Titles and Abstracts for the workshop
Probabilistic Aspects of Multiple Ergodic Averages

V. BERGELSON

ASPECTS OF UNIFORMITY IN THE THEORY OF MULTIPLE RECURRENCE

Abstract. We will discuss several generalizations of Szemerédi's theorem on arithmetic progressions and formulate various results and conjectures focusing on the abundance of configurations in sets of positive density.

K. DAJANI

FROM SYMMETRIC DOUBLING MAPS TO ALPHA-CONTINUED FRACTIONS

Abstract. We introduce a parametrized family of maps S_α, the so called symmetric doubling maps, defined on $[-1,1]$ by $S_\alpha(x) = 2x - d\alpha$, where $d \in \{-1,0,1\}$ and $\alpha \in [1,2]$. Each map S_α generates binary expansions with digits -1,0 and 1. The transformations S_α have a natural invariant measure μ_α that is absolutely continuous with respect to Lebesgue measure. We show that for a set of parameters of full measure, the invariant measure of the symmetric doubling map is piecewise smooth. We also study the frequency of the digit 0 in typical expansions, as a function of the parameter α. In particular, we investigate the self similarity displayed by the function $\alpha \to \mu_\alpha([-1/2,1/2]$, where $\mu_\alpha([-1/2,1/2]$ denotes the measure of the cylinder where digit zero occurs. This is done by exploiting a relation with another family of maps, namely the alpha-continued fraction maps.

A. FAN

SOME REMARKS ON WEIGHTED BIRKHOFF AVERAGES

Abstract: We consider weighted Birkhoff sum of the form $\sum_{n=0}^{N-1} w_n f(T^n x)$ and related objects. The weights (w_n) can be q-multiplicative sequences, Thue-Morse sequence being a special example, and oscillating sequences, Möbus function being a special example. One of remarks is that for a full oscillating sequence (w_n), the multiple ergodic sum

$$\sum_{n=0}^{N-1} w_n f_1(T^n x) f_2(T^{2n}) \dots f_k(T^{kn} x)$$

is $o(N)$ for any x, any continuous functions $f_1, \dots, f_k$ $(k \geq 1)$ and any topological dynamical system T with quasi-discrete spectrum in the sense of Hahn-Parry. Another remark is that Davenport-Erdös-LeVeque argument can prove the weighted Birkhoff theorem when (w_n) shares the Davenport property of the Möbus function.

M. HAASE

☞ **OPERATORS IN ERGODIC THEORY** [Mini course]

The titles of the of the individual lectures are:

1. Operators Dynamics versus Base Space Dynamics.
2. Compact Semigroups and Splitting Theorems.
3. Dilations and Joinings.

Y. KIFER

ERGODIC AND LIMIT THEOREMS FOR GENERALIZED NONCONVENTIONAL SUMS AND ANOTHER EXTENSION OF THE SZEMERÉDI'S THEOREM

I will speak about ergodic and limit theorems for generalized nonconventional sums of the form

$$\sum_{n=1}^{N} F\big(X(n), X(2n), \dots, X(\ell n); X(N-n), X(2(N-n)), \dots, X(\ell(N-n)\big)$$

(and more general ones), where $X(1), X(2), \dots$ is a sequence of random variables which, in particular, can be generated by a dynamical system T^n so that $X(n) = T^n f$ for a function f. I will exhibit also a related extension of the Szemerédi theorem on arithmetic progressions in sets of integers having positive upper density.

B. KRA

☞ **MULTIPLE ERGODIC THEOREMS: OLD AND NEW** [Mini course]

The classic mean ergodic theorem has been extended in numerous ways: multiple averages, polynomial iterates, weighted averages, along with combinations of these extensions. I will give an overview of these advances and the different techniques that have been used, focusing on convergence results and what can be said about the limits.

M. POLLICOTT

MULTIPLE MIXING AND HIGHER CORRELATIONS FOR HYPERBOLIC SYSTEMS

Abstract: In an interesting paper from 2001, Kotani and Sunada related higher order correlations for hyperbolic maps to derivatives of pressure. We will discuss the relationship between these results and determinants of transfer operators, and the the advantages of such formulations. We will also discuss the scope for generalizations to hyperbolic (semi-)flows.

S. SEURET

RANDOM SPARSE SAMPLING IN A GIBBS WEIGHTED TREE

Consider a Gibbs measure on the dyadic tree, and the associated Ç Gibbs È dyadic tree where each finite word w carries the weight $\mu([w])$. The length of w is denoted by $|w|$. Fix a parameter $\eta \in (0,1)$. We perform a sampling of the Gibbs tree, by keeping each value $\mu([w])$ with probability $2^{-|w|\eta}$, otherwise we replace this value by 0. We study the possibility of reconstructing the initial Gibbs tree from the sampled tree, and perform the multifractal analysis of the remaining structure (which can be viewed as a capacity on the dyadic tree). Various phase transitions occur, both in the reconstruction process and in the multifractal spectra.

EVGENY VERBITSKIY

PROBABILISTIC PROPERTIES OF NONCONVENTIONAL ERGODIC AVERAGES

Nonconventional ergodic theorems establish convergence of expressions like

$$S_N^{(k)} = \frac{1}{N}\sum_{n=1}^{N} f_1(T^n x)\dots f_k(T^{nk} x),$$

where $T : X \to X$ is a measure-preserving transformation.
Recently, there has been a surge of new results establishing finer probabilistic properties of non-conventional averages. I will review some of these results: namely, Central Limit theorems, Large Deviations results, and results on the validity of multifractal formalism for non-conventional averages.

B. WEISS

LIMIT THEOREM FOR MULTIPLE AVERAGES AND STRICT ERGODICITY

Abstract: Recently some new pointwise limit laws were established by Huang Shao and Ye using some results on the interface between ergodic theory and topological dynamics. These results concern refinements of the classic theorem of Jewett-Krieger which showed that any ergodic transformation has a strictly ergodic model. I will explain these results and indicate how they are employed.

M. WU

ON THE INTERSECTIONS OF TIMES 2 AND TIMES 3 INVARIANT SETS

Abstract: For a natural number m, let T_m denote multiplication by $m \bmod 1$ on the unit interval $[0,1]$. Two compact sets E, F of the real line are said to be strongly transverse if for all real numbers u and t, the Hausdorff dimension (dim) of the intersection of $E$and $uF + t$ is bounded by $\dim(E) + \dim(F) - 1$ or 0, whichever is larger. In the later 60's, Furstenberg conjectured that if E is a T_2-invariant closed set and F is a T_3-invariant closed set, then E and F are strongly transverse. In this talk, we will recall some recent progress on this conjecture and present a solution.

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