Surfaces in Luminy

3-7 october 2016

Research interests

Jérôme Buzzi

I study the ergodic theory of smooth maps, and in particular the entropy of smooth diffeomorphisms with some hyperbolicity and their symbolic dynamics.

With Mike Boyle, I have a shown that surface $C^{1+\epsilon}$-diffeomorphisms are "almost-Borel" conjugate to countable state Markov shifts "modulo zero entropy". This leads to a classification by the following entropy and periodic data : for each $p \geq 1$:

1. the supremum of the entropies of invariant probability measures with period p (i.e., there is a cyclically permuted partition modulo a negligible set with cardinality p) ;
2. the cardinality of the set of invariant probability measures that achieve the above supremum when it is nonzero

Problem. Which values of these date are realized by C^∞-smooth surface diffeomorphisms ?

Joint results with S. Crovisier and O. Sarig, give qualitative restrictions.

I'd like to go beyond with a little help from my topologist friends.

Jonathan Conejeros

My area of research is the topological dynamics. I use the rotation set in order to describe some properties of the dynamics of a homeomorphism of surface that is isotopic to the identity. For example in the local case, i.e. around a fixed point, we proved that the local rotation set is an interval. We proved the analogous result in the case of the open annuls. Actually, using the "forcing theory", we study the conditions that ensure the existence of non-contractible periodic orbits of arbitrarily high periodic for a homeomorphism of the open annulus that is isotopic to the identity and preserves a probability measure with full support. This result has consequence for rational pseudo-rotation of the open annulus, namely its lift having rotation set null is non-wandering.

Luis Hernandez Corbato

My research focuses in low–dimensional discrete dynamical systems. Formerly, my dynamical world was purely topological but I started to learn some contact and symplectic geometry a few months ago and now I search for nice dynamical problems in this area.

In recent times I also became interested to annular dynamics, in particular to the question of how similar is the rotational behavior of an accessible point of an annular continua to the expected one given by the prime end rotation number. It can be proven that accessible points do not "significantly" drift away in both backward and forward time.

Previously I worked in some questions on planar dynamics (with Ruiz del Portal, see his talk) also related to prime ends and to some "irrational" dynamics which appear cannot be made area–contracting.

The Conley index is a topological invariant of isolated invariant sets. In few words, it encapsulates some local information which can be read from its unstable manifold/set W^u. To understand it more, we are trying to grab some insight in the local topology of W^u (what is a branch ?) in terms of Cech homology or cohomology.

Martin Leguil

I am a PhD student working under the supervision of Artur Avila and Julie Déserti. I have been working on interval exchange transformations and translation flows, especially on the question of weak mixing ; by a previous result of Avila and Forni, we know that for an irreducible permutation which is not a rotation, this property holds almost surely. With Artur Avila, I have strengthened this result to show that the set of exceptions does not have full Hausdorff dimension.

I have also worked on partially hyperbolic dynamics, more precisely, on the accessibility property. With Zhiyuan Zhang, we have worked on the C^r-density of this property for certain classes of partially hyperbolic diffeomorphisms.

More recently, I have studied with Julie Déserti the properties of a family of polynomial automorphisms of $\mathbb{C}^3$; from a geometric perspective, we get results on the centralizer, the rational foliations preserved by such maps. We have also studied it from a dynamical point of view ; in particular, we show that a transition happens for a certain value of the parameter.

I have also worked on the spectral theory of one-dimensional quasi-periodic Schrödinger operators. With Qi Zhou, Jiangong You and Zhiyan Zhao, we have studied the asymptotics of the size of spectral gaps. This question is linked to the dry Martini problem, and also has applications to homogeneity of the spectrum of such operators.

Frédéric Le Roux

With Sobhan Seyfaddini and Vincent Humilière, I study the dynamics of hamiltonian diffeomorphisms on surfaces. We would like to understand the dynamical meaning of some invariants that are constructed using generating functions or Floer homology. We are especially interested in the Viterbo conjugacy-invariant norm, and in the Entov-Polterovitch quasi-morphism.

I am also interested in the algebra of transformation groups. One of my favorite open question, related to the first theme, is that of the algebraic simplicity of the group of area and orientation preserving homeomorphisms of the 2-sphere. With Katie Mann, we answered an old question of Schrier by proving that many transformation groups share the following property : every sequence of elements belong to a finitely generated subgroup. This is related to properties called strong boundedness and strong distorsion.

Finally, I quasi-priodically come back to the study of Brouwer homeomorphisms (fixed point free, orientation preserving homeomorphisms of the plane). The dynamics of a Brouwer homeomorphism is very dissipative, since every point is wandering. In particular, after compactifying by adding a point at infinity, the entropy vanishes. Recently, Louis Hauseux (in his master thesis) has computed the polynomial entropy of Brouwer homeomorphisms, and shown that it can take every real value from 2 to infinity. This apparently provides the first "natural" examples with non integer polynomial entropy. This phenomena seems to be related to the "oscillating set" that was introduced some years ago by François Béguin and myself in order to construct Brouwer homeomorphisms that preserve every leaves of a given foliation, but have no square root.

Kathryn Mann

I study actions of infinite groups on manifolds and the moduli spaces of such actions. As a related special case, I also study spaces of *left-invariant linear* or *circular orders* on groups.

One part of this project is motivated by the general Zimmer program : classify all actions of a given group G on a manifold M. Even when $M = S^1$ this is very challenging. Two projects I am working on now are 1. to study $\mathrm{Hom}(G, \mathrm{Homeo}(S^1))$ via the easier *space of circular orders on* G, and 2. identifying rigidity phenomena for group actions that come from something geometric, for instance, surface groups as lattices in $\mathrm{PSL}(2,\mathbb{R}) \subset \mathrm{Homeo}(S^1)$ have a rigidity property.

Another side of my work involves understanding the groups $\mathrm{Homeo}(M)$ and $\mathrm{Diff}(M)$ themselves (for any manifold M). These groups have a very rich algebraic structure, which is closely related to their topological structure and also to the topology of M. Recently, I've also studied the *large scale geometry* of $\mathrm{Homeo}(M)$, and related ideas of *distortion* (of elements and subgroups) and *boundedness*, and their dynamical consequences.

Emmanuel Militon

I am interested in groups of homeomorphisms and diffeomorphisms of manifolds (especially of surfaces). I try to study them with tools from topological dynamics. More recently, with Sebastian Hurtado, I was interested in groups of diffeomorphisms of surfaces which preserve a Cantor set. Here are some topological-dynamics-related questions I am interested in (which arised either from my research or out of pure curiosity) :

1. Take a homeomorphism of the torus with one-point rotation set. Is it true that it can be conjugated to a homeomorphism arbitrarily close to a rotation ?
2. Take a homeomorphism of the torus isotopic to the identity. Can you write it as a product of a uniformly bounded number of diffeomorphisms with a small rotation set (say a rotation set with diameter bounded above by one).
3. Take a homeomorphism of the torus with an invariant minimal set K. What does the set of rotation vectors of points of K look like ?

Hiromichi Nakayama

I am strongly interested in the topological dynamics of 2-dimensional manifolds with 1-dimensional minimal sets. For the topological dynamics of 1-dimensional manifolds, many great results have been obtained. I think it may be possible to raise the dimension of the manifolds. The first target is their minimal sets.

Recently, I constructed a C^∞ diffeomorphism of a 2-dimensional manifold with connected but not locally connected minimal sets (containing arcs). In 1991, Walker constructed such an example of C^1 diffeomorphisms using the graph of $\sin 1/x$ infinitely many times. I used the inverse limit of circles in order to raise the differentiability of the dynamical systems.

Thirty years ago, I started my study from codimension one foliations of three manifolds (transversely affine foliations). Then I was interested in minimal flows of 3-dimensional manifolds, and my current target is the dynamical systems of surfaces.

Andres Navas

There are two concrete problems for group actions on 2-dimensional manifolds that interest me.

The first is a little "shame" : there is no example in the literature of a finitely-generated, torsion-free group that has no faithful action on a 2 manifold. Perhaps there is an easy solution to this (these groups should certainly exist), yet this seems to be an open problem.

The second concern growth of groups of diffeomorphisms. It is known that no finitely-generated group of $C^{1+alpha}$ diffeomorphisms of the interval/circle/real line can have intermediate growth (faster than polynomial but slower than exponential), yet examples of groups of C^1 diffeomorphisms with this property are known (see Navas, GAFA 2008). The two-dimensional case of this is unknown, even for real-analytic diffeomorphisms.

Justyna Signerska-Rynkowska

My scientific interests concentrate on dynamical systems theory, chaos theory and their applications to modeling of biological and physical phenomena, especially to neurosciences. These include study of hybrid neuron models, coupling continuous dynamics given by ordinary differential equations with discrete events (the reset mechanism). Due to the connection of these (periodically/almost-periodically driven) models with circle/interval maps or maps with almost periodic displacement, I am particularly interested in low dimensional dynamics such as e.g. rotation theory and different almost periodic structures.

Recently, I have also pursued my interests in the direction of topological methods in nonlinear analysis, such as e.g. Lefschetz and Nielsen theory. Theoretical part of my research contains also study of curlicues, curves in complex plane generated by iterations of maps.

Michele Triestino

My recent interests join a long-term project started by Deroin, Kletpysn (also attending the conference) and Navas. Originating from understanding the relationship between minimality and ergodicity in (co)dimension 1, the problem for me has taken the form of understanding all finitely generated groups of circle diffeomorphisms (well, at least as much extensively as possible). Up to now, the strongest results have been obtained in real-analytic regularity : modulo one (conjecturally) very patological case, we understand the dynamics and the algebraic structure of most such groups.

When passing to weaker regularity, more difficulties arise, and this is the direction I am starting getting interested in : the example to keep in mind is a group like Thompson's T.

This can be defined as the group of all piecewise linear circle diffeomorphisms with derivatives that are powers of 2, and discontinuity points for derivatives and their images that are dyadic rationals. It has been proved long ago by Ghys and Sergiescu that T acts smoothly on the circle. As far as I know, this group is the only interesting source of examples of veritable non-real analytic smooth dynamics. It has nice algebraic properties, however it has a property that we do not like : stabilizers of points are huge!

Somehow understanding groups like T is important : for example it has been conjectured in the 70s by Dippolito that groups acting with a minimal Cantor set must be conjugated to a piecewise affine action in restriction to the Cantor set.

Jian Wang

I am interested in symplectic geometry and dynamical systems. My PhD thesis was an intersection of these two fields. I am working on problems as continuation of my PhD but more symplectic geometry oriented. I am currently interested in

-Rigidity of a pseudo-rotation diffeomorphism of T^2 whose rotation vector ϵ is not of Brjuno type;
-Transversality of the symplectic matrices in convexity Hamiltonian dynamical systems;
-Simplicity of the group Hamiltonian homeomorphisms of closed oriented surfaces with genus ≥ 1 and the simplicity of the group of homeomorphisms of 2-sphere or 2-disk.

RESEARCH INTERESTS

JAN P. BOROŃSKI

The following topics are among my research interests. More details are suggested by the titles of select publications.
(A) Strange attractors, exotic minimal sets and dynamics on cofrontiers[1-4]
(B) Fixed Point Theory [5-7]
(C) Continua in topology and dynamics [8]

Select Recent Work

[1] *Boroński J.P.&Clark A.&Oprocha P.,* New exotic minimal sets from pseudo-suspensions of Cantor systems, **arXiv** 2016
[2] *Boroński J.P.&Oprocha P.* Rotational chaos and strange attractors on the 2-torus, **Math. Zeit.** 2015
[3] *Boroński J.P.&Oprocha P.* On entropy of graph maps that give hereditarily indecomposable inverse limits, **J. Dyn. Diff. Eq.** 2015
[4] *Boroński J.P.&Oprocha P.* On indecomposability in chaotic attractors, **Proc. Am. Math. Soc.** 2015
[5] *Boroński J.P.* On a generalization of the Cartwright-Littlewood fixed point theorem for planar homeomorphisms, **Erg. Th. Dyn. Syst.** 2015
[6] *Boroński J.P.* A fixed point theorem for the pseudo-circle, **Top. Appl.** 2011
[7] *Boroński, J.P.,* Fixed points and periodic points of orientation-reversing planar homeomorphisms, **Proc. Am. Math. Soc.** 2010
[8] *Boroński J.P.&Oprocha P.* On dynamics of the Sierpiński Carpet, **arXiv** 2015