Using machine learning methods in geometric modeling

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Contents

- **Part 1: Learning knot placements** for b-spline curve approximation and t-spline surface skinning
  - Using support vector machines (SVMs) to learn good knot vectors for spline curve/surface approximation.

Contents

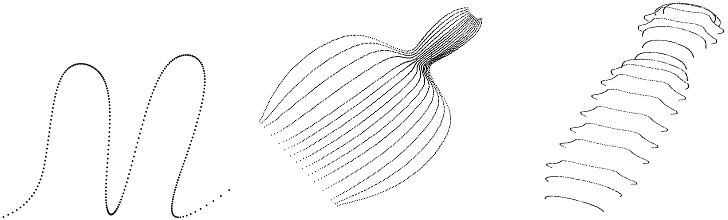
- **Part 1: Learning knot placements** for b-spline curve approximation and t-spline surface skinning
  - Using support vector machines (SVMs) to learn good knot vectors for spline curve/surface approximation.
- **Part 2: Learning surface primitive classification** from point clouds
  - Using SVMs to classify point clouds to geometric primitive classes.
  - Using stacked auto-encoders (SAEs) to learn geometric features used for classification.

Part 1: Learning knot placements for b-spline curve and t-spline surface approximation

2d-input Given a sequence of points $\mathbf{p}_i$ for curve approximation.

3d-input Given an array of sequences of points for surface skinning (lofting).

Output: Good knot vector(s) for spline approximation.



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Curve approximation

Curves: Compute the control points c_j of a b-spline curve of degree l

$$C(t) = \sum_{j=0}^{J} c_j \cdot N_j^l(t)$$

minimizing

$$\sum_{i=0}^{m} \|\mathbf{p}_i - C(t_i)\|^2.$$

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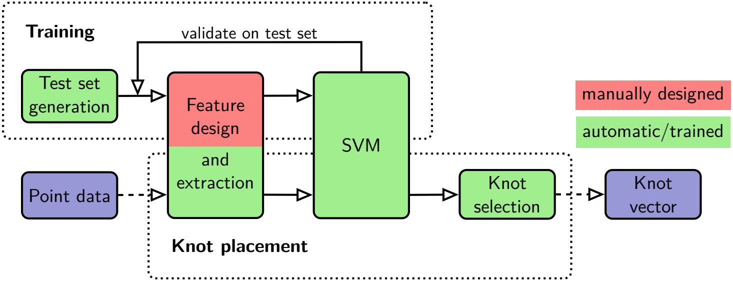
Skinning: Later...

Approach

- Design geometric point features for knot placement.
- Train a support vector machine (SVM) to learn the quality of a candidate knot.
  - For validation use approximation of test data.
- Use the trained SVM to generate knot vectors for new approximation tasks.



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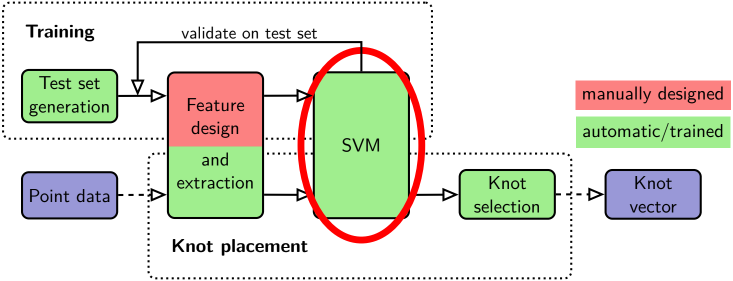
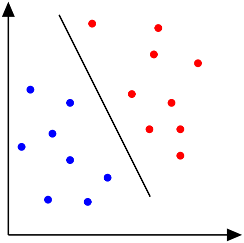


Support vector machines (SVMs) (1)

1. Map data points to feature vectors $\mathbf{x}_i$.
   - For training label the data with correct class y_i.



Support vector machines (SVMs) (1)

1. Map data points to feature vectors $\mathbf{x}_i$.
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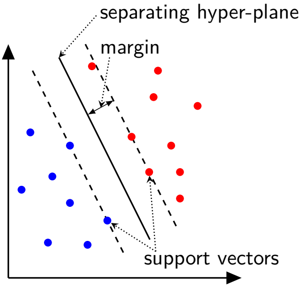
2. For a linear decision function $\mathrm{sign}(\omega^t \mathbf{x} + b)$, maximize the margin between the classes by minimizing

$$\omega^t \omega / 2 \qquad \text{s.t.} \qquad y_i(\omega^t \mathbf{x}_i + b) \geq 1.$$



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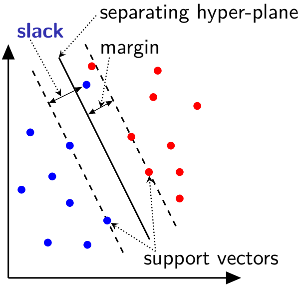
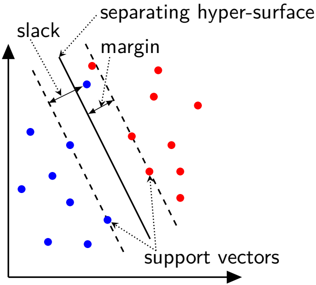
3. For noisy data add **slack** ξ_i, i.e. minimize

$$\omega^t \omega / 2 + C \sum \xi_i \qquad \text{s.t.} \qquad y_i(\omega^t \mathbf{x}_i + b) \geq 1 - \xi_i.$$



Support vector machines (SVMs) (2)

4. For non-linearly-separable data, replace the scalar product with a kernel $K(\mathbf{x}_i, \mathbf{x}_j)$, i.e. use the decision function
$$\operatorname{sign}\left(\sum \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b\right).$$
   - This is solved using Lagrange multipliers and dualization with $\omega = \sum \alpha_i y_i \mathbf{x}_i$.



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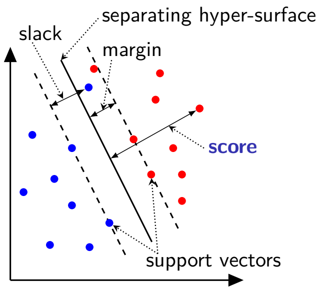
5. The **score** measures the signed distance to the separating hyper-surface.

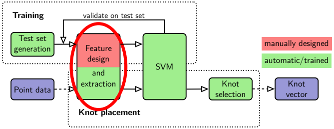


Geometric features for curve knot placement (1)

What are good **geometric features for knot placement?**

See e.g. [Park/Lee2007; Piegl/Tiller2000+2012; Razdan; Yuan/Chen/Zhou2013; etc.].

- Point distances.



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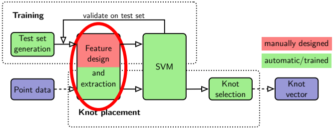

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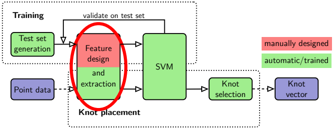
$$\kappa_i > \max(\kappa_{i-1}, \kappa_{i+1}) \quad \text{and} \quad \kappa_i \geq \kappa_{\text{average}}.$$



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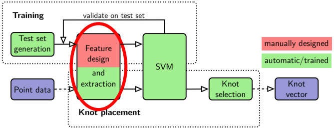
- Features derived from the LCM
  - Angles to closest LCM.
  - Euclidean, arc-length, or parametric point distances to the closest LCM.



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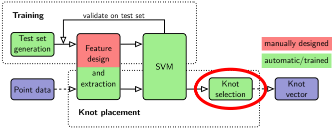
Geometric features for curve knot placement (2)

Subsequent **knot selection**

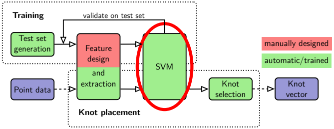
- Use salient features as knots and, in regions without salient features, sample uniformly,
- ... or use averages of uniform parameter averages...



Score-based knot placement

How well do points with **salient geometric features perform for approximation?**

- **Score maxima** of a trained SVM.
  - Choose peaks by prominence: Highest value you have to at least descend to to reach a higher peak.



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- **Score maxima** of a trained SVM.
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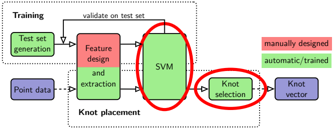
Subsequent **knot selection**

- For knot insertion, find closest score peak to middle point of knot segment with maximum score-sum. .
- In regions without score peaks, sample uniformly or score-weighted.



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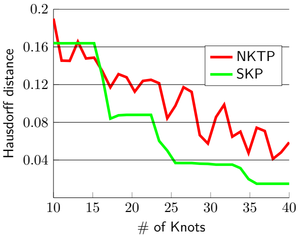
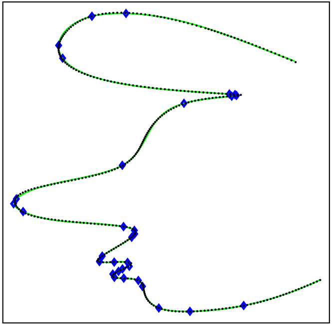
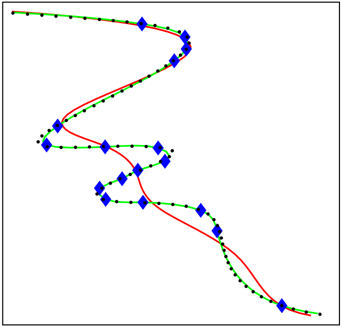
Results for SVM curves knot placement

- Score from high score (red) to low score (blue)

Results for b-spline curve approximation







Surface skinning

Approximation and approach analog to curves.

Curves: ... done.

Skinning: Compute the control points $c_{i,j}$ of a t-spline surface of degree (k, l)

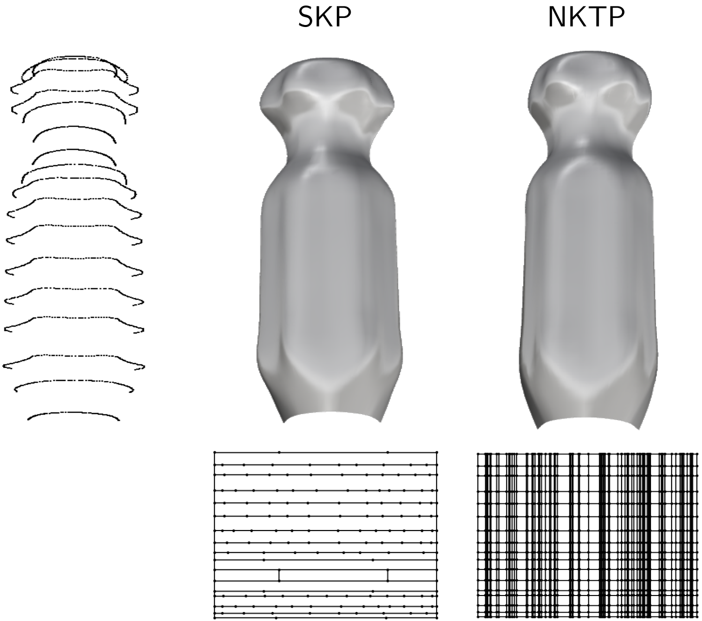
$$S(u, v) = \sum_{j=0}^{J} \sum_{i=0}^{I_j} c_{i,j} \cdot N_{i,j}^k(u) \cdot N_j^l(v)$$

minimizing

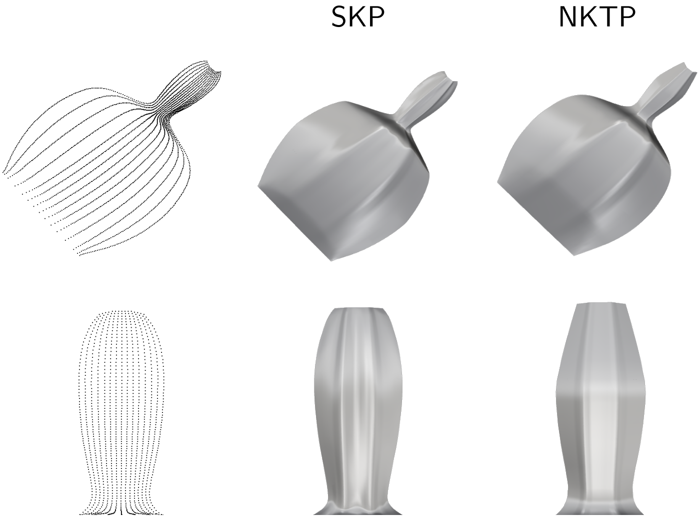
$$\sum_{j=0}^{m} \sum_{i=0}^{n_j} \|\mathbf{p}_{i,j} - S(u_i, v_j)\|^2$$

where each $N_{i,j}^k$ has its own knot vector $\mathbf{u}^{(j)}$.

Results for t-spline skinning (1)

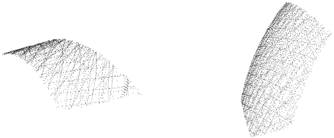


Results for t-spline skinning (2)

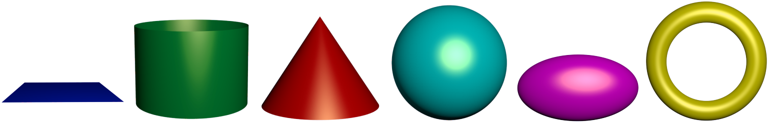


Part 2: Learning surface primitive classification

Input: Point clouds sampled from a patch of a primitive surface.



Output: Classification to the correct primitive class.

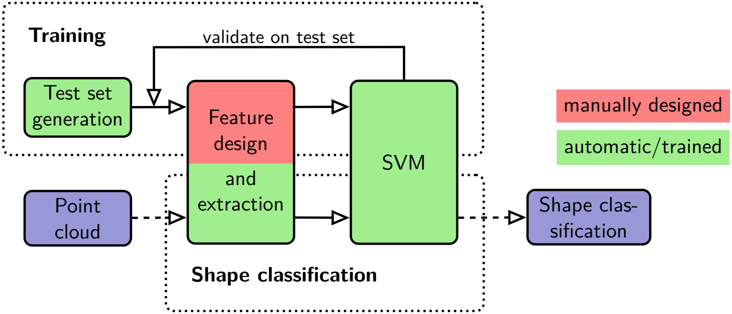


Planes Cylinders Cones Spheres Ellipsoids Tori

SVM primitive classification (1)

General approach analog to knot-placement.

- Design geometric point cloud features for primitive classification.
- Train a support vector machine (SVM) to learn the corresponding primitive class.
  - For validation use pre-labeled of test data.
- Use the trained SVM to classify new point clouds.



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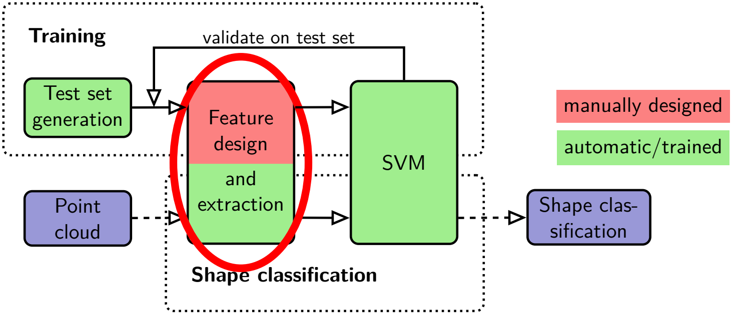


SVM primitive classification (2)

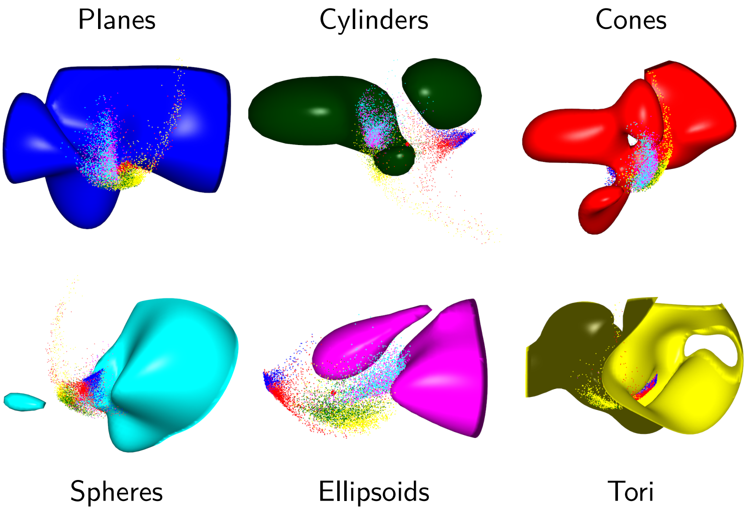
Point clouds are mapped to feature vectors.

See e.g. [Koenderink/van Doorn1992; Osada et al. 2002; Wahl et al. 2003; etc.].

- Point relation features:
  - Point distances, point angles, triangle areas, etc.
- Normal based features:
  - Normal directions, normal angles, etc.
- Curvature based features:
  - Curvature directions, curvature angles, shape index, etc.
- Hybrid features:
  - Surflet pairs, etc.

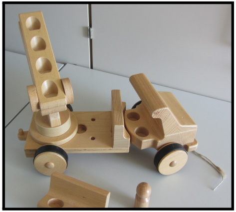
Results of SVM primitive classification (1)

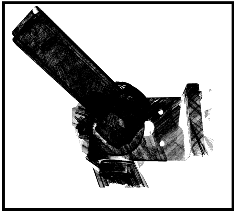
- Separating hyper-surfaces projected to 3d.

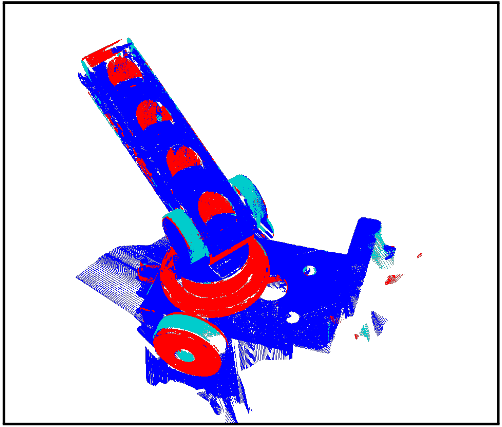


Results of SVM primitive classification (2)

- Classification of a manual 3d-scan.



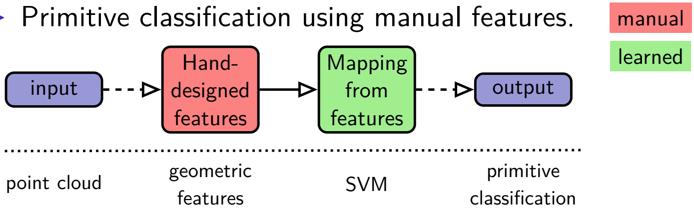




planes, cones, spheres.

Learning geometric features using SAEs

- Primitive classification using manual features.



Learning geometric features using SAEs

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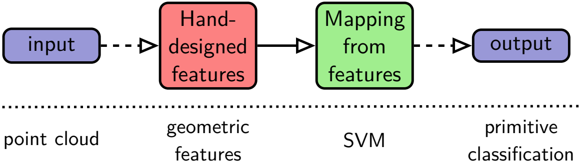
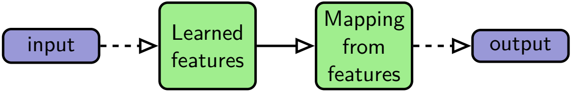
manual

learned



- Learning geometric features for classification.



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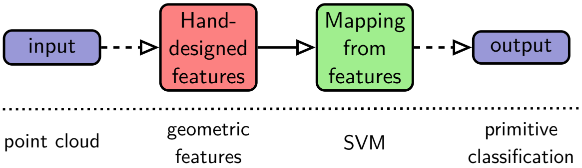

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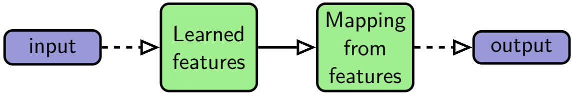

- Stacked auto-encoders (SAEs).

Learning geometric features using SAEs

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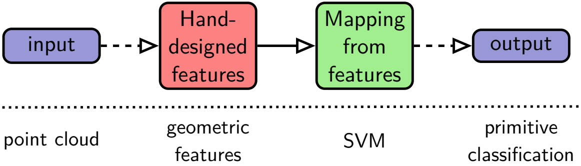

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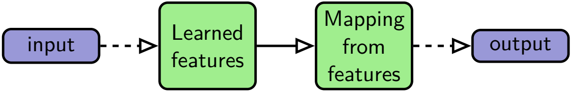

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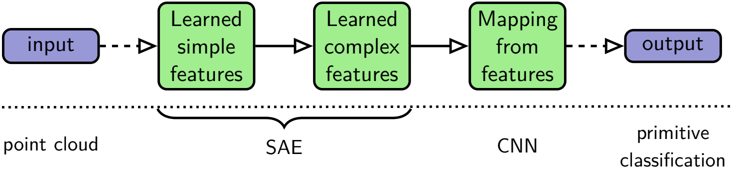

Feature engineering using SAEs

- A stacked auto-encoder (SAE) is
  - a multi-layer perceptron
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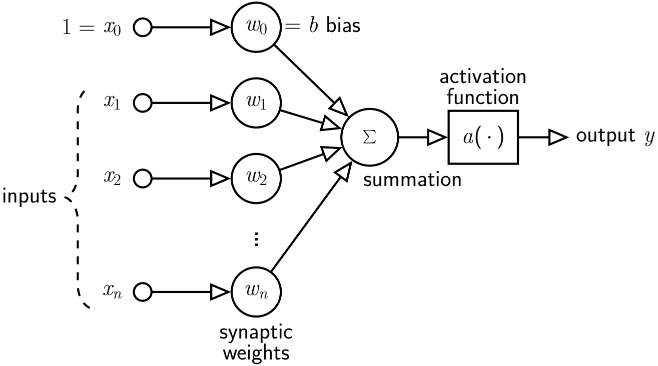
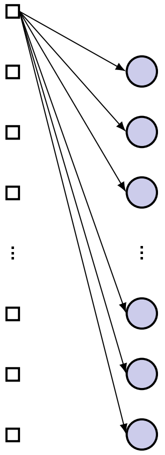
① Perceptron

- A **perceptron** implements for the inputs $x_1, \ldots, x_n$ the function
$$y = a\left(b + \sum_{i=1}^{n} w_i \cdot x_i\right).$$



② Deep neural nets (DNNs)

- **Deep neural nets**: multiple layers of perceptrons.



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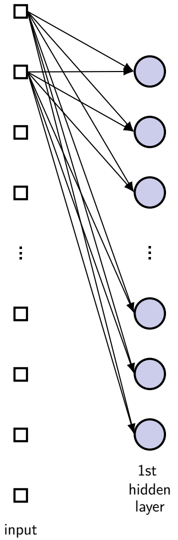

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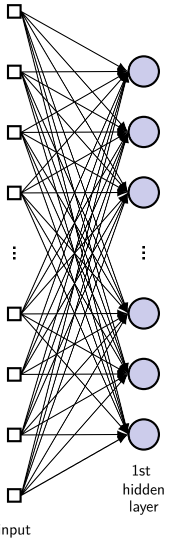

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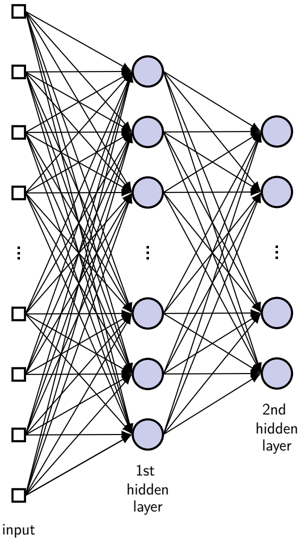

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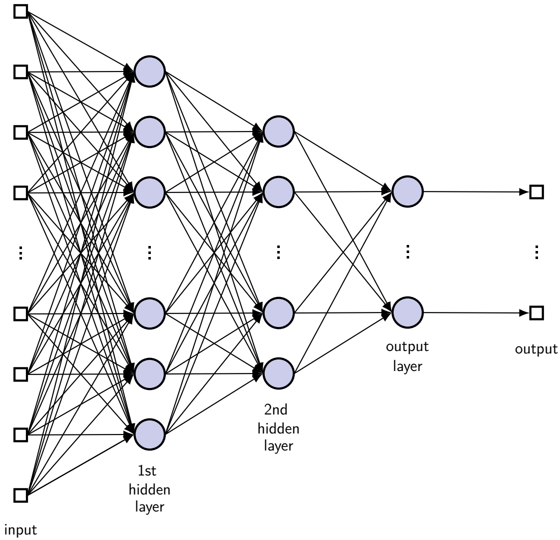
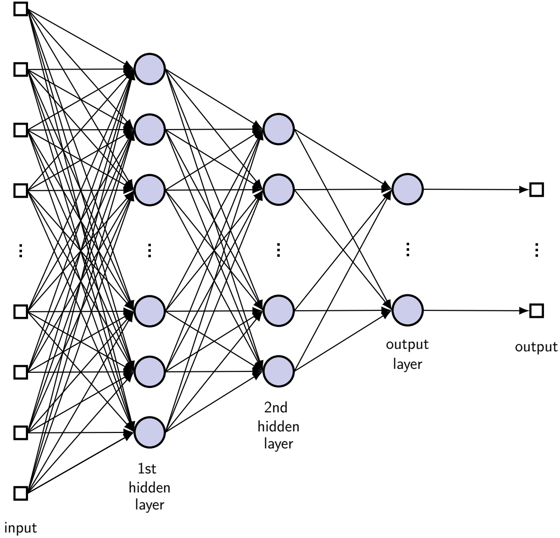


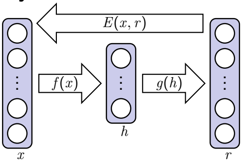
② Deep neural nets (DNNs)

- **Deep neural nets**: multiple layers of perceptrons.
- Training via back-propagation and gradient descent.



③ Stacked auto-encoders (SAEs)

- A auto-encoder minimizes an error $E(x,r)$ on each layer.



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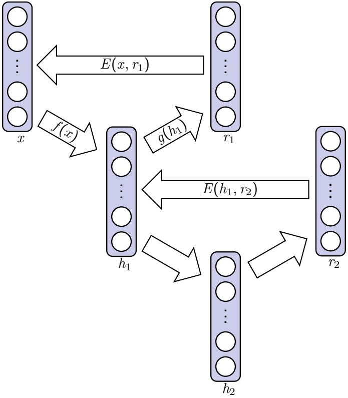

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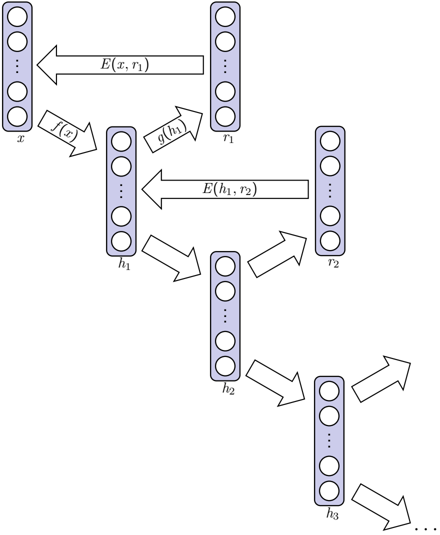

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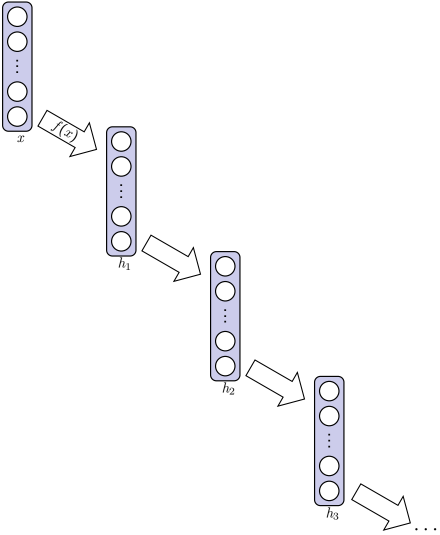
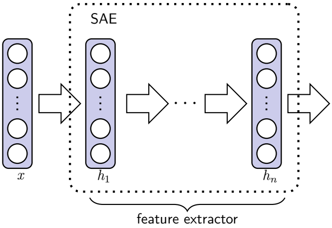


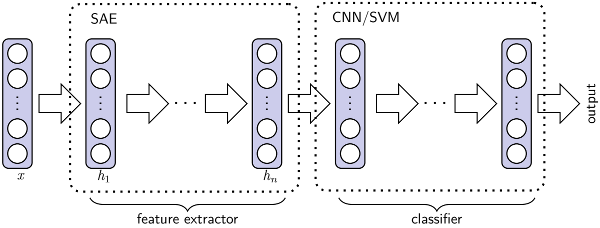
③ Stacked auto-encoders (SAEs)

- A **stacked auto-encoder** minimizes an error $E(x, r)$ on each layer.
- The trained SAE can be used as feature extractor.



③ Stacked auto-encoders (SAEs)

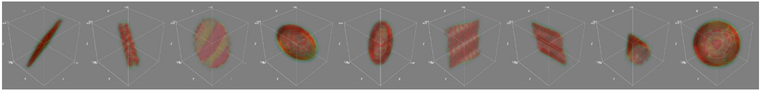
- A **stacked auto-encoder** minimizes an error $E(x, r)$ on each layer.
- The trained SAE can be used as feature extractor.
- The features learned by the SAE are used to train a second DNN to classify the data.



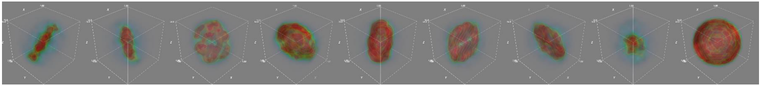
Learned geometric features (1)

- The auto-encoder is capable of representing geometric primitives.

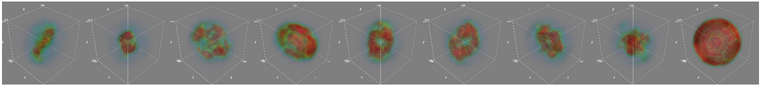
original



DAE, 4100 neurons



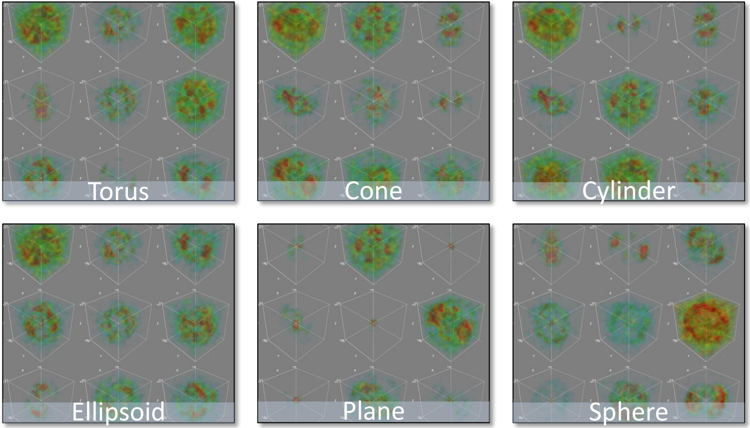
SDAE, 3 layers, 4100 neurons



Learned geometric features (2)

- How can we interpret the un-supervised learned geometric features?

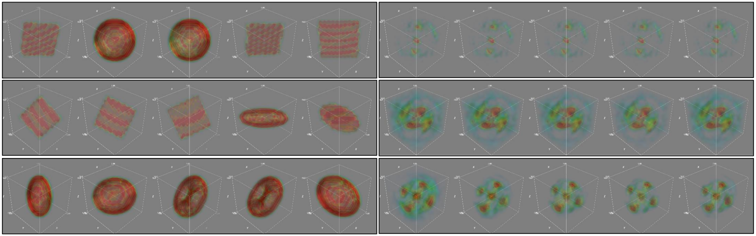
SDAE 3x4100 (Gaussian noise = 0.5)



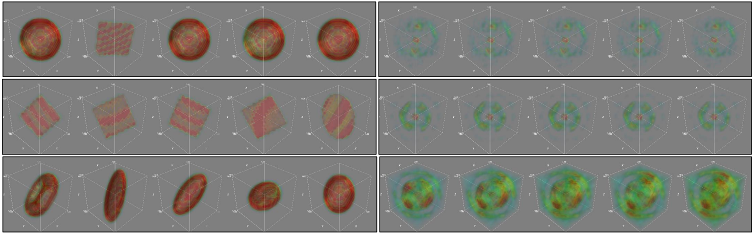
Learned geometric features (2)

- Saliency maps.

SAE 3x4100 (no noise)



SDAE 3x4100 (Gaussian noise = 0.5)



Thank you!

Questions?