Local and Global Invariants of Singularities
CIRM, 23-27 February 2015

Titles and abstracts

• **1. Paolo Aluffi**

Title: *Segre classes of monomial schemes and Segre zeta functions*

Abstract: Several invariants of singularities may be expressed in terms of Segre classes, a key ingredient in Fulton-MacPherson intersection theory. We will quickly review several applications of Segre classes, and present a formula computing them for schemes that are 'monomial' with respect to a collection of possibly singular hypersurfaces meeting along complete intersections. The formula is expressed as a formal integral over a Newton polytope associated with the scheme. For subschemes of projective space, this formula is a simple instance of a 'Segre zeta function', defined for homogeneous ideals. We conjecture that this zeta function is always rational, with poles corresponding to the degrees of some of the ideal's generators.

• **2. Nero Budur**

Title: *Cohomology jump loci and singularities*

Abstract: Cohomology jump loci of local systems generalize the Milnor monodromy eigenvalues. We address recent progress on the local and global structure of cohomology jump loci. More generally, given an object with a notion of cohomology theory, how can one describe all its deformations subject to cohomology constraints? We give an answer in terms of differential graded Lie algebra pairs. This is joint work with Botong Wang.

• **3. George Comte**

Title: *Motivic Real Milnor fibre*

Abstract: Joint work with G. Fichou. We define a Grothendieck ring for basic real semialgebraic formulas, that is, for systems of real algebraic quations and inequalities. In this ring the class of a formula takes into consideration the algebraic nature of the set of points satisfying this formula and this ring contains as a subring the usual Grothendieck ring of real algebraic formulas. We give a realization of our ring that allows us to express a class $Z[1/2]$ a linear combination of classes of real algebraic formulas, so this realization gives rise to a notion of virtual Poincaré polynomial for basic semialgebraic formulas. We then define zeta functions with coefficients in our ring, built on semialgebraic formulas in arc spaces. We show that they are rational and relate them to the topology of real Milnor fibers.

• **4. Alex Dimca**

Title: *Hodge Theory and syzygies of the Jacobian ideal*

Abstract: Let f be a homogeneous polynomial, defining a principal Zariski open set $D(f)$ in some complex projective space $\mathbb{P}^n$ and a Milnor fiber $F(f)$ in the affine space $\mathbb{C}^{n+1}$. Let $f_0, \ldots, f_n$ denote the partial derivatives of f

with respect to $x_0, \ldots, x_n$ and consider syzygies $a_0 f_0 + a_1 f_1 + \ + a_n f_n = 0$, where a_j are homogeneous polynomials of the same degree k.

Using the mixed Hodge structure on $D(f)$ and $F(f)$, one can obtain information on the possible values of k.

• 5. Karl Fieseler

Title: *Combinatorial Intersection Cohomology for toric varieties and fans*

Abstract: In this talk we describe the intersection cohomology of toric varieties in terms of the combinatorics of the underlying fan and related problems. It is based on joint work with J.-P. Brasselet, G. Barthel and L. Kaup as well as results obtained by K-Karu, P. Bressler and V. Lunts.

• 6. Xavier Gomez-Mont

Title: *Grothendieck Residue in the Jacobian Algebra and Cup Product in Vanishing cohomology*

Abstract: The Jacobian algebra, obtained from the ring of germs of functions modulo the partial derivatives of a function f with an isolated singularity, has a non-degenerate bilinear form, Grothendieck Residue, for which multiplication by f is a symmetric nilpotent operator. The vanishing cohomology of the Milnor Fibre has a bilinear form induced by cup product for which the nilpotent operator N, the logarithm of the unipotent part of the monodromy, is antisymmetric. Using the nilpotent operators we obtain primitive parts of the bilinear form and we compare both bilinear forms. In particular, over $\mathbb{R}$, we obtain signatures of these primitive forms, that we compare.

• 7. Nivaldo Grulha

Title: *The Euler obstruction and generalizations*

Abstract:

• 8. June Huh

Title: *Positivity of Chern classes of Schubert cells and varieties*

Abstract: Chern-Schwartz -MacPherson class is a functorial Chern class defined for any algebraic variety. I will give a geometric proof of a positivity conjecture of Aluffi and Mihalcea that Chern classes of Schubert cells and varieties in Grassmannians are positive. While the positivity conjecture is a purely combinatorial statement, a combinatorial 'counting' proof is known only in very special cases. In addition, the current geometric argument do not work for Schubert varieties in more general flag varieties. Should we expect the same positivity for Chern classes of Schubert varieties in G/P?

• 9. Dmitry Kerner

Title: Recombination formulas for the spectrum of plane curve singularities

Abstract: The spectrum of (complex, isolated) plane curve singularity admits a variety of additive formulas. These are often obtained by cut&paste operations on the resolution.

Consider the abelian group generated by all the possible such spectra. We demonstrate a neat class of generators. As an application we prove several inequalities expressing the idea that "most of the spectral values are concentrated near zero".

• 10. André Legrand

Title: *Isolated conical singularities : Hochschild homology and intersection complex, intersection K-theory*

Abstract: In the first part, with J.P. Brasselet, we generalize the Hochschild-Kostant-Rosenberg theorem for varieties with isolated singularities. Following the Teleman localisation process we show that the closed Hochschild homology of totally bounded functions corresponds to the intersection complex.

In a second part, starting from the Karoubi multiplicative K-theory, we construct a kind of Chern-Weil theory adapted to isolated conical singularities. The Chern character takes its values in the intersection cohomology of Goresky-MacPherson. An integer intersection K-theory for such singularities is also proposed (join work with D. Poutriquet)

• 11. Ursula Ludwig

Title: *A de Rham type result for singular spaces*

Abstract: An important topological invariant for singular spaces is their intersection homology, introduced in the 80ties by Goresky and MacPherson. The Cheeger-Goresky-MacPherson Theorem establishes an isomorphism between the $\mathcal{L}^2$-cohomology of a singular space with isolated cone-like singularities and its intersection cohomology. De Rham Theorems for singular spaces have been studied in more generality since. However the only case in which a quasi-inverse to the integration map can be described explicitly is for a polyhedron embedded in a Euclidean space. This case is due to Brasselet, Goresky and MacPherson and involves the concept of shadow forms. The aim of this talk is to present a de Rham type result for singular spaces with isolated cone-like singularities using radial Morse functions. First we will give a construction of a complex generated by smooth critical points of the radial Morse function and harmonic forms (in high degree) on the link manifolds of singular points. This complex computes the intersection homology of the space. Moreover it can be compared explicitly with a (finite dimensional) complex of forms, which computes the $\mathcal{L}^2$-cohomology of the space.

• 12. Laurentiu Maxim

Title: *Motivic infinite cyclic covers*

Abstract: To an infinite cyclic cover of a punctured neighborhood of a simple normal crossing divisor on a complex quasi-projective manifold we associate (assuming certain finiteness conditions are satisfied) an element in the equivariant Grothendieck ring of varieties, called motivic infinite cyclic cover, which satisfies birational invariance. Our construction provides a unifying approach for the Denef-Loeser motivic Milnor fibre of a complex

hypersurface singularity germ, and the motivic Milnor fiber of a rational function, respectively. This is joint work with M. Gonzalez Villa and A. Libgober.

• 13. Clint McCrory

Title: *Real intersection homology*

Abstract: Thirty years ago Goresky and MacPherson asked if there is a self-dual generalization of intersection homology for real algebraic varieties. They proposed that it would have coefficients in the integers mod 2, that it would not be a purely topological invariant, and that it would agree with the homology of a small resolution. I will report on a proposed definition of this invariant that was developed with Adam Parusinski. We have a conjectural general position theorem for the cycles of the theory. Recent work by Greg Friedman and James McClure on products and duality for classical intersection homology suggests a way to prove that the resulting intersection pairing is nonsingular.

• 14. Nguyen Thi Bich Thuy

Title: *On a singular set associated to a polynomial mapping* $G : \mathbb{C}^n \to \mathbb{C}^{n-1}$

(Joint work with Maria Aparecida Soares Ruas)

Abstract: We construct a real pseudo-manifold $\mathcal{V}_G$ associated to a polynomial mapping $G : \mathbb{C}^n \to \mathbb{C}^{n-1}$ $(n \geq 2)$ such that the character $B(G) = \emptyset$ is characterized by the vanishing of the intersection homology of $\mathcal{V}_G$, where $B(G)$ is the Bifurcation set of G.

• 15. Markus Pflaum

Title: *Invariants of singularities coming from noncommutative geometry*

Abstract: The sheaves of smooth and of Whitney functions on a subanalytic set can be studied by methods coming from noncommutative geometry. For example, localization of the Hochschild complex of the their global section algebras leads to invariants of singularities. In another direction, the complex of forms over the algebra of Whitney functions is known to determine the real homotopy type of the underlying space, at least in the semianalytic case. In the talk these ideas will be explained in more detail, and shown that they form a natural continuation of previous joint work with Jean-Paul Brasselet.

• 16. Michel Raibaut

Title: *Motivic nearby cycles*

Abstract: In this talk we will recall first the notions of motivic nearby cycles and motivic Milnor fibers of a polynomial map introduced by Denef and Loeser and developped by Bittner, Guibert, Loeser and Merle. These objects are motives, elements of a Grothendieck of varieties, which contain usual additive and multiplicative invariants of the corresponding Milnor fibrations. Then, we will give two applications of this construction : the

motivic Milnor fiber at infinity of a polynomial map and the motivic Milnor fibers of a rational function.

• 17. Marcello Saia

Title: *A Lefschetz coincidence theorem for intersection homology Quelle coïncidence !*

A. Joint work [3]. That is a joint work of J.-P. Brasselet with T. Suwa. For two maps of $\mathcal{C}^\infty$ manifolds (oriented, possibly non-compact and of possibly different dimensions) one defines the global coincidence class in the homology of the source manifold and the local coincidence class in the homology of each connected component of the coincidence set, which is in the source manifold. Then we have a general coincidence point theorem. We give an explicit expression of the local class in the case the component is a reasonable space, for instance a pseudo-manifold. One of the key ingredients for this is the presentation of the Thom class in the Cech-de Rham cohomology. In the case of two maps of compact manifolds of the same dimension with only isolated coincidence points, this viewpoint gives a simple direct proof of the classical Lefschetz coincidence point formula.

We also take up the case of several maps, for which there is a work of C. Biasi, A.K.M. Libardi and T.F.M. Monis (Forum Math. 2013). They define the global Lefschetz coincidence cohomology class and prove that the non-vanishing of the class implies the existence of a coincidence point. In this case also, we define the global and local coincidence homology classes, relate the global class to their class and give explicit expressions of local classes.

This is a summary of some parts of the joint work of C. Bisi, F. Bracci T. Izawa and T. Suwa [1] and the joint work of J.-P. Brasselet with T. Suwa [3].

B. Joint work [2]. Goresky and MacPherson proved in [5] the Lefschetz fixed point theorem in the context of a class of "placid" self maps of singular spaces, by using intersection homology. In fact they showed that both the graph of f and the diagonal carry fundamental classes in the intersection homology of $X \times X$, and that the Lefschetz number $IL(f)$ is the intersection number of these two classes. This result leads us naturally to the question of coincidence, already pointed out in [5] although not explicitly. Our main goal is to explicit the formula of the Lefschetz coincidence number for placid maps $f, g : X \to Y$ between oriented compact $\mathbb{Q}$-Witt spaces of dimension n and prove the Lefschetz coincidence theorem for this setting.

We show some formulas for the *coincidence Lefschetz numbers* in intersection homology of a class of pair of "placid" maps of singular Witt spaces. The main difference between these results and the result of Goresky-MacPherson is that there are no analogous results for the ordinary homology of a smooth variety.

We understand that the Lefeschtz numbers for coincidences obtained using intersection homology in Witt spaces is relevant since in this case we

can use the Poincaré duality for these spaces, therefore we can follow the approach of Lefeschtz for coincidence theory using ordinary homology of smooth varieties see [6] or [4].

References

[1] Cinzia Bisi, Filippo Bracci, Takeshi Izawa and Tatsuo Suwa, *Localized intersection of currents and the Lefschetz coincidence point theorem*, http://arxiv.org/abs/1406.0607v1.

[2] Jean-Paul Brasselet, Thais F. M. Monis, Eliris C. Rizziolli and Marcelo Saia, *A Lefschetz coincidence theorem for intersection homology*, in preparation.

[3] Jean-Paul Brasselet, and Tatsuo Suwa, *Quelle coïncidence !,* in preparation. .

[4] A. Dold, *Lectures in Algebraic Topology*, Springer-Verlag, New York, (1972).

[5] M. Goresky and R. MacPherson, *Letschetz fixed point theorem for intersection homology*, Comment. Math. Helvetici 60 (1985) 366–391.

[6] S. Lefeschtz, *Topology* Amer. Math. Soc. Colloq. Publ. **XII**, A. M. S. Providence, R. I. (1930).

- **18. Jose Seade**

  Title: *Indices of vector fields on Singular varieties and the Milnor number*
  Abstract:

- **19. Dirk Siersma**

  Title: *Projective hypersurfaces with 1-dimensional singularities*

  Abstract: The homology of projective hypersurfaces is classically known for smooth hypersurfaces. Due to results of Dimca the homology of a singular hypersurface with isolated singularities is related to the homology of the smooth case as follows: the difference is concentrated in one dimension and related to the direct sum of the Milnor lattices of the singular points. In the talk we will treat 1-dimensional singularities. By using a one parameter smoothing of an n-dimensional hypersurface we can compare with a smooth hypersurface. We call this the vanishing homology of the smoothing. We will show that this (relative) homology is concentrated in two dimensions only: $n+1$ and $n+2$. Moreover we will give precise information and bounds for the Betti numbers of the vanishing homology in terms of properties of the singular set, the generic transversal singularities, the 'special non-isolated singularities and (if they occur) the isolated singularities. As an example: the $n+2$ Betti number is bounded by the sum of (generic) transversal Betti numbers on each irreducible component of the 1-dimensional singular set. In several cases this Betti number is zero. We discuss several examples. Similar method applies to deformations of local singualarities. This is joint work with Mihai Tibar, and a preprint can be found on http://arxiv.org/abs/1411.2640

- **20. Bernard Teissier**

  Title: *On the invariants born from the dynamics of intersections*

  Abstract: A presentation of some of those invariants born from measuring the vanishing rates rates with the deformation parameter of the sizes or

positions of intersections obtained by deforming intersections to a general position situation.

• **21. Alexandre Varchenko**

Title: *Critical points of master functions and hypergeometric differential equations*

Abstract: The Jacobi ring of the critical set of a holomorphic function is an important object of singularity theory. Master functions are holomorphic functions that govern the multidimensional hypergeometric integrals. The Jacobi ring of the critical set of a master function has additional interesting combinatorial structures. I will review these structures and applications to differential equations satisfied by the hypergeometric integrals.

• **22. Wim Veys**

Title: *Semigroup and Poincaré series for a finite set of divisorial valuations*

Abstract : This is joint work with my student Leen Van Langenhoven. We consider a finite set V of divisorial valuations coming from a modification of K^d, where K is a field, and we assume that V admits a finite generating sequence. If K is infinite, we show that the semigroup of values of V is finitely generated, and we propose an efficient way to calculate it. For arbitrary K, we show that the Poincar series associated to V is a rational function, whose denominator can be expressed in terms of the valuation vectors of the elements in the generating sequence. However, a finite generating sequence does not always exist. We give an example of a modification whose semigroup of values is not finitely generated.

• **23. Shoji Yokura**

Title: *Characteristic classes of singular varieties (around works of J.-P. Brasselet)*

Abstract: