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Effects of host diversity and complex foraging on the dynamics of trophically transmitted parasites

Virgile BAUDROT, Clémentine FRITSCH, Antoine PERASSO and Francis RAOUL

UMR CNRS 6249 Chrono-environnement – Université Bourgogne Franche-Comté (Besançon, France)









What flow in food webs?

- Matter and energy (Lindeman, 1942; Loreau 2010)
 Occurrence and strength of links
- Parasites (Lafferty et al., 2006)
 40 75 % of food web links (Dobson et al., 2008)

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Multi-host parasites: *Echinococcus multilocularis*





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How prey **biodiversity** influence the **dynamics** of trophically transmitted **parasites**?

Part 1 - Baudrot et al. (2016), Ecology

How predators adapt their foraging to changes in the prey community?



Multi-Species Functional Responses – MSFR: $\Phi_i(\vec{x})$

Rate of ingestion of a prey *i* depending on the prey community (Oaten and Murdoch, 1975; Gentleman et al., 2003)

A general mechanism...



Main previous studies: Poggiale et al. (1998) ; Jeschke (2002); Leeuwen et al. (2007)

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... giving a general formulation...

$$\Phi_i(\vec{x}) = \underbrace{p_i(\vec{x})}_{\Theta(\vec{x})} \times \Theta(\vec{x})$$

... several phenomenological properties...

(Holling 1959; Murdoch, 1972; Oaten and Murdoch, 1975; Tilman 1980, Holt 1983; Chesson, 1984; Gentleman et al., 2003; Van Leeuwen et al., 2007; Morozov and Petrovskii, 2013; Vallina et al., 2014)







Data collection and fitting

Frequency-dependent and density-dependent switching are critical properties inclusion in the epidemiological model...







Population growthPredationEpidemic
$$\frac{dz_S}{dt} = b_z z - \left(m_z + (b_z - m_z)\frac{z_S + z_I}{k_z}\right) z_S$$
 $+\mu z_I$ $\frac{dx_{iS}}{dt} = bx_i - \left(m + (b - m)\frac{\sum_j x_{jS} + x_{jI}}{k}\right) x_{iS}$ $-p_i(x_1, x_2)\Theta(x_1, x_2)\frac{x_{iS}}{x_i} z$ $\frac{dz_I}{dt} = -\left(m_z + (b_z - m_z)\frac{z_S + z_I}{k_z}\right) z_I$ $-\mu z_I$ $\frac{dx_{iI}}{dt} = -\left(m + (b - m)\frac{\sum_j x_{jS} + x_{jI}}{k}\right) x_{iI}$ $-p_i(x_1, x_2)\Theta(x_1, x_2)\frac{x_{iI}}{x_i} z$

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$$\frac{dz_S}{dt} = b_z z - \left(m_z + (b_z - m_z)\frac{z_S + z_I}{k_z}\right) z_S$$
 $-z_S \sum_i \eta_i \Phi_i(x_1, x_2)\frac{x_{iI}}{x_i} + \mu z_I$ $\frac{dx_{iS}}{dt} = bx_i - \left(m + (b - m)\frac{\sum_j x_{jS} + x_{jI}}{k}\right) x_{iS}$ $-p_i(x_1, x_2)\Theta(x_1, x_2)\frac{x_{iS}}{x_i} z$ $\frac{dz_I}{dt} = -\left(m_z + (b_z - m_z)\frac{z_S + z_I}{k_z}\right) z_I$ $+z_S \sum_i \eta_i \Phi_i(x_1, x_2)\frac{x_{iI}}{x_i} - \mu z_I$ $\frac{dx_{iI}}{dt} = -\left(m + (b - m)\frac{\sum_j x_{jS} + x_{jI}}{k_z}\right) x_{iI}$ $-p_i(x_1, x_2)\Theta(x_1, x_2)\frac{x_{iI}}{x_i} z$ $\frac{dx_{iI}}{dt} = -\left(m + (b - m)\frac{\sum_j x_{jS} + x_{jI}}{k_z}\right) x_{iI}$ $-p_i(x_1, x_2)\Theta(x_1, x_2)\frac{x_{iI}}{x_i} z$



How biodiversity of IH may influence the dynamic of the parasite?



Measure of disease risk : Basic reproductive number = \mathcal{R}_0

Expected number of secondary cases per primary case in a disease-free population

Disease Free Equilibrium - DFE

Total pop. = \sum clinical state \implies DFE = $(z_S = z^*, x_{iS} = x_i^*, z_I = 0, x_{iI} = 0)$

- Math: DFE is stable (LAS) if $0 < \mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$
- Bio: If $\mathcal{R}_0 > 1 \Rightarrow$ Epidemic, if $0 < \mathcal{R}_0 < 1 \Rightarrow$ No epidemic

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Explicit formulation of the DFE

Change of variables:

- Total prey density: $y = x_1 + x_2$
- Proportion of prey 1: $\lambda_1 = x_1/(x_1 + x_2)$

Assumptions:

- Consistency of MSFR: $p_i = p_{iS} + p_{iI}$ (Morozov and Petrovskii, 2013)
- Overall feeding: $\Theta(x_1, x_2) = \theta(y)$
- Preference: $p_1(x_1, x_2) = \rho_1(\lambda_1, y)$

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Conditions for the DFE

$$\lambda_1^* = \rho_1(\lambda_1^*, y^*)$$

$$z^* = k_z$$

$$ry^* \left(1 - \frac{y^*}{k}\right) = \theta(y^*)z^*$$

Switching (or no preference)

 $\rho_{1}(\lambda_{1}^{*}, y^{*})$ Proportion
of prey *i*ingested
Proportion of prey *i* available λ_{1}

Measure of disease risk : Basic reproductive number = \mathcal{R}_0

Next Generation Matrix (Diekmann et al., 1990 and 2009, Driessche and Watmough, 2002)

Foraging behavior (MSFR)

$$\mathcal{R}_{0} = \sqrt{\frac{\eta z^{*}}{b(b_{z} + \mu)}} \times (\Gamma_{1} \Phi_{1}(x_{1}^{*}, x_{2}^{*}) + \Gamma_{2} \Phi_{2}(x_{1}^{*}, x_{2}^{*}))$$
"Introduction" of susceptibles

With the change of variables + condition at DFE

$$\mathcal{R}_{0} = \sqrt{\frac{\eta z^{*} \theta(y^{*})}{b(b_{z} + \mu)}} \times (\Gamma_{2} + \lambda_{1}^{*} \times (\Gamma_{1} - \Gamma_{2}))$$

$$\bullet \quad \text{Diversity of IH}$$

$$\bullet \neq \text{competences}$$

Overall feeding + Different competences



Overall feeding + Different competences



- The ingestion rate influences the speed of dilution/amplification of the disease
- A limited overall feeding emphasizes the dilution/amplification effect

Different competences + Limited ingestion + Rank switching



Different competences + Limited ingestion + Rank switching





- Develop MSFR
 Apply on multi-host parasite
 Assess the disease risk
- Frequency-dependent and total-density dependent switching in red foxes voles
- Foraging × Competence New tools/ideas on dilution/amplification of disease risk

Perspectives

- Increase food web complexity (more species, competition, mutualism, migration)
- Integration of variability (Random Matrix Theory)

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Why not for a post-doctoral proposal? 🕲

Thank you







