Mathematical Modelling of Plankton-Oxygen Dynamics under the Climate Change

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Contents of the Talk

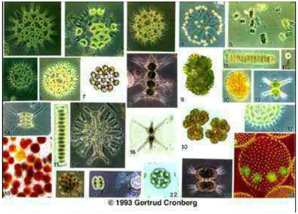
- Introduction: what it is all about and why it is important
- Two-species model and first insights
- Oxygen-phyto-zooplankton model
- Effect of the global warming
- Long term transients and complex dynamics
- Discussion, open problems, future work

Plankton: general information

Plankton is the most abundant taxa in the ocean. It is divided into two groups such as phytoplankton (algae) and zooplankton (animals).

Phytoplankton

- Marine food chain is based on phytoplankton.
- Phytoplankton contains chlorophyll and hence produces oxygen.



Zooplankton

- Zooplankton feed on phytoplankton (and is consumed by larger animals)
- Zooplankton can respond rapidly to algal blooms

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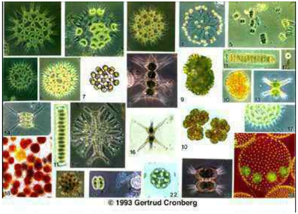

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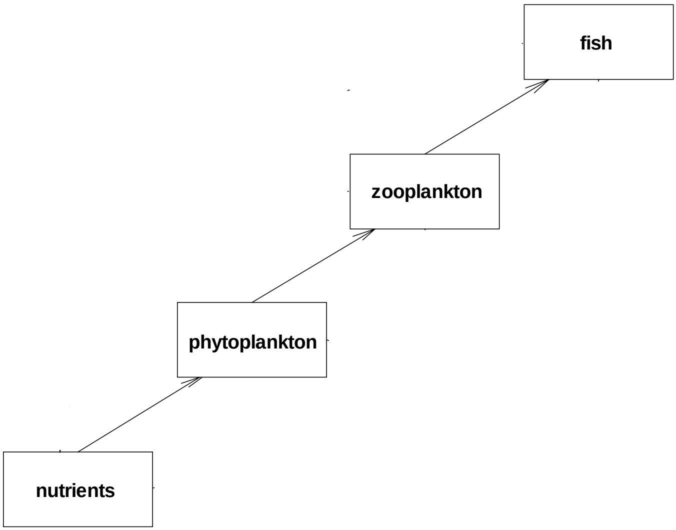
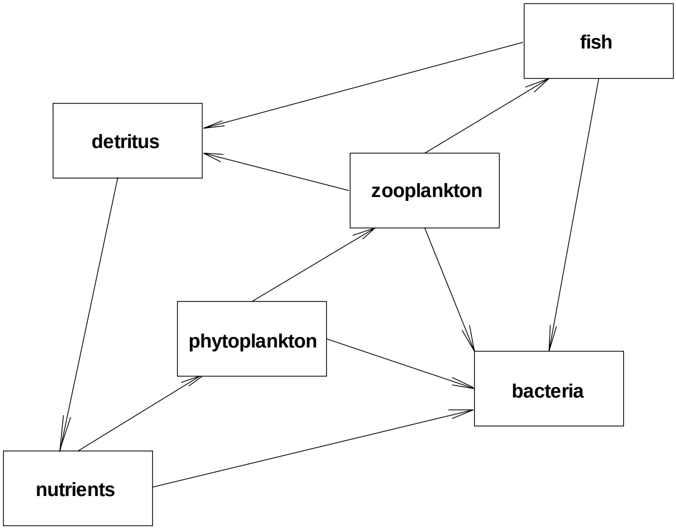
Marine food chain

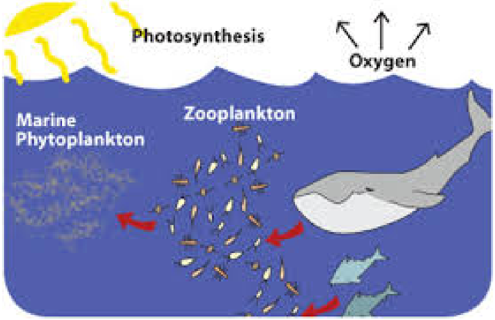


...or, rather, a food web (simplified)



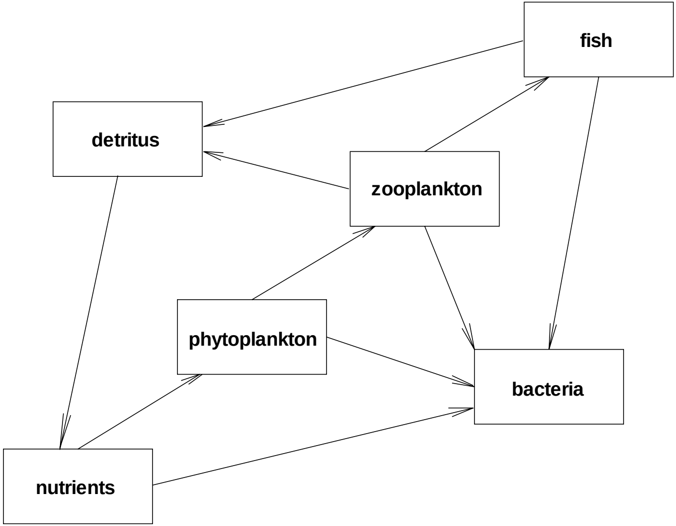
Importance of the marine systems

- 70% of the earth's surface is covered by the oceans
- 70% of atmospheric oxygen is produced by phytoplankton in the oceans due to the photosynthetic activity of phytoplankton



http://www.marinephytoplankton.com/shop/

Oxygen is a vital component of marine ecosystem



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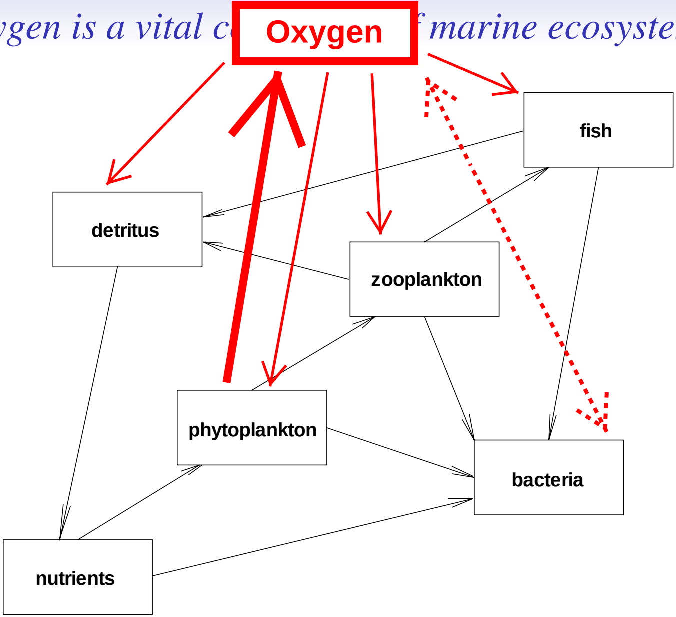


Dynamical systems approach to ecological modelling

Consider an ecosystem that consists of n species, $u_1, \ldots, u_n$.

A generic model is

$$\begin{aligned}
\frac{du_1}{dt} &= f_1(u_1, \ldots, u_n), \\
&\ldots \\
\frac{du_n}{dt} &= f_n(u_1, \ldots, u_n).
\end{aligned}$$

A steady state (equilibrium) is an array $(\bar{u}_1, \ldots, \bar{u}_n) = \bar{\mathbf{u}}$ such that

$$f_1(\bar{\mathbf{u}}) = \ldots = f_n(\bar{\mathbf{u}}) = 0.$$

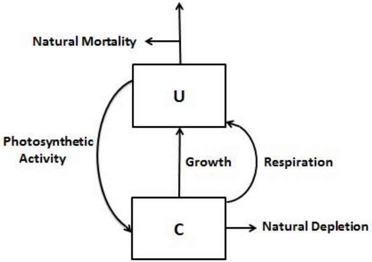
Of particular relevance are **stable** steady states.

Part I: Plankton-Oxygen Model

Oxygen-phytoplankton model

$$\frac{dc}{dt} = \frac{Au}{c+1} - \frac{uc}{c+c_2} - c,$$

$$\frac{du}{dt} = \left(\frac{Bc}{c+c_1} - u\right) u - \sigma u.$$



c $\rightarrow$ Oxygen, u $\rightarrow$ Phytoplankton

$A \rightarrow$ is the rate of oxygen production,

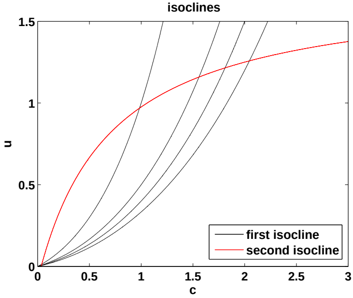
$c_2 \rightarrow$ is half saturation constant,

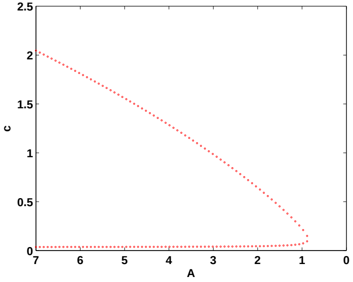
$B \rightarrow$ is growth rate of phytoplankton,

$\sigma \rightarrow$ is natural mortality rate of phytoplankton.

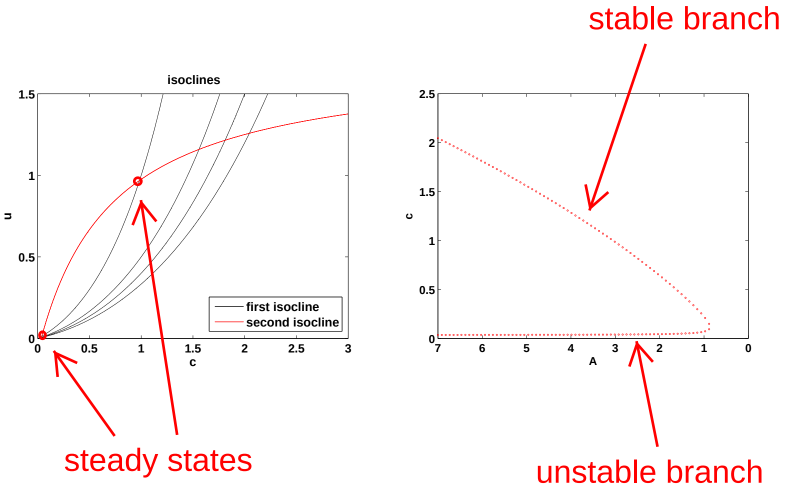
Sekerci & Petrovskii (2015) Bull. Math. Biol. 77, 2325-2353.

System steady states

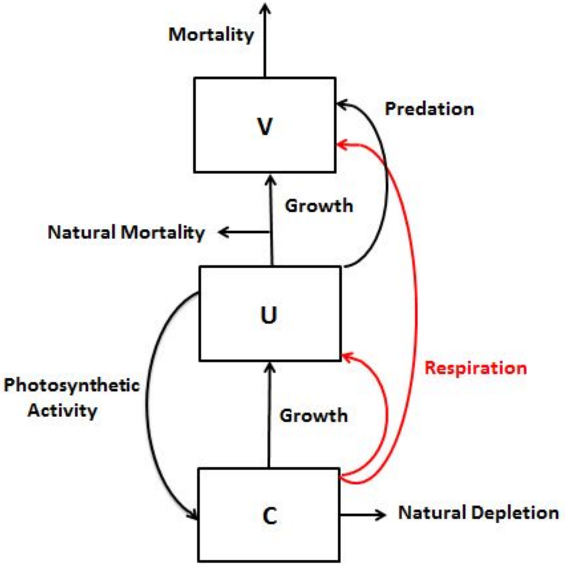




System steady states



Oxygen-phytoplankton-zooplankton model: flowchart



Oxygen-phyto-zooplankton model: equations

$$\frac{dc}{dt} = A(1 - \frac{c}{c+1})u - \frac{uc}{c+c_2} - c - \frac{\nu cv}{c+c_3},$$

$$\frac{du}{dt} = (\frac{Bc}{c+c_1} - u)\ u\ - \sigma u - \frac{\beta uv}{u+h},$$

$$\frac{dv}{dt} = \left(\frac{\eta c^2}{c^2 + {c_4}^2}\right) \frac{uv}{u+h} - \mu v.$$

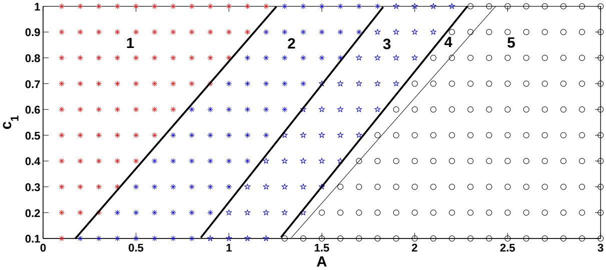
where

$\frac{\nu cv}{c+c_3}$ describes the oxygen consumption by zooplankton,

$\frac{\beta uv}{u+h}$ describes the zooplankton feeding on phytoplankton.

Sekerci & Petrovskii (2015) Bull. Math. Biol. 77, 2325-2353.

Stability of system's steady states



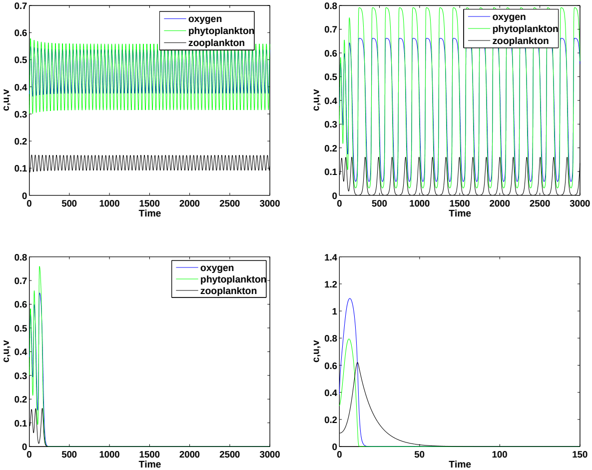
A map in the parameter plane (A, c_1) where different domains correspond to different system's stability.

Domain 1 $\longrightarrow$ extinction I: disappearance of the positive steady state,

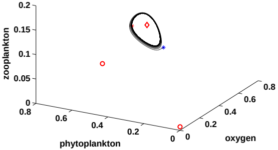
Domain 2 $\longrightarrow$ $E_2^{(2)}$ and E_3 are stable,
Domain 3 $\longrightarrow$ E_3 is stable,
Domain 4 $\longrightarrow$ E_3 is unstable focus,
Domain 5 $\longrightarrow$ extinction II: no stable attractor.

Sekerci & Petrovskii (2015) Bull. Math. Biol. 77, 2325-2353.

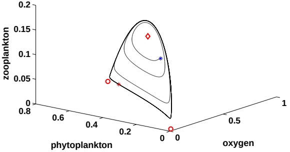
Examples of temporal (local) dynamics



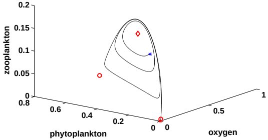
The system is unsustainable when A is too large, i.e. for $A > A_{cr}$.


zooplankton
phytoplankton
oxygen

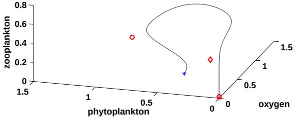
(a)


zooplankton
phytoplankton
oxygen

(b)


zooplankton
phytoplankton
oxygen

(c)


zooplankton
phytoplankton
oxygen

(d)

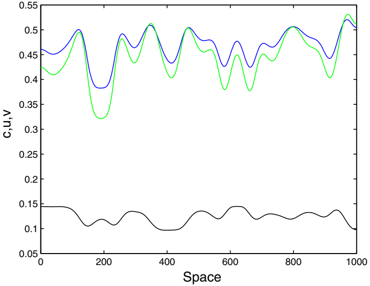
How to turn local dynamics to global – include space

Spatial transport & mixing of plankton in the ocean is mostly due to the ocean turbulence:

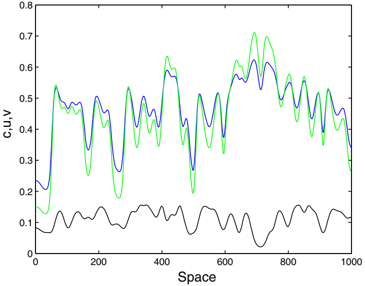
$$\begin{aligned}
\frac{\partial c}{\partial t} &= D_T \nabla^2 c + \frac{Au}{c+1} - \frac{\delta uc}{c+c_2} - \frac{\nu cv}{c+c_3} - c, \\
\frac{\partial u}{\partial t} &= D_T \nabla^2 u + \left(\frac{Bc}{c+c_1} - u \right) u - \frac{uv}{u+h} - \sigma u, \\
\frac{\partial v}{\partial t} &= D_T \nabla^2 v + \frac{\eta c^2}{c^2 + {c_4}^2} \, \frac{uv}{u+h} - \mu v,
\end{aligned}$$

where D_T is the turbulent diffusion coefficient

In the parameter range where the coexistence state is unstable, a generic property of this system is pattern formation, in 1D:



$A = 2.02$



$A = 2.054$

as well as in 2D: ($A = 2.09$) ($A = 2.3$)

Sekerci & Petrovskii (2015) Bull. Math. Biol. 77, 2325-2353.

Malchow, Petrovskii & Venturino (2008) Spatiotemporal Patterns in Ecology & Epidemiology. CRC Press.

Part II: Effect of the Global Warming

Effect of the global warming: *biological evidence*

Net oxygen production (by phytoplankton) is the difference between the amount of oxygen produced in photosynthesis and the amount of oxygen consumed due to phytoplankton breathing

Oxygen production rate and consumption rate depend on water temperature differently (e.g. Jones 1977, Robinson 2000, Hancke & Glud 2004)

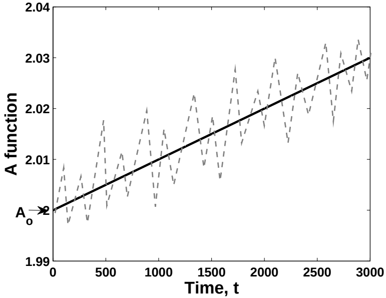
As a result, the net oxygen production is a function of water temperature. For some plankton species, it can drop to zero if water temperature increases by about 6 C (Robinson 2000).

Whether it typically increase or decrease remains unclear

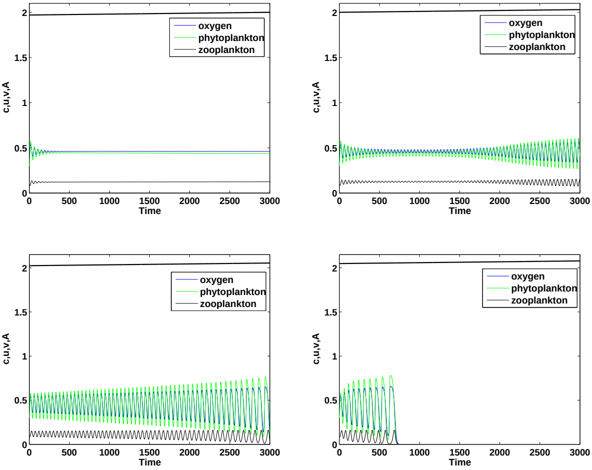
Effect of the global warming: *how to model?*

$$A = A_0 \quad \text{for} \quad t < t_1, \qquad A = A_0 + \omega\,(t - t_1) \quad \text{for} \quad t \geq t_1.$$

$t_1 \rightarrow$ is the moment when global warming started,
$A_0 \rightarrow$ is the rate of net O_2 production before changes,
$w \rightarrow$ quantifies the rate of global warming.



Response to warming - local dynamics



(a) $A_0 = 1.97$, (b) $A_0 = 2$, (c) $A_0 = 2.024$, (d) $A_0 = 2.048$, and $\omega = 10^{-5}$.

Global warming in the spatially explicit model

$$
\begin{aligned}
\frac{\partial c}{\partial t} &= D_T \nabla^2 c + \frac{A(t) u}{c+1} - \frac{\delta u c}{c + c_2} - \frac{\nu c v}{c + c_3} - c, \\
\frac{\partial u}{\partial t} &= D_T \nabla^2 u + \left(\frac{Bc}{c + c_1} - u \right) u - \frac{uv}{u+h} - \sigma u, \\
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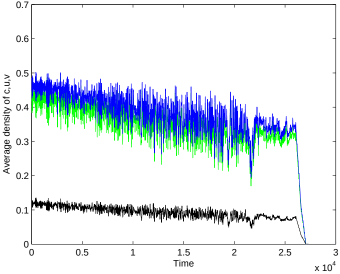
\end{aligned}
$$

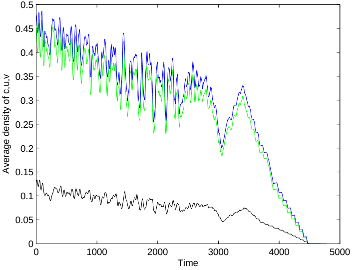
With an increase in A, spatiotemporal patterns are evolving with time to eventually result in extinction

Spatial system's response to global warming

Spatially average O_2 concentration and plankton densities over time:



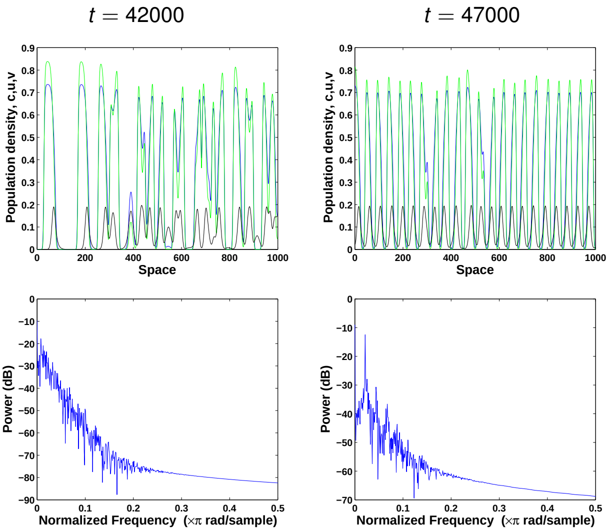
$\omega = 10^{-5},\ A_0 = 2.048$



$\omega = 10^{-4},\ A_0 = 2.05.$

Global warming eventually results in **complete depletion of oxygen**

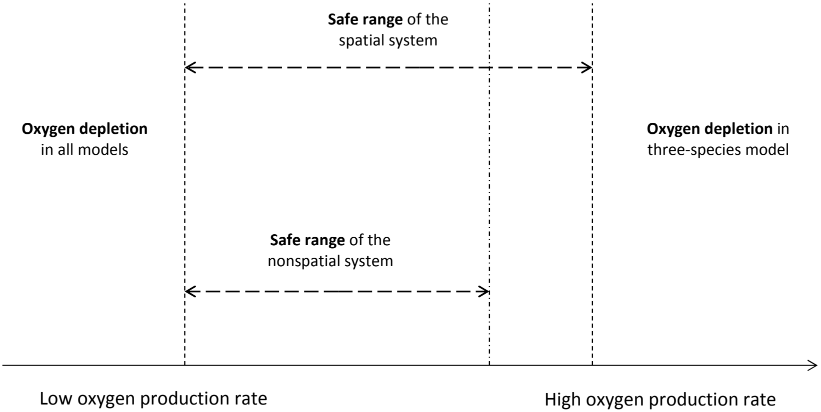
Spatial distribution changes when the system is approaching the catastrophe:



Regularity of the spatial distribution as an **early warning signal**?

Synopsis

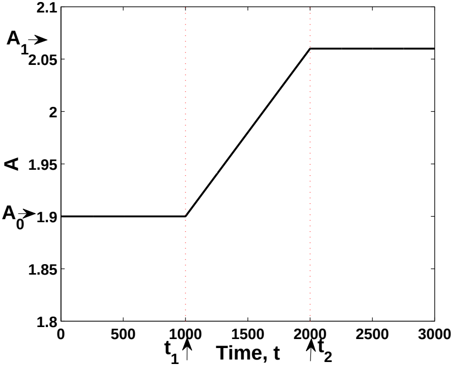
The prediction of global oxygen depletion appears to be robust to the model choice:



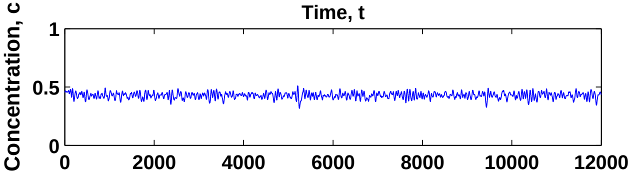
Part III: Scenarios of the Global Warming

What if the global warming stops?

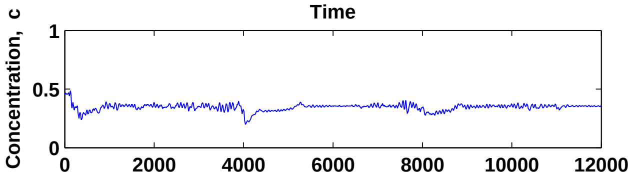
$$T(t) = \begin{cases} T_0, & 0 \leq t \leq t_1, \\ T_0 + \frac{T_1 - T_0}{t_2 - t_1}(t - t_1), & t_1 \leq t \leq t_2, \\ T_1, & t_2 \leq t. \end{cases}$$



The outcome is sensitive to the final value of A...



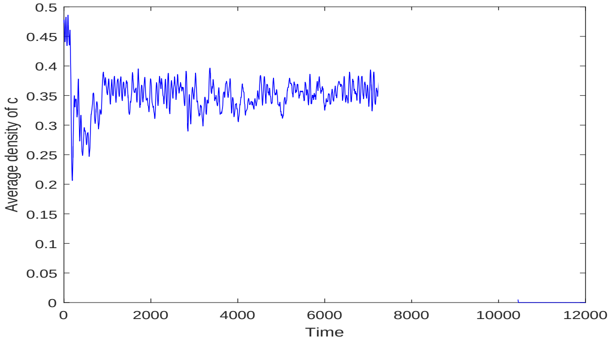
$A_1 = 2.1$



$A_1 = 2.2779$

Other parameters $A_0 = 2.05$, $t_1 = 100$ and $t_2 = 270$.

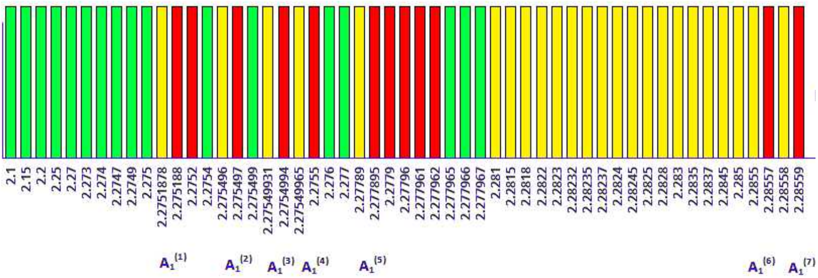
...so that a small change in A_1 can turn persistence to extinction, but only after a **long-living transient dynamics**:



$A_1 = 2.2752$

Other parameters $A_0 = 2.05$, $t_1 = 100$ and $t_2 = 270$.

The system exhibits intermittency with regards to parameter A_1:



The long-term outcome of the oxygen-plankton system:

green bar - existence, irregular spatial pattern,
yellow bar - existence, “regular” spatial pattern,
red bar - extinction.

Summary & Conclusions

- Net oxygen production by phytoplankton depends on water temperature, hence it can be expected that oxygen production can disrupted by the global warming

- Our models show that oxygen production can stop suddenly, i.e. without the O_2 concentration necessarily dropping down prior to the disaster.

  Increased regularity of the spatial pattern is an early warning signal of the approaching catastrophe

- Sustainable oxygen production is only possible in an intermediate range of the O_2 production rate

- For a more complex scenario of global warming, extinction can be preceded by long-term transient dynamics

Open questions & future work

- To make the model more realistic, e.g. to include more ecosystem components (nutrients, bacteria, etc.) and/or to take into account details of ocean hydrodynamics

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- Laboratory study an a relevant plankton-oxygen system?

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- Laboratory study an a relevant plankton-oxygen system?

- To examining palaeontological records?
  - The important role of oxygen in the geological history has been recognized (e.g. Berner et al.: Oxygen and Evolution, Science (2007) 316:557-558)
  - Five events of mass extinction during the last 550 My, at least some of them are thought to by caused by the drop in atmospheric oxygen concentration – but the specific mechanism remains unclear

Acknowledgement

- Yadigar Sekerci Firat (Leicester/Malatya)
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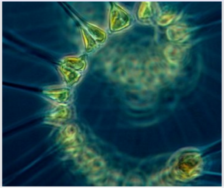
- Ezio Venturino (Torino)

Thanks for listening



An increase in the water temperature of the world's oceans of around six degrees Celsius -- which some scientists predict could occur as soon as 2100 -- could stop oxygen production by phytoplankton by disrupting the process of photosynthesis.

Credit: NOAA MESA Project.

Falling oxygen levels caused by global warming could be a greater threat to the survival of life on planet Earth than flooding, according to researchers from the University of Leicester.

Urban decay: Humanity faces an uncertain future

In the study, published in the Bulletin of Mathematical Biology , the professor and his team developed a new model of ocean-based oxygen production.

The researchers suggested global warming could interrupt the process of photosynthesis - the natural process by which phytoplankton produce up to half of the world's oxygen supply.

Emerging threats: Plankton peril

Depletion of ocean phytoplankton could suffocate life on planet Earth

Published 4 December 2015

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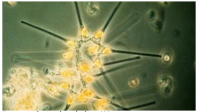
About two-thirds of the planet's total atmospheric oxygen is produced by ocean phytoplankton — and therefore cessation would result in the depletion of atmospheric oxygen on a global scale, which could threaten the mortality of animals and humans.



Oxygen-producing phytoplankton // Source: noaa.gov

Scientists reveal how Earth's oxygen could dramatically fall due to change in ocean temperature of just several degrees.

Falling oxygen levels caused by global warming could be a greater threat to the survival of life on planet Earth than flooding,

Global warming could suffocate the planet before it floods, say scientists from Leicester

By PA_Warzynski | Posted: December 02, 2015