its bicovariograms, echoing a similar relation between the covariogram and the perimeter, established by Bruno Galerne. These findings are applied to derive first or second order results for excursions of random Gaussian fields or Poisson shot noise fields.

PARTICLE TRANSPORT IN STOCHASTIC TESSELLATIONS : APPLICATION TO REACTOR PHYSICS

Coline Larmier (CEA, France),
`coline.larmier@cea.fr`

Abstract: In nuclear reactor physics, the analysis of heterogeneous media with quenched disorder is key to many applications : assessing the probability of re-criticality after severe accidents leading to core melt-down, or evaluating the impact of steam-water dispersion on neutron slowing-down and diffusion. We assume that such disordered systems can be modelled by stochastic tessellations, which form a prototype model of random partition of space. We investigate the statistical properties of d-dimensional isotropic Poisson tessellations of finite size by resorting to Monte Carlo simulation. We characterize finite-size effects and determine the percolation threshold, the strength of the percolating cluster and the average cluster size. Moreover, we analyze the features of particle transport through an ensemble of such random tessellations, for simple benchmark configurations. In this framework, we determine the ensemble-averaged reflection and transmission probabilities for d-dimensional boxes with different initial conditions.

Joint work with Andrea Zoia, Fausto Malvagi, Eric Dumonteil, Alain Mazzolo

EXISTENCE AND ABSENCE OF PERCOLATION FOR OUT-DEGREE ONE RANDOM GRAPHS

Simon Le Stum (USTL Lille 1, France),
`simonlestum@hotmail.fr`

Abstract: In this talk, we will discuss about existence of Poisson out degree one random graph obtained like a stopped germs/grains dynamic. We will focus on a stopped line segment dynamic and a stopped Brownian dynamic. We obtain the existence of these two models consequently of a general result of Percolation which generalize the Peter Hall famous Theorem about subcritical regime in Poisson boolean model. The second part of the talk concerns the absence of percolation results about some out degree one models built thanks to the Existence result. Precisely, a general $[0, 2\pi]$ -line segment model in the plane with random speed V does not percolate if $\mathbf{E}[V^3] < +\infty$.

Joint work with David Coupier and David Dereudre

CONSTRUCTING PSEUDO-STATIONARY COVARIANCE FUNCTIONS ON GRAPHS WITH EUCLIDEAN EDGES AND WITH RESPECT TO THE GEODESIC OR THE RESISTANCE METRIC

Jesper Møller (Aalborg University, Denmark),
jm@math.aau.dk

Abstract: We are motivated by the problem on how to define (Gaussian) random fields and point process models (such as log Gaussian Cox processes) on linear networks and more generally the set given by the union of the vertices and edges (considered as sets) of a graph. In particular we aim at constructing covariance functions which only depend on either the geodesic (shortest path distance) or the resistance metric. Briefly, the resistance between two vertices of an electrical network is the voltage drop when a current of one ampere flows between the two vertices, and we show how this metric can be extended to the union of the vertices and edges of a general graph. In general the resistance metric provides the shortest distances but the two metrics agree for tree graphs, and we argue that the resistance metric may be relevant in connection to e.g. road networks. Combining methods from probability theory, graph theory and harmonic analysis we show that different classes of covariance functions can more easily be constructed for the resistance metric than for the geodesic metric.

Ongoing joint work with Ethan Anderes and Jakob G. Rasmussen.

A CENTRAL LIMIT THEOREM FOR LIPSCHITZ–KILLING CURVATURES OF GAUSSIAN EXCURSIONS

Dennis Müller (Karlsruhe Institute of Technology),
dennis.mueller@kit.edu

Abstract: In this talk, we study the excursion set of a real stationary isotropic Gaussian random field $X = \{X(t) \mid t \in \mathbb{R}^d\}$ above a fixed level $u \in \mathbb{R}$. We show that the standardized Lipschitz–Killing curvature \mathcal{L}_m , $m = 0, \dots, d-1$, of the intersection of the excursion set with an observation window converges in distribution to a normal distribution as the window grows to the d -dimensional Euclidean space, that is

$$\frac{\mathcal{L}_m(B_N^d \cap X^{-1}([u, \infty))) - \mathbb{E}[\mathcal{L}_m(B_N^d \cap X^{-1}([u, \infty)))]}{\mathcal{H}^d(B_N^d)^{\frac{1}{2}}} \xrightarrow{\mathcal{D}} \mathcal{N}(0, \sigma_m^2)$$

as $N \rightarrow \infty$, where $\mathcal{H}^d(B_N^d)$ denotes the volume of the centered ball of radius N . Moreover a lower bound for the asymptotic variance σ_m^2 is presented.

✦ A SLIVNYAK-MECKE-TYPE FORMULA FOR STIT TESSELLATION PROCESSES AND SOME APPLICATIONS

Werner Nagel (Friedrich-Schiller-Universität Jena, Germany),
`werner.nagel@uni-jena.de`

Abstract: We consider random processes where the states are tessellations of the plane or a higher-dimensional Euclidean space. The transition to a new state is caused by a random division of individual cells, where each cell has a random life time, and at the end of its life it is randomly divided. Among these models, the STIT tessellation (stochastically STable under the operation of IIteration of tessellations) appears to be the most promising one from a theoretical point of view, and also a potential reference model for applications. The STIT model shares some properties with Poisson hyperplane tessellations. In the talk we explain a version of the Slivnyak-Mecke formula which holds for STIT. Some corollaries, providing features of STIT illustrate the usefulness of this proposition.

Joint work with Christoph Thäle, Viola Weiss and Linh Ngoc Nguyen

✦ CLUSTER COUNTING IN THE RANDOM CONNECTION MODEL

Franz Nestmann (KIT, Germany),
`franz.nestmann2@kit.edu`

Abstract: The classical random connection model (RCM) can be obtained by a Poisson process η on \mathbb{R}^d and a connection function $\varphi : [0, \infty) \rightarrow [0, 1]$. Each pair of points $(x, y) \in \eta^2$, with $x \neq y$, is connected via an edge with probability $\varphi(|x_1 - x_2|)$, independently of all other points and connections. This model can be extended by adding a random and independent mark in $[0, \infty)$ to each Poisson point. In this marked model, the probability for an edge between two Poisson points also depends on the marks of the two points. This marked model includes the classical Gilbert model. In the Gilbert model the points are marked with random radii and two points are connected if the balls around them intersect. In this talk we are interested in the number of clusters of the classical and the marked RCM that are isomorphic to a connected finite graph and are located in an observation window. We will use the Stein-Malliavin method to derive bounds for the normal approximation of this random number in the Wasserstein and the Kolmogorov distance.

This talk is based on joint work with Günter Last (Karlsruhe) and Matthias Schulte (Bern).

✦ ASYMPTOTICS FOR RANDOM MARKED CLOSED SETS

Zbyněk Pawlas (Charles University, Czech Republic),
`pawlas@karlin.mff.cuni.cz`

Abstract: A random marked closed set (RMCS) is defined as a random upper semi-continuous function on a random domain (random closed set). An example could be the Boolean model with the marking function given by the sum of kernel functions defined on individual grains. Another example is provided by the excursion set of a continuous random field. We consider non-parametric estimators of basic summary characteristics of stationary RMCSs. The second-order characteristics are of the main interest. We study the asymptotic behaviour of these estimators as the observation window is expanding.

OPTIMAL CUTS OF RANDOM GEOMETRIC GRAPHS

Mathew Penrose (University of Bath, UK),
m.d.penrose@bath.ac.uk

Abstract: Given a ‘cloud’ of n points sampled independently uniformly at random from a Euclidean domain D , one may form a geometric graph by connecting nearby points using a distance parameter $r(n)$. We consider the problem of partitioning the cloud into two pieces to minimise the number of ‘cut edges’ of this graph, subject to a penalty for an unbalanced partition. The optimal score is known as the Cheeger constant of the graph. We discuss convergence of the Cheeger constant (suitably rescaled) for large n with suitably chosen $r(n)$, towards an analogous quantity defined for the original domain D .

VOLUMES OF CONVEX HULLS OF $n \leq d + 1$ POINTS IN d -DIMENSIONAL CONVEX BODIES

Benjamin Reichenwallner (Salzburg University, Austria),
benjamin.reichenwallner@sbg.ac.at

Abstract: For a convex body K , we consider the volume of a random simplex whose vertices are chosen uniformly in K . We denote its volume by V_K . Rademacher gave a surprising answer to a question of Meckes whether $K \subseteq L$ would imply $\mathbb{E}V_K \leq \mathbb{E}V_L$, only leaving the expected volume of a random tetrahedron in dimension three as an open task. We use analytic methods to give a counterexample to this monotonicity of inclusion in dimension three. Moreover, we state similar results for higher moments as well as for convex hulls of $n < d + 1$ points in d -dimensional convex bodies.

Joint work with Matthias Reitzner and Stefan Kunis

ANISOTROPY OF HÖLDER GAUSSIAN RANDOM FIELDS : CHARACTERIZATION, ESTIMATION, AND APPLICATION TO IMAGE TEXTURES.

Frédéric Richard (Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France),
frederic.richard@univ-amu.fr

Abstract: The characterization and estimation of the Hölder regularity of random fields has long been an important topic of Probability theory and Statistics. This notion of regularity has also been widely used in Image Analysis to measure the roughness of textures. However, such a measure is often not sufficient to characterize textures as it does account for their directional properties (*e.g.* isotropy and anisotropy). In this talk, I will present an approach to further characterize directional properties associated to the Hölder regularity of random fields. Using the spectral density, I will define a notion of asymptotic topothesy which quantifies directional contributions of field high-frequencies to the Hölder regularity. This notion is related to the topothesy function of the so-called anisotropic fractional Brownian fields, but is defined in a more generic framework of intrinsic random fields. I will then propose a method based on multi-oriented quadratic variations to estimate this asymptotic topothesy. Eventually, I will present an application of this method to the characterization of historical photographic papers.

STOCHASTIC 3D MODELING OF AMORPHOUS MICROSTRUCTURES

Volker Schmidt (Ulm University, Germany),
volker.schmidt@uni-ulm.de

Abstract: We consider the concept of stochastic 3D modeling of geometrically complex microstructures as a tool for virtual materials testing, including applications to organic solar cells, fuel cells as well as Li-ion batteries. Using 3D models from stochastic geometry, one can generate a large variety of stochastically simulated microstructures with little computational effort. These virtual microstructures can be used as data basis to elucidate microstructure-property relationships. In this way, e.g., charge transport in disordered media can be investigated, where effective conductivity can be expressed by morphological parameters such as volume fraction, tortuosity (windedness of transport paths) and constrictivity (bottleneck criterion) of the considered material phase. Using simple analytical formulas and data mining techniques, we were able to accurately predict effective conductivities for given values of volume fraction, tortuosity and constrictivity. Moreover, we developed stochastic 3D models for further kinds of particle-based materials, like granular aggregates or polycrystalline alloys, and established links between mechanical and microstructure properties of these materials.

WIRELESS NETWORK SIGNALS WITH MODERATELY CORRELATED SHADOWING STILL APPEAR POISSON

Dominic Schuhmacher (Universität Göttingen),
schuhmacher@math.uni-goettingen.de

Abstract: The strengths of signals emitted from transmitters in a wireless network, such as Wi-Fi or mobile phone, are observed by a user at a fixed position. We model transmitter locations as an orderly spatial point process and assume that signals are subjected to power law path loss and correlated log-normal shadowing effects. Using tools from Stein's method, we give limit theorems and rates in Wasserstein distance for the convergence of the point process of received signal strengths towards a Poisson process. The rate of convergence depends essentially on a combination of the degree of orderliness of the transmitter point process and the decay of the shadowing correlation in the distance between transmitters.

Joint work with Nathan Ross

NORMAL APPROXIMATION FOR STABILIZING FUNCTIONALS

Matthias Schulte (University of Bern, Institute of Mathematical Statistics and Actuarial Science),
matthias.schulte@stat.unibe.ch

Abstract: In this talk functionals of Poisson or binomial point processes are considered which are sums of scores of the points of the underlying point process. Moreover, it is assumed that these scores stabilize in the sense that they are determined by the point itself and the points in a random neighborhood given by a so-called radius of stabilization. If the radius of stabilization decays exponentially fast and some moment assumptions are satisfied, Berry-Esseen bounds for the normal approximation of stabilizing functionals can be shown. These new bounds lead to presumably optimal rates of convergence and improve existing results for the normal approximation of stabilizing functionals. As applications quantitative central limit theorems for several problems from stochastic geometry such as random graphs, Voronoi approximation and convex hulls of random points in a smooth convex body are derived. The general bounds are proven by using the Malliavin-Stein method.

Joint work with Raphael Lachièze-Rey and Joseph Yukich.

FINDING PATTERNS IN BROWNIAN MOTION

Hermann Thorisson (University of Iceland, Iceland),
hermann@hi.is

Abstract: We consider the problem of finding particular patterns in a realisation of a two-sided standard Brownian motion. Examples include two-sided Skorohod imbedding, the Brownian bridge and several other patterns, also in planar Brownian motion. The key tool here are recent allocation results in Palm theory.

Joint work with Guenter Last, Peter Moerters and Wenpin Tang

DENSITIES OF MIXED VOLUMES FOR BOOLEAN MODELS

Wolfgang Weil (Karlsruhe Institute of Technology, Germany),
wolfgang.weil@kit.edu

Abstract: The classical Miles formulas express the densities $\bar{V}_j(Z)$ of the intrinsic volumes $V_j, j = 0, \dots, d$, of a stationary and isotropic Boolean model Z in \mathbb{R}^d in terms of the intensity γ and the mean values $\bar{V}_i(Y)$ of the underlying particle process Y . Since they comprise a triangular array, γ can be expressed in terms of $\bar{V}_j(Z), j = 0, \dots, d$, which leads to natural estimators for the mean number of particles (per unit volume). For stationary, non-isotropic Boolean models Z , the functionals V_j are not sufficient anymore, since they are motion invariant. Instead, direction dependent functionals would be necessary like the mixed volumes $V(Z[j], Kd - j)$ for suitable test bodies K (compact convex sets in \mathbb{R}^d). In Weil (2001), formulas for corresponding densities $\bar{V}(Z[j], Kd - j)$ were obtained which are in close analogy to the Miles formulas, but contain mixed functionals $\bar{V}_{m_1, \dots, m_k, d-j}(Y, \dots, Y, K)$ of the particles in Y (and K) arising from translative integral geometry. It was shown that for dimensions $d = 2$ and $d = 3$ the densities of Z still determine γ , this unified some previous approaches with local functionals (the method of moments). A corresponding attempt for $d = 4$ was incomplete and the cases $d \geq 5$ remained open.

We now have a general result for all dimensions : The densities $V(Z[j], Kd - j)$, for $j = 0, \dots, d$ and all convex test bodies K , determine γ uniquely. The proof is based on a recent flag representation of mixed functionals $V_{m_1, \dots, m_k}(K_1, \dots, K_k)$ (Hug-Rataj-Weil, 2017) and uses a deep result of Alesker (2001) on the representation of translation invariant valuations.

Joint work with Daniel Hug

INTEGRAL GEOMETRIC FORMULAE FOR TENSORIAL CURVATURE MEASURES

Jan Weis (KIT, Germany),
`jan.weis@kit.edu`

Abstract: The tensorial curvature measures are the natural tensor-valued generalizations of the curvature measures of convex bodies in Euclidean space. On convex polytopes, there exist further generalizations some of which also have continuous extensions to arbitrary convex bodies. The talk provides two complete sets of integral geometric formulae, so called kinematic formulae and Crofton formulae, for such (generalized) tensorial curvature measures. These formulae treat the intersection of a convex body with a second geometric object (in the kinematic formulae this is another convex body; in the Crofton formulae this is an affine subspace) which is uniformly moved by a proper rigid motion. The proofs, which are also sketched in this talk, proceed in a more direct way than the classical proofs of the corresponding integral formulae for curvature measures.

Joint work with Daniel Hug

CONVEX HULLS OF LÉVY PROCESSES

Florian Wespi (University of Bern, Switzerland),
`florian.wespi@stat.unibe.ch`

Abstract: Let $X(t)$, $t \geq 0$, be a Lévy process in \mathbb{R}^d starting at the origin. We study the closed convex hull Z_s of $\{X(t) : 0 \leq t \leq s\}$. In particular, we provide conditions for the integrability of the intrinsic volumes of the random set Z_s and find explicit expressions for their means in the case of symmetric α -stable Lévy processes. If the process is symmetric and each its one-dimensional projection is non-atomic, we establish that the origin a.s. belongs to the interior of Z_s for all $s > 0$. Limit theorems for the convex hull of Lévy processes with normal and stable limits are also obtained.

This talk is based on a joint work with Ilya Molchanov (University of Bern).

GEOMETRIC FUNCTIONALS OF FRACTAL PERCOLATION

Steffen Winter (Karlsruhe Institute of Technology, Germany),
`steffen.winter@kit.edu`

Abstract: Fractal percolation is a family of random sets suggested by Mandelbrot in the seventies as a model for clotting phenomena. They are known to undergo a sharp phase transition from a totally disconnected to a percolating regime, but exact percolation thresholds are unknown. Motivated by the desire to find bounds on percolation thresholds, we study the expectations of some geometric functionals of these sets which are closely related to fractal curvatures. We obtain explicit formulas for some of these expectations including some rescaled Euler characteristic. Our results suggest that simulation of such fractal phenomena can be quite misleading.

Joint work in progress with Michael Klatt

MARKED POINT PROCESSES FOR OBJECT DETECTION AND TRACKING IN HIGH RESOLUTION IMAGES : APPLICATIONS TO REMOTE SENSING AND BIOLOGY

Josiane Zerubia (Université Côte d'Azur, INRIA, France),
`josiane.zerubia@inria.fr`

Abstract: In this talk, we combine the methods from probability theory and stochastic geometry to put forward new solutions to the multiple object detection and tracking problem in high resolution remotely sensed image sequences. First, we present a spatial marked point process model to detect a pre-defined class of objects based on their visual and geometric characteristics. Then, we extend this model to the temporal domain and create a framework based on spatio-temporal marked point process models to jointly detect and track multiple objects in image sequences. We propose the use of simple parametric shapes to describe the appearance of these objects. We build new, dedicated energy based models consisting of several terms that take into account both the image evidence and physical constraints such as object dynamics, track persistence and mutual exclusion. We construct a suitable optimization scheme that allows us to find strong local minima of the proposed highly non-convex energy. As the simulation of such models comes with a high computational cost, we turn our attention to the recent filter implementations for multiple objects tracking, which are known to be less computationally expensive. We propose a hybrid sampler by combining the Kalman filter with the standard Reversible Jump MCMC. High performance computing techniques are also used to increase the computational efficiency of our method. We provide an analysis of the proposed framework. This analysis yields a very good detection and tracking performance at the price of an increased complexity of the models. Tests have been conducted both on high resolution satellite and microscopy image sequences.

Joint work with Paula Craciun and Mathias Ortner

THINNING SELF-DIVISIBLE POINT PROCESSES

Sergei Zuyev (Chalmers, Sweden),
`sergei.zuyev@chalmers.se`

Abstract: Consider a classical limit scheme of superposition of independent point processes (PPs) which are subject to independent thinning to converge. It has been known for a few decades that all possible limit laws in the triangular array scheme constitute the class of *infinitely-divisible* PPs. When the superposed PPs are i.i.d., the weak limits are the so-called *thinning-stable* PPs which are Cox processes driven by a stable parameter measure. Their full characterisation has recently been obtained [1]. If the superposed thinned PPs are independent, but not necessarily identically distributed, the limit law is that of a *thinning selfdivisible* (TSD) PP which satisfy the following equality in distribution : a PP Φ is TSD if for any $c \in (0, 1)$ there is a point process Ψ_c independent of Φ such that $\Phi = c \circ \Phi + \Psi_c$, where $c \circ$ stands for the operation of independent thinning with retention probability c .

We present the first results on the structure of TSD PPs and show possible extensions to a *general branching* operation on the PPs including the independent thinning as a particular case.

[1] Yu. Davydov, I. Molchanov and S. Zuyev. Stability for random measures, point processes and discrete semigroups. *Bernoulli*, **17**, 1015–1043

Joint work with Michel Davydov

Posters

CHARACTERISATION OF TWO-PHASE FLOWS THROUGH IMAGE PROCESSING METHODS AND STOCHASTIC GEOMETRY

Mathieu de Langlard (Commissariat à l'Énergie Atomique, DEN/DMRC/SA2I, F-30207 Bagnols-sur-Cèze, France),
`mathieu.delanglard@cea.fr`

Abstract: Image analysis is a widespread tool for the characterization of particulate systems in chemical engineering. However, for two-phase flows, due to the wide range of particles size and the appearance of large clusters, which are a consequence of particle projections overlapping, automatic particle detection remains a challenge. Stochastic geometry then appears as a new promising approach. After presenting briefly a developed image processing algorithm to determine the 2D size distribution of the ellipse-like particles, the presentation will focus on the 3D modelling of the particles population using an adapted version of Matérn point process. The aim is to define and study the orthogonal projection of the 3D process. Besides, an exploratory analysis is proposed to compare second-order moment related characteristics of the projected point process and the real point patterns (centers of the detected ellipses on the real images).

Joint work with Fabrice Lamadie, Sophie Charton and Johan Debayle

REGULARITY AND RECONSTRUCTION

Andrew du Plessis (CSGB, Institut for matematik, Aarhus University),
`matadp@math.au.dk`

Abstract: Let L be a cubic lattice in \mathbf{R}^3 . For any subset $X \subset \mathbf{R}^3$, the *digital image* $D_L(X)$ of X with respect to L is just the sampling $L \cap X$ of X by L . A much-studied problem in computer vision is to determine under which conditions on X the geometry, or at least the embedded topology, of X can be recovered from such digital images.

A closed subset $X \subset \mathbf{R}^3$ is said to be *r-regular* if it is the union of all the closed balls of radius r contained in it. We show that the embedded isotopy type of a compact *r-regular* subset X of \mathbf{R}^3 can be reconstructed algorithmically from $D_L(X)$, for any cubic lattice L whose side length d satisfies $r/d > 0.95571$. Previous work (published in 2006) had claimed a reconstruction only when $r/d > \sqrt{3}$.

Joint work with Sabrina Tang Christensen

UNIQUENESS OF THE MEASUREMENT FUNCTION IN CROFTON'S FORMULA WITH LINES

Rikke Krog Eriksen (CSGB, Aarhus University, Denmark),
rke@math.au.dk

Abstract: For a convex body (compact convex set) K in \mathbb{R}^n , Crofton's intersection formula states that the $(n - k + j)$ th intrinsic volume can be written as an invariant integral of the j th intrinsic volume of the section $K \cap E$, where E is a k -plane, $0 \leq j \leq k < n$.

Motivated by results in stereology, we ask if there are other functionals ϕ , replacing the j th intrinsic volume, with this property. The answer is affirmative if no additional assumptions are imposed on ϕ . The answer is negative, due to Hadwiger's theorem, when we assume in addition that ϕ is a continuous (w.r.t. Hausdorff metric) and motion invariant valuation. Strengthening this, we show that the assumption of motion invariance is sufficient for uniqueness when $k = 1$, i.e. when intersecting with lines.

Joint work with Markus Kiderlen

QUANTIFICATION OF ENDOCYTOSIS AS PHARMACOLOGICAL RESPONSE BY FLUORESCENCE MICROSCOPY

Ana I. Gómez (Universidad de Cantabria, Spain),
gomezanab@gmail.com

Abstract: Endocytosis processes can be observed in images obtained from living cells. Quantifying internalizing vesicles or endosomes, that result from pharmacological responses, provides a method to evaluate pharmacological properties of different compounds.

Fluorescence signal from endosomes has been characterized by a 2D-Gaussian shape of a given size distribution. Specific techniques are needed to discriminate from the foreground, that in this case is constituted by the signal observed at the plasma membrane of the living cells.

We propose an image analysis algorithm to evaluate the fluorescent signal originated at endosomes over image stacks acquired in an inverted epifluorescence microscope. This work improves in characterization of endosome signal with respect to time and aims to provide a comparison based on the pharmacological response between different agonists.

Joint work with Marcos Cruz, Juan F. López-Giménez and Victor M. Campa

STABLE TRANSPORTS BETWEEN STATIONARY RANDOM MEASURES

Ali Khezeli (Institute for Research in Fundamental Sciences (IPM), Tehran, Iran),
alikhhezeli@ipm.ir

Abstract: We give an algorithm to construct a translation-invariant transport kernel between two ergodic random measures on \mathbb{R}^d , given that they have equal intensities. The (non-constructive) existence of such a transport kernel was proved in [G. Last, H. Thorisson 09]. Our algorithm is a generalization of [C. Hoffman, A. E. Holroyd & Y. Peres 06], in which a construction is provided for the Lebesgue measure and the Poisson point process. In the general case, we limit ourselves to what we call *constrained transport densities and transport kernels*. We give a definition of stability of constrained transport densities and introduce our construction inspired by the Gale-Shapley stable marriage algorithm. For stable constrained transport densities, we study existence, uniqueness, monotonicity w.r.t. the measures and boundedness.

Joint work with Mir-Omid Haji-Mirsadeghi

IMPROVED CAVALIERI-TYPE ESTIMATION FOR PERTURBED SYSTEMATIC SAMPLING

Mads Stehr (CSGB, Aarhus University, Denmark),
mads.stehr@math.au.dk

Abstract: The volume of a 3-dimensional solid can be approximated by means of the unbiased Cavalieri estimator from area measurements on parallel hyperplanes intersecting the solid with equidistant spacing. In applications however, the assumption of exactly equidistant planes is rarely fulfilled, leading to a considerable variance inflation compared to the equidistant case; see Baddeley *et al.* (2006). Kiderlen and Dorph-Petersen (2017) therefore suggest to use quadrature rules when the actual distances between consecutive planes are available. Under mild assumptions estimators based on the trapezoid rule or Simpson's rule are still unbiased, and simulations indicate variance reduction in both cases.

We present an explicit variance formula when applying the trapezoid rule in the case of randomly perturbed sampling positions. Also, we will illustrate that this theoretical variance is indeed smaller than that of the existing Cavalieri estimator when estimating the volume of the unit ball. By simulations we will illustrate the variance reduction of the trapezoid rule and Simpson's rule for different area-functions with varying smoothness properties.

A.J. Baddeley, K.A. Dorph-Petersen, and E.B.V. Jensen. (2006) : *A Note on the Stereological Implications of Irregular Spacing Sections*. J. Microsc. **222**, 177-181.

M. Kiderlen, K.A. Dorph-Petersen. (2017) : *The Cavalieri Estimator with Unequal Section Spacing Revisited*. Centre for Stochastic Geometry, University of Aarhus, Research Report 2017 **04**.

Joint work with Markus Kiderlen

GENERALIZATIONS OF SOME ASYMPTOTIC RESULTS FOR MARKED POINT PROCESSES

Daniela Novotná (Charles University, Czech Republic),
dnlnt@gmail.com

Abstract: Gibbs point processes form one of the most important classes of point processes that enable us to incorporate dependence among the points. Usually, Gibbs point processes are defined on the Euclidean space \mathbb{R}^d . We introduce the definition on more general space that includes for instance Gibbs segment processes. Using Stein's method, we obtain upper bounds for the Wasserstein distance between the distribution of an innovation of a Gibbs point process with pair potential and standard normal distribution. Based on this result, we induce central limit theorem for a functional of a planar segment process with pair potential.

REACH OF REPULSION FOR DETERMINANTAL POINT PROCESSES IN HIGH DIMENSIONS

Eliza O'Reilly (University of Texas at Austin, USA),
eoreilly@math.utexas.edu

Abstract: Determinantal point processes offer useful point patterns that exhibit repulsion between points, resulting in more regularly spaced point patterns than Poisson point processes. In this research we examine stationary and isotropic determinantal point processes as space dimension tends to infinity. At distances growing with the square root of the dimension, we quantify the effect of the repulsion of a typical point of the point process. Under certain conditions, we can show a finite reach at this scaling where the repulsive effect concentrates as dimension goes to infinity. Using these results, examples of large classes of specific DPP models exhibiting this behavior are found.

Joint work with François Baccelli

MODELLING POLYCRYSTALLINE MICROSTRUCTURES BY TESSELLATIONS

Ondřej Šedivý (Ulm University, Germany),
ondrej.sedivy@uni-ulm.de

Abstract: Parametric tessellation models have proven to be a suitable concept for the description of the highly complex geometries of grain boundaries observed in polycrystalline microstructures. Here, each grain is represented by a small number of parameters that — through their interaction with parameters of other grains — determine its size and shape. An immediate advantage of this representation is significant data reduction, as a large voxel-based dataset can be reduced to a much smaller set of parameters. Fitting a great variety of tessellation models, being modifications of the basic Voronoi concept, to empirical image data is possible with the use of a universal optimization technique based on simulated annealing. This algorithm will be introduced in detail. Further, evaluation of some geometric features, notably curvatures and dihedral angles, from parameters of the tessellations will be presented, where a particular focus will be given to locally anisotropic tessellations generated by ellipsoids.

TOPOLOGICAL RECONSTRUCTION OF r -REGULAR SETS

Helene Svane (CSGB, Aarhus University, Denmark),
helenesvane@math.au.dk

Abstract: Let $r > 0$. An r -regular set X is a closed set such that both X and $\overline{X^c}$ are unions of closed balls of radius r .

Let $d > 0$ and let X be an r -regular set. In our work, we have considered the lattice cubes of a cubic lattice $(d\mathbb{Z})^3$ that are entirely contained in X . We call this set of black voxels the black voxel reconstruction of X .

We have showed that it is possible to reconstruct the topology of an r -regular set X from its black voxel reconstruction if $d\sqrt{3} < r$.

Joint work with Andrew du Plessis

CENTRAL LIMIT THEOREMS FOR INTRINSIC VOLUMES OF RANDOM POLYTOPES

Nicola Turchi (Ruhr Universität Bochum),
nicola.turchi@rub.de

Abstract: Consider a convex body K in \mathbb{R}^n , which boundary is twice differentiable with positive Gaussian curvature everywhere. Pick now $N > n + 1$ points X_1, \dots, X_N uniformly at random in K . We indicate with K_N the convex hull of such points. We focus our interest on the intrinsic volumes $V_j(K_N)$ of K_N , for $j \in \{1, \dots, n\}$. The importance of these functionals is given by the fact that they form a basis of the vector space of all motion-invariant and continuous valuations on convex bodies. Central limit theorems for $V_j(K_N)$ as $N \rightarrow \infty$ are shown, for every $j \in \{1, \dots, n\}$. This result is achieved combining different tools, such as estimates for volumes of floating bodies and normal approximation bounds due to Chatterjee and Lachièze-Rey, Peccati, developed from Stein's method.

Joint work with Christoph Thäle and Florian Wespi

MICTOMECH : FROM MICROSTRUCTURES TO MECHANICAL PROPERTIES

Martina Vittorietti (Delft University of Technology, The Netherlands),
m.vittorietti@tudelft.nl

Abstract: There are intricate relations between 3D features of multi-phase metallic microstructures and the mechanical properties of the material. Therefore, a first aim in this project is to model this relation statistically. As most data on microstructures actually consist of 2D sections, obtaining information on the 3D microstructure based on sectional data is another important aim. Voronoi diagrams are a popular model for microstructure characterization. Starting from the most basic case, Poisson-Voronoi (PV) diagrams, we use a sophisticated simulation program to construct a close Monte Carlo based approximation for the distribution of the main geometrical characteristics of the 2D sectional and 3D cells. Using the results, we can test whether a PV-model is consistent with observed sections. This will probably rarely hold. Therefore, we aim to study more realistic models, like Multi-Level Voronoi diagrams.

Joint work with Geurt Jongbloed, Jilt Sietsma and Wei Li

ANALYSIS OF IMAGE TEXTURE HETEROGENEITY USING ANISOTROPIC MULTIFRACTAL BROWNIAN FIELDS

Thi Lan Huong Vu (Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France),
`thi-lan-huong.vu@univ-amu.fr`

Abstract: In this poster, we present a model of anisotropic multifractional Brownian field which accounts for local directional properties of image textures. We describe a method based on quadratic variations to estimate main parameters of this model and state an asymptotic normality result about the estimates. We present an application of this method to test the heterogeneity of image textures.

Joint work with Frédéric Richard