MULTIVARIATE APPROXIMATION AND INTERPOLATION WITH APPLICATIONS

September 19 - 23, 2016

Mohamed ADDAM: B-spline finite element method for dynamic deflection of beam deformation model.

In this work, we solve the dynamic deflection of beam deformation model. We use the frequency-domain method to solve the considered transport problem, as a partial differential equation with non-homogeneous boundary conditions. The method employs the Fourier transform and consists of two stages. In the first stage the equations are transformed into an elliptic problem for the frequency variables. The numerical solutions of this problem are approximated using a Galerkin projection based on the B-spline finite element method. In the second stage a Gauss-Hermite quadrature procedure is proposed for the computation of the solution of the inverse Fourier transform. The frequency domain method avoids the discretization of the time variable in the considered system and it accurately resolves all time scales in deflection of beam deformation regimes. A similarly frequency-domain method and B-spline finite element analysis are used to solve the coupled Timoshenko transverse vibrating equations with non-homogeneous boundary conditions. Finally, several test examples are presented to verify high accuracy, effectiveness and good resolution properties for smooth and discontinuous solutions.
(Joint work with Abdarrahman Bouhamidi).

Keywords. Beam deformation model, Timoshenko transverse vibrating equations, B-spline finite element analysis. Gauss-Hermite quadrature method. Coupled partial differential equations and numerical analysis.

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Carolina BECCARI, Mike NEAMTU: Rational Geometric Splines: construction and applications in the representation of smooth surfaces.

Rational Geometric Splines (or RAGS) are piecewise functions that can be used to represent smooth parametric surfaces of arbitrary topological genus and arbitrary order of continuity. The main advantage of RAGS surfaces in comparison to alternative representations, for example those based on so-called manifold splines, subdivision methods and T-splines, is that they mimic the standard bivariate splines on planar domains and their constructive aspects, thus they are closer in spirit to the traditional NURBS representation. After reviewing the basics of RAGS, we will discuss how to construct these splines by suitable association with a homogeneous geometry. We will provide computational examples considering direct analogs of the Powell-Sabin macro-elements and also spline surfaces of higher degrees and higher orders of continuity obtained by minimizing an energy functional.

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Len BOS: Some Bivariate Generalizations of Berrut's Rational Interpolants.

We discuss some bivariate generalizations of Berrut's rational interpolants to the case of "equally spaced" points on a triangle.

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Vittoria BRUNI: Some applications of the wavelet transform with signal-dependent dilation factor.

Time-scale transforms play a fundamental role in the compact representation of signals and images [1]. Non linear time representation provided a significant contribution to the definition of more flexible and adaptive transforms. However, in many applications signals are better characterized in the frequency domain. In particular, frequency distribution in the frequency axis is strictly dependent on the signal under study. On the contrary, frequency axis partition provided by conventional transforms obeys more rigid rules. It would be then desirable to have a transform able to adapt to the frequency content of the signal under study, i.e. having a changing Q factor. The rational dilation wavelet transform [2, 3] (RDWT) is a flexible tool that allows to change the dilation factor at each step of the transform as well as the analyzing window function, by maintaining the structure and properties of the classical wavelet transform, which is implemented through perfect reconstruction filter banks. Some examples concerning the way of selecting significant scales, i.e. central frequencies and bandwidths of the filter bank, in different applications, including image denoising, deblurring and fusion, will be shown. The properties of the

corresponding adaptive transform will be also discussed.
(Joint work with Domenico Vitulano).

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Maria CHARINA: Multigrid and subdivision.

Multigrid is an iterative method for solving large linear systems of equations whose Toeplitz system matrix is positive definite. One of the crucial steps of any Multigrid method is based on multivariate subdivision. We derive sufficient conditions for convergence and optimality of Multigrid in terms of trigonometric polynomials associated with the corresponding subdivision schemes.
(This is a joint work with Marco Donatelli, Lucia Romani and Valentina Turati).

Ole CHRISTENSEN: The unitary extension principle and its generalizations.

The unitary extension principle (UEP) by Ron & Shen yields a convenient way of constructing tight wavelet frames in $L^2(\mathbf{R})$. Since its publication in 1997 several generalizations and reformulations have been obtained, and it has been proved that the UEP has important applications within image processing. In the talk we will present a recent extension of the UEP to the setting of generalized shift-invariant systems on $\mathbf{R}$ (or more generally, on any locally compact abelian group). For example, this generalization immediately leads to a discrete version of the UEP.
(The results are joint work with Say Song Goh).

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Oleg DAVYDOV: Error bounds for conditionally positive definite kernels without polynomial terms.

We discuss new error bounds for the interpolation by conditionally positive definite kernels where the usual polynomial term is omitted. It has been proved in a famous paper by C. A. Micchelli that radial basis function interpolants $s(x) = \sum \lambda_j \phi(\|x - x_j\|)$ exist uniquely for the multiquadric radial function $\phi(r) = -\sqrt{r^2 + c^2}$. Since ϕ is only conditionally positive definite, usual error bounds are nevertheless only applicable to the multiquadric interpolant if the sum is augmented by a constant function. Recent work [1] that fills this gap will be presented in detail. It provides in particular error bounds for all functions in Pontryagin spaces [2] associated with any conditionally positive definite kernels of order one that are negative on the diagonal. The bounds are in terms of the growth function as in [3] and imply in particular the global spectral convergence orders as well as the local consistency estimates for the numerical differentiation and related generalized finite difference methods. It is also shown that the Pontryagin native spaces of such kernels contain the traditional semi-Hilbert native spaces.

References

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Stefano DE MARCHI: On the rescaled method for RBF approximation.

In the recent paper [1], a new method to compute stable kernel-based interpolants has been presented. This *rescaled interpolation* method combines the standard kernel interpolation with a properly defined rescaling operation, which smooths the oscillations of the interpolant. Although promising, this procedure lacks a systematic theoretical investigation.

Through our analysis, this novel method can be understood as standard kernel interpolation by means of a properly rescaled kernel. This point of view allow us to consider its error and stability properties.

First, we prove that the method is an instance of the Shepard's method, when certain weight functions are used. In particular, the method can reproduce constant functions.

Second, it is possible to define a modified set of cardinal functions strictly related to the ones of the not-rescaled kernel. Through these functions, we define a Lebesgue function for the rescaled interpolation process, and study its maximum - the Lebesgue constant - in different settings.

A preliminary theoretical result on the estimation of the interpolation error is also presented.

Among the possible applications discussed in [2], we present the one that couples the method with a partition of unity algorithm. This setting seems to be the most promising, and we illustrate its behavior with some experiments.

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[2] Andrea Idda: *A comparison of some RBF interpolation methods: theory and numerics*, Master's degree thesis, Department of Mathematics, University of Padova (2015).

Shai DEKEL: Deep learning on Manifolds.

Deep learning (DL) methods have become very popular in recent years due to their success in a variety of "human like" tasks in areas such as computer vision and natural language processing. However, there are many data science problems in which data is collected by sensors that, unlike pixels of a camera, are located on geometric structures. In these cases, conventional DL architectures such as the Convolutional Neural Networks (CNN) are unusable, because the notion of training localized convolutions on the data is not obvious. We show how modelling DL architectures based on harmonic analysis on graphs and manifolds provide better classification and estimation than previous work on several data sets.

Chongyang DENG: A unified interpolatory subdivision scheme for quadrilateral meshes.

For approximating subdivision schemes, there are several unified frameworks for effectively constructing subdivision surfaces generalizing splines of an arbitrary degree. In this paper, we present a similar unified framework for interpolatory subdivision schemes. We first decompose the $2n$-point interpolatory curve subdivision scheme into repeated local operations. By extending the repeated local operations to quadrilateral meshes, an efficient algorithm can be further derived for interpolatory surface subdivision. Depending on the number n of repeated local operations, the continuity of the limit curve or surface can be of an arbitrary order C^L, except in the surface case at a limited number of extraordinary vertices where C^1 continuity with bounded curvature is obtained. Boundary rules built upon repeated local operations are also presented.
(Joint work with Weiyin Ma).

Nira DYN: Reconstruction of 2D shapes and 3D objects from their 1D parallel cross-sections by "geometric piecewise linear interpolation.

The talk presents first a method for the reconstruction of 2D shapes from their 1D parallel cross-sections, and then a method for the reconstruction of 3D objects from a similar type of data, with the latter method related to the first. Both methods are based on regarding the object/shape as a set-valued function and the cross-sections as its samples. The reconstructed object/shape is a continuous set-valued function, which interpolates the data. It consists of pieces which are polytopes/polygons. The two methods and their properties will be discussed, and some examples of reconstruction will be given.
This talk is based on a joint work with Elza Farkhi and our joint student Shirley Kainan.

Nora ENGLEITNER: Partially Nested Hierarchical B-Splines.

The established construction of hierarchical B-splines [1] starts from a given sequence of nested spline spaces, and hence it is not possible to pursue independent refinement strategies in different parts of a model. However, this possibility would be highly useful for designing surfaces with creases or similar features and in isogeometric analysis for using independent refinement techniques (such as h- and p-refinement) in different areas of the computational domain.

In order to overcome the limitation of the hierarchical B-splines, we generalize Kraft's selection mechanism to obtain sequences of partially nested spline spaces. We generalize earlier work, which was limited to constant polynomial degrees and single knots [2], to the case of varying degrees and different knot multiplicities. We introduce assumptions that enable us to define a hierarchical spline basis, to establish a truncation operation, to obtain the partition of unity property, and to derive a completeness result. In addition we discuss an automatic refinement algorithm for such spline spaces and present its application to a least-squares approximation problem. (Joint work with Bert Jüttler and Urška Zore.)

References

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Paul ESCANDE: Estimation of linear integral operator from scattered impulse reponses.

The problem of the restoration of images degraded by spatially varying blurs appear in various domains such as computer vision, astronomy and microscopy. Blurs can be modeled through linear integral operators. Once discretized an operator can be viewed as a matrix. In these applications the huge number of pixels N of images leads to very large matrices $N \times N$. It prohibits most naive approaches, this is called the curse of dimensionality. Moreover, in many settings, the operator is incompletly known and must be estimated prior to the restoration of images.

The problem of estimating linear integral operators has a long history and has known a growing interest in the last decade. Most of the methods developed do not scale with the dimensionality of the problem. In this work we propose the construction of a robust-to-noise estimator from a set of scattered impulse responses that is tractable in huge dimension. It is based on a row by row interpolation of the time varying impulse reponse (TVIR) that handles the noise. Furthermore we provide a complete theoretical analysis of the performance of the estimator. This analysis inherits from the theory of splines, radial basis functions and sampling inequalities in Sobolev spaces. Finally we illustrate the method on the estimation of a spatially varying blur and restorations of images.
(Joint work with Jérémie Bigot and Pierre Weiss).

Greg FASSHAUER: Some Recent Insights into Computing with Positive Definite Kernels.

In this talk I will discuss recent joint work with Mike McCourt (SigOpt, San Francisco) that has led to progress on the numerically stable computation of certain quantities of interest when working with positive definite kernels to solve scattered data interpolation (or kriging) problems.

In particular, I will draw upon insights from both numerical analysis and modeling with Gaussian processes which will allow us to connect quantities such as, e.g., (deterministic) error estimates in terms of the power function with the kriging variance. This provides new kernel parametrization criteria as well as new ways to compute known criteria such as MLE. Some numerical examples will illustrate the effectiveness of this approach.

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Brigitte FORSTER: Directional time-frequency analysis via continuous frames.

Grafakos and Sansing [GS08] have shown how to obtain directionally sensitive time-frequency decompositions in $L^2(\mathbb{R}^n)$ based on Gabor systems in $L^2(\mathbb{R}^d)$; the key tool is the "ridge idea," which lifts a function of one variable to a function of several variables. We generalize their result in two steps: First by showing that similar results hold starting with general frames for $L^2(\mathbb{R})$, both in the setting of discrete frames and continuous frames, and second by extending the representations to Sobolev spaces. The first step allows to apply the theory for several other classes of frames, e.g., wavelet frames and shift-invariant systems, and the second one significantly extends the class of examples and applications. We will consider applications to the Meyer wavelet and complex B-splines. In the special case of wavelet systems we show how to discretize the representations using ϵ-nets. (This is joint work with Peter Massopust (TU Munich, Germany) and Ole Christensen (DTU Lyngby, Denmark)).

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Karlheinz GRÖCHENIG: Sampling for solutions of the heat equation.

Let $u(x,t)$ be the solution of the heat equation on $\mathbb{R}^d$ with initial condition $u(x,0) = f$. Assume that f has compact support K and that u is sampled at a fixed time t, i.e., a finite number of samples $u(x_j,t), j = 1,\dots,r$, is known. What can be said about $u(x,t)$? What can be said about the initial condition?

(i) We derive an error estimate for how accurately $u(x,t)$ can be approximated from finitely many (non-uniform) samples taken from a neighborhood of K.

(ii) We then study the random sampling of $u(x,t)$ for fixed t.

(iii) Whereas the estimate of the initial condition f from a solution $u(\cdot,t)$ at a given time t is one of the most ill-posed problems, it is possible to estimate the L^p-norm of the initial condition with stability guarantees.
(This is joint work with Richard Bass, formerly University of Connecticut).

Philipp GROHS: Stable Phase Retrieval in Infinite Dimensions.

Phase retrieval seeks to reconstruct a signal $f \in \mathcal{H}$, a Hilbert space, from phaseless measurements $(|\langle f, \varphi_\lambda\rangle|)_{\lambda\in\Lambda}$, w.r.t. a measurement system $(\varphi_\lambda)_{\lambda\in\Lambda} \subset \mathcal{H}$. Such problems can be seen in a wide variety of applications, ranging from X-ray crystallography, microscopy to audio processing and deep learning algorithms. The most classical phase retrieval problem arises in X-ray crystallography where a compactly supported function f is reconstructed from intensity measurements of its Fourier transform $\widehat{f}$, whereas in audio processing one often aims to reconstruct a function $f \in L^2(\mathbb{R})$ from its spectrogram, e.g. the absolute values of its Gabor transform $V_g f(x,y) := \int_{\mathbb{R}} f(t)g(t-x)e^{2\pi i y t}dt$.

While we still do not have a precise theoretical understanding of when a phase-retrieval problem is well-posed (in the sense that the absolute values $|\langle f, \varphi_\lambda\rangle|)_{\lambda\in\Lambda}$ determine any $f \in \mathcal{H}$ up to a global phase factor), the crucial question of whether the reconstruction is *stable*, in the sense that bounds of the form

$$\inf_{\varphi\in\mathbb{R}} \|f - e^{i\varphi}g\| \leq C\|(|\langle f, \varphi_\lambda\rangle|)_{\lambda\in\Lambda} - (|\langle g, \varphi_\lambda\rangle|)_{\lambda\in\Lambda}\|,$$

with C a moderate constant and suitable norms, hold true, is even less understood.

We present three contributions towards such an understanding.

(1) Phase retrieval is always stable if $\mathcal{H}$ is finite-dimensional [1, 3].
(2) Phase retrieval is always unstable whenever $\mathcal{H}$ is infinite-dimensional and the constant C may deteriorate superexponentially for natural fine-grained finite-dimensional approximations of an infinite-dimensional problem [1, 3]
(3) Faced with the negative result in Item 2 we propose a novel stability paradigm for phase retrieval which turns out to be sufficient for several applications. In this novel sense, we are able to prove stability results, even in the infinite-dimensional setting [2].

(This is a joint work with R. Al-Aifari and I. Daubechies).

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Evelyne HUBERT: A moment matrix approach to computing symmetric cubatures.

A quadrature is an approximation of the definite integral of a function by a weighted sum of function values at specified points, or nodes, within the domain of integration. Gaussian quadratures are constructed to yield exact results for any polynomials of degree $2r-1$ or less by a suitable choice of r nodes and weights. Cubature is a generalization of quadrature in higher dimension.

Constructing a cubature amounts to find a linear form $p \mapsto a_1 p(x_1) + \ldots + a_r p(x_r)$ from the knowledge of its restriction to polynomials of degree d or less. The unknowns are the weights a_j and the nodes x_j. An approach based on moment matrices was proposed in [2]. We give a basis-free version in terms of the Hankel operator H associated to a linear form. The existence of a cubature of degree d with r nodes boils down to conditions of ranks and positive semidefiniteness on H. We then recognize the nodes as the solutions of a generalized eigenvalue problem. Standard domains of integration are symmetric under the action of a finite group. It is natural to look for cubatures that respect this symmetry [1]. Introducing adapted bases obtained from representation theory, the symmetry constraint allows to block diagonalize the Hankel operator H. The size of the blocks is explicitly related to the orbit types of the nodes. From the computational point of view, we then deal with smaller-sized matrices both for securing the existence of the cubature and computing the nodes.
(Joint work with Mathieu Collowald).
For more details see `https://hal.inria.fr/hal-01188290`.

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B. Ali IBRAHIMOGLU: On Computing the Derivative of the Lebesgue Function of Barycentric Rational Interpolation.

Polynomial interpolation at equidistant nodes can be very ill-conditioned and deliver a very bad approximant. Fortunately, rational interpolation with preassigned poles seems to be a promising alternative.

In [1] we give a fine analysis, obtaining the precise growth formula

$$\frac{2}{\pi}\left(\ln(n+1) + \ln 2 + \gamma\right) + o(1)$$

for the Lebesgue constant, with γ being the Euler constant, in the case of barycentric rational interpolation at equidistant interpolation points. This formulation was only possible after computing the lower bound estimate for the Lebesgue constant in terms of the digamma function $\Psi(x)$. In a similar way, we give an expression to compute the derivative of the Lebesgue function of barycentric rational interpolation using the digamma function $\Psi(x)$. This expression allows us to evaluate it at the midpoints for very high values of n.

References

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Byeongseon JEONG: Interpolatory and noninterpolatory Hermite subdivision schemes reproducing polynomials

In this study, we present a large family of Hermite subdivision schemes with tension parameters. The proposed schemes are quasi-interpolatory because they reproduce polynomials up to certain degrees. Depending on the choice of tension parameters, the corresponding schemes become interpolatory. The smoothness analysis has been performed by using the factorization framework of subdivision operators. Also, the approximation order of the proposed schemes is discussed. Some numerical examples are presented in order to demonstrate the performance of the proposed Hermite schemes.
(Joint work with Jungho Yoon).

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Bert JÜTTLER: Low Rank Spline Surfaces.

It has been observed recently that tensor-product spline surfaces with low rank coefficients provide advantages for efficient numerical integration, which is important in the context of matrix assembly in isogeometric analysis [1]. By exploiting the low-rank structure one may efficiently perform multivariate integration by a executing a sequence of univariate quadrature operations. This fact has motivated us to study the problem of creating such surfaces from given boundary curves.

On the one hand, we reconsider the classical constructions, which include Coons surfaces [2]. We analyze the rank of the resulting parameterizations. On the other hand, we propose a new coordinate-wise rank-2 interpolation algorithm and discuss its extension to the case of parametric boundary curves. Here we discuss the properties of the new construction, which include a permanence principle and the reproduction of bilinear surfaces. Special attention is paid to the property of affine invariance.

This is joint work with Dominik Mokriš.

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Angela KUNOTH: 25+ Years of Wavelets for PDEs.

Ingrid Daubechies' construction of orthonormal wavelet bases with compact support published in 1988 started a general interest to employ these functions also for the numerical solution of partial differential equations (PDEs). Concentrating on linear elliptic and parabolic PDEs, I will start from theoretical topics such as the well-posedness of the problem in appropriate function spaces and regularity of solutions and will then address quality and optimality of approximations and related concepts from approximation theory. We will see that wavelet bases can serve as a basic ingredient, both for the theory as well as for algorithmic realizations. Particularly for situations where solutions exhibit singularities, wavelet concepts enable adaptive appproximations for which convergence and optimal algorithmic complexity can be established. I will describe corresponding implementations based on biorthogonal spline-wavelets.

Moreover, wavelet-related concepts have triggered new developments for efficiently solving complex systems of PDEs, as they arise from optimization problems with PDEs.

Jeremy LEVESLEY: Error estimates for multilevel Gaussian quasi-interpolation on the torus.

In an earlier paper [1] has shown that multilevel quasi-interpolation of periodic functions, using Gaussian's, converges. However, the theoretical results are far from the observed numerical results, in which a doubling of the number of data points results in a reduction in error of 0.8. In this paper we develop a more sophisticated analysis in which we observe theoretical rates closer to the observed rates, and extend the results from the circle to the torus.
(Joint work with Brad Baxter, Xingping Sun and Simon Hubbert).

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Tom LYCHE: Simplex spline bases on the Powell-Sabin 12 split.

For C_1 quadratics on the Powell-Sabin 12-split (PS-12) it is possible to construct an extremely good basis using Simplex splines with 5 knots chosen among the vertices and midpoints of the edges. This basis reduces to a B-spline basis on each edge, have a positive partition of unity, a Marsden identity that splits into real linear factors, and an intuitive domain mesh. The basis is stable in the L_∞ norm with a condition number independent

of the geometry, have a well-conditioned Lagrange interpolant at the domain points, and a quasi-interpolant with local approximation order 3, ([1]). A similar construction has recently been published for C^3 quintics on (PS-12), ([4], [5]). In this talk we investigate the possibility of using other piecewise polynomial spaces on PS-12, ([2], [3]).
(Joint work with Georg Muntingh).

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Carla MANNI: Spline spaces over planar T-meshes and Extended complete Tchebycheff spaces.

Tchebycheff spaces, or more precisely extended Tchebycheff spaces, are natural generalizations of algebraic polynomial spaces. They are a popular tool in approximation theory, especially because they form a very flexible substitute for algebraic polynomial spaces to solve Hermite interpolation problems. Besides algebraic polynomial spaces, important examples of extended Tchebycheff spaces are the null spaces of differential operators with real constant coefficients. Univariate Tchebycheffian splines are smooth piecewise functions with sections in extended Tchebycheff spaces. Multivariate extensions of Tchebycheffian splines can be obtained via the tensor-product approach or by considering more general T-mesh structures. We consider Tchebycheffian spline spaces over planar T-meshes and we analyze its dimension, extending the results presented in [1, 2]. In particular, we focus on extended complete Tchebycheff spaces and we show the full similarity with the classical polynomial case, [3], of the resulting spline spaces. As in the polynomial case, it is of particular interest to understand when the dimension only depends on combinatorial quantities of the T-mesh (such as number of vertices, edges and faces), on the given smoothness, and on the componentwise dimensions of the underlying extended Tchebycheff section spaces.
(This is joint work with Cesare Bracco, Tom Lyche, Fabio Roman and Hendrik Speleers).

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Peter MASSOPUST: B-Splines and Clifford Algebra.

Recently, ideas and methods from the theory of Clifford algebra and Clifford analysis have been applied to image and signal analysis. One prominent example is the analysis of four-component seismic data (consisting of a hydrophone (scalar part) and three orthogonally oriented geophones (vector part)) by Olhede et al. using quaternionic wavelets. For the purposes of multichannel signal and image analysis, one naturally needs one direction for each channel. This suggests to consider extensions of complex-valued transforms to higher dimensions, i.e., to quaternion- and Clifford-valued bases and their associated transforms.

We employ Clifford algebraic and analytic methods to define quaternionic B-splines. Several algebraic and analytic properties of these quaternionic B-splines are presented. We establish refinability for these novel B- splines and show that they define a dyadic multiresolution analysis of $L^2(\mathbb{R}, \mathbb{H})$.
(Joint work with Jeff Hogan).

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Caroline MOOSMÜLLER: Smoothing of vector and Hermite subdivision schemes.

Curve subdivision schemes are algorithms which iteratively refine discrete input data and produce smooth curves in the limit. When studying their properties, the regularity of the limit curve is a topic of high interest.

In scalar subdivision, it is well-known that a subdivision scheme which produces C^m ($m \geq 0$) limit curves can be transformed to a new scheme producing C^{m+1} limit curves via the additional insertion of midpoints in every subdivision step, see e.g. [3]. In terms of the symbol, this procedure can be described by multiplying the symbol with the *smoothing factor* $\frac{z+1}{2}$.

A prominent example is the Lane-Riesenfeld (L-R) algorithm which produces the $m-$th B-Spline curve in the limit. Its mask is given by $\left(\frac{z+1}{2}\right)^m (z+1)$ (doubling of points, followed by m rounds of inserting midpoints). The L-R algorithm is obtained from the C^0 scheme $\frac{(z+1)^2}{2}$ (linear interpolant) by applying $(m-1)$ smoothing steps. This results in limit curves of regularity C^{m-1}.

In this talk we investigate if a similar smoothing procedure exists in the case of vector and Hermite subdivision schemes. It is known that the de Rham transform [2] in Hermite subdivision increases the regularity by 1, see [1]. In these papers, the authors use geometrical ideas, such as corner cutting. We choose an algebraic approach to this problem which, in analogy to the scalar case, manipulates the symbol of a given vector scheme or Hermite scheme.

(Joint work with Nira Dyn).

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Bernard MOURRAIN: Sparse multivariate polynomial-exponential representation and interpolation.

Many problems and applications related to functional analysis involves series representations, which are sums of polynomial-exponential functions. This is for instance the case of Fourier transforms of spikes or signals which are the superposition of vibrations or waves. An important problem is to recover the sparse decomposition of the signal as a sum of polynomial-exponential terms, using few information computed from the signal. Compressing sensing is a celebrated solution to adress this problem, based on the computation of a sparse solution of an underdetermined linear system and its relaxation into a convex optimization problem. But it involves a predefined set of basis functions.

In this presentation, we will describe an alternative approach which does not require a predefined dictionary of basis functions. We will show that series, which are finite sums of polynomial-exponential functions are in correspondence with Artinian Gorenstein algebras. This correspondence is given explicitly by Hankel operators of finite rank. We will exploit the properties of these operators and describe a new algorithm to compute the sparse decomposition of multivariate series. This method uses linear algebra tools for solving multivariate polynomial systems, namely eigenvalue and eigenvector computation of operators of multiplication. It can be seen as a generalization of Prony's method to multivariate series [4, 3]. We will illustrate its performance and its numerical behavior on different problems such as wave identifications, multivariate interpolation and vanishing ideals of points [2], tensor decomposition [1].

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Olga MULA: Dictionary data assimilation for recovery problems.

The approximation of physical systems by coupling measurement data and parametric PDE models has led to a new class of problems in optimal recovery. The setup is the following. Let $\mathcal{M}$ be a compact set of a Hilbert space $\mathcal{H}$ and let V be a finite dimensional subspace of $\mathcal{H}$ which approximates the elements of $\mathcal{M}$ at ε accuracy. Given a function $u \in \mathcal{M}$, the goal is to find the optimal reconstruction of u with the knowledge of V and of m measurements of u, $\ell_i(u),\ i = 1, \dots, m$, where the ℓ_i are linear functionals of $\mathcal{H}'$. Physically speaking the ℓ_i should be understood as sensors to install in the real system.

In this setting, the optimal map understood in a sense described in [1] was first found in [2]. However, its stability strongly depends on the measurement subspace $W = \ w_1, \dots, w_m\} \subset \mathcal{H}$, where the w_i are the Riesz representations of the ℓ_i. In this talk, we first discuss whether the use of subspaces of W could help stabilize the approximation and retrieve better accuracy. Since there are many physical situations in which there is some freedom to choose the sensors, we present a greedy strategy to select appropriate ones from a dictionary and measure its "deviation" from the optimal choice via convergence rate results.

This is an ongoing work in collaboration with P. Binev from the University of South Carolina.

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Mike NEAMTU: Recent Progress on RAGS.

In this talk we will discuss recent results concerning rational geometric splines (RAGS), which are splines defined on 3D triangulations. We will mainly focus on results establishing approximation rates of such splines.
(This is joint work with Carolina Beccari and Maria van der Walt).

Anthony NOUY: Adaptive hierarchical low-rank approximation of multivariate functions using statistical methods.

Tensor methods are among the most prominent tools for the numerical solution of high dimensional problems where functions of multiple variables have to be approximated. Such high-dimensional approximation problems naturally arise in stochastic analysis and uncertainty quantification. In many practical situations, the approximation of high-dimensional functions is made computationally tractable by using rank-structured approximations [1].

In this talk, we present statistical methods for the approximation of a multivariate function $u(x_1, \dots, x_d)$ in tree-based (hierarchical) tensor format, using random evaluations of the function. We particularly focus on the tensor-train format, yielding an approximation of the form

$$u(x_1, \dots, x_d) \approx \sum_{i_1=1}^{r_1} \dots \sum_{i_{d-1}=1}^{r_{d-1}} v_{1,i_1}^{\sigma(1)}(x_{\sigma(1)}) v_{i_1,i_2}^{\sigma(2)}(x_{\sigma(2)}) \dots v_{i_{d-1},1}^{\sigma(d)}(x_{\sigma(d)}),$$

where $v_{i_{\nu-1},i_\nu}^{\nu}(x_\nu)$ are univariate functions represented on suitable functional bases, and where σ is a permutation of $\{1, \dots, d\}$ depending on the chosen tree for the tensor-train representation. Adaptive strategies using statistical error estimates are proposed for the selection of the functional bases, the ranks $(r_1, \dots, r_{d-1})$ and also the permutation σ (i.e. the tree).

Numerical examples illustrate the ability of the proposed algorithms to approximate high-dimensional functions using only a very few evaluations of the functions, which is of particular importance when the function represents a complex numerical code. In this context, the examples illustrate the interest of going from a simple canonical tensor format [2] to more sophisticated hierarchical tensor formats, in order to better exploit the available information on the function.

This is a joint work with M. Chevreuil, L. Giraldi and P. Rai.

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Juan M. PEÑA: Recent advances on Accuracy and Stability in Approximation and C.A.G.D.

Recent results on accurate and stable computational methods for Approximation Theory and Computer Aided Geometric Design are discussed.

The relationship with numerical methods for some structured classes of matrices related to positivity is analyzed. In fact, for these classes of matrices many computations can be performed with high relative accuracy. Among these classes of matrices, we include Vandermonde matrices, Cauchy matrices, Cauchy-Vandermonde matrices, Bernstein-Vandermonde matrices, rational Bernstein collocation matrices, rational Said-Ball collocation matrices, Jacobi-Stirling matrices, Pascal matrices and q-Bernstein collocation matrices.

Another considered problem is the stability of evaluation algorithms for curves and surfaces. We present results for the corresponding evaluation of univariate and multivariate functions.

Valérie PERRIER: Helmholtz-Hodge decomposition, Divergence-free wavelets and applications.

Vector field analysis is ubiquitous in numerous domains, and often the solution of problems should satisfy some conservation property: this is the case in the numerical simulation of incompressible flows where the velocity field is divergence-free.

The Helmholtz-Hodge decomposition [2], under certain smoothness assumptions, allows to separate any vector field into the sum of three uniquely defined components: divergence-free, curl-free and gradient of a harmonic function. Thus, the Helmholtz-Hodge decomposition provides a powerful tool for several applications such as the resolution of partial differential equations [5], and more recently for optimal transport [3]. Therefore, it is important to have at hand decompositions which preserve divergence-free and curl-free properties, and efficient algorithms associated.

Recently, works have been done for the design of efficient divergence free wavelets on square and cubic domains, and their application to the practical Helmholtz-Hodge decomposition computation; first with periodic boundary conditions [1], then with general boundary conditions including Dirichlet or Neumann boundary condition [4].

In this presentation we will first present the divergence-free wavelet bases setting and the last developments [5, 6]. Such bases can be constructed from suitable one dimensional biorthogonal multiresolution analyses on $[0, 1]$, linked by differentiation and integration, with wavelets satisfying homogeneous boundary condition. Then the second part will be devoted to applications concerned with the Helmholtz-Hodge decomposition and divergence free wavelets: in particular we will focus to the optimal transport problem and application to image interpolation. Indeed, the dynamic optimal transport formulation provided by Benamou-Brenier searches for the solution of a minimization problem, satisfying a divergence-free condition. Reformulating the problem using the Helmholtz Hodge decomposition first improves the convexity property of the functional, and second leads to faster implementations, using divergence-free wavelets or primal dual-algorithms.

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Francesca PITOLLI: Less is enough: localizing neural sources by the random sampling method.

Keywords: Neuroimaging; Magnetoencephalography; Electroencephalography inverse problem; random sampling.

Magnetoencephalography (MEG) and Electroencephalography (EEG) allow to reconstruct the neuroelectric current distribution in the brain from non-invasive measurements. In fact, the electric current generated by the activity of the neural sources, produces both a magnetic field outside the head and an electric potential on the skull that can be measured by M/EEG devices. The reconstruction of the neuroelectric activity map underlining a given set of data results in an ill-posed and ill-conditioned inverse problem that requires regularization techniques to be solved. These techniques are often time-consuming, computationally and memory storage demanding. In this talk, we use a slimmer procedure, the random sampling inversion method [2-3], as a regularization technique and show that this inversion method achieves good localization accuracy. Some numerical tests on real M/EEG data are also displayed.

Acknowledgements: Joint work with Cristina Campi and Annalisa Pascarella

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Gerlind PLONKA: Sparse approximation by modified Prony method.

The classical Prony method works with exactly sampled data of the exponential sum

$$h(x) := \sum_{j=1}^{M} c_j \, \mathrm{e}^{f_j x}\,, \quad x \geq 0\,, \tag{1}$$

in the case of known order M. Following an idea of G.R. de Prony from 1795, we can recover all parameters c_j, f_j of the exponential sum (1), if sampled data $h(k)$, $k = 0, \ldots, 2M-1$ are given, where $z_j := \mathrm{e}^{f_j}$ are distinct values in $\mathbb{D} := \{z \in \mathbb{C} : \, 0 < |z| \leq 1\}$. If the number of terms M is unknown, we can use numerical methods, like the ESPRIT method, to evaluate M by considering the numerical rank of a suitable Hankel matrix that is build by the equidistant function values $h(k)$, see [2].

Recently, we extended the Prony-method to a reconstruction technique for sparse expansions of eigenfunctions of suitable linear operators, see [1]. This more general approach provides us with a tool to unify all Prony-like methods on the one hand and establishes a much broader field of applications of the method on the other hand. In particular, it can be shown that all well-known Prony-like reconstruction methods for exponentials and polynomials known so far, can be seen as special cases of this approach. The new insight into Prony-like methods enables us to derive also new reconstruction algorithms for orthogonal polynomial expansions and for expansions in finite-dimensional settings.

However, in many applications for sparse approximation, the function to be recovered is only approximatively of the form (1), and we may want to approximate h by an exponential sum with an a priory fixed number M of exponentials. The open questions are now:
Does the Prony method provide a good approximation of h if the number of terms M is underestimated? Can the error caused by the Prony approximation be exactly quantified in a suitable norm? Do we have to modify the Prony method in order to achieve better estimates? Can such a modified Prony method for sparse approximation be generalized to sparse expansions of eigenfunctions of linear operators?
(This is joint work with Vlada Pototskaia).

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Hartmut PRAUTZSCH: Spherical Splines.

The Bézier representation of homogenous polynomials has little and not the usual geometric meaning if we consider the graph of these polynomials over the sphere. However the graph can be seen as a rational surface and has an ordinary rational Bézier representation. As I will show, both Bézier representations are closely related. Further I consider rational spline constructions for spherical surfaces and other closed manifolds with a projective or hyperbolic structure.

Vladimir Yu. PROTASSOV: Applications of subdivision schemes to combinatorics and to number theory.

Subdivision schemes have found lots of applications in engineering, approximation, interpolation, curve and aid design, etc. They provide an efficient end easily implementable method for approximating and generating smooth surfaces by values in a given mesh. Recently it became clear that subdivision schemes also have some purely theoretical applications, in particular, in combinatorics and in number theory. In particular, the well-developed theory of subdivisions provides a powerful tool in the classical problem of asymptotic of the Euler binary partition function. This problem in various formulations was studied previously by K.Mahler, N.G.De Bruijn, L. Carlitz, B.Reznick, and others. We answer several open questions about the asymptotic behaviour of the binary partition function. In particular, we show that the rate of growth of that function can be expresses in terms of the limit function of a special subdivision schemes. The case or polynomial growth correspond to a subdivision scheme that converges in the space L_1, while the case of regular power growth corresponds to the case when the limit function is a refinable spline. By this observation we classify both these cases.

Moreover, the use of subdivisions allows us to go further and to obtain some generalizations, including multivariate ones. Several open problems will be formulated and discussed.

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Christophe RABUT: Variational Bézier or B-spline curves and surfaces.

Given a control polygon, we use some least-square criterion, to derive “B-curves” (i.e. Bernstein, B-spline, hyperbolic or circular spline curves...) which are closer the control polygon, still being in the same vectorial space as the original one. We usually loose the convex hull property, and better preserve the general shape of the control polygon. The idea is simple : we minimize some L^2-distance between the curve and the control polygon.

Furthermore, by increasing (resp. decreasing) the degree of the parametric polynomial curve, in the same way we derive a curve still closer (resp. further) the control polygon. Similarly we obtain the same type of results by increasing (resp. decreasing) the number of knots of the spline curve

Actually the so-obtained curves (or surfaces, or any multi-dimensional object) are in the space generated by the original B-functions and some “new control points” which are easily derived. The obtained curve (or surface) is the quasi-interpolation, by using the original B-functions, of the so-obtained “new control polygon”. We so keep all the known properties of the original quasi-interpolation (including convex hull property), express in this “new control polygon”.

We give ways to derive new B-functions which are linear combinations of the original B-functions (i.e. Bernstein polynomials, B-splines, hyperbolic or circular B-spline curves...), such that the associated B-curve is closer the control polygon than the usual one (still being in the same functional space, but possibly not in the convex hull of the control polygon), and better preserving the general shape of the control polygon. We also give ways to derive associated basis functions such that the so-obtained curve is further form the control polygon (more cutting the angles), while being in the same functional space.

Finally we propose to mix this least-square criterion together with a least-square distance between some points of the curve and the control points, and with a variational criterion aiming to cut down the oscillations of the curve. Various curves are presented to show the interest of these new curves.

The same strategy is possible for surfaces by using corresponding B-surfaces, such as tensorial product of Bernstein polynomials, of B-splines, or by using polyharmonic B-splines.

Ulrich REIF: Approximation and Modeling with Ambient B-Splines.

The approximation of data given on some manifold is a difficult task since, in general, it is not clear how to construct finite-dimensional function spaces with favorable properties on that manifold. A natural idea is to extend the given data to the ambient space of the manifold, for instance by requesting constant values along lines perpendicular to the manifold. In this way, a new function is defined, which can be approximated by standard tensor product splines in the ambient space. Restricting these splines to the manifold yields the desired approximation. We show that this method has optimal approximation power and illustrate its properties by practical examples. The method can also be used to define parametrizations of free-form surfaces of arbitrary topology. Unlike other approaches, like geometric continuity or subdivision, the ambient B-spline method yields any desired order of smoothness easily.

In the second part of the talk, we discuss the potential of the method for the solution of intrinsic partial differential equation on manifolds. Here, not a given function but functionals like the Laplace-Beltrami operator must be extended to the ambient space of the manifold. Once this is done, the resulting higher-dimensional PDE can be approximated by tensor product B-splines, and again, the actual solution is found by restricting the solution to the manifold. Theorems on the existence and uniqueness of solutions are provided, while convergence results are not available, yet.

Lucia ROMANI: Convergence of corner cutting algorithms refining points and nets of functions.

Carl de Boor showed the convergence of corner cutting algorithms for points in a very general case [1]. In this talk we consider the same assumptions of Gregory and Qu [2], and we give a very simple proof of corner cutting convergence based on results from approximation theory. Moreover, we extend this new elementary proof to the bivariate setting, both in the case of algorithms refining points and nets of functions. As to the latter, we show that, if the initial net consists of univariate continuous functions that are Lipschitz continuous on grid intervals, then corner cutting converges to a bivariate continuous function.
(This is joint work with Costanza Conti and Nira Dyn).

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Milvia ROSSINI: Applications of variably scaled kernels.

Within radial basis functions, handling scale parameters is a well-known and open problem and a proper scaling of the basis function plays a crucial role in applications.

A generalization is given in [1] where new transformed kernels based on classical RBFs are defined by using a scale function.

In this talk, we show different roles of these variably scaled kernel. In an interpolation process, the variable scale parameter may affect both stability and accuracy and an its appropriate choice can significantly improve the recovery quality. In particular here we discuss their application for the recovery of function with particular features such as discontinuities.

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Gabriele SANTIN: Non-symmetric kernel-based greedy approximation.

We analyze kernel-based recovery problems defined by general, and possibly distinct, trial and test spaces.

This setting allows to analyze in a common framework two notable situations, namely point-based interpolation with kernel centers different from the data sites, and non-symmetric collocation. While the second is frequently observed to give better approximations than symmetric collocation, the first one allows to construct interpolants which are not bounded to be centered on the given data. For some kernel, both of them can be also extended to deal with variable shape parameters.

After discussing error and stability properties of this recovery procedure in an abstract setting, and comparing it with the symmetric methods, we will specialize our analysis to the interpolation case. In particular, we discuss a greedy algorithm to adaptively construct data-dependent test and trial spaces. This algorithm is proven to be an extension of the method introduced in [5], and in particular it is treated by means of bi-orthonormal bases which generalize the Newton basis [4].

Different greedy-selection criteria will be presented, and we will discuss their properties. Some of them will be shown to be extensions of the criteria proposed in [1, 3, 6]. Experimentally, we demonstrate their potential on artificial examples as well as on a real world application of a human spine model [7].
(This is joint work with Bernard Haasdonk).

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Tomas SAUER: Prony's problem and superresolution in several variables: structure and algorithms

Prony's problem consists of recovering a function

$$f(x) = \sum_{\omega \in \Omega} f_\omega \, e^{\omega^T x}, \qquad \Omega \subset (\mathbb{R} + i\mathbb{T})^s$$

from samples on an integer grid $\Gamma \subset \mathbb{Z}^s$. In the univariate case, this problem is classical [5] and there exist numerical methods that compute the set Ω by means of eigenvalue methods and the then the coefficients f_ω by solving a linear system. Best known are the MUSIC [7] and ESPRIT [6] algorithm which have been studied and improved recently, see, for example [4].

Recently, also the multivariate version of Prony's problem has been considered, either by projection methods [2] or more direct attempts [3]. As in the univariate case, a *Hankel matrix*

$$F_{A,B} := \left[f(\alpha + \beta) : \begin{array}{c} \alpha \in A \\ \beta \in B \end{array} \right], \qquad A, B \subset \mathbb{N}_0^s,$$

or, equivalently, a *Toeplitz matrix*

$$T_{A,B} := \left[f(\alpha - \beta) : \begin{array}{c} \alpha \in A \\ \beta \in B \end{array} \right], \qquad A, B \subset \mathbb{N}_0^s,$$

play a crucial role as their kernels are isomorphic to the ideal

$$I_\Omega = \{p \in \mathbb{C}[x] : p(X_\Omega) = 0\}, \qquad X_\Omega := e^\Omega = \{e^\omega : \omega \in \Omega\},$$

provided that the index set A is sufficiently rich: the monomials defined by it must span an interpolation space for X_Ω. This information can be used for symbolic or symbolic/numeric hybrid algorithms that compute Ω by means of the eigenvalues of multiplication tables, where the multiplication tables can be efficiently extracted from $F_{A,B}$ or $T_{A,B}$.

The *superresolution* problem as stated and popularized in [1] boils down to Prony's problem as well, but with the additional requirement that the sampling set Γ consists of "small" multiindices. This raises the question how to find small, ideally minimal sets A, B such that the respective monomials allow for interpolation. Under certain additional constraints this turns out to be the well–known hyperbolic cross.

The talk starts from the algebraic structure of the problem and shows how to construct good bases for the Prony ideal and a good interpolation space directly from the Hankel or Toeplitz matrices. All these are *algebraic* methods that would work exactly in exact arithmetic and do not rely on any separations of the frequencies. Nevertheless, numerical implications will be discussed as well.

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Nada SISSOUNO: Adaption of tensor product spline spaces to approximation on domains.

We address the approximation of functions given over a bounded domain $\Omega \subset \mathbb{R}^2$ by tensor product splines. Obviously, this works perfectly for rectangles. In applications, naturally more general domains and, with this, problems for the spline approximation arise.

An open issue is that constants in the error estimates depend on the shape of Ω or the tensor product grid, respectively its knots. We will focus on anisotropic estimates as considered in [1] where the error is bounded by pure partial derivatives of the approximated function which are linked to the maximal knot distance of the grid in the corresponding coordinate direction.

In this talk we introduce a constructive modification of non-uniform tensor product spline spaces which equips the spaces with all properties necessary to get rid of the unwanted dependence of constants in the error estimates. The method is based on a uniform approach presented in [2].

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Hendrik SPELEERS: Local approximation methods using hierarchical splines

Hierarchical spline spaces provide a flexible framework for local refinement coupled with a remarkable intrinsic simplicity. They are defined in terms of a hierarchy of locally refined meshes, reflecting different levels of refinement. The so-called *truncated hierarchical basis* is an interesting basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical basis: its elements form a convex partition of unity, they are locally supported and strongly stable [1, 2].

In this talk we discuss a general approach to construct local quasi-interpolants in hierarchical spline spaces expressed in terms of the truncated hierarchical basis [3, 4]. The main ingredient is the property of *preservation of coefficients* of the truncated hierarchical basis representation. Thanks to this property, the construction of the hierarchical quasi-interpolant is basically effortless. It is sufficient to consider a local quasi-interpolant in each space associated with a particular level in the hierarchy, which will be referred to as a one-level quasi-interpolant. Then, the coefficients of the proposed hierarchical quasi-interpolant are nothing else than a proper subset of the coefficients of the one-level quasi-interpolants. No additional manipulations are required. Important properties – like polynomial reproduction – of the one-level quasi-interpolants are preserved in the hierarchical construction. We also discuss the local approximation order of the hierarchical quasi-interpolants in different norms, and we illustrate the effectiveness of the approach with some numerical examples.

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Joachim STÖCKLER: Methods for constructing multivariate tight wavelet frames.

We give a survey on the construction of multivariate tight wavelet frames with compact support and higher order of vanishing moments. For the unitary extension principle, methods of real algebraic geometry and linear system theory were combined in [1, 2] to provide parameterizations of classes of tight wavelet frames. In essence, the matrix extension problem can be reduced to a scalar "sum-of-squares" problem for multivariate trigonometric polynomials. For the oblique extension principle, a similar construction method was advised in [1], but leads to rational trigonometric functions providing tight wavelet frames with unbounded support. We report about obstacles and recent progress in resolving this defect and explain a new method that is influenced by linear system theory.

(This is joint work with M. Charina, M. Putinar and C. Scheiderer).

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Valentina TURATI: Multigrid and Subdivision: grid transfer operators.

In this work, we solve the dynamic deflection of beam deformation model. We use the frequency-domain method to solve the considered transport problem, as a partial differential equation with non-homogeneous boundary conditions. The method employs the Fourier transform and consists of two stages. In the first stage the equations are transformed into an elliptic problem for the frequency variables. The numerical solutions of this problem are approximated using a Galerkin projection based on the B-spline finite element method. In the second stage a Gauss-Hermite quadrature procedure is proposed for the computation of the solution of the inverse Fourier transform. The frequency domain method avoids the discretization of the time variable in the considered system and it accurately resolves all time scales in deflection of beam deformation regimes. A similarly frequency-domain method and B-spline finite element analysis are used to solve the coupled Timoshenko transverse vibrating equations with non-homogeneous boundary conditions. Finally, several test examples are presented to verify high accuracy, effectiveness and good resolution properties for smooth and discontinuous solutions.

Keywords. Beam deformation model, Timoshenko transverse vibrating equations, B-spline finite element analysis. Gauss-Hermite quadrature method. Coupled partial differential equations and numerical analysis.

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Alberto VISCARDI: Irregular Tight Wavelet Frames: Matrix Approach.

The Unitary Extension Principle is a powerful tool for constructing compactly supported tight wavelet frames in the shift-invariant setting. It is well-known that it leads to matrix extension problems. The entries of the corresponding matrices are trigonometric polynomials. In the univariate setting, explicit expressions for such extensions are known for all trigonometric polynomials arising from refinement equations. Our first goal is to reduce such matrix extension problems to factorizations of positive semi-definite matrices with real entries. In the non shift-invariant setting, Chui, He and Stöckler showed how to construct tight frame elements via the factorization of global positive semi-definite matrices derived from B-splines over irregular knot sequences ([1], [2]). Our second goal is to construct such global matrices in a more general setting and to simplify their factorizations. Our simplification leads to factorizations of few positive semi-definite matrices of much smaller size.

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Elena VOLONTÈ: Anisotropic Diagonal Scaling Matrices and Subdivision Schemes in Dimension d.

In the fields of subdivision schemes and multiresolution analysis, scaling matrices are fundamental because they fix how to refine the given data. Among the classical scaling matrices, the family of shearlet matrices use parabolic matrices where the diagonal elements are one the square of the others. Combining these parabolic matrices with shear matrices we obtain matrices that are capable to treat anisotropic problems. The drawback of shearlet matrices is the quite huge determinant of the parabolic matrix. In fact the absolute value of the scaling matrix determinant gives the number of disjoint cosets

that represents the number of filters used in the multiresolution analysis. An high determinant increases the computational complexity for the analysis and reconstruction of a signal. In this talk we study a family of anisotropic diagonal scaling matrices with the same good properties of shearlet matrices, such as handling directionality and generating a multiple multiresolution analysis, but with lower determinant. In particular we discuss the case $d = 3$ that is important for the applications.
(This is joint work with Mira Bozzini, Milvia Rossini andThomas Sauer).

Holger WENDLAND: Kernel-based Discretisation for Solving Matrix-valued PDEs.

In this talk, I will discuss numerical techniques for solving certain matrix-valued partial differential equations. Such PDEs arise, for example, when constructing a Riemannian contraction metric for a dynamical system given by an autonomous ODE.

The method for solving these PDEs uses a new meshfree discretisation scheme based upon kernel-based approximation spaces. However, since these approximation spaces have now to be matrix-valued, the kernels we need to use are fourth order tensors. Hence, I will start with reviewing and extending recent results on even more general reproducing kernel Hilbert spaces. I will then apply this general theory to solve a typical matrix-valued PDE and derive error estimates for the approximate solution. I will also show some illustrative examples from dynamical systems.

(This is joint work with Peter Giesl).

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Hyoseon YANG: Sixth-order Weighted essentially non-oscillatory schemes based on exponential polynomials.

The aim of this study is to develop a novel sixth-order weighted essentially non-oscillatory (WENO) finite difference scheme. To design new WENO weights, we present two important measurements: a discontinuity detector (at the cell boundary) and a smoothness indicator. The interpolation method is implemented by using exponential polynomials with tension parameters such that they can be tuned to the characteristics of the given data, yielding better approximation near steep gradients without spurious oscillations, compared to the WENO schemes based on algebraic polynomials at lower computational cost. A detailed analysis is performed to verify that the proposed scheme provides the required convergence order of accuracy. Some numerical experiments are presented and compared with other sixth-order WENO schemes to demonstrate the new algorithm's ability.

(This is joint work with Youngsoo Ha, Chang Ho Kim and Jungho Yoon).

Jungho YOON: Univariate Non-linear Approximation Scheme for Piecewise Smooth functions.

The aim of this study is to construct a non-linear scheme to approximate piecewise smooth data on uniform grids. The proposed method is obtained by amending the quasi-interpolation method based on the B-splines. Although quasi-interpolation methods are very useful for data approximation, they often suffer from ringing artifacts when approximating near discontinuities. In order to overcome this drawback, this paper constructs a non-linear approximation method which prevents spurious oscillations around discontinuities, while achieving high order accuracy in smooth regions. To this end, we first present a measurement which estimates the local smoothness of the given data and then construct a local non-linear approximation scheme to approximate data with singularities. Some numerical results are provided to demonstrate the ability of the proposed non-linear method.
(This is joint work with Hyoseon Yang).

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Evening Meeting on Teaching and Pedagogy

Organising Committee of the MAIA conference[1]

Abstract

The organising committee propose to all interested people to meet together on Tuesday night (say from 21:00 to 22:30) in order to talk on teaching and pedagogy. This meeting will be moderated by Christophe Rabut

The details of the subject will be completely open, but we propose to focus on two points:

1. Are lectures as efficient as we would like? If not, which are the modifications that people in the audience use to improve them?

2. The heterogeneity of the level of the students seems to some of us to be growing... is it the impression in the audience? What is done and what to do in order to improve the situation?

Depending on the evolution of the discussion, I might tell you some tips (or more... two strategies) I use for teaching more efficiently. They are mainly focused on collaborative learning, which will be explained and detailed depending on the interest of the audience and of the evolution of the debate.

Christophe Rabut