SURFACES IN LUMINY

October 3–7, 2016

Begoña Alarcón: Global dynamics for symmetric planar maps.

We consider suficient conditions to determine the global dynamics of maps of the plane with a unique fixed point which is also hyperbolic, with emphasis in the equivariant case. The study of global dynamics has long been of interest. Particular attention has been given to the question of inferring global results from local behaviour, when a unique fixed point is either a local attractor or repellor. However, the problem of characterizing the existence of a global saddle for planar diffeomorphisms is still open.

When the map is equivariant under the action of a subgroup of the orthogonal group $O(2)$, it is possible to describe the local dynamics and - from this - also the global dynamics. In particular, if the group contains a reflection, then there is a line invariant by the map. This allows us to use results based on the theory of free homeomorphisms to describe the global dynamical behaviour.

The presence of symmetry in a dynamical system creates special features that may be used to obtain global results. Planar dynamics with symmetry, when the fixed point is either an attractor or a repellor, has been addressed in [3]-[5]. These results do not treat the important case when the fixed point is a local saddle. However, in [4] it is shown that the only symmetry groups that admit a local saddle are $\mathbb{Z}_2(\langle -Id \rangle)$, $\mathbb{Z}_2(\langle \kappa \rangle)$ and $D_2 = \mathbb{Z}_2^a \oplus \mathbb{Z}_2^b$. The superscripts a and b indicate that the groups $\mathbb{Z}_2^a$ and $\mathbb{Z}_2^b$ are generated by two reflections, a and b, on orthogonal lines.

We obtain suficient conditions under which the local saddle is -actually- a global saddle, in a topological sense. We also address the special case of D_2–symmetric maps and some applications to differential equations.

References

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Jerôme Buzzi: The almost-Borel structure of surface diffeomorphisms.

In a joint work with Mike Boyle, we defined an invariant combining entropies and periods which leads to a classification of surface diffeomorphisms respecting all hyperbolic measures and their conjugacy with Markov shifts. We aim to give more concrete interpretation of this invariant (in particular using joint work with Crovisier and Sarig) and, ultimately, to characterize the values it can achieve.

André de Carvalho: Super generalized pseudo-Anosov maps.

The purpose of this talk is not to set a new record for the number of noun modifiers in mathematical definitions, but to present a construction which applies to graph maps in general and yields:

1) a pseudo-Anosov map if the graph map is a train track map; 2) a generalized pseudo-Anosov map if the graph map is post-critically finite and has an irreducible aperiodic transition matrix; 3) an interesting type of surface homeomorphisms which generalizes both the previous classes otherwise.

In particular, this produces a unified construction of surface homeomorphisms whose dynamics mimics that of the tent family of interval endomorphisms, completing an earlier construction of unimodal generalized pseudo-Anosov maps in the post-critically finite case.

At the end of the talk I'll take suggestions for better names for type 3).

This is joint work with Phil Boyland and Toby Hall.

Grzegorz Graff: Shub conjecture for smooth self-maps of the sphere.

Estimating of the growth rate of the number of periodic points for smooth self-maps of compact manifolds is a challenging problem. Let us consider C^1 self-map f of m-dimensional sphere $\mathbb{S}^m$ with isolated fixed points of f^n for each n and degree $\mathrm{Deg}(f)$ such that $|\mathrm{Deg}(f)| \geq 2$. In 1974 Michael Shub conjectured that the growth rate of the number of periodic points is at least (asymptotically) exponential (Problem 4 in [4]):

$$\limsup_{n\to\infty} \frac{\log \#\mathrm{Fix}(f^n)}{n} \geq \log|\mathrm{Deg(f)}|.$$

There was no progress in solving Shub conjecture in the whole generality within the last decades, and the same question was repeated by Shub as an open problem during International Congress of Mathematicians in Madrid in 2006 [5] (Problem 3). Recently, Shub and Pugh [3] and Misiurewicz [2] proved Shub conjecture for C^1 self-maps of $\mathbb{S}^2$ which preserve latitudes.

The aim of the talk is to sketch the known partial results related to Shub conjecture and discuss possible prospects for its solution. In particular, we prove the conjecture for C^1 self-maps of $\mathbb{S}^2$ which preserve longitudes and estimate the number of periodic points for C^1 self-maps of higher dimensional spheres.

This is joint work with Michał Misiurewicz and Piotr Nowak-Przygodzki.

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Pierre-Antoine Guihéneuf: Generic homeomorphisms and rotation sets.

This talk will deal with rotation sets of generic homeomorphisms, in both the general and the conservative setting. In particular, a previous work of A. Passeggi states that the rotation set of a generic homeomorphism is a polygon with rational vertices. In a recent small work with A. Koropecki, we give an alternative proof of this statement, that allows us to get the same result in the conservative setting. One of the main arguments of this proof is the genericity of shadowing, that I obtained recently in a small work in collaboration with T. Lefeuvre.

Alejandro Kocsard: On the dynamics of minimal homeomorphisms of $\mathbb{T}^2$

In this talk we will discuss some rigidity results about minimal 2-torus homeomorphisms which are isotopic to the identity.

The rotation sets of such systems have empty interior and hence, they are either a point or a line segment. We shall concentrate on the last case and show that they always exhibit a certain "generalized invariant foliation." Then we will use this topological structure to obtain some dynamical information of these systems, showing that they are much more rigid than minimal pseudo-rotations.

Andres Koropecki: A Poincaré Bendixson theorem for translation lines and applications to invariant continua.

Given an orientation-preserving homeomorphism f of the sphere, a translation line is the image L of a continuous injection $\mathbb{R} \to \mathbb{S}^2$ such that $f|_L$ has no fixed points. We show that there are two possibilites for such a line: either its ω-limit contains a fixed point, or its *filled* ω-limit is a "rotational attractor". This is in analogy with the behavior of flow orbits described by the Poincaré-Bendixson theorem.

Among the applications of this result, we give a general description of the dynamics in a neighborhood of the boundary of an open invariant topological disk U whenever there are fixed prime ends which are not realized as fixed points in the boundary, obtaining a version on the ambient space of a description of the dynamics on the prime ends compactification given by Cartwright-Littlewood and Matsumoto-Nakayama. Namely, we show that ∂U is contained in the union of the basins of a finite set of rotational attractors and repellors.

As a consequence, if an invariant circloid K contains a fixed point but its boundary does not, then the dynamics in a neighborhood of K is topologically semiconjugate to a Morse-Smale system in the circle.

We also obtain a description of the dynamics of invariant continua without fixed points, which in particular implies the following: if a continuum is contained in the chain recurrent set of f and has no fixed points, then its complement has exactly two invariant connected components.

This is a joint work with Alejandro Passeggi.

Kathryn Mann: Orderability and groups of homeomorphisms of the circle.

As a counterpart to Deroin's minicourse, we discuss actions of groups on the circle in the C^0 setting. Here, many dynamical properties of an action can be encoded by the algebraic data of a left-invariant circular order on the group. I will highlight rigidity and flexibility phenomena among group actions, and discuss new work with C. Rivas relating these to the natural topology on the space of circular orders on a group.

Shigenori Matsumoto: Nontrivial attractor-repellor maps of S^2 and rotation numbers.

An orientation preserving homeomorphism h of S^2 which satisfies the following conditions is called a *nontrivial attractor-repellor map* (NAR map). Choose two points from S^2 and name them $\pm\infty$.

(1) $-\infty$ is an attractor of h with basin $W_{-\infty}$ and ∞ a repellor with basin W_∞. ($W_{\pm\infty}$ is homeomorphic to an open disc.)
(2) $Z = S^2 \setminus (W_{-\infty} \cup W_\infty)$ is nonempty. Equivalently, h is not a North-South map.
(3) $W_{-\infty} \cap W_\infty \neq \emptyset$. Equivalently, there is no continuum separating $\pm\infty$.

The study of such maps was innitiated in [1] and [2]. We fix once and for all a lift $\tilde{h}$ of h to the universal covering space of $S^2 \setminus \{\pm\infty\}$. Then the rotation number $\mathrm{rot}(\tilde{h}, \mu) \in \mathbb{R}$ is defined for any h-invariant probability measure μ on $S^2 \setminus \{\pm\infty\}$. ($\mu$ is necessarily supported on Z.) Besides, we have two prime end rotation numbers $\mathrm{rot}(\tilde{h}, \pm\infty)$ defined as follows. The map h extends to a homeomorphism of the prime end compactification $W^*_{\pm\infty}$ of $W_{\pm\infty}$. The choice of the lift $\tilde{h}$ as above gives a lift of $h : W^*_{\pm\infty} \setminus \{\pm\infty\} \to W^*_{\pm\infty} \setminus \{\pm\infty\}$ to the universal cover. Its restiction to the boundary line defines $\mathrm{rot}(\tilde{h}, \pm\infty)$.

Theorem 1. *For any $\alpha_\pm \in \mathbb{R}$, there is a NAR maps h and its lift $\tilde{h}$ such that* $\mathrm{rot}(\tilde{h}, \pm\infty) = \alpha_\pm$.

The following is a refinement of the results in [1] and [2].

Theorem 2. *(1) If* $\mathrm{rot}(\tilde{h}, +\infty) = \alpha$ *is rational, then there is a periodic orbit whose rotation number is α.*

(2) If $-\infty$ is accessible from $W_{+\infty}$, then all the rotation numbers $\mathrm{rot}(\tilde{h}, \pm\infty)$ *and* $\mathrm{rot}(\tilde{h}, \mu)$ *coincide. If moreover it is irrational, there are no periodic orbits.*

Since our map h resembles gradient-like maps, it is natural to consider the chain recurrent set. It is contained in $\{\pm\infty\} \cup Z$, and is partitioned into a disjoint union of chain transitive classes: each, closed and h-invariant. For any chain transitive class C_0 contained in Z, we have a version of the Poincaré-Birkhoff theorem. Unfortunately at this moment, we only have a considerably weaker statement. Let us denote $[\alpha, \beta] = \{\mathrm{rot}(\tilde{h}, \mu) \mid \mathrm{Supp}(\mu) \subset C_0\}$.

Theorem 3. *(1) If $[a, b] \subset [\alpha, \beta]$ and if a, b are rationals realized by periodic points, then any rationals in $[a, b]$ are realized by periodic points.*

(2) If α, β are rationals such that $\alpha < \beta$, then they are realized by periodic points.

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Alejandro Passeggi: Franks-Misiurewicz conjecture for extensions of irrational rotations

We present a proof of the Franks-Misiurewicz conjecture for extensions of irrational rotations. In particular, making use of a result by T. Jäger and a recent result by A. Kocsard, this implies that the rational slope case in the conjecture is true for minimal homeomorphisms. This is in contrast to the counter example announced by A. Avila for the irrational slope case.

Francisco Ruiz del Portal: Some dynamical applications of Carathéodory's prime ends theory.

There are many papers in the literature where prime ends theory has been useful to solve dynamical problems or to provide new theoretical results. Our approach has many points in common with some of them but in this talk our goal will be somehow different. We shall place the emphasis on results about global attraction and estimation of basins of attraction looking for their applicability to differential equations.

Coathors: L. Hernández-Corbato, R. Ortega.

Sobhan Seyfaddini: C^0 Hamiltonian dynamics and the Arnold conjecture.

After introducing Hamiltonian homeomorphisms and recalling some of their properties, I will focus on fixed point theory for this class of homeomorphisms. The main goal of this talk is to present the outlines of a C^0 counter example to the Arnold conjecture in dimensions four and higher. This is joint work with Lev Buhovsky and Vincent Humiliere.

Gioia M. Vago: The Ogasa invariant for homology spheres in dimension 3

The Ogasa invariant $\nu(M)$ of a manifold M is obtained by a minimax procedure on attaching handles.

Its dynamical meaning is the following. On the one hand, any Morse function on M must have a regular level N such that the sum of the Betti numbers of N is at least $\nu(M)$. On the other hand, there exists a Morse function on M such that the sum of the Betti numbers of any regular level N does not exceed $\nu(M)$.

The computation of such an invariant is straightforward in dimension 2. With Michel Boileau (Aix-Marseille Univ, France) we have understood the meaning of this invariant, as well as its behaviour, in dimension 3. Its exact computation is not always explicit, and is intrinsically very technical indeed. Starting from dimension 4, the question of its meaning (and of its computation) is open.

In this talk, we shall discuss in details the case of arborescent graph manifolds, for which the value of the Ogasa invariant admits a nice characterisation in terms of the underlying graph.