FRENCH-AMERICAN CONFERENCE ON NONLINEAR DISPERSIVE PDES

CIRM, LUMINY, JUNE 12-16, 2017

Plenary Speakers:

Hajer Bahouri

Dispersion phenomena on the Heisenberg group when the vertical frequency tends to 0

Abstract: In the first part of the talk, we built a frequency space on the Heisenberg and establish asymptotic description of the Fourier transform when the "vertical" frequency tends to 0. This construction is based on a new approach of the Fourier transform on the Heisenberg group which is classically defined as a one parameter family of bounded operators on $L^2(\mathbb{R}^d)$.

In the second part, we take advantage of this relevant approach to analyze dispersion phenomena on the Heisenberg group when the vertical frequency tends to 0.

(from a joint work with Jean-Yves Chemin and Raphael Danchin).

Valeria Banica

Dynamics of almost parallel vortex filaments

Abstract: We consider the 1-D Schrödinger system with point vortex-type interactions that was derived by R. Klein, A. Majda and K. Damodaran and by V. Zakharov to modelize the dynamics of N nearly parallel vortex filaments in a 3-D incompressible fluid. We first prove a global in time result and display several classes of solutions. Then we consider the problem of collisions. In particular we establish rigorously the existence of a pair of almost parallel vortex filaments, with opposite circulation, colliding at some point in finite time. These results are joint works with E. Faou and E. Miot.

Christophe Besse

Numerical schemes for nonlinear Schrödinger equation

Abstract: In this talk, we will describe and compare usual numerical schemes for nonlinear Schrödinger equation (usual splitting method, ERK and Lawson schemes, IMEX, ...). We will focus and analyse numerical schemes which preserve conserved quantities (Crank-Nicolson and relaxation scheme). Usually, conservative schemes are implicit (or linearly implicit) and we will comment the specific treatment to allow efficiency.

Thomas Duyckaerts

Bound from below of the exterior energy for the wave equation and applications

Abstract: In this talk (from works with Hao Jia, Carlos Kenig and Frank Merle), I will present classification results for solutions of critical nonlinear wave equations in the spirit of the soliton resolution conjecture. The proofs are based on bounds from below of the energy of solutions of the linear wave equation whose I will give several versions.

Gadi Fibich

Continuations beyond the singularity, loss of phase, stochastic interactions, and universality

Abstract: The question on how to continue NLS solutions beyond the singularity has been open for many years. In the first part of this talk, I will discuss several potential continuations. A common feature of all these continuations is that the solution phase is lost after the singularity. Recently, we showed that "loss of phase" can occur even if the NLS solution does not collapse (e.g., in the subcritical case). Therefore, if two NLS solutions travel a sufficiently long distance (time) before interacting, it is not possible to predict whether they would intersect in- or out-of-phase. Hence, if the underlying propagation model is non-integrable, a deterministic prediction of the interaction outcome becomes impossible. "Fortunately", because the relative phase between the two solutions becomes uniformly distributed in $[0, 2\pi]$, the statistics of the interaction outcome becomes universal, and can be efficiently computed using a polynomial-chaos approach, even when the distribution of the noise source is unknown. Joint work with Moran Klein, Amir Sagiv, and Adi Ditkowski.

Stephen Gustafson

Ground states and dynamics for perturbed critical NLS

Abstract: Ground state solitary waves of perturbed versions of the 3D, focusing, energy-critical nonlinear Schrodinger equation can be found either by a variational method, or by perturbing the well-known (static) ground states of the critical equation. We show the two constructions agree, and use the variational characterization to classify the radially-symmetric dynamics below the perturbed ground states. Joint work with Matt Coles.

Christian Klein

Numerical study of solitons, dispersive shock waves and blow-up

Abstract: We present a detailed numerical study of the effects of nonlinearity and dispersion in various nonlinear dispersive PDEs: the formation of rapid modulated oscillations known as dispersive shock waves, the stability of solitons and blow-up. As examples we study dispersive regularizations of the Hopf equation, the generalized Korteweg-de Vries (KdV) and Kadomtsev-Petviashvili equations, fractional KdV and Whitam equations, and from the family of nonlinear Schrödinger equations as the Davey-Stewartson equations. We discuss asymptotic descriptions of dispersive shocks and self similar blow-up.

Evgeny Kuznetsov

Solitons vs Collapses

Abstract: This talk is devoted to solitons and wave collapses which can be considered as two alternative scenarios pertaining to the evolution of nonlinear wave systems describing by a certain class of dispersive PDEs (see, for instance, review [1]). For the former case, it suffices that the Hamiltonian be bounded from below (or above), and then the soliton realizing its minimum (or maximum) is Lyapunov stable. The extremum is approached via the radiation of small-amplitude waves, a process absent in systems with finitely many degrees of freedom. The framework of the nonlinear Schrodinger equation, the ZK equation and the three-wave system is used to show how the boundedness of the Hamiltonian H, and hence the stability of the soliton minimizing H can be proved rigorously using the integral estimate method based on the Sobolev embedding theorems. Wave systems with the Hamiltonians unbounded from below must evolve to a collapse, which can be considered as the fall of a particle in an unbounded potential. The radiation of small-amplitude waves promotes collapse in this case.

This work was supported by the Russian Science Foundation (project no. 14-22- 00174).

References

[1] V.E. Zakharov and E.A. Kuznetsov, *Solitons and collapses - two scenarios of the evolution of nonlinear wave systems* , Physics Uspekhi **55**, 535 - 556 (2012).

Felipe Linares

On the periodic Zakharov-Kuznetsov equation

Abstract: In this talk we will consider the Cauchy problem associated to Zakharov-Kuznetsov equation in a general torus. We will establish the local well-posedness for data in H^s, $s > 3/2$ via a compactness method. Then we show that for some choices of the periods the problem is not semi-linear in sharp contrast with the cases $\mathbb{R}^2$ and $\mathbb{R} \times \mathbb{T}$.

Joint work with M. Panthee (UNICAMP, Brazil) and N. Tzvetkov (Université de Cergy Pontoise).

Pavel Lushnikov

Stokes wave and dynamics of complex singularities in 2D hydrodynamics with free surface

Abstract: The Stokes wave is the fully nonlinear periodic gravity wave propagating on a fluid fluid representing a particular example of 2D hydrodynamics of ideal fluid with free surface. The Stokes wave of greatest height has the limiting form with 120 degrees angle on the crest as found by Stokes in 1880. A time-dependent conformal transformation is used which maps a free fluid surface into the real line with fluid domain mapped into the lower complex half-plane. 2D fluid dynamics is fully characterized by the motion of complex singularities in the upper complex half-plane of the conformal map and the complex velocity. The only singularity in the physical sheet of Riemann surface of non-limiting Stokes wave is the square-root branch point located on the imaginary axis. The second sheet has a singularity in lower complex half-plane. We found the infinite number of square root singularities in infinite number of non-physical sheets of Riemann surface. As the height of the Stokes wave increases, all these singularities simultaneously approach the real line from different sheets of Riemann surface and merge together forming 2/3 power law singularity of the limiting wave.

Yvan Martel

Strongly interacting solitary waves for NLS

Abstract: I will present two cases of strong interactions between solitary waves for the nonlinear Schrödinger equations (NLS). In the mass sub- and super-critical cases, a work by Tien Vinh Nguyen proves the existence of multi-solitary waves with logarithmic distance in time, extending a classical result of the integrable case (1D cubic NLS equation). In the mass-critical case, a work by Yvan Martel and Pierre Raphaël gives a new class of blow up multi-solitary waves blowing up in infinite time with logarithmic rate.

These special behaviours are due to strong interactions between the waves, in contrast with most previous works on multi-solitary waves of (NLS) where interactions do not affect the general behaviour of each solitary wave.

References: https://arxiv.org/abs/1512.00900 https://arxiv.org/abs/1611.08869

Luc Molinet

A rigidity result for the Camassa-Holm equation

Abstract: The Camassa-Holm equation possesses peaked solitary waves called peakons. We prove a Liouville property for uniformly almost localized (up to translations) H^1-global solutions of the Camassa-Holm equation with a momentum density that is a non negative finite measure. More precisely, we show that such solution has to be a peakon. As a consequence, we prove that peakons are asymptotically stable in the class of H^1-functions with a momentum density that belongs to $\mathcal{M}_+(\mathcal{R})$.

Andrea Nahmod

Randomization and dynamics in nonlinear PDE: an overview

Abstract: In this talk we give an overview on some basic probabilistic methods and questions in the study of the local and long time dynamics by focusing on 3 prototypes: i) Probabilistic well posedness for periodic nonlinear Schrödinger (NLS) equation; ii) probabilistic well posedness for derivative nonlinear wave equation with null form on $\mathbb{R}^2$ (DNLW) and iii) long time solutions to some fluid equations (periodic): Navier-Stokes (NS) and modified surface quasigeostrophic equation (mSQG).

Natasa Pavlovic

Infinitely many conserved quantities for the cubic Gross-Pitaevskii hierarchy in 1D

Abstract: The derivation of nonlinear dispersive PDE, such as the nonlinear Schrödinger (NLS) from many particle quantum dynamics is a central topic in mathematical physics, which has been approached by many authors in a variety of ways. In particular, one way to derive NLS is via the Gross-Pitaevskii (GP) hierarchy, which is an infinite system of coupled linear non-homogeneous PDE that describes the dynamics of a gas of infinitely many interacting bosons, while at the same time retains some of the features of a dispersive PDE.

In this talk we will look into what the nonlinear PDE such as the NLS can tell us about the GP hierarchy, and will present recent results on infinitely many conserved quantities for the cubic GP hierarchy in 1D that are obtained with Mendelson, Nahmod and Staffilani.

Fabrice Planchon
The wave equation on a model convex domain revisited

Abstract: We detail how the new parametrix construction that was developped for the general case allows in turn for a simplified approach for the model case and helps in sharpening both positive and negative results for Strichartz estimates.

Benjamin Schlein
Dynamical and spectral properties of Bose-Einstein condensates

Abstract: We consider systems of N bosons interacting through a repulsive potential $N^{3\beta-1}V(N^{\beta}x)$ that scales with N. For $\beta = 1$, we recover the well-known Gross-Pitaevskii regime. We present new techniques that allow us to prove the convergence towards the time-dependent Gross-Pitaevskii equation with optimal rate. Furthermore, we explain how, for small potentials, this approach can be used to show complete Bose-Einstein condensation (with a uniform bound on the number of excitations), for the ground state and, more generally, for states with small excitation energy. For $\beta < 1$, the same method can be used to establish the validity of Bogoliubov theory for the low-lying excitation spectrum. This talk is based on joint works with C. Boccato, C. Brennecke and S. Cenatiempo.

Catherine Sulem
Soliton Resolution for Derivative NLS equation

Abstract: We consider the Derivative Nonlinear Schrödinger equation for general initial conditions in weighted Sobolev spaces that can support bright solitons (but exclude spectral singularities). We prove global wellposedness and give a full description of the long- time behavior of the solutions in the form of a finite sum of localized solitons and a dispersive component. Our analysis provides explicit formulae for the multi-soliton component as well as the correction dispersive term. We use the inverse scattering approach and the nonlinear steepest descent method of Deift and Zhou (1993) revisited by the $\overline{\partial}$-analysis of Dieng-McLaughlin (2008) and complemented by the recent work of Borghese-Jenkins-McLaughlin (2016) on soliton resolution for the focusing nonlinear Schrödinger equation. This is a joint work with R. Jenkins, J. Liu and P. Perry.

Nicola Visciglia
On the growth of Sobolev norms for NLS in a compact setting

Abstract: We study the growth of higher order Sobolev norms for solutions to NLS posed in suitable compact settings. First we consider NLS posed on 2d and 3d compact manifolds and we provide in this framework a-priori bounds on the growth of the Sobolev norms H^s with $s > 1$. We also present some recent extensions and improvements in the context of NLS perturbed by an harmonic oscillator. In this situation we take advantage of suitable bilinear estimates, that as far as we know are not available in the framework of a compact manifold. The talk is based on joint works with F. Planchon and N. Tzvetkov.

Michael Weinstein

Dispersive waves in novel 2d media; Honeycomb structures, Edge States and the Strong Binding Regime

Abstract:

We discuss the 2D Schroedinger equation for periodic potentials with the symmetry of a hexagonal tiling of the plane. We first review joint work with CL Fefferman on the existence of Dirac points, conical singularities in the band structure, and the resulting effective 2D Dirac dynamics of wave-packets. We then focus on periodic potentials which are superpositions of localized potential wells, centered on the vertices of a regular honeycomb structure, corresponding to the single electron model of graphene and its artificial analogues. We prove that for sufficiently deep potentials (strong binding) the lowest two Floquet-Bloch dispersion surfaces, when appropriately rescaled, are uniformly close to those of the celebrated two-band tight-binding model, introduced by PR Wallace (1947) in his pioneering study of graphite. We then discuss corollaries, in the strong binding regime, on (a) spectral gaps for honeycomb potentials with PT symmetry-breaking perturbations, and (b) topologically protected edge states for honeycomb structures with ”rational edges. This is joint work with CL Fefferman and JP Lee-Thorp. Extensions to Maxwell equations (with Y Zhu and JP Lee-Thorp) will also be discussed.

Vladimir Zakharov

Unresolved problems in the theory of integrable systems

Abstract:

In spite of enormous success of the theory of integrable systems, at least three important problems are not resolved yet or are resolved only partly. They are the following:

1. The IST in the case of arbitrary bounded initial data.

2. The statistical description of the systems integrable by the IST. Albeit, the development of the theory of integrable turbulence.

3. Integrability of the deep water equations.

These three problems will be discussed in the talk.

Shorts Talks:

André De Laire

The Sine-Gordon regime of the Landau-Lifshitz equation with a strong easy-plane anisotropy

Abstract: It is well-known that the dynamics of biaxial ferromagnets with a strong easy-plane anisotropy is essentially governed by the Sine-Gordon equation. In this talk, we provide a rigorous justification to this observation. More precisely, we show the convergence of the solutions of the Landau-Lifshitz equation for biaxial ferromagnets towards the solutions of the Sine-Gordon equation in the regime of a strong easy-plane anisotropy.

Our result holds for solutions to the Landau-Lifshitz equation in high order Sobolev spaces. We provide an alternative proof for local well-posedness in this setting by introducing high order energy quantities with better symmetrization properties. We then derive the convergence from the consistency of the Landau-Lifshitz and Sine-Gordon equations by using well-tailored energy estimates. As a by-product, we also obtain a further derivation of the free wave regime of the Landau-Lifshitz equation.

This is joint work with Philippe Gravejat (Université de Cergy-Pontoise).

Luiz Gustavo Farah Dias

Instability of solitary waves for the critical KdV equation

Abstract: We revisit the classical result of Martel-Merle about instability of solitary waves for the critical KdV equation. We give another proof of this result without relying in a pointwise estimate for the solution. This is a joint work with Justin Holmer and Svetlana Roudenko.

François Genoud

Stable solitons in the 1D cubic-quintic NLS with a delta-function potential

Abstract: I will speak about the one-dimensional nonlinear Schrödinger equation with a combination of cubic focusing and quintic defocusing nonlinearities, and an attractive delta-function potential. This model comes from nonlinear optics, and the delta-function potential describes the interaction of a broad solitonic beam with a narrow defect. I will show that all spatial solitons can be determined explicitly in terms of elementary functions. Using bifurcation and spectral-theoretic arguments, I will then prove that all these solutions are (nonlinearly) stable. A noteworthy feature of the model is a regime of 'bistability', where two solitons with same propagation constant coexist. This is a joint work with Boris Malomed and Rada Weishäupl.

Cristi Guevara

Nonlinear Effects in the Exciton-Polariton System

Abstract: Exciton-polaritons are half-light, half-matter quantum quasiparticles arising from a strong coupling of quantum wells (excitons) and photon cavities (photons). The quasiparticles emerging from these mixing obey the Bose-Einstein statistics, condensate at high temperatures due to their ultra-small mass; their interaction is described by a quantum-mechanical system, the photon field provides dispersion and the exciton is responsible for a nonlinear behavior similar to the cubic nonlinear Schrödinger (NLS) and Gross-Pitaevskii equations.

Short-time analysis of the lossless system shows that, when the photon field is excited, the time required for that field to exhibit nonlinear effects is longer than the time required for the nonlinear Schrödinger equation, in which the photon field itself is nonlinear. For fixed initial data, nonlinear effects of order ϵ are observed at time $t = \epsilon/5$, as compared to NLS, for which nonlinear effects are observed at time ϵ. These power laws are generalized to initial data of order ϵ^{α} and nonlinearity power p.

Lysianne Hari

Scattering for Nonlinear Klein-Gordon equations posed on product spaces

Abstract: We will talk about the large time behavior of nonlinear dispersive PDEs, and will focus on the particular case of energy-subcritical Klein-Gordon equations. The behavior of such equations is well-known on the euclidean space $\mathcal{R}^d$: one can make a comparison, in some sense, with solutions of the linear equation (scattering). On the other hand, this kind of phenomenon does not appear on compact riemannian manifolds such as the kâ??dimensional flat torus $\mathbb{T}^k$. Our aim is to understand the leading behavior when one mixes the spaces, namely, if the equation is posed on a product space of the form $\mathbb{R}^d \times \mathbb{T}^k$.

In this first part of the talk, we will prove that some Strichartz estimates are available and will deduce small data scattering. Then, the case of large data scattering with defocusing nonlinearities will be handled and it will rely on a concentration-compactness and rigidity scheme.

Nikolaos Tzirakis

Dispersive partial differential equations on the half-line

Abstract: I will discuss recent developments on the well-posedness theory of the initial and boundary value problem for dispersive partial differential equations on the half line.