Surfaces in Luminy

3-7 october 2016

Poster abstracts

On Sublinear displacement of torus homeomorphisms
Guilherme Silva Salomão and Fabio Armando Tal (USP)

In this poster we want to present our current work in progress: let $f : \mathbb{T}^2 \to \mathbb{T}^2$ be a torus homeomorphism homotopic to the identity such that the rotation set $\rho(f)$ is a line segment with irracional slope of the form $\rho(f) = \{t\,\vec{v} \mid 0 \leq t \leq 1\}$. We will show that it is not possible that the orbits have unlimited displacement in the direction of $v^{\perp}$. For this we will use the Brouwer-Le Calvez foliation (see [1]) and the forcing theory for transverse paths developed in [2].

[1] Le Calvez, P., *Une version feuilletée équivariante du théorème de translation de Brouwer*, Publications Mathématiques de l'IHÉS, 102, 2005, no. 1, 1–98.

[2] Le Calvez, P., Tal, F. A., *Forcing theory for transverse trajectories of surface homeomorphisms*, arXiv:1503.09127.

Transition density probability function of a markovian perturbation of a probability measure preserving homeomorphism
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In this work proof we study markovian perturbation of a dynamics system $f : M \to M$, where M is a compact manifold and f preserves a probability measure of full support. It is shown that, if the perturbated dynamics has a transition density probability function $p : M \times M \to [0, \infty)$ that is continuous and satisfies $p(x, f(x)) > 0$, then there exist $N > 0$ and $b > 0$ such that the N- transition probability density p_n is bounded below by b, and the process has a single stationary invariant measure with exponentially fast convergence. Furthermore, we give estimates for the values of b depending on the modulus of continuity of p, f and the minimal value of $h(x) = p(x, f(x))$. Finally, we provide similar estimates for a class of stochastic perturbations of convex billiards with C^2 boundary by a similar method.

Rotation theory in analysing hybrid neuron models

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Integrate-and-fire (IF) models are popular models widely used in applications. They combine excitable nature of nerve cells membranes with the discrete nature of spike emission and thus belong to the so-called hybrid dynamical systems, coupling continuous dynamics given by ordinary differential equations with the reset mechanism. Since the theory of hybrid systems is relatively new, rigorous mathematical analysis of this class of models is still a challenging issue, yet the methods of analysing them have noticeably developed during recent years.

A number of works ([Keener, Hoppensteadt, Rinzel 1981; Coombes, Bressloff 1999; Carrillo, Ongay 2001; Tiesinga 2002; Brette 2004; Gedeon, Holzer 2004,...]) already used circle mappings and rotation theory in order to analyze effectively the qualitative dynamics of one-dimensional periodically driven IF models. This was achieved by describing the sequence of consecutive spikes via iterations of a real map, called *firing map*, which could be seen as a lift of a degree-one circle map. Due to the development of some further results on the displacement sequence for trajectories of circle homeomorphisms/diffeomorphisms ([Marzantowicz, Signerska 2014]), not only the qualitative nature of spike trains, but also some characteristics of interspike-intervals can be rigorously described ([Marzantowicz, Signerska 2015; Signerska-Rynkowska 2015]).

In comparison to one-dimensional IF models, bidimensional IF models ([Izhikevich 2003; Brette, Gerstner 2005; Touboul, Brette 2009;...]) are in turn very versatile representations of neuronal dynamics and have a rich phenomenology but their mathematical study is more complicated. Interestingly, in the presence of subthreshold singularities these systems are able to exhibit mixed-mode oscillations (MMOs), in which consecutive spikes or bursts are alternated by small subthreshold oscillations ([Rubin, Signerska-Rynkowska, Touboul,Vidal 2016]), as observed in different cell types and brain areas. However, in this case the emerging *adaptation map* is no longer continuous and moreover shows divergence of the derivative in the neighbourhood of discontinuity points, which prevents one from using classical theories of circle diffeomorphisms of Poincaré and Denjoy, as for one-dimensional models. Notwithstanding, such a map, cut to the proper invariant interval, can still be seen as a discontinuous circle map, falling into the framework of either the non-overlapping lifts ([Rhodes, Thompson 1986, 1991; Brette 2003]) or the so-called "old heavy maps" ([Misiurewicz 1986]), with the univocal bidirectional link between the rotation number of the trajectory and the signature of the generated MM(B)Os.

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