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Anticipating critical transitions of chaotic attractors through boundary crises

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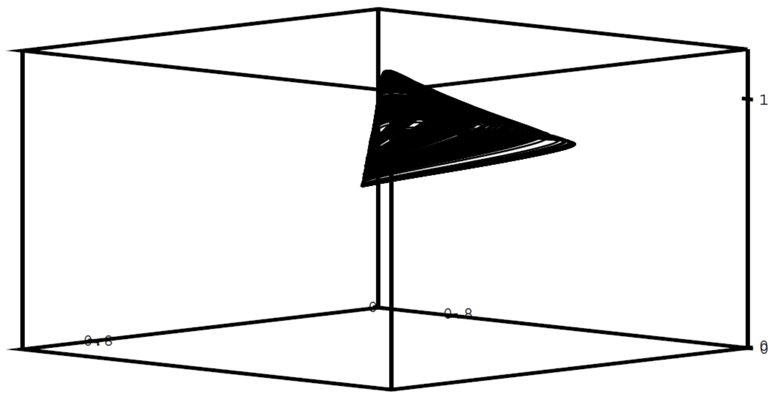
Motivation: Erratic population collapses

- Lots of work on early warning signals for critical transitions, but this mostly focuses on local bifurcations.
- There are also many critical transitions in nature featuring erratic population collapses.
- The most likely explanation for these collapses is the collision of a growing chaotic attractor with the boundary of another attractor.

Boundary crises of chaotic attractors

- Often, chaotic attractors aren‘t globally attracting and the system is bistable.
- Basins of attraction are separated by some boundary manifold.



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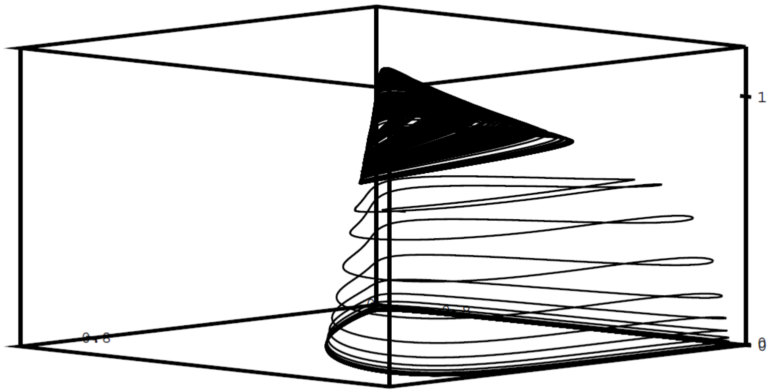

Boundary crises of chaotic attractors

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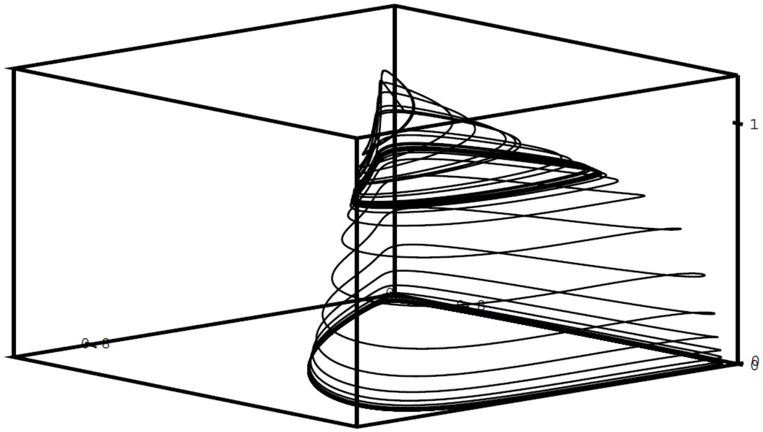
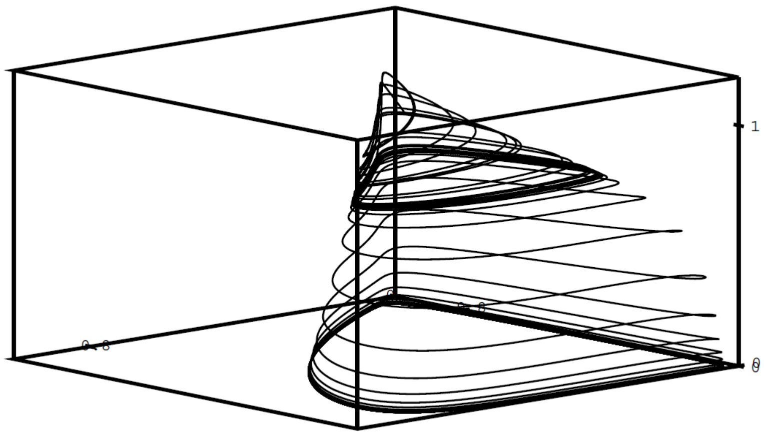
- Basins of attraction are separated by some boundary manifold.
- As parameters change, the chaotic attractor can collide with this boundary and disappear.



Anticipating boundary crises?

Anticipating boundary crises is challenging compared to other critical transitions:

- They‘re nonlocal and inherently nonlinear.
- Analysis tends to be system-specific. So it‘s difficult to obtain generic methods of prediction.
- So is it worth trying?



Collapses in Fisheries from 1955-2005

No collapse (3/4 of fisheries)

Plateau-shaped collapse, 21%

Erratic collapse, 45%

Gradual collapse, 33%

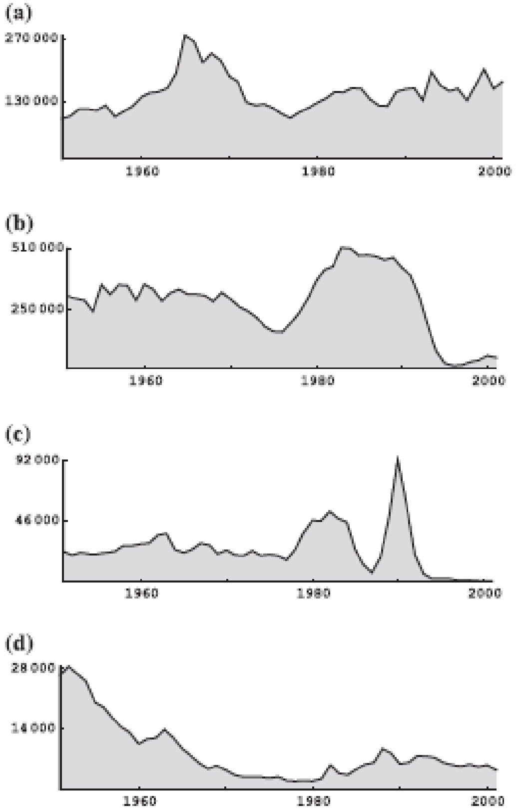


Figure 2 Typical observed patterns of catch time series: (a) no collapse: Atlantic herring in Sweden, (b) plateau-shaped collapse: Atlantic cod in Canada, (c) erratic collapse: Atlantic cod in Greenland, (d) smooth collapse: European hake in the UK.

Mullon, Fréon and Cury (2005), Fish and Fisheries

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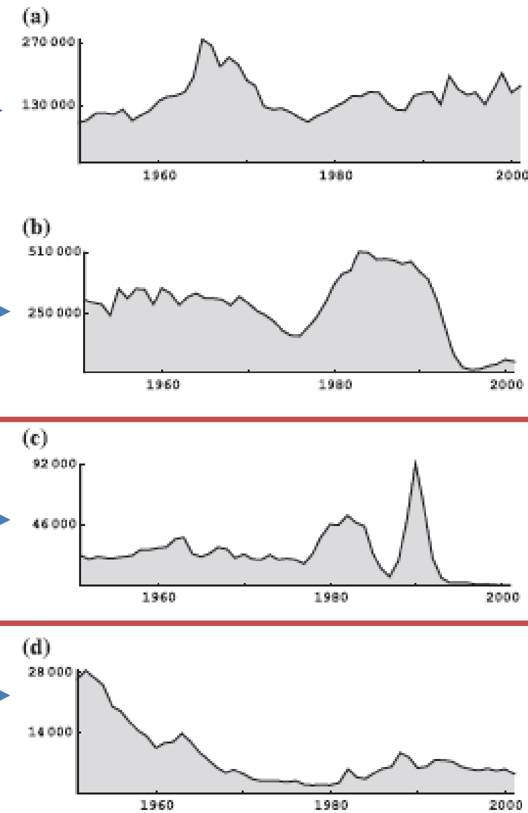

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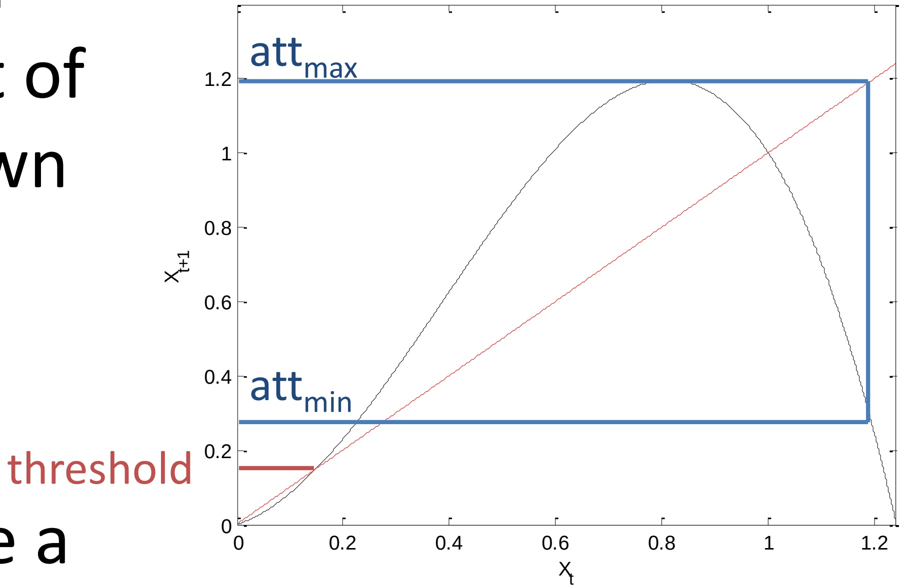
Hump shaped 1D map: Paradigm boundary crisis

- The simplest case of a boundary crisis is that of the discrete map shown here.
- If the map changes so that att_{min} is below the threshold we have a boundary crisis.



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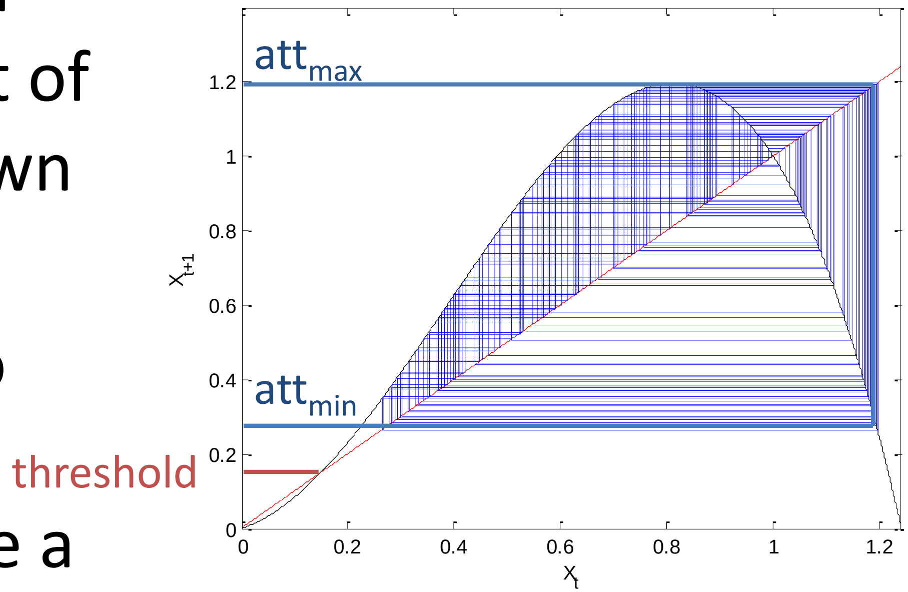

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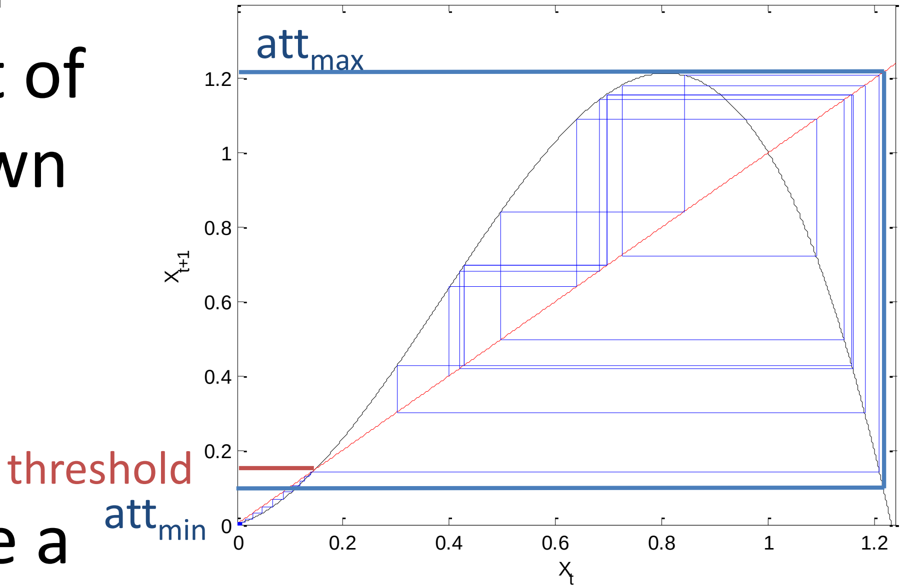

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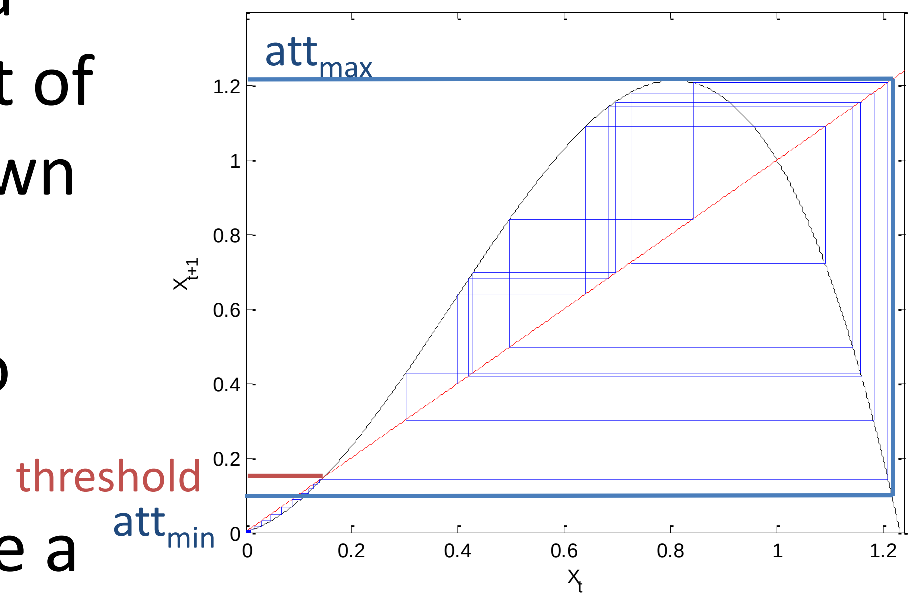
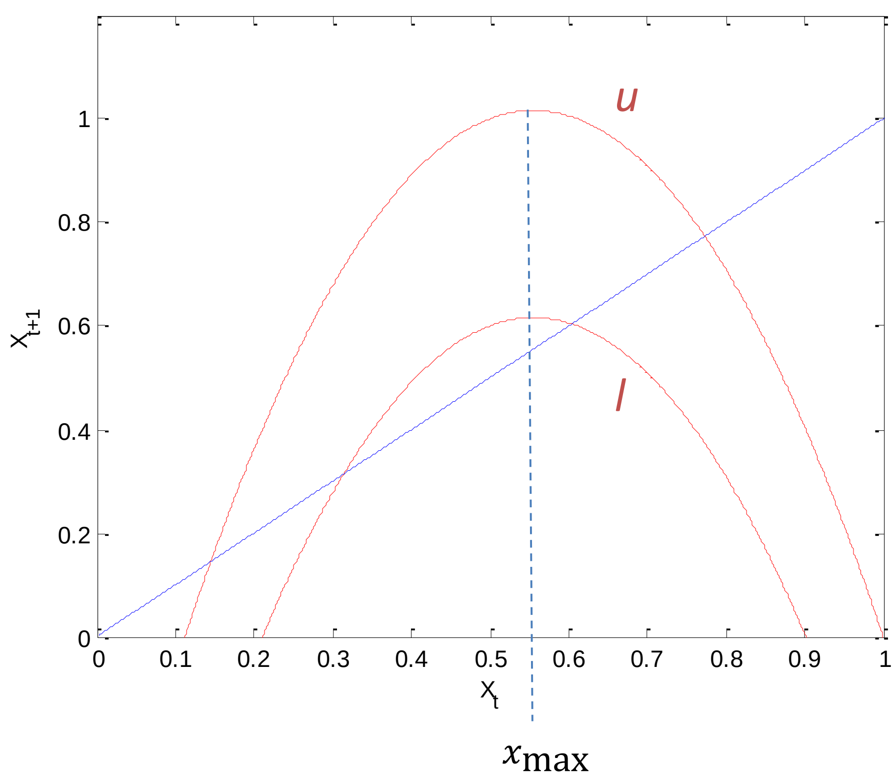


- So far so good, but in a generic method for anticipating boundary crises, we can't rely on particular functional forms...

Partially specified humped discrete time models

So we want to consider 1D maps $x_{t+1} = f(x_t)$ where:

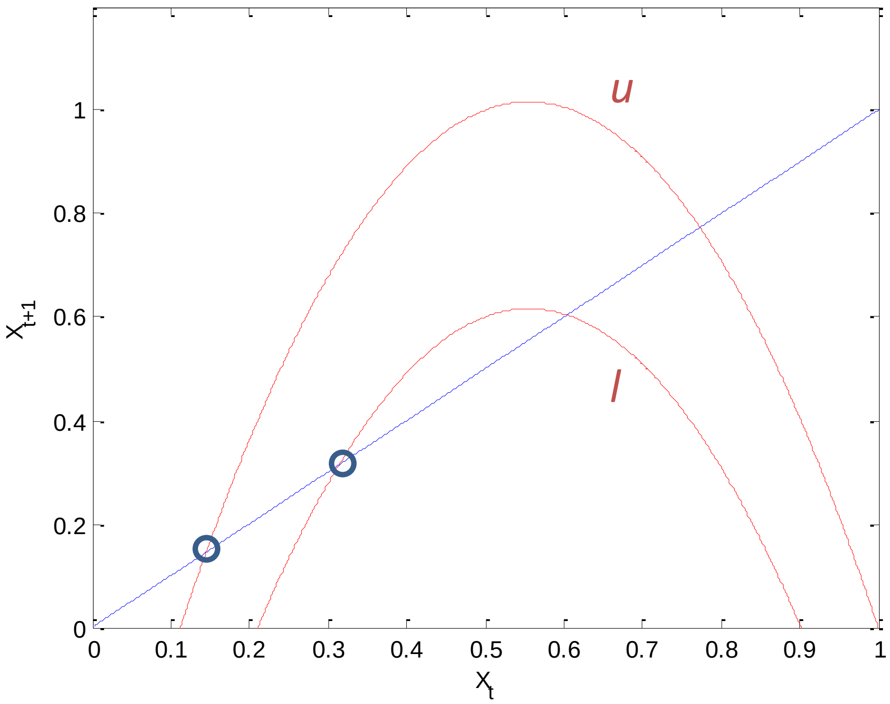
- $l(x) < f(x) < u(x)$ for some bounds l and u,
- $f'(x) > 0,\ x < x_{\max}$
- $f'(x) < 0,\ x > x_{\max}$



Bounds of the critical threshold

It‘s easy to find the minimum and maximum boundary for any function passing between $u(x)$ and $l(x)$.

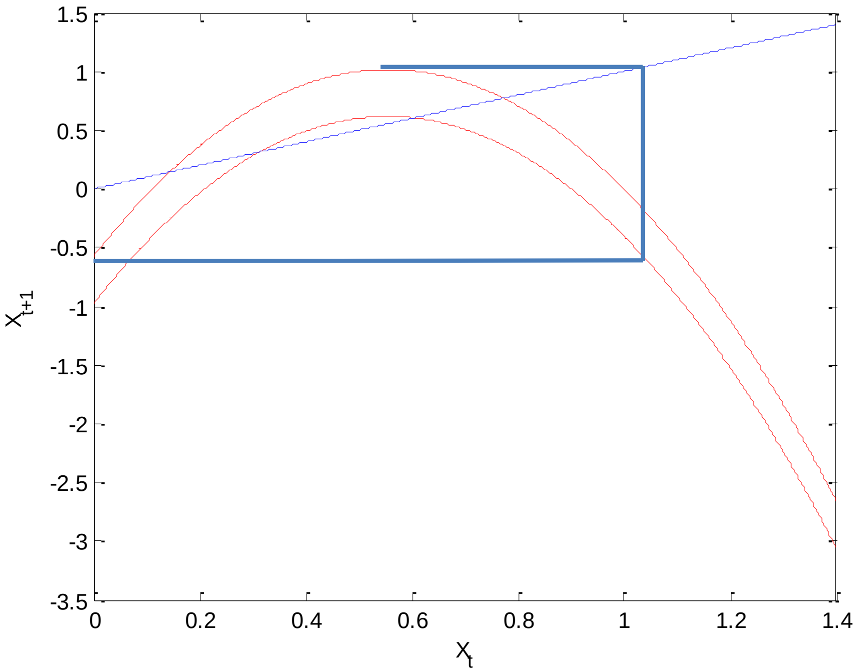
They are the lowest intersections of the identity line and u and l, respectively.



Bounds of the lowest possible attractor value

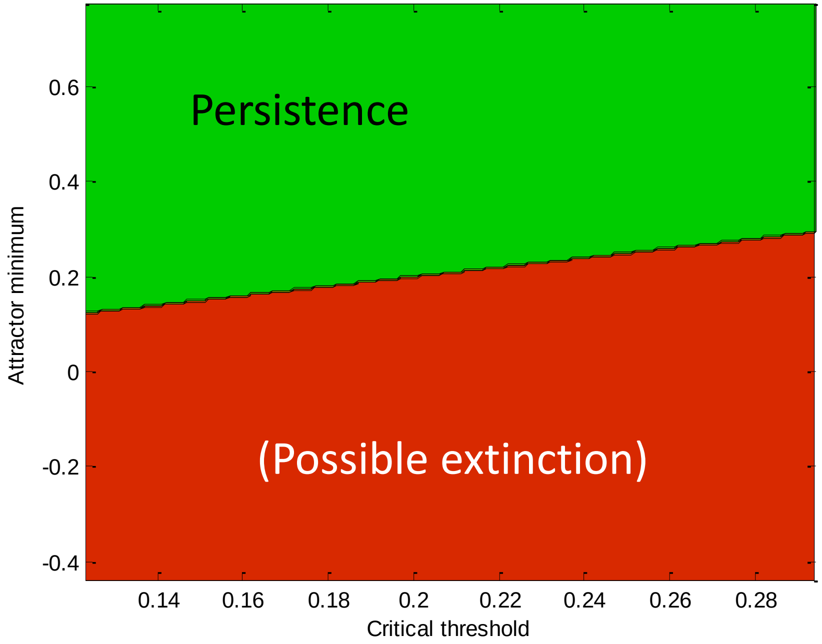
It‘s not much harder to find the minimum and maximum lower bound on an attractor.

This just involves `reflecting‘ points in the identity line and considering the extreme cases.



Plots of the probability of persistence

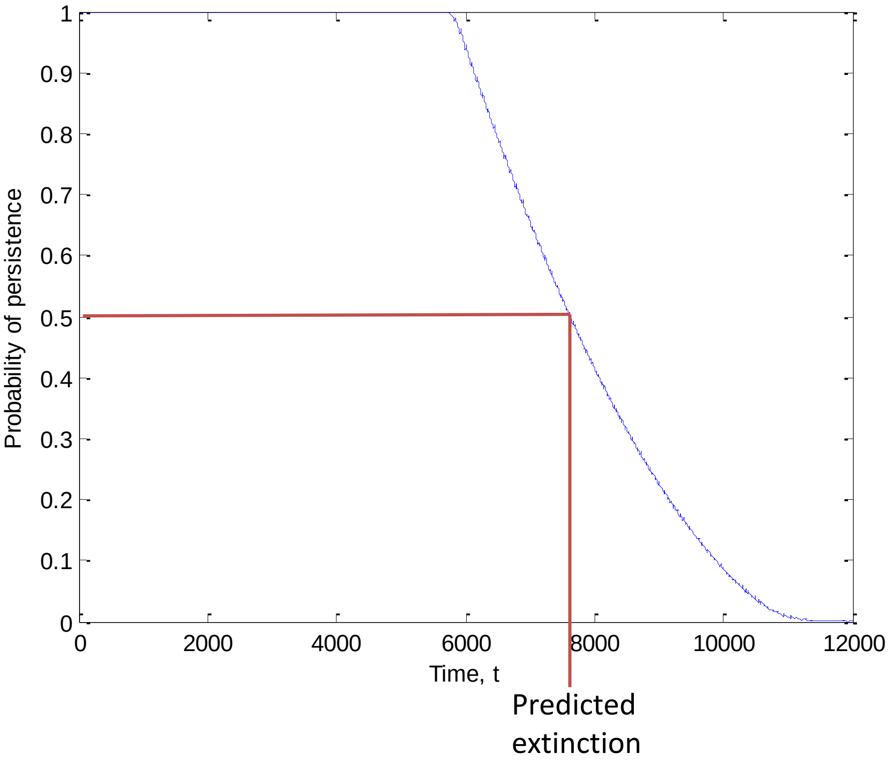
- The max and min values of the boundary value and the attractor lower bound form a rectangle.
- We have regions of possible extinction or necessary persistence.
- The relative areas serve as an estimate of the extinction possibility.



Allowing negative numbers prevents information loss

Probability of a system changing in time

- We can construct upper and lower bounds, u and l that change in time.
- Then see how the probability of persistence changes with time.
- When it drops to 50% we should expect extinction.



Demonstration: Tritrophic Rosenzweig-MacArthur model

To test the method, we‘ve applied it to the following tritrophic food chain model, with a slow increase in the carrying capacity, K.

$$\frac{dx_1}{dt} = rx_1\left(1 - \frac{x_1}{K}\right) - \frac{a_2 x_1}{b_2 + x_1} x_2 \quad (1)$$

$$\frac{dx_2}{dt} = c_2 \frac{a_2 x_1}{b_2 + x_1} x_2 - d_2 x_2 - \frac{a_3 x_2}{b_3 + x_2} x_3 \quad (2)$$

$$\frac{dx_3}{dt} = c_3 \frac{a_3 x_2}{b_3 + x_2} x_3 - d_3 x_3 \quad (3)$$

x_1: Prey
x_2: Predator
x_3: Top Predator

For information on this system and the boundary crisis involved see e.g.:

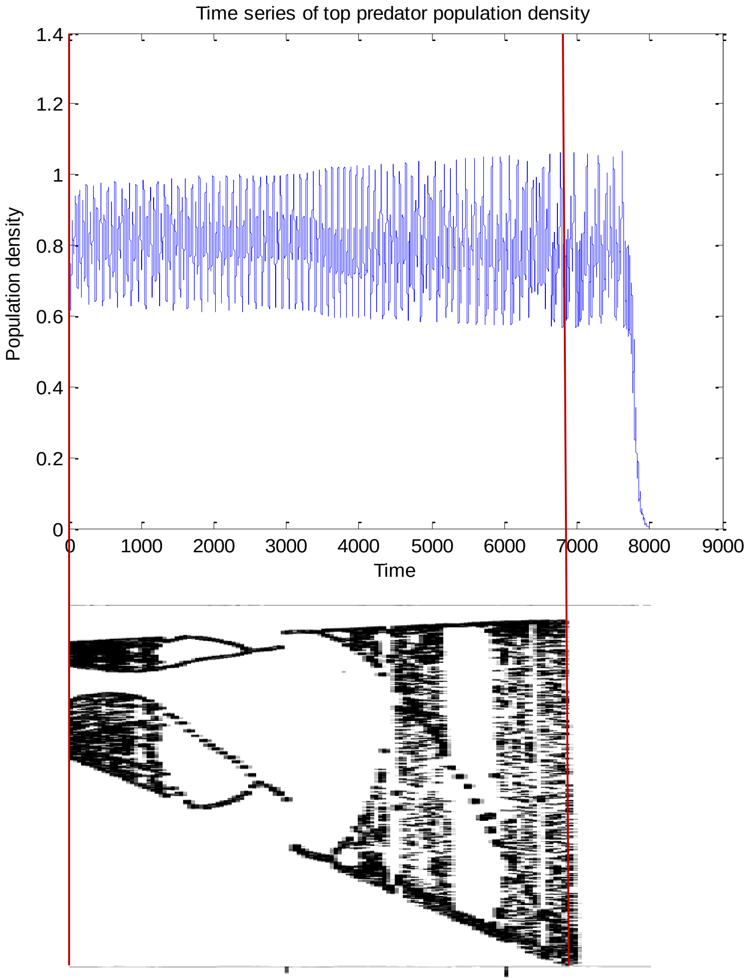
- Hastings, A., Powell, T., 1991. Chaos in a Three-Species Food Chain. **Ecology**
- McCann, K., Yodzis, P., 1994. Nonlinear Dynamics and Population Disappearances. **The American Naturalist**

Demonstration: Deterministic time series

The resulting time series is shown here with the bifurcation diagram at the corresponding time.

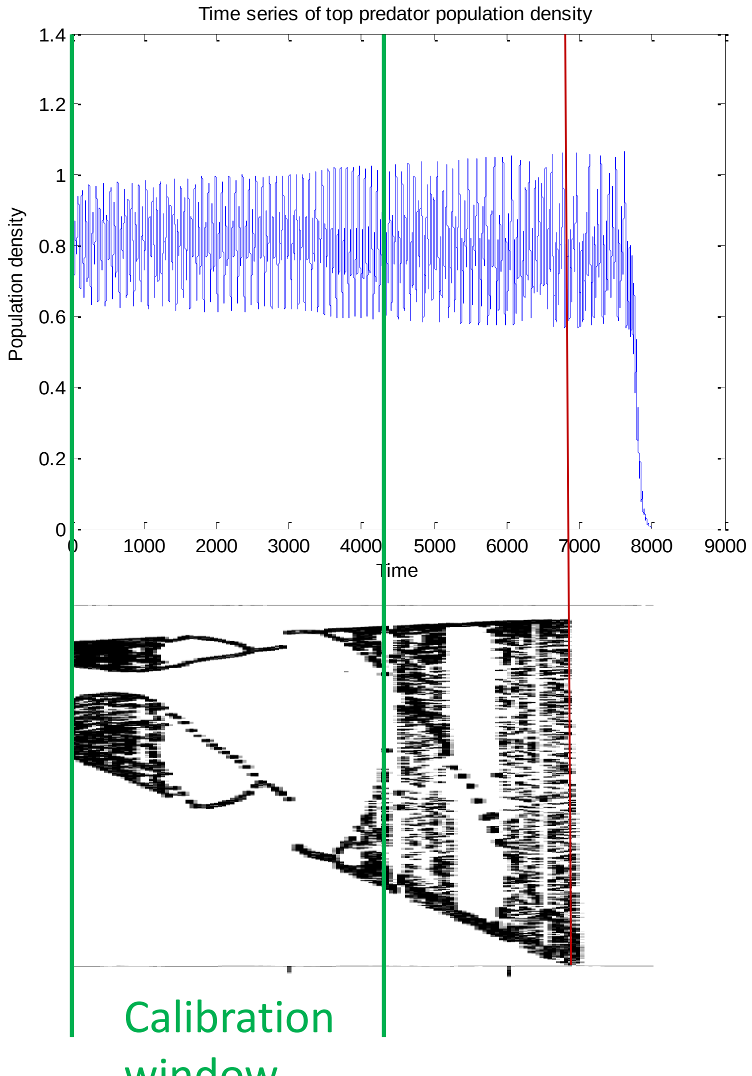
Note two challenges:

- The periodic windows throughout much of the time series.
- The transient dynamics after the boundary crisis.

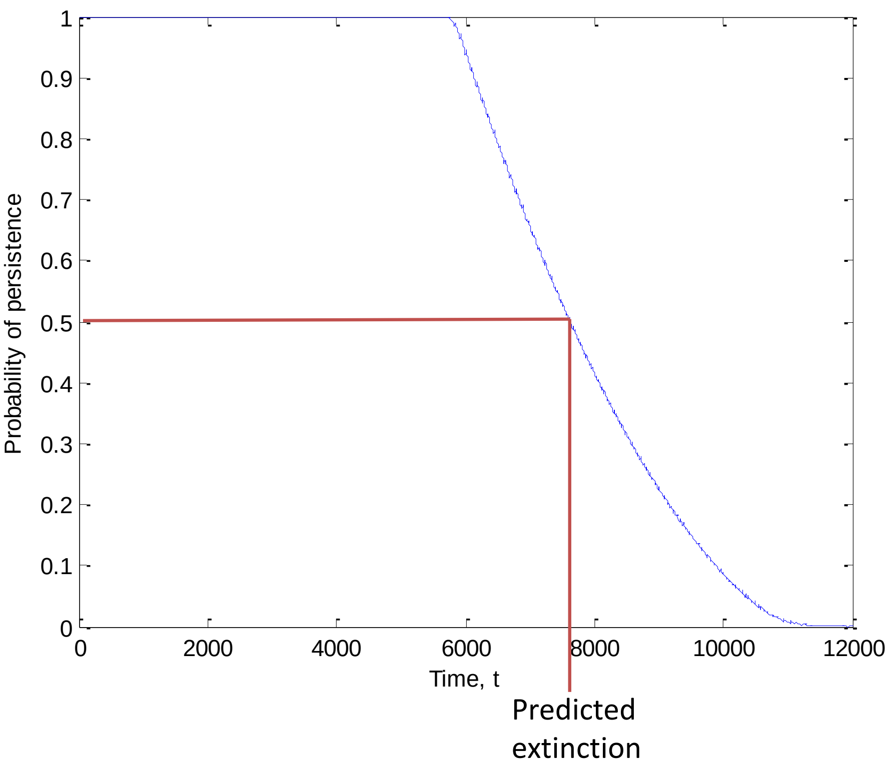


McCann & Yodzis, 1994, Am. Nat.

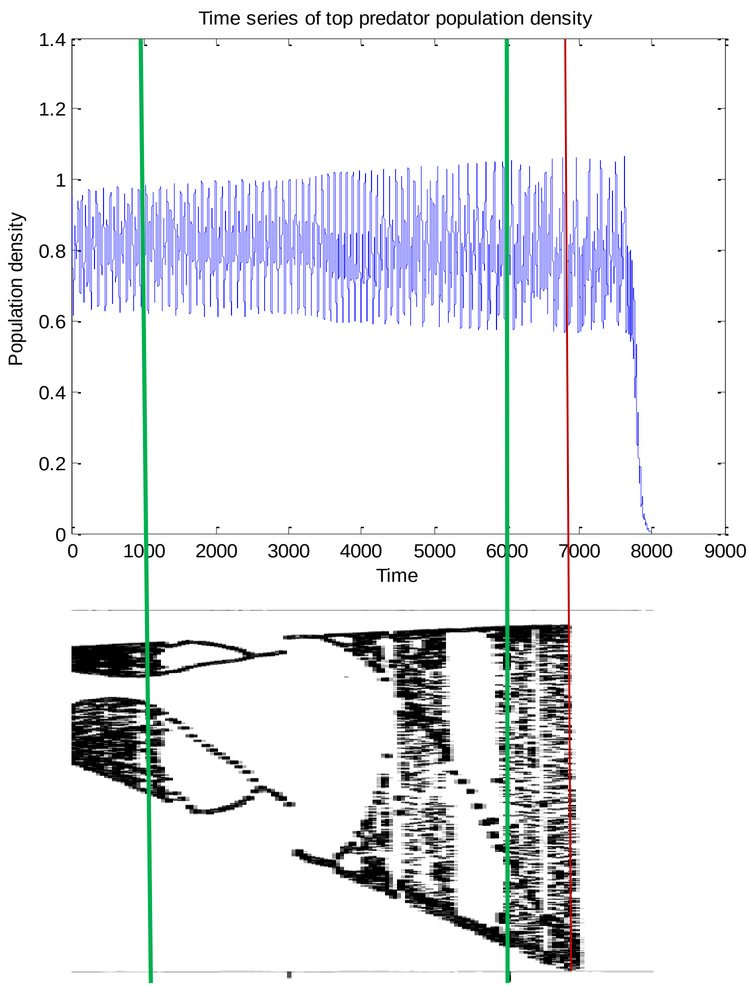
Successful prediction



Predicted probability of persistence

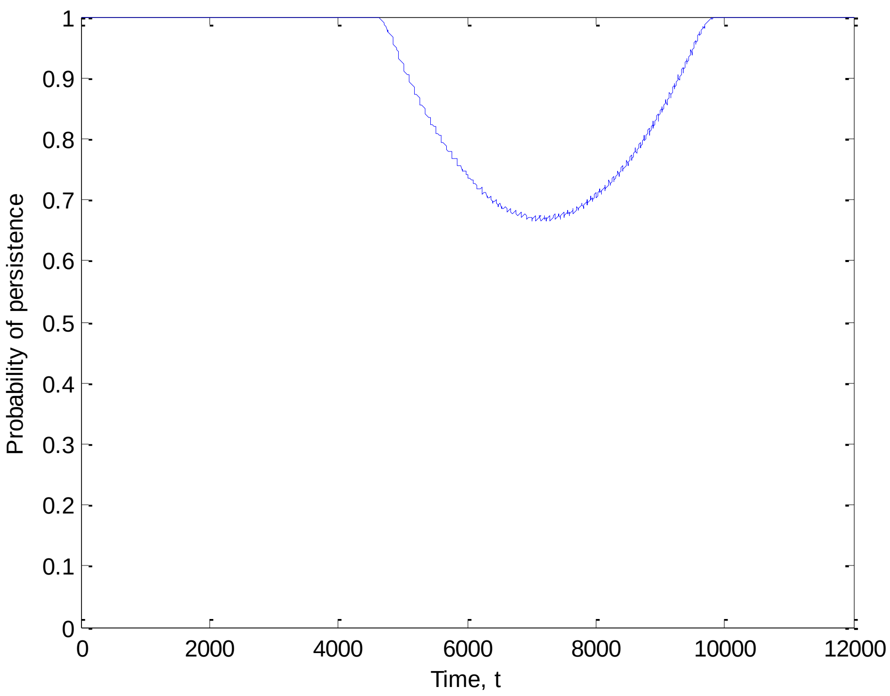


Less successful prediction



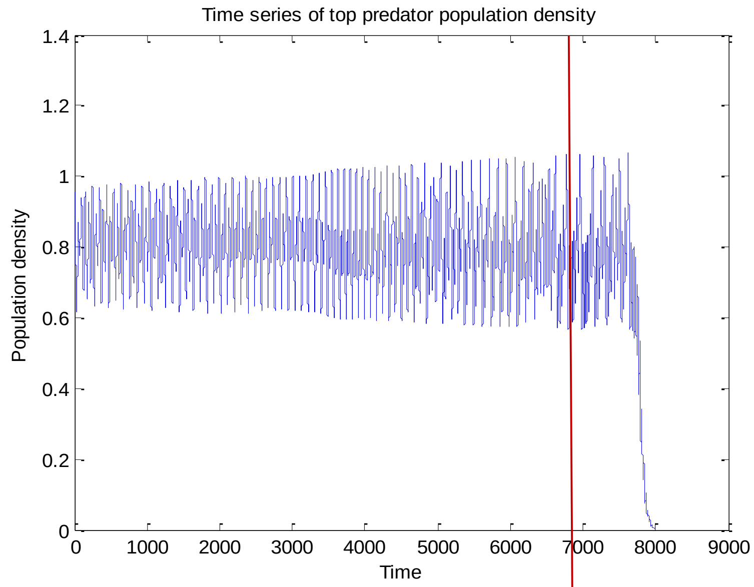
Calibration window

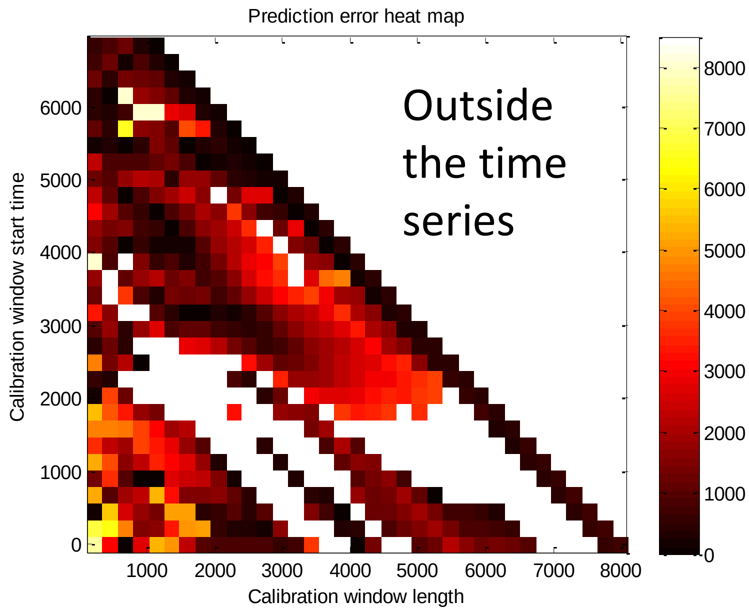
Predicted probability of persistence

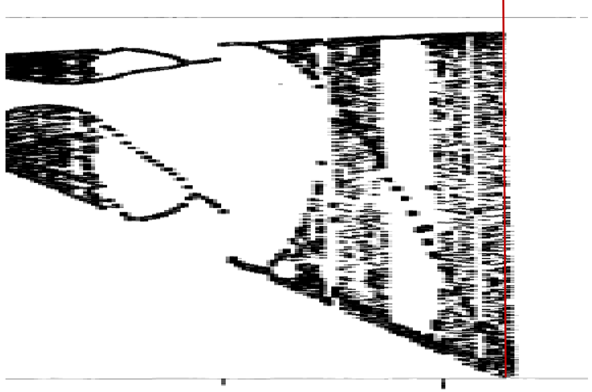


No extinction predicted!

Predictive success vs calibration window

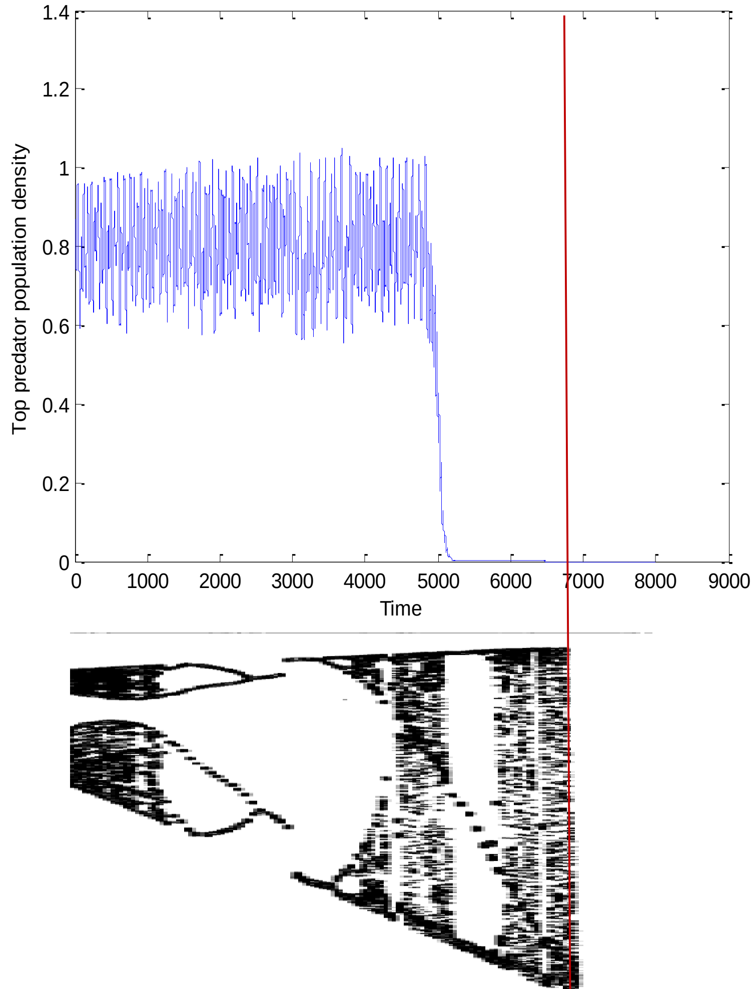


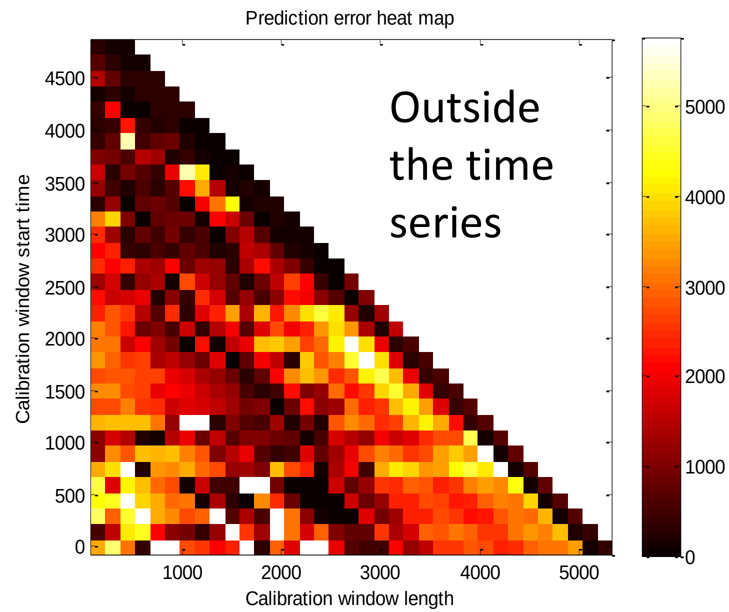




White regions: no extinction predicted
Conclusion: The periodic windows seem to be causing the method to fail, but not in a very straightforward way.

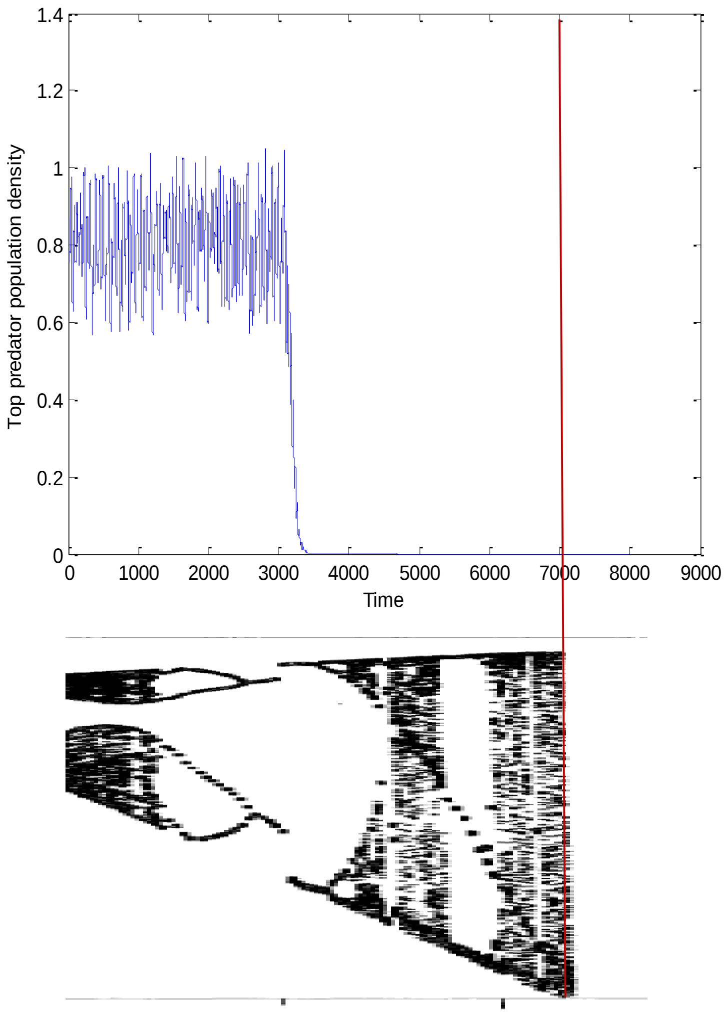
Effects of 0.5% noise

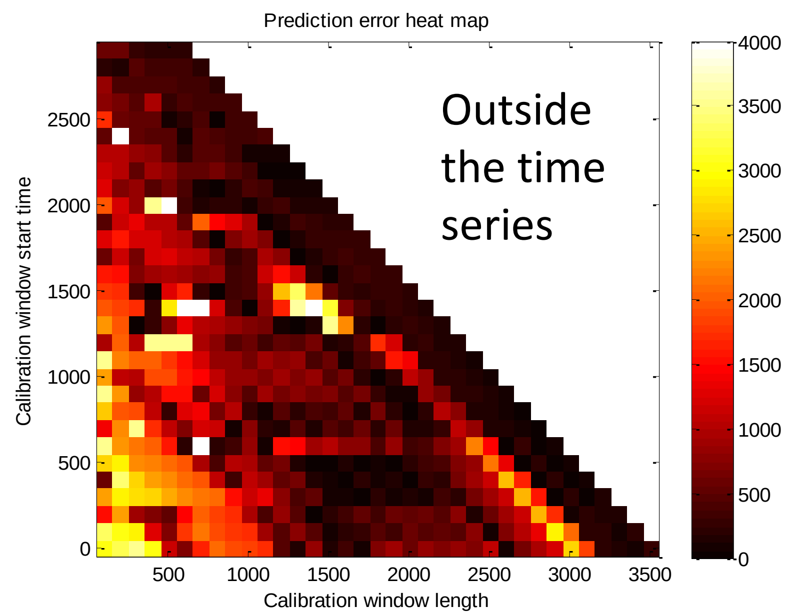




Conclusion: Noise generally seems to stop the method from missing the extinction completely.

Effects of 1% noise





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Conclusions

- Method aims to predict boundary crises of chaotic attractors by representing chaotic time-series by uncertain, evolving, hump-shaped maps.
- We’ve proved the concept works by predicting a collapse in a continuous time model.
- The method struggles with periodic windows, but noise seems to help (for once!)