

# Telomere length and senescence heterogeneity: when size matters!

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06.07.16

**PDE and Probability for Life Sciences  
CIRM, Marseille**

# Outline of the talk

Luria-Delbrück experiment: The foundation of modern biology is a concerted effort in maths and biology.

Telomere length and senescence

Escaping senescence: an interdisciplinary approach?

# The Luria-Delbrück experiment (1943): an early fruitful interaction between mathematics and biology



*Photo courtesy of AP/Wide World Photos*

Salvador E. Luria in his Massachusetts Institute of Technology Laboratory October 16, 1969, after word that he shared the 1969 Nobel Prize with two other bacteriologists for research on viruses.



*Photo courtesy of AP/Wide World Photos*

King Gustaf Adolf, right, presents the Nobel Prize in Physiology or Medicine to German-born American biologist Max Delbrück in Stockholm, Sweden, December 10, 1969. Delbrück, of the California Institute of Technology, shares the prize with American biologist Alfred D. Hershey and Italian-American biologist Salvador E. Luria for their discoveries concerning the replication mechanism of viruses and their genetic structure.

**Mutations** in bacteria that confer survival advantages arise over time.

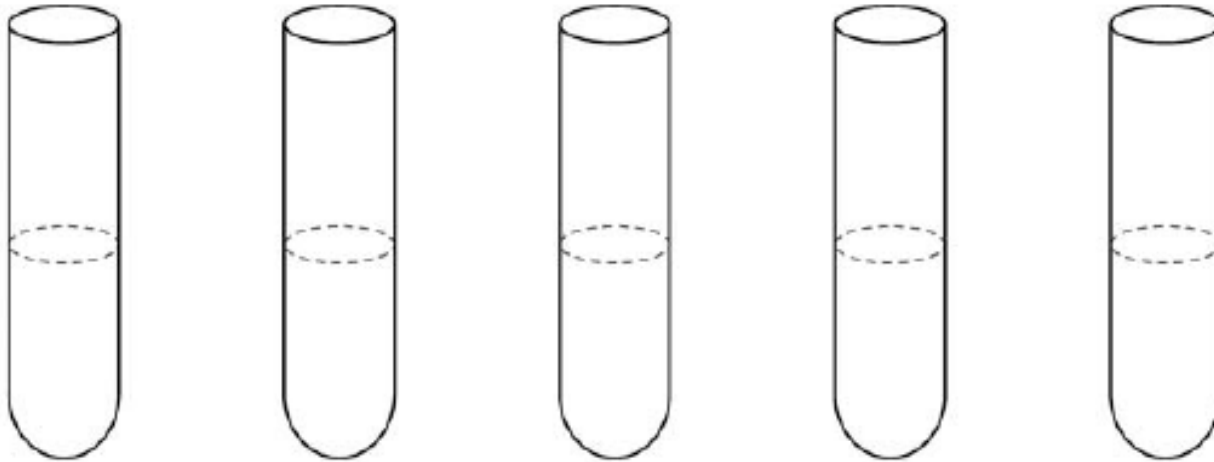
Two hypotheses:

1. Directed/induced mutations

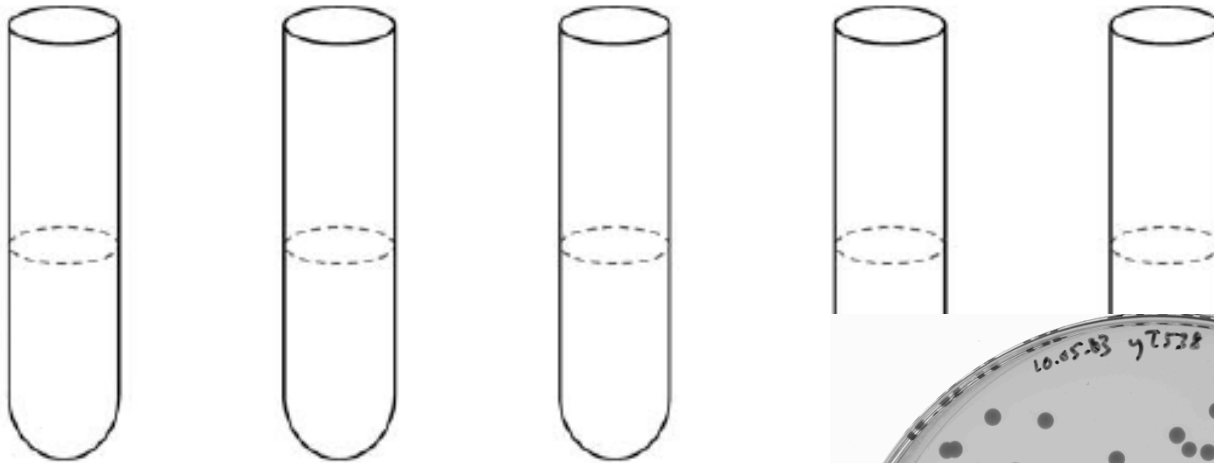
2. Random/spontaneous mutations  
= Darwinism



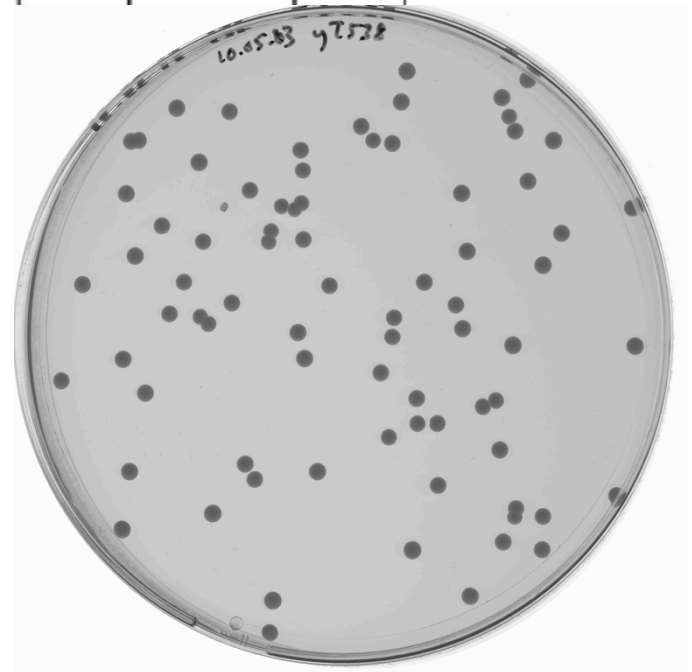
N independent cultures starting with one normal (wild-type) cell

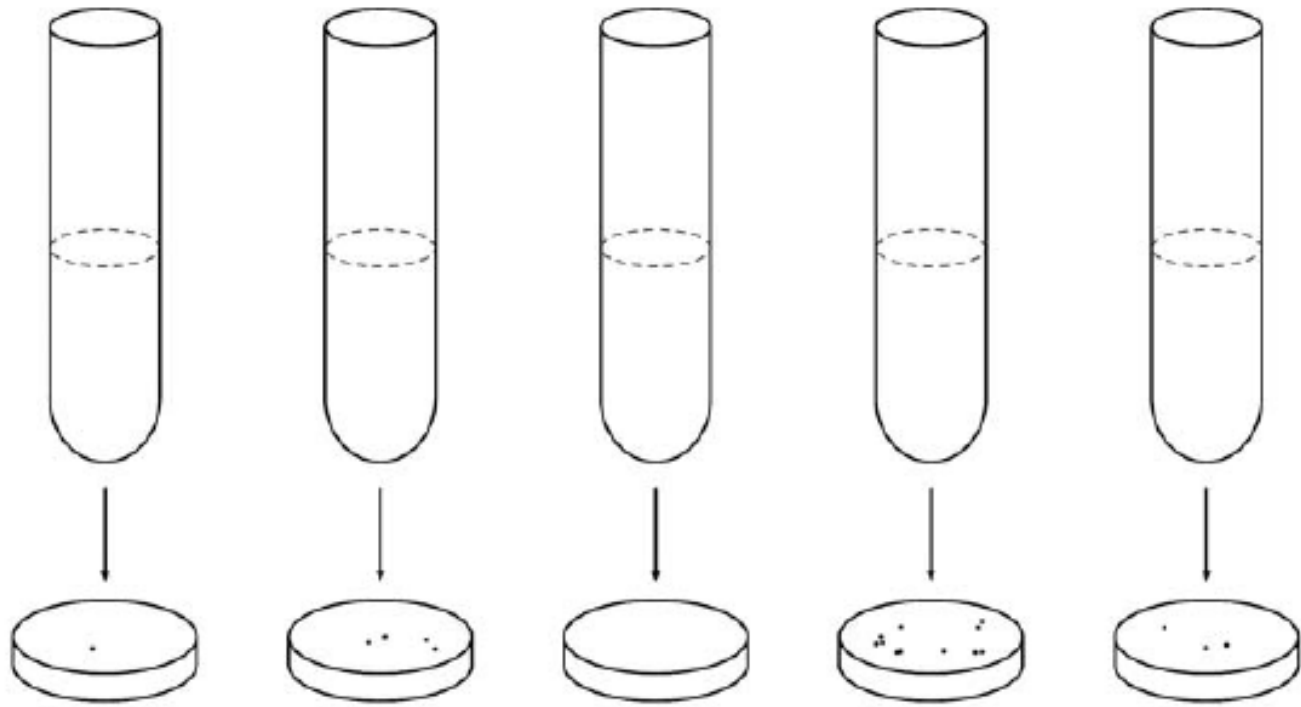


N independent cultures starting with one normal (wild-type) cell



After growth to  $\sim 10^9$  cells,  
plating on **selective** medium to  
count the number of mutants

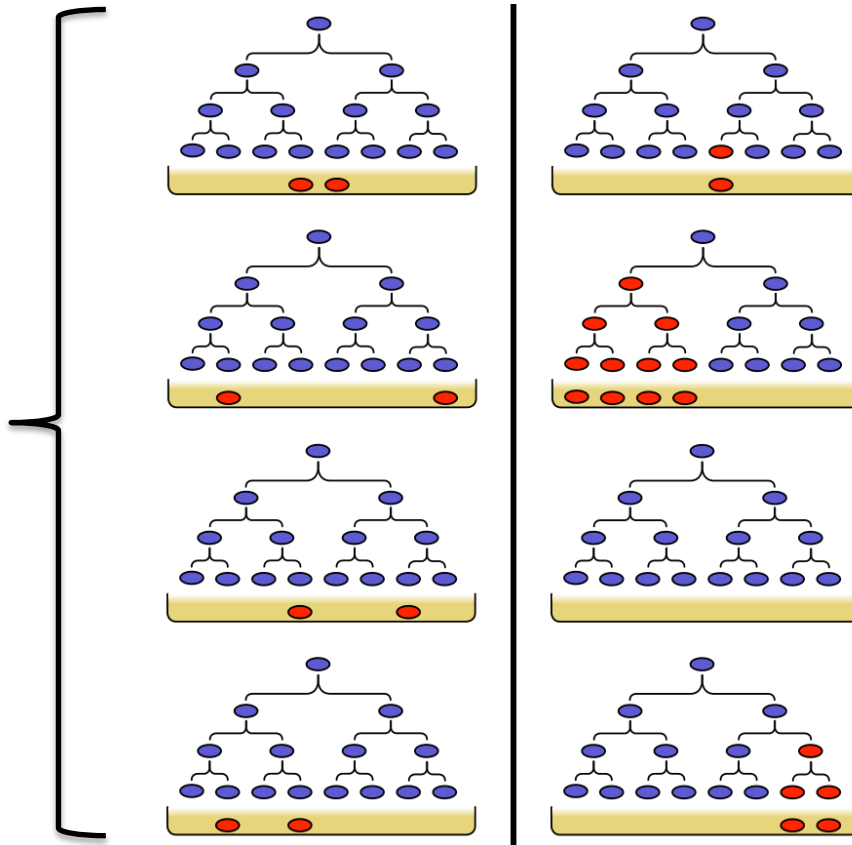




# 1. Directed/induced mutations

# 2. Random/spontaneous mutations = Darwinism

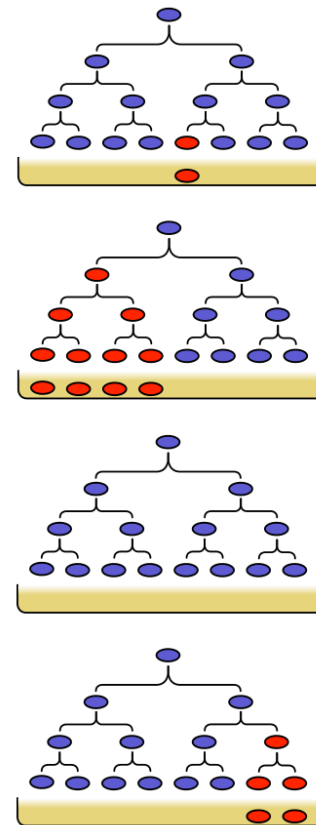
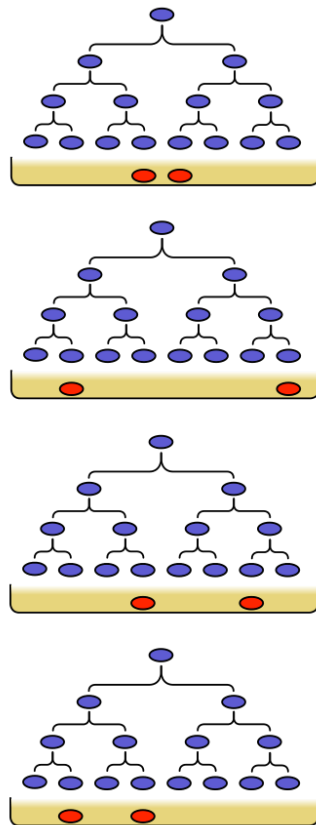
N independent cultures starting with one cell



1. Directed/induced mutations

2. Random/spontaneous mutations  
= Darwinism

Poisson distribution



Distribution with much greater variance  
= Luria-Delbrück distribution

# 70 years of progress in the study of the Luria-Delbrück distribution

Assumptions in the Luria-Delbrück model:

- The process starts with one wild-type cell and no mutant,
- Deterministic growth for wild-type and mutant cells,
- Exponential growth (to infinity),
- Mutations occur randomly at a rate proportional to the population size.

Luria and Delbrück

Mean and variance

Lea and Coulson

Approximate pgf, stochastic model

Haldane

Combinatorial approach

Armitage

Exact pgf

Kendall

Arbitrary distribution for cell cycle

Bartlett

Exact pgf, fully stochastic model

Crump and Hoel

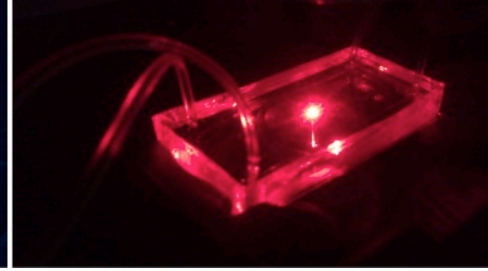
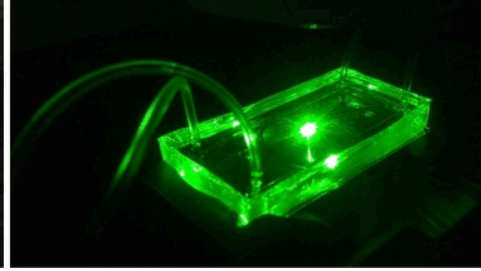
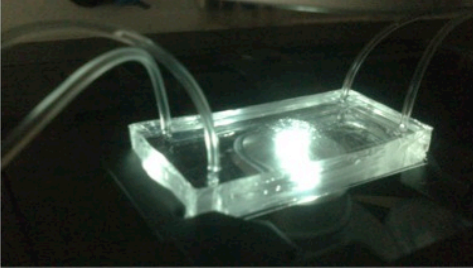
Filtered Poisson process theory

Ma and Sarkar

Algorithm for probability function

Many others...

But no closed expression!



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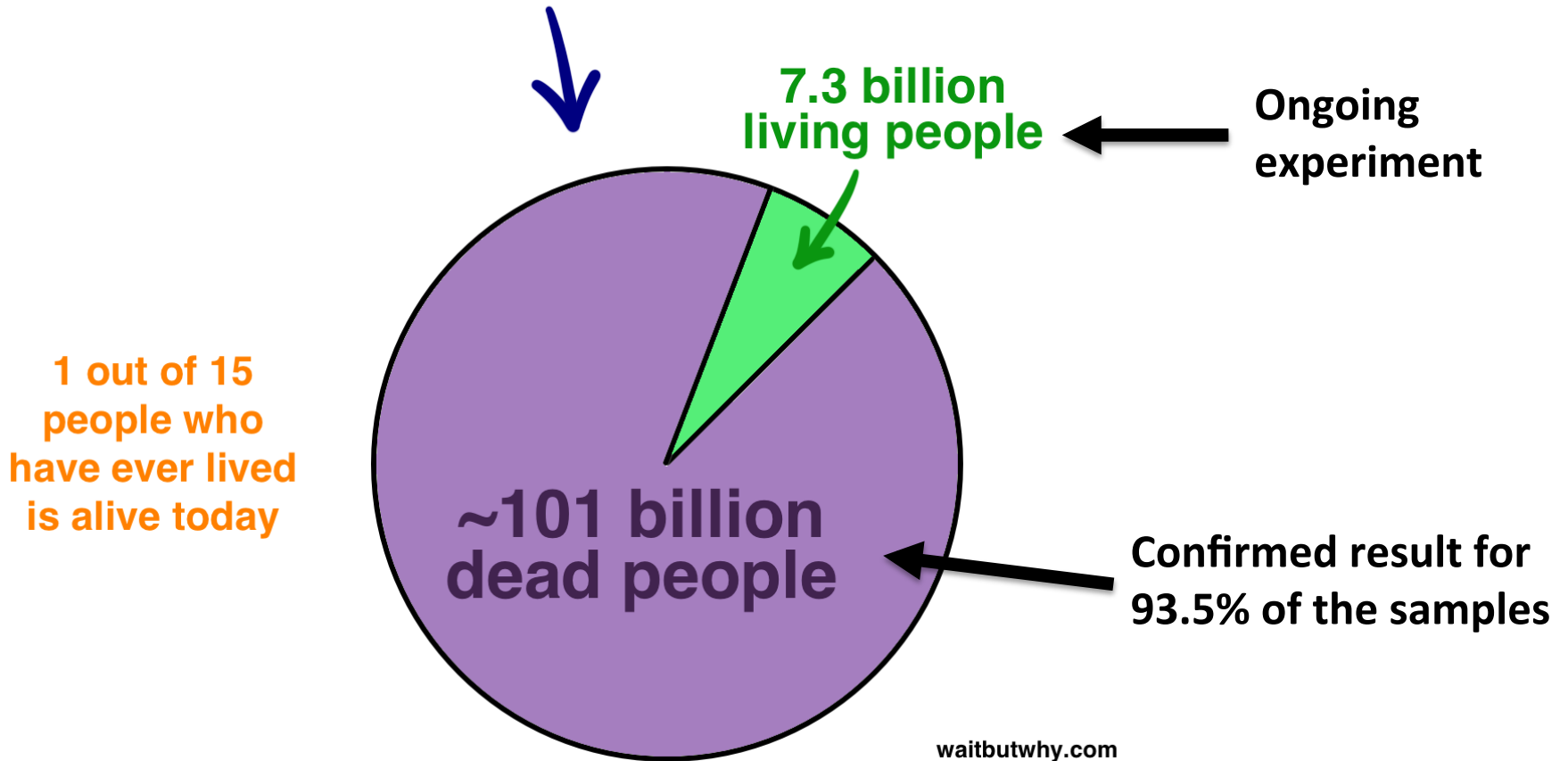
Institut de Biologie Physico-Chimique

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**PDE and Probability for Life Sciences  
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All of us will die one day...

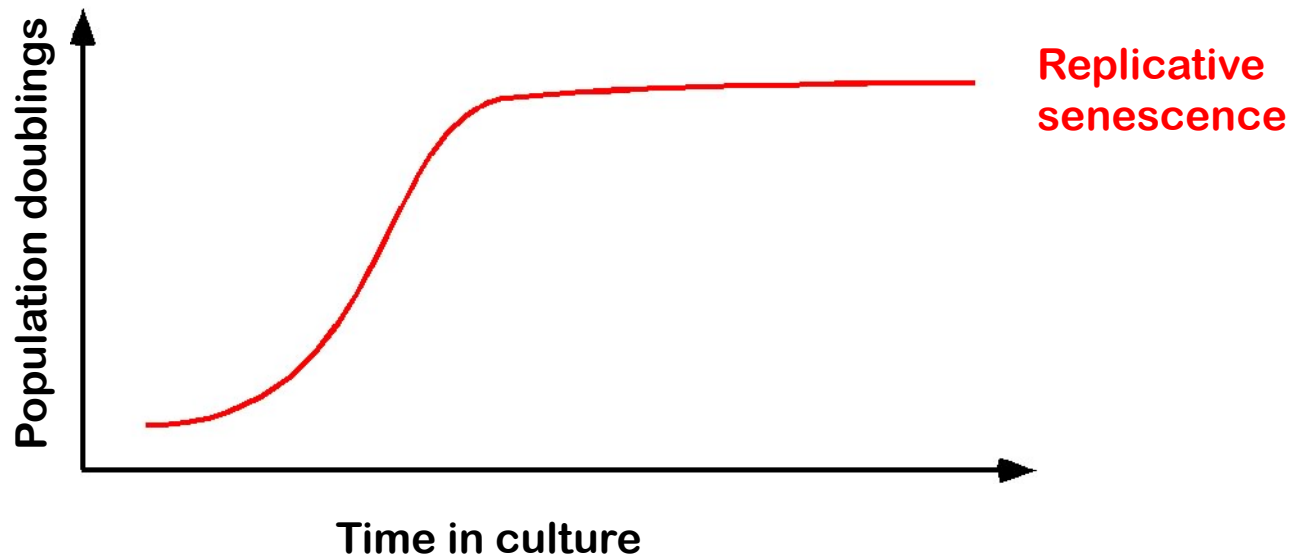
## All Humans Who Have Ever Lived (~108 Billion)





An intrinsic barrier to cell  
proliferation:  
**Replicative Senescence**

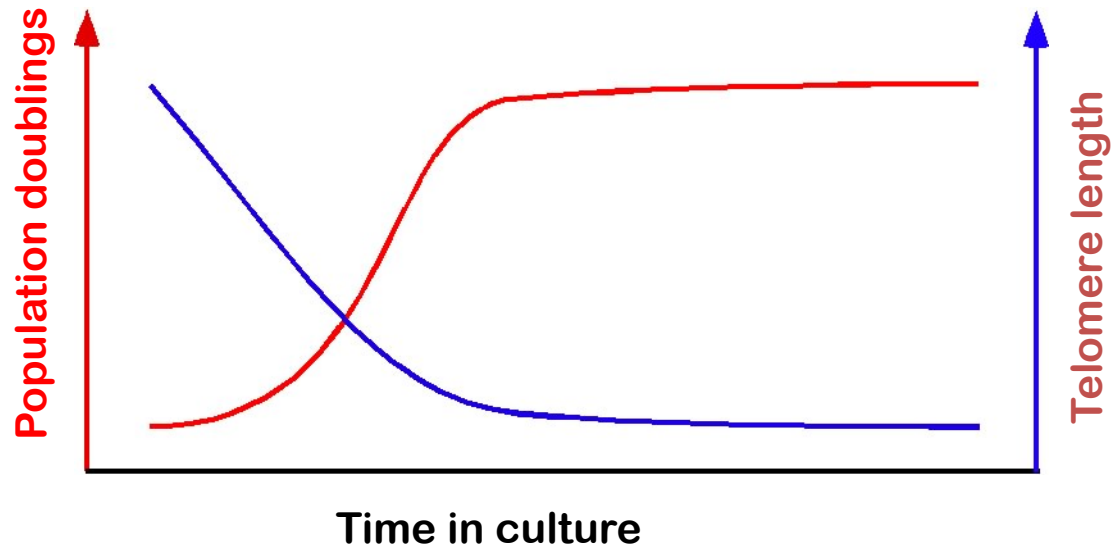
# What is replicative senescence?



**Replicative senescence is the ultimate and irreversible loss of replicative capacity occurring in primary somatic cell culture**

Hayflick and Moorehead 1961 The serial cultivation of human diploid cell strains  
Exp. Cell Res.

# Replicative senescence correlates with telomere attrition

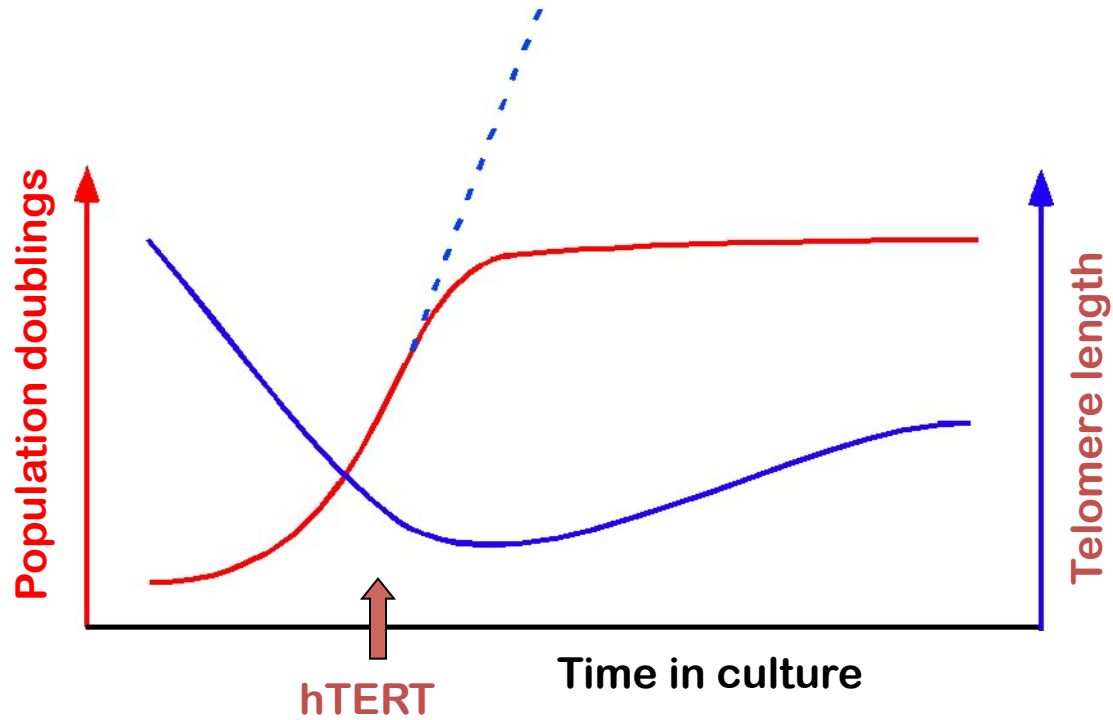


**Telomeres shorten during ageing of human fibroblasts**

C. B. Harley, A. B. Futcher and C. W. Greider (1990)

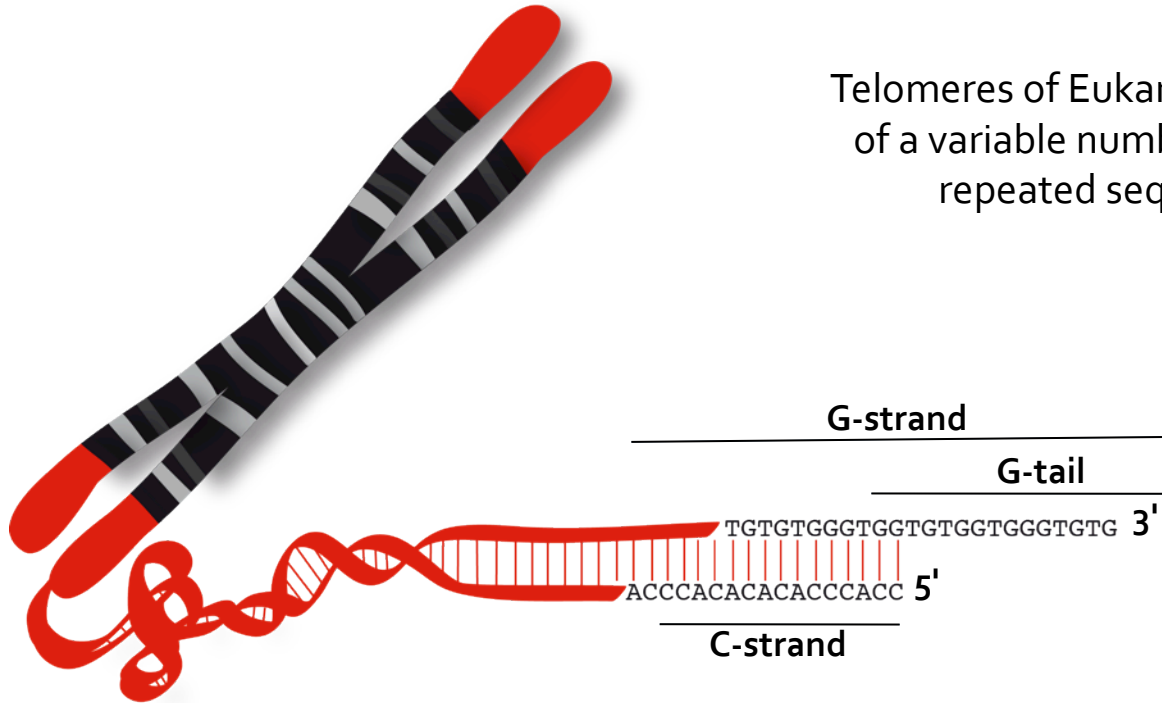
**Nature** 345(6274): 458-460

# Telomere elongation is required for unlimited cell proliferation

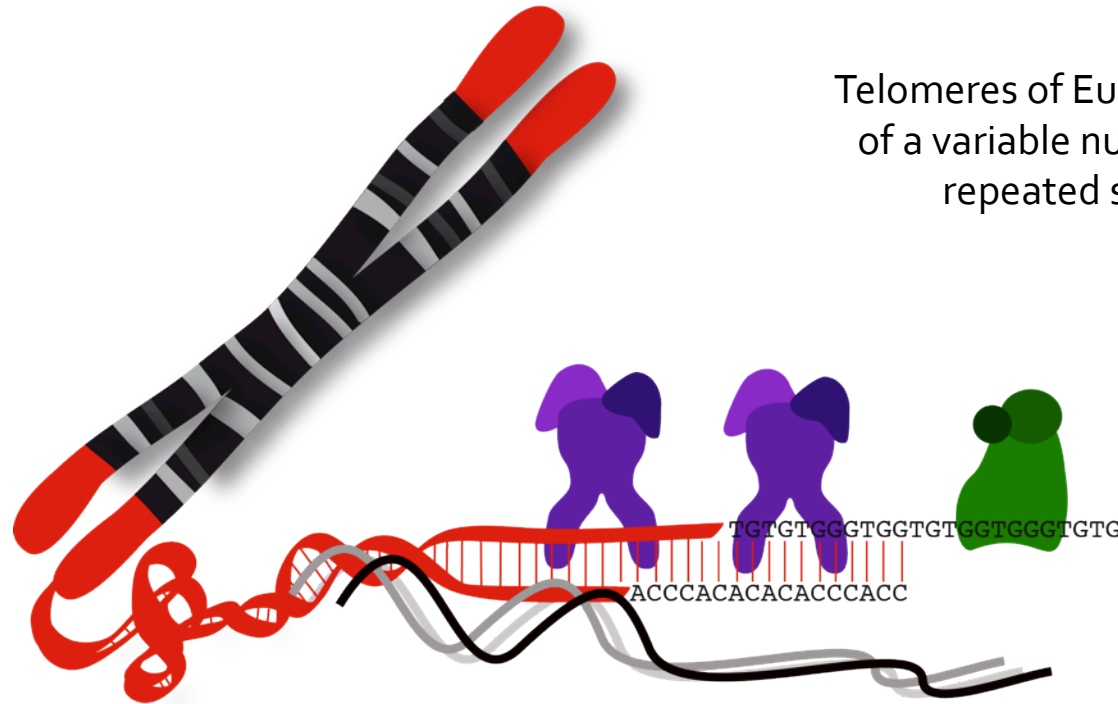


# Telomeres

Telomeres of Eukaryotes consist of a variable number of G-rich repeated sequences



# Telomeres



Telomeres of Eukaryotes consist of a variable number of G-rich repeated sequences

Telomeric repeats bind specific proteins

Telomeres also bind a telomeric encoded RNA - TERRA

# Structure

## ➤ Proteins on the double-stranded and single-stranded parts of telomeres

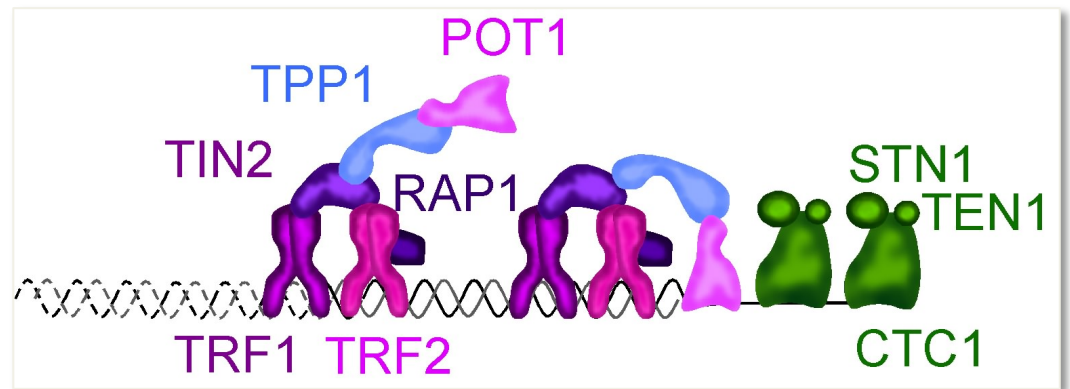
### Yeast telomeres:



250-400 bp of TG1-3 repeats

Overhang <10 nt

### Mammalian telomeres:



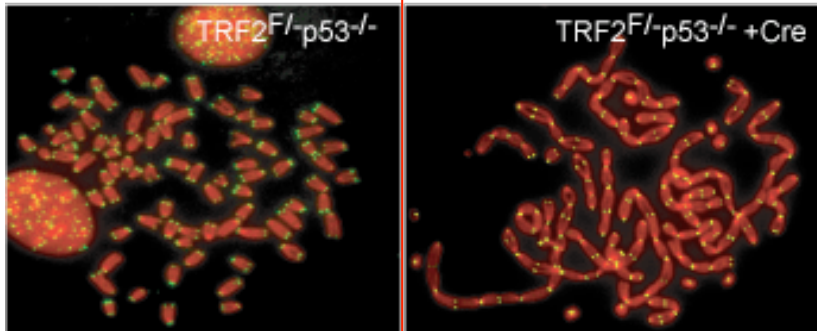
2-100 kb of TTAGGG repeats

Overhang 50-500 nt

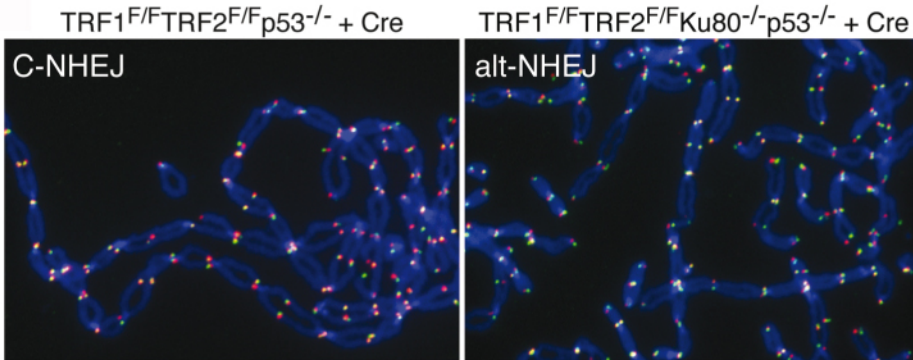
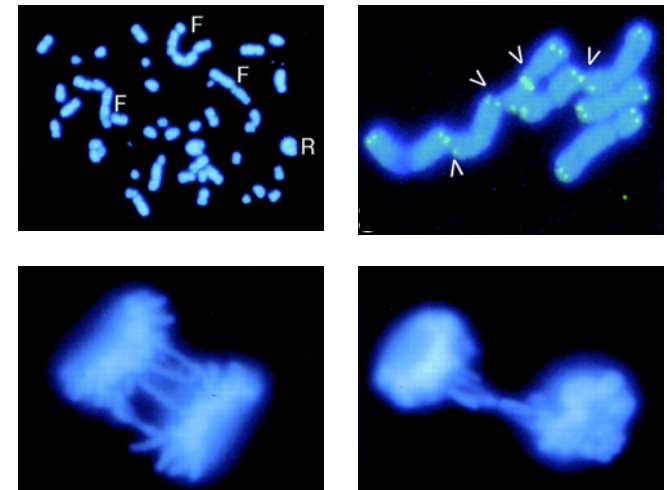
# Functions of telomeres

- If dysfunction of telomeric proteins, GENOME INSTABILITY!

Chromosome of a normal cell



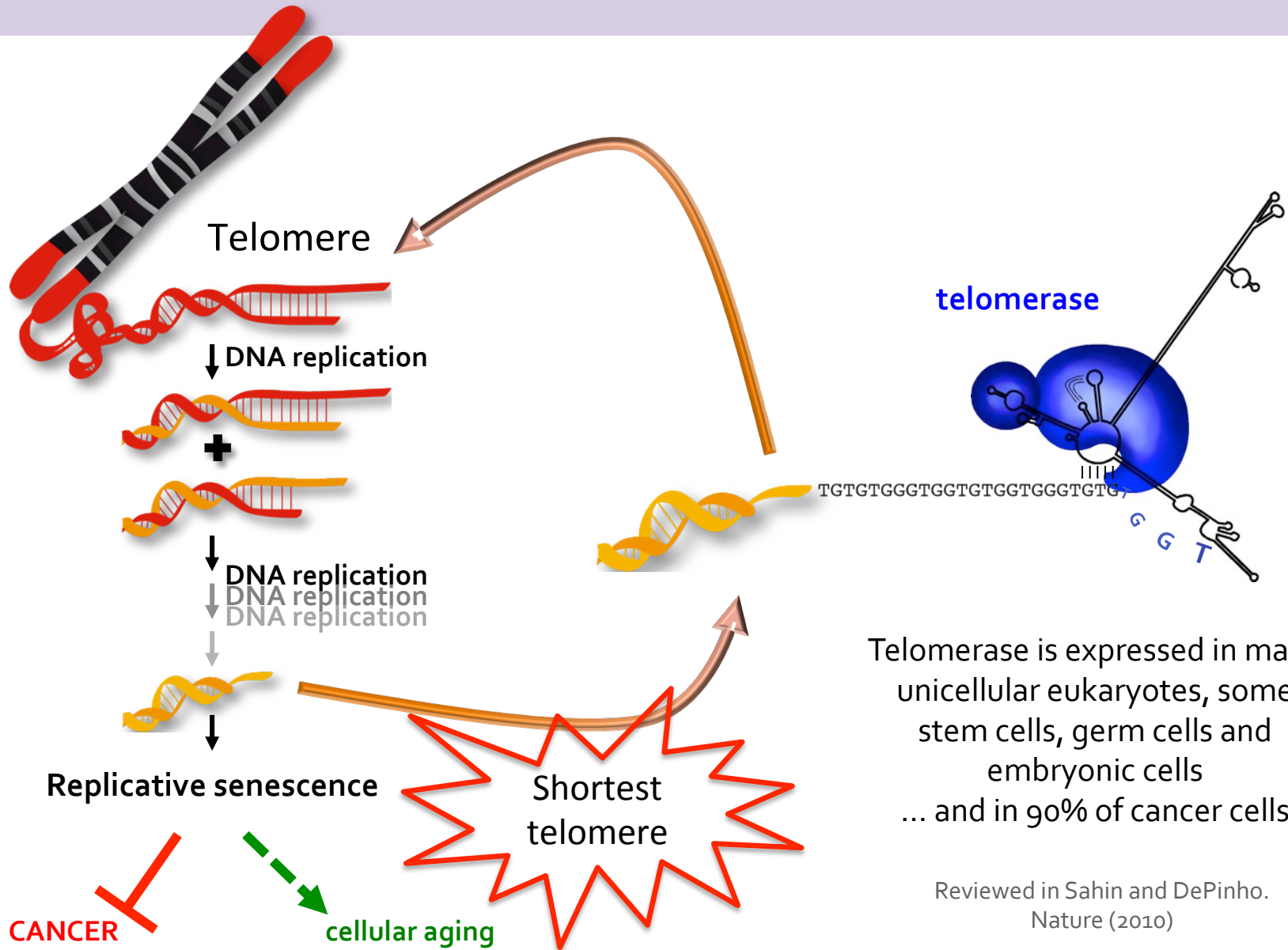
Titia de Lange's work



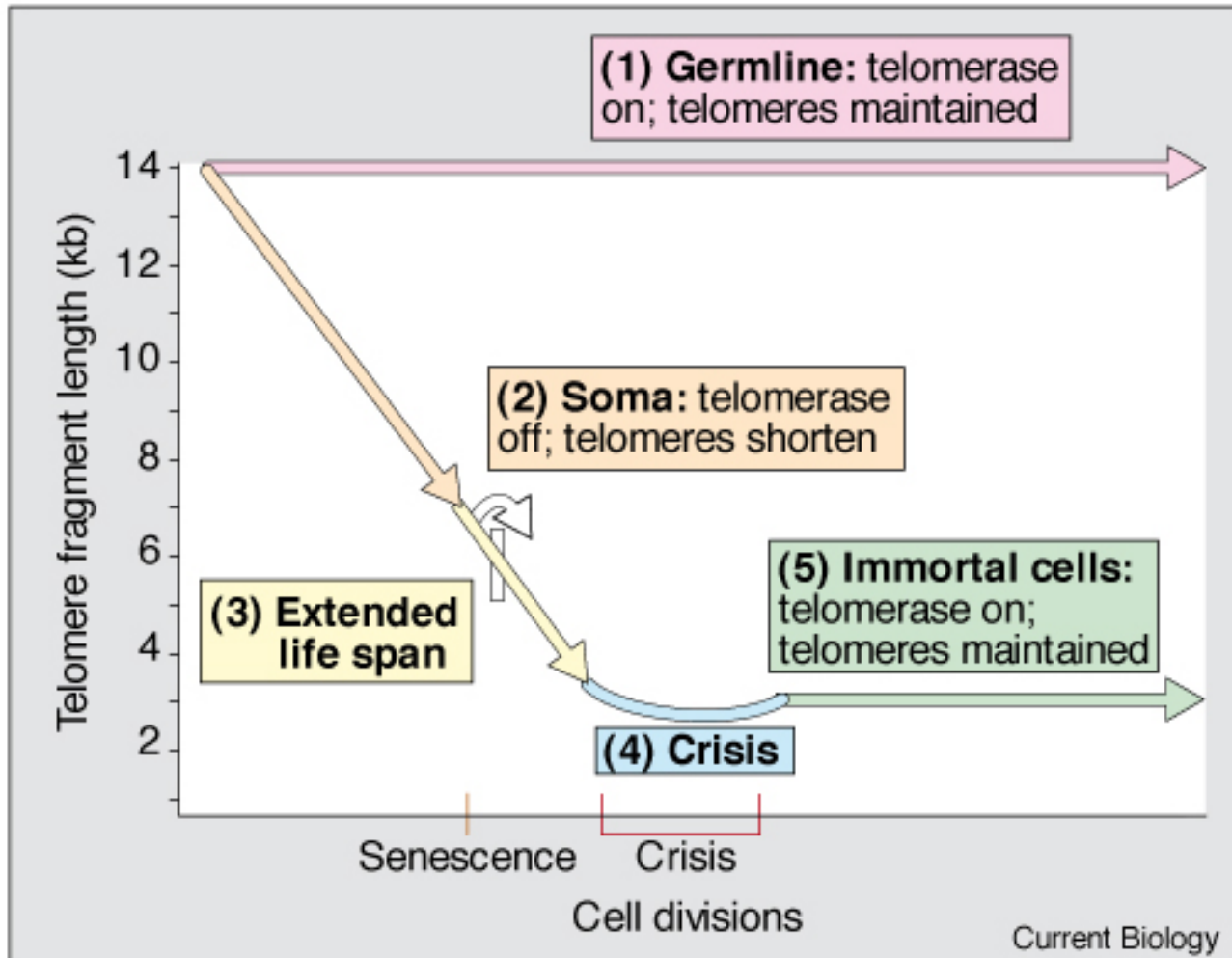
Anaphase bridge



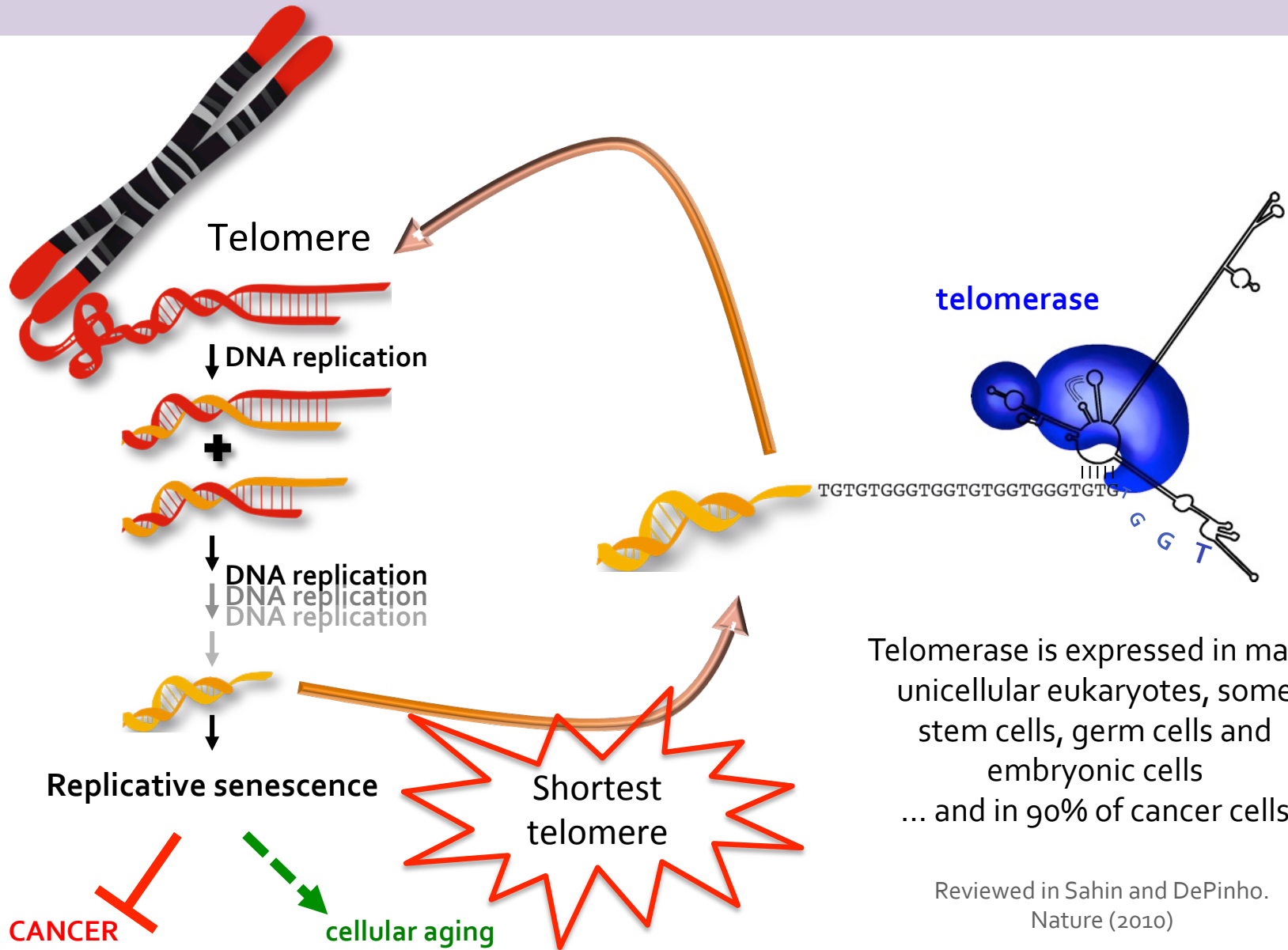
# Telomeres shorten and can be elongated



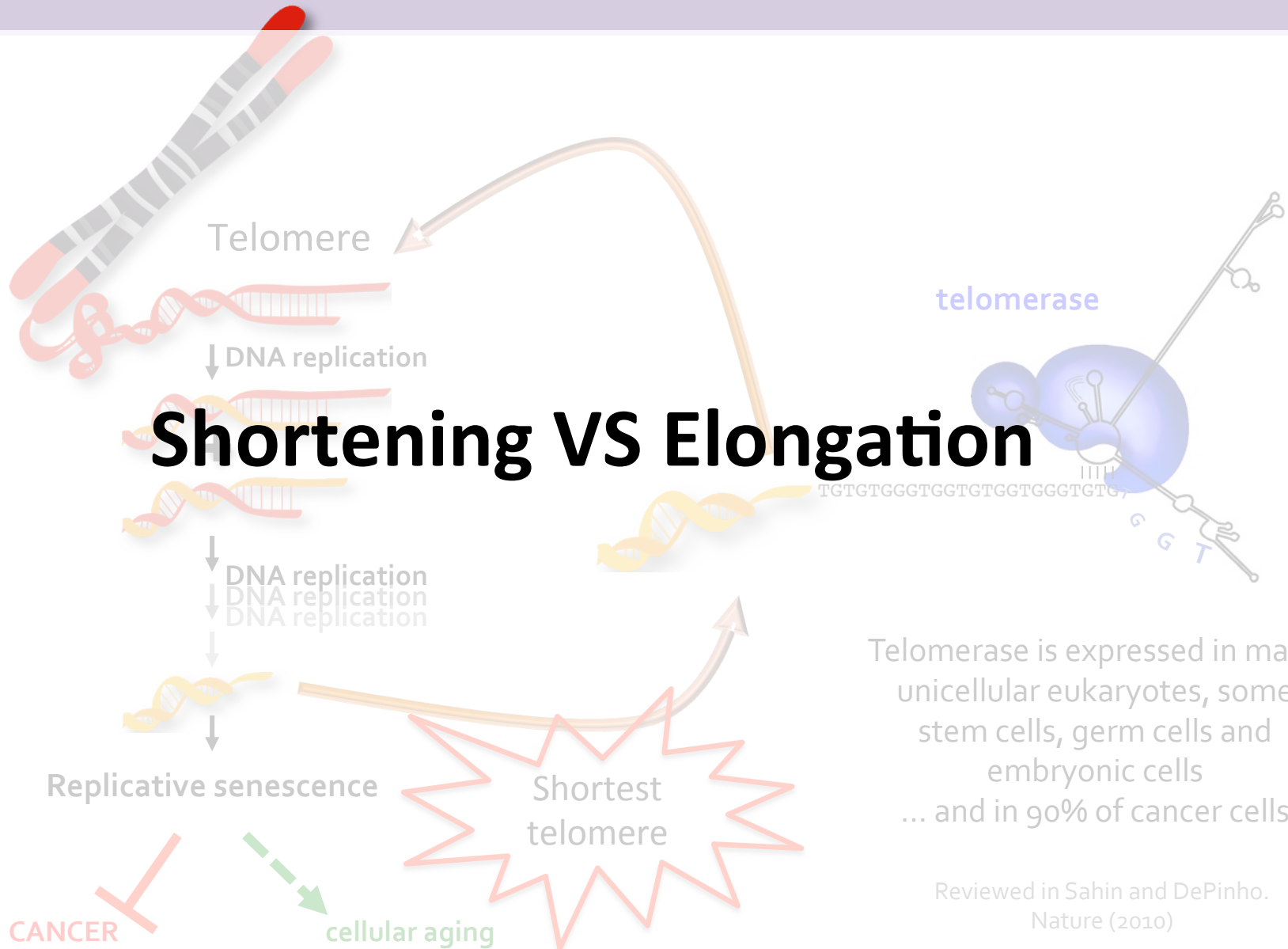
# Senescence, crisis and escape



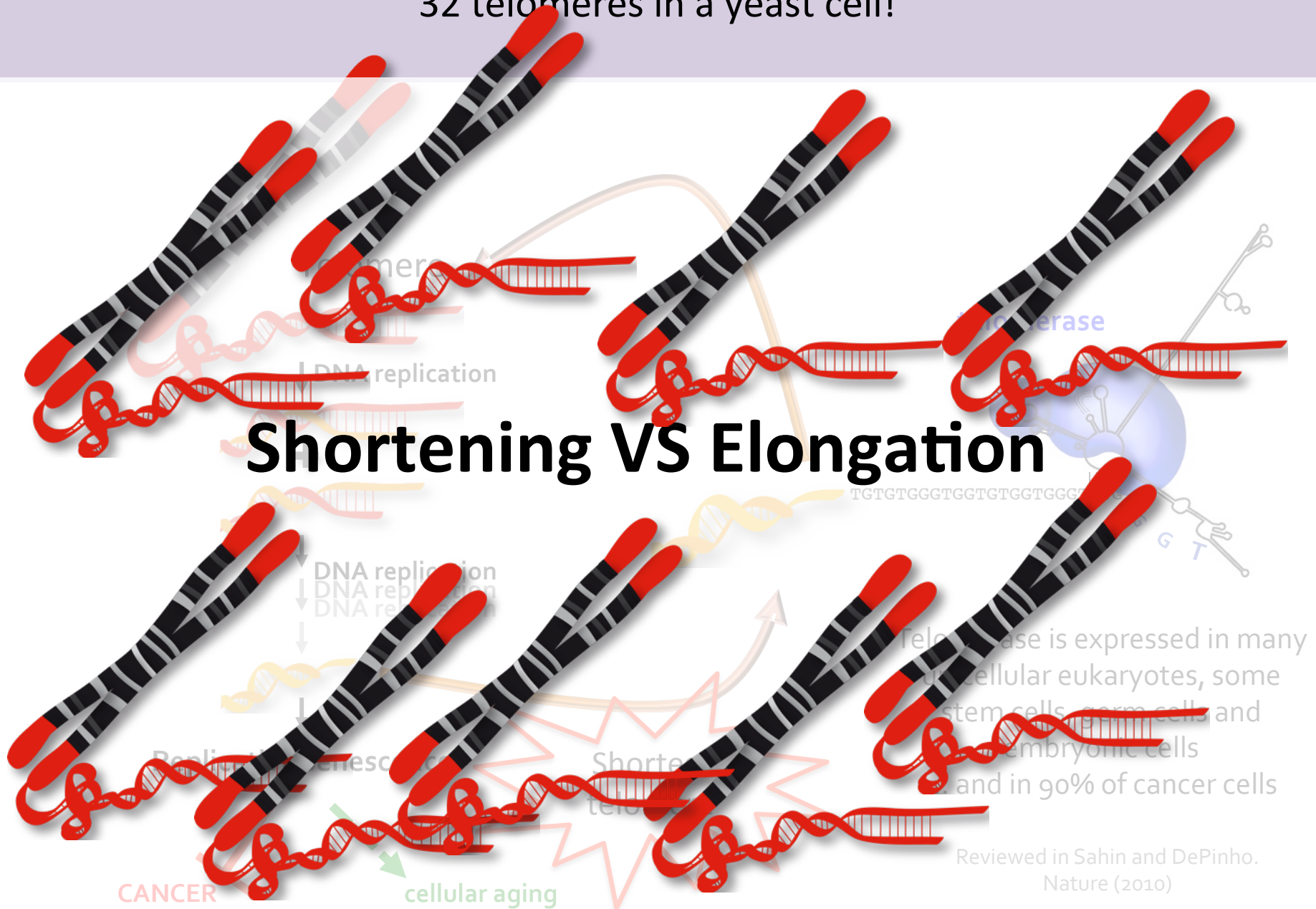
# Telomeres shorten and can be elongated



# Telomeres shorten and can be elongated



32 telomeres in a yeast cell!



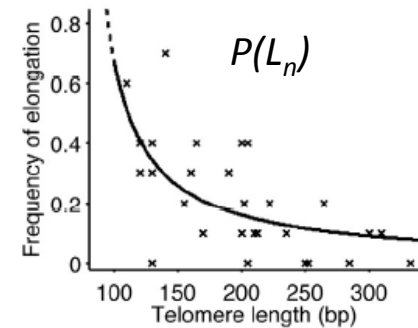
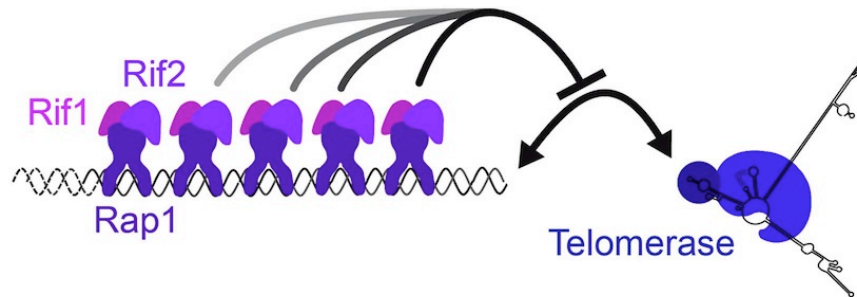
What is the  
**steady-state**  
**distribution** of  
telomere lengths?

# Mathematical model of telomere distribution

Collaboration with Khanh Dao Duc and David Holcman (ENS Paris)



Protein-counting mechanism



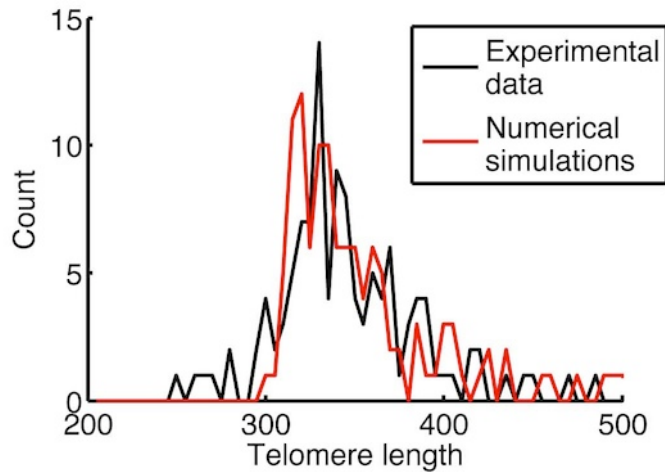
Teixeira *et al.* 2004 Cell

At each cell division,  $L_{n+1} = \begin{cases} L_n - a \\ L_n - a + b \end{cases}$ , with probability  $\begin{matrix} 1 - P(L_n) \\ P(L_n) \end{matrix}$   $\begin{matrix} = \text{shortening} \\ = \text{elongation} \end{matrix}$

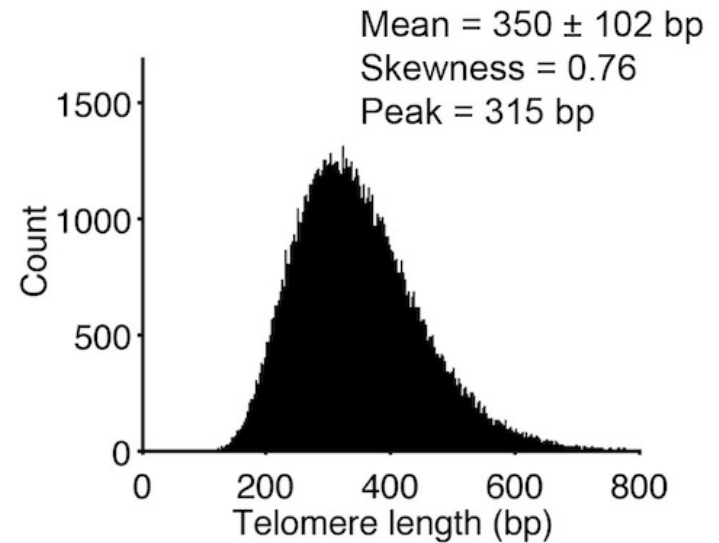
➤ Model of telomere dynamics based on the protein-counting mechanism

# Telomere length distribution

## Comparison with experimental data



## Modeled distribution



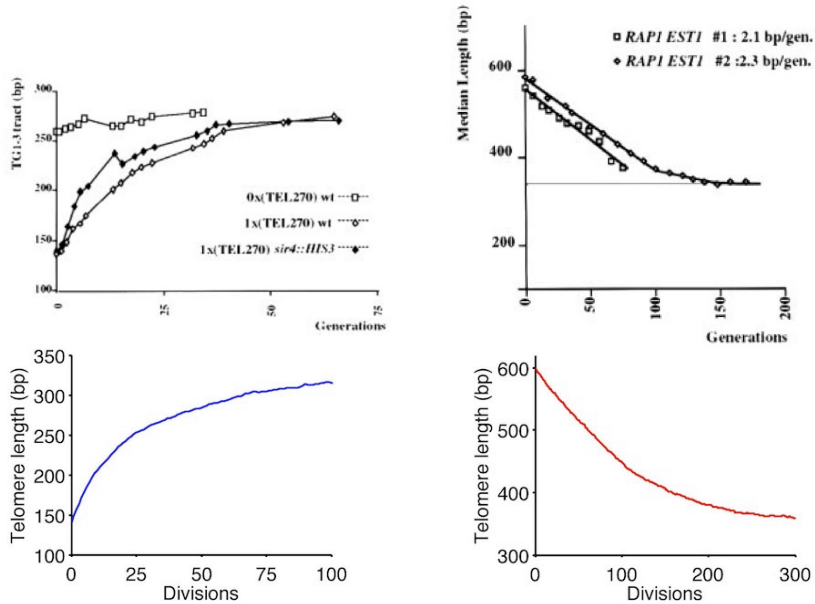
➤ Consistent with experimental measurements of telomere length



# The dynamics of telomere length results from the protein-counting mechanism

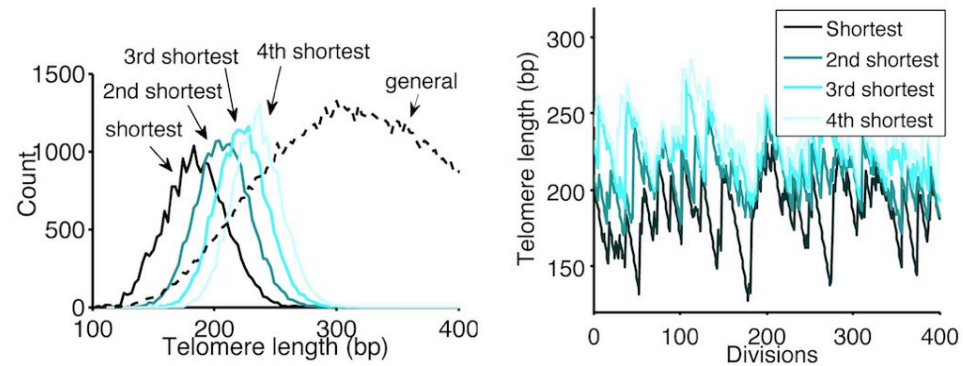
## Return to equilibrium after perturbation

Marcand *et al.*, 1999



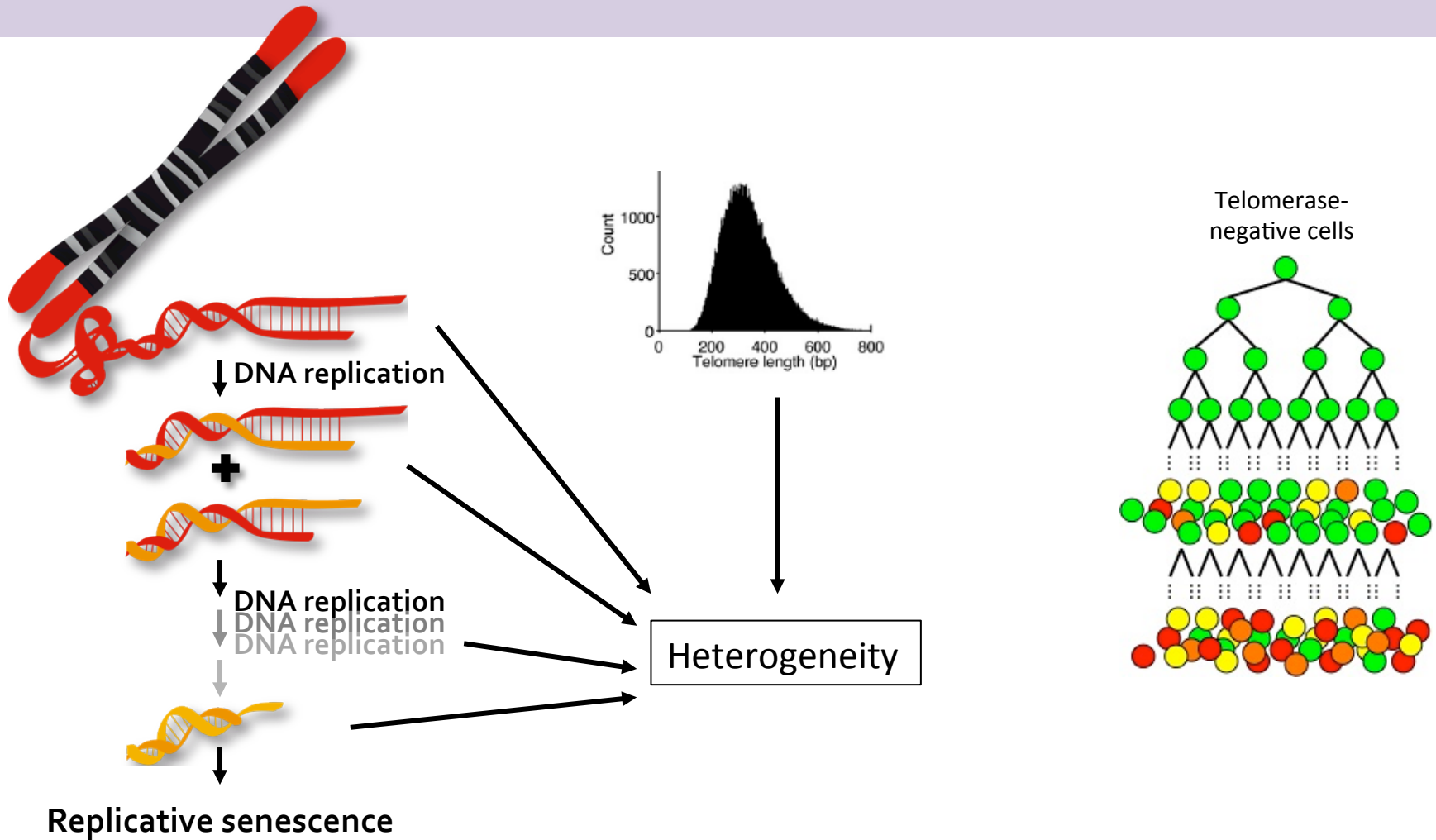
Our work

## Dynamics of the shortest telomeres



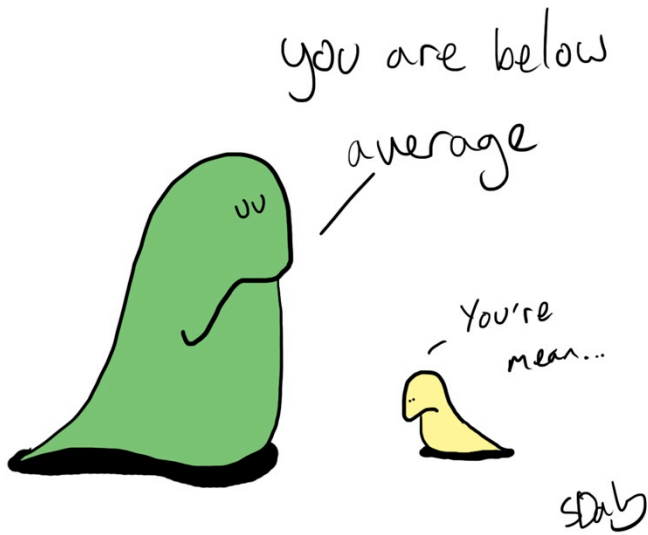
➤ The model reproduces the dynamic behaviour of telomeres

# Replicative senescence: a heterogeneous process.



# Two caveats of population studies

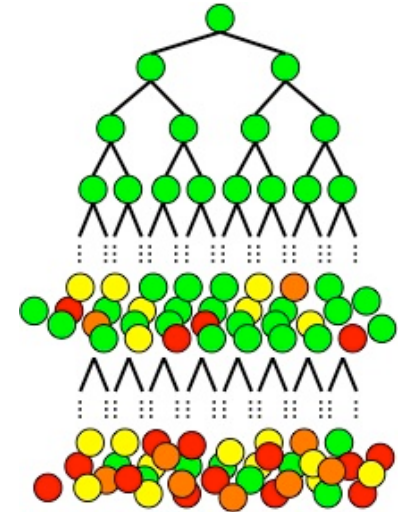
Average



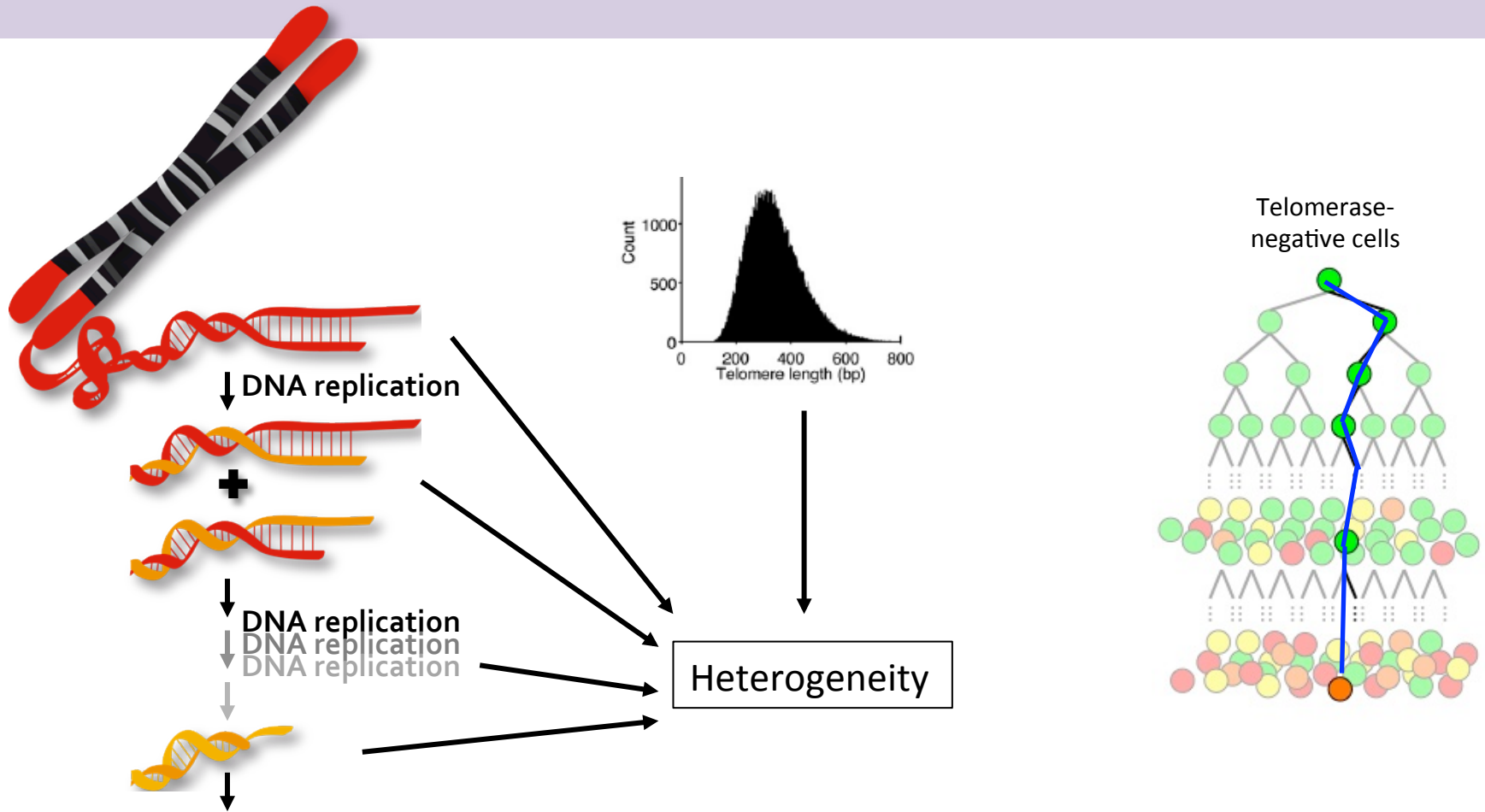
Competition



Telomerase-negative cells



# A change of scale to study a complex phenotype

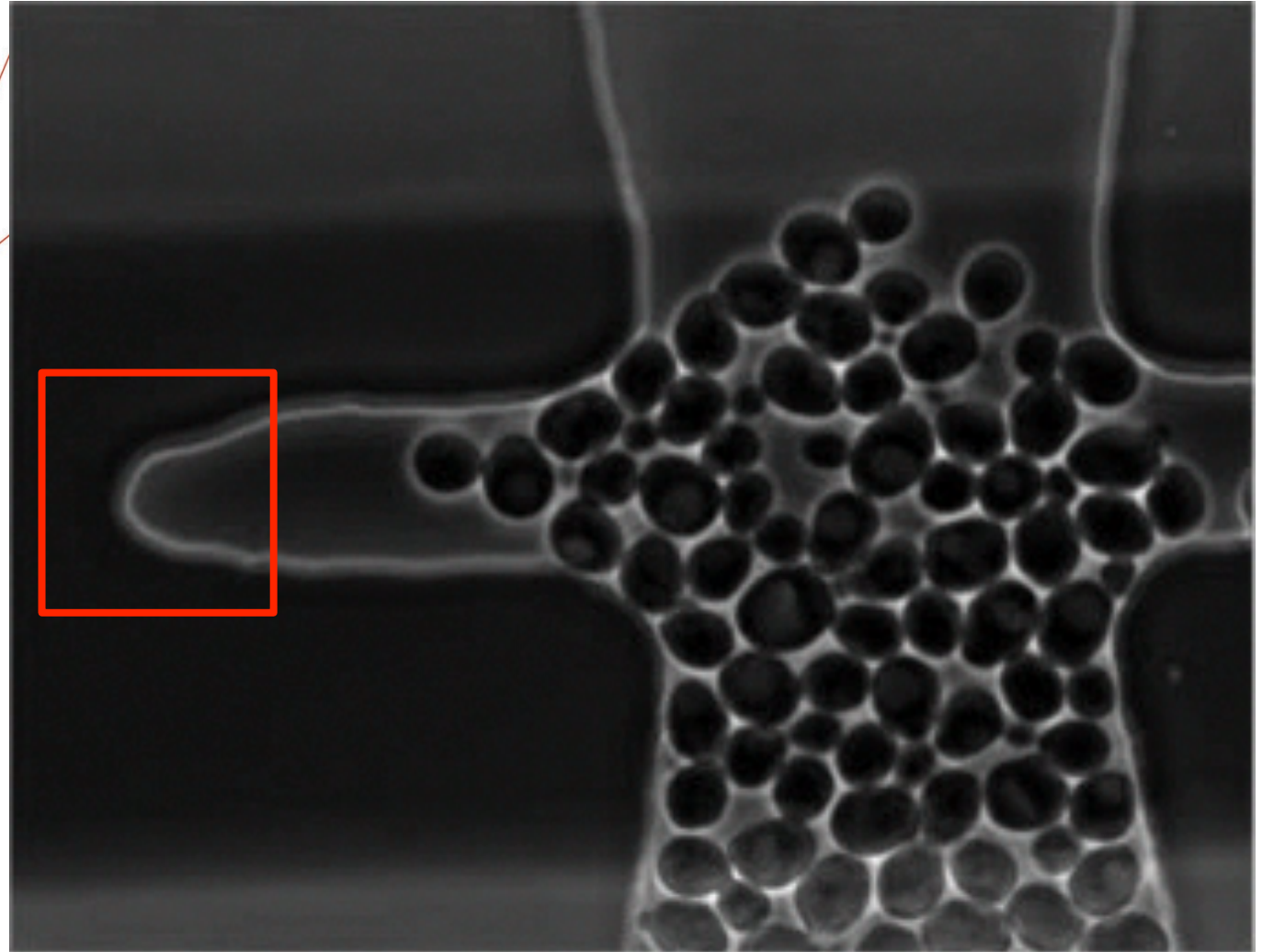
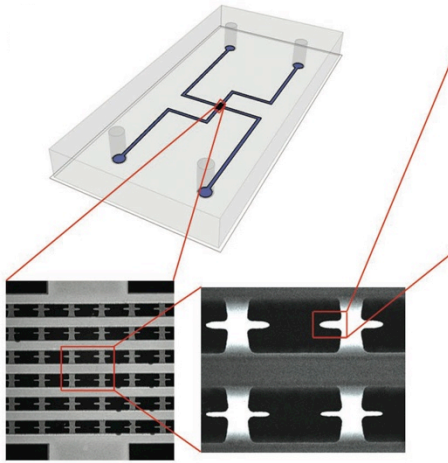


Replicative senescence

- Sequence of events leading to senescence?
- Dynamics of entry into senescence? Progressive? Sharp?

Study at the level of **individual lineages**.

Experimental strategy:  
**Real-time single-cell analysis by microfluidics  
coupled with live-imaging microscopy**



Microfluidics:

**Gilles Charvin**

(IGBMC, Strasbourg)

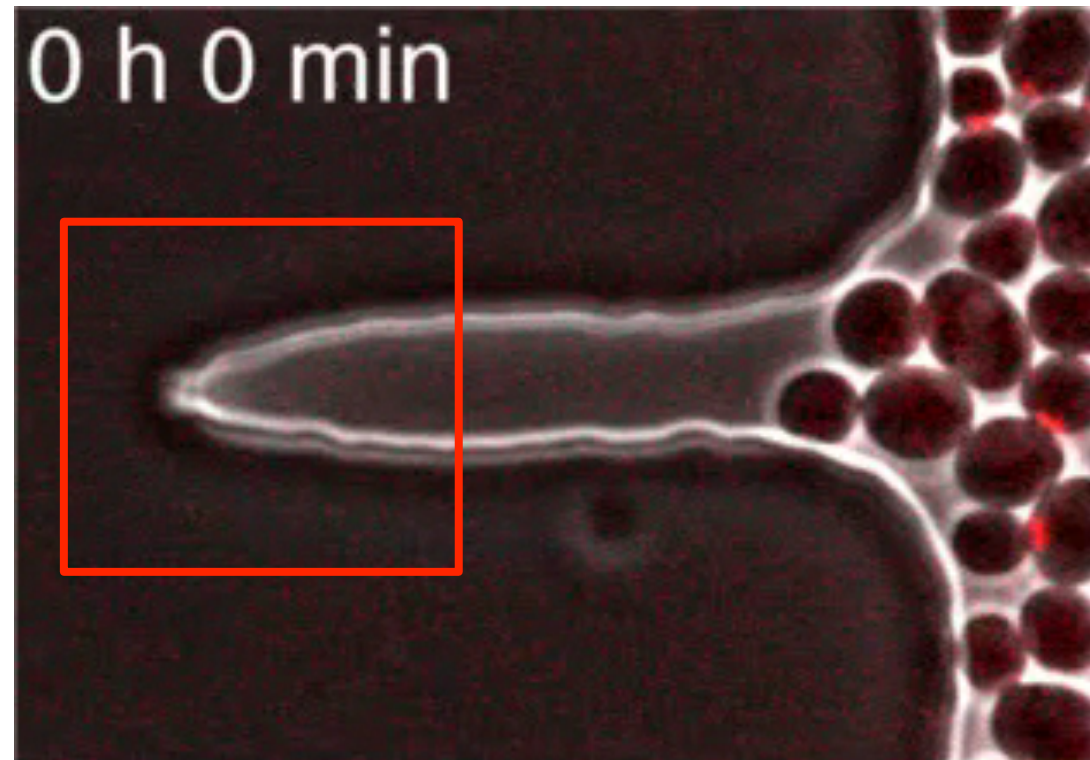
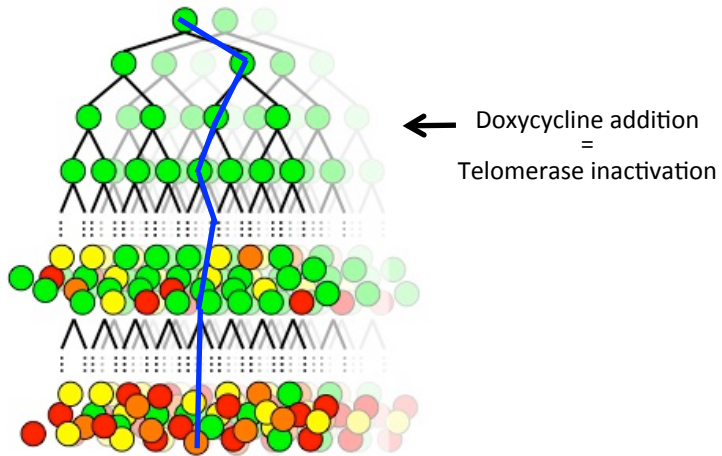
Fehrmann *et al.* 2013

Cell Rep

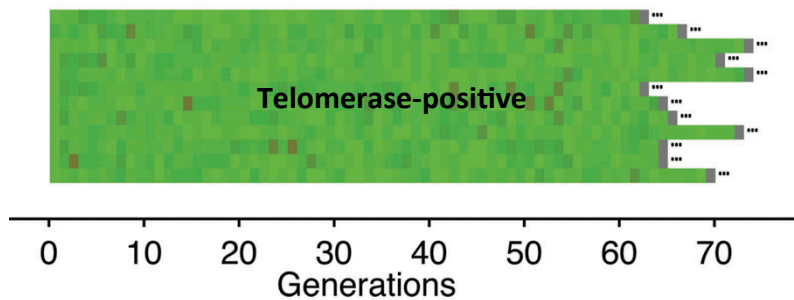
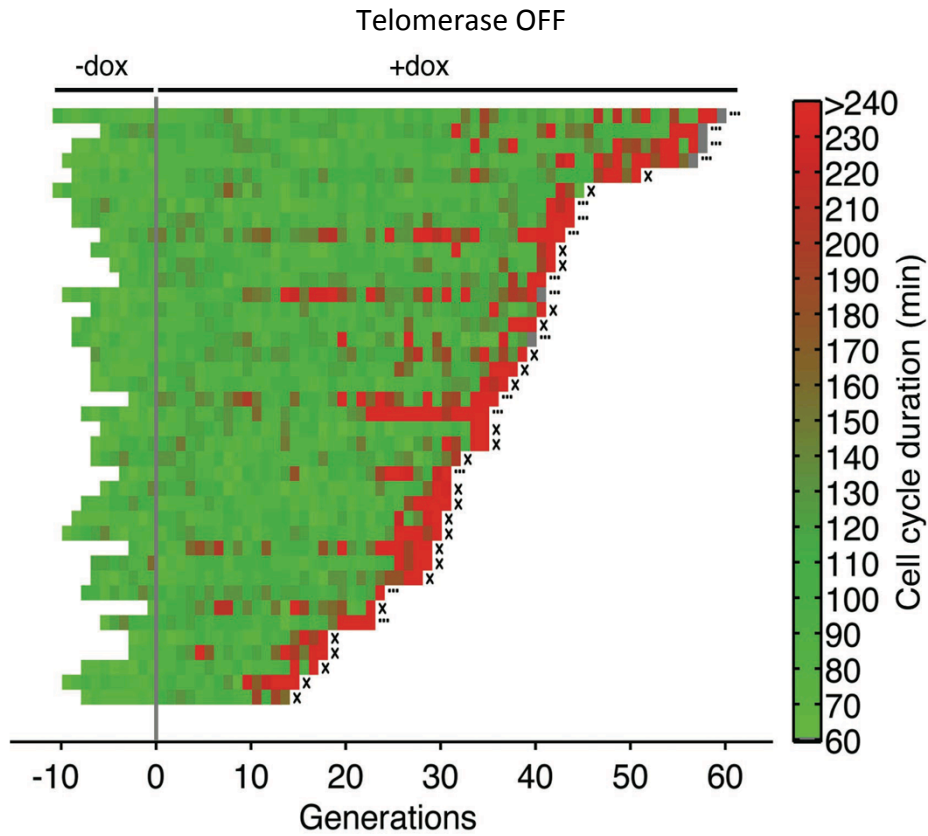


# Experimental strategy

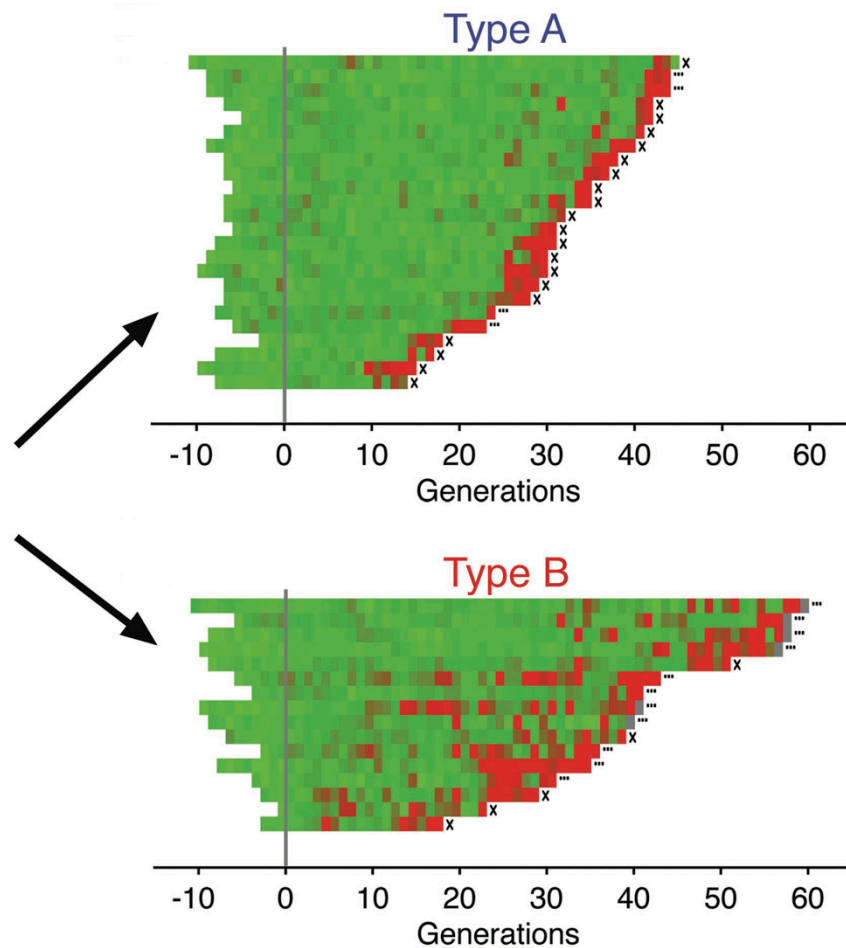
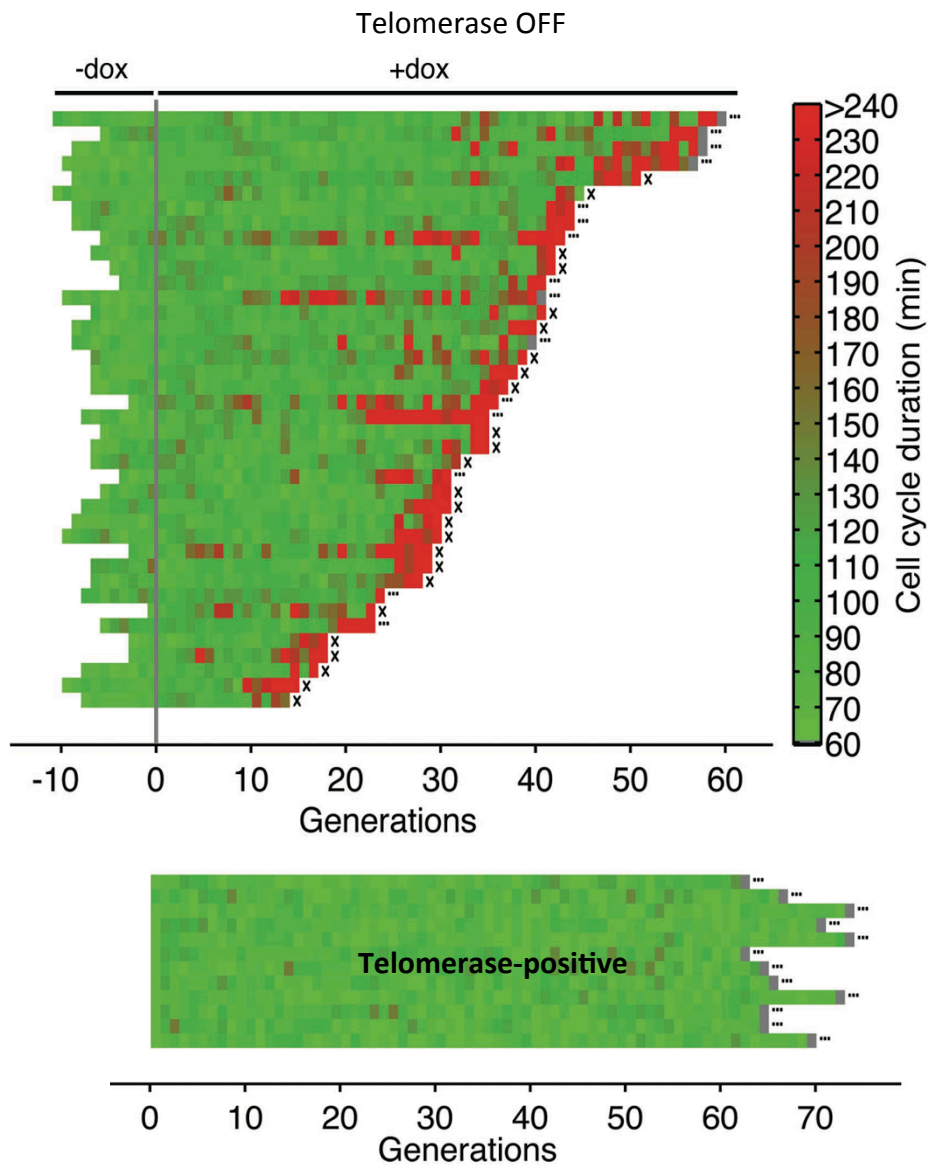
*TetO2-TLC1*: Repressible telomerase expression by doxycycline addition, to induce senescence.



# Senescence dynamics in individual lineages

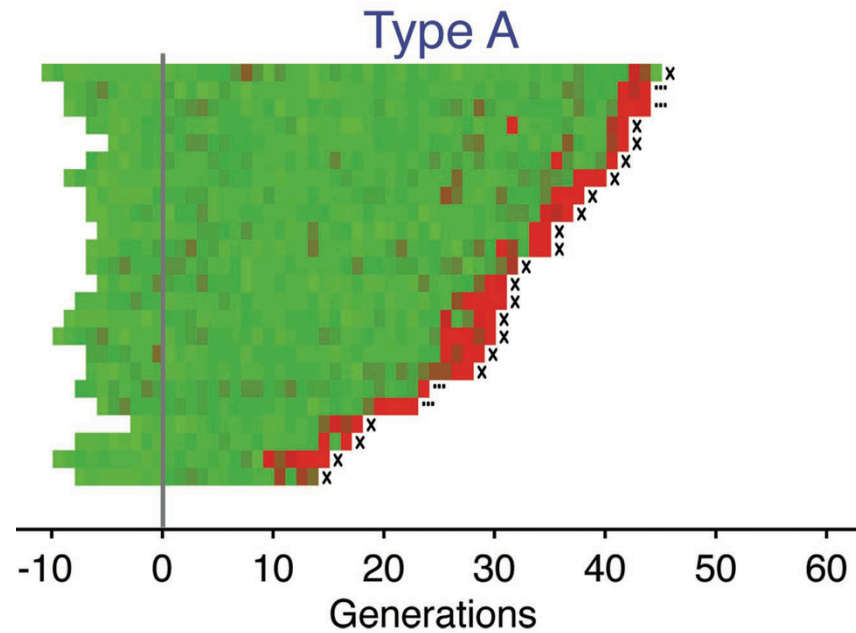


# Two types of senescence dynamics



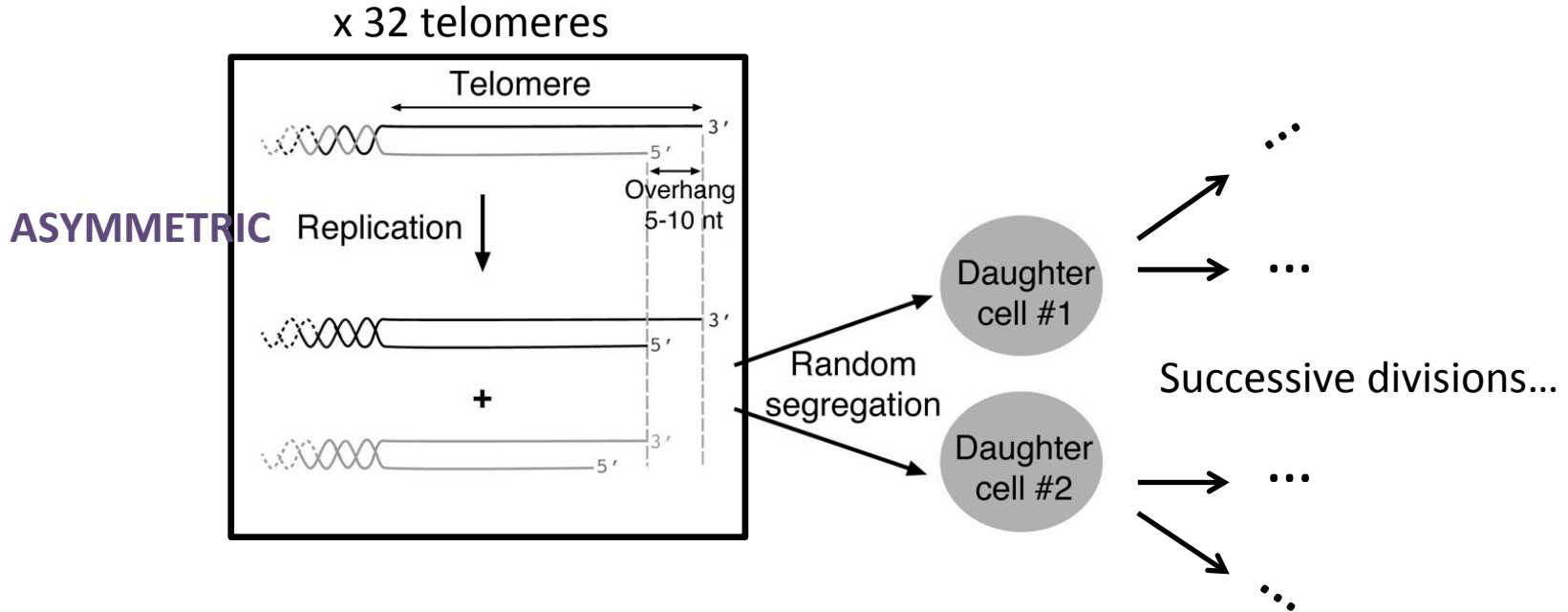


Type A lineages correspond to a canonical model of telomere shortening



# Model of senescence: from the molecular structure of telomeres to senescence onset

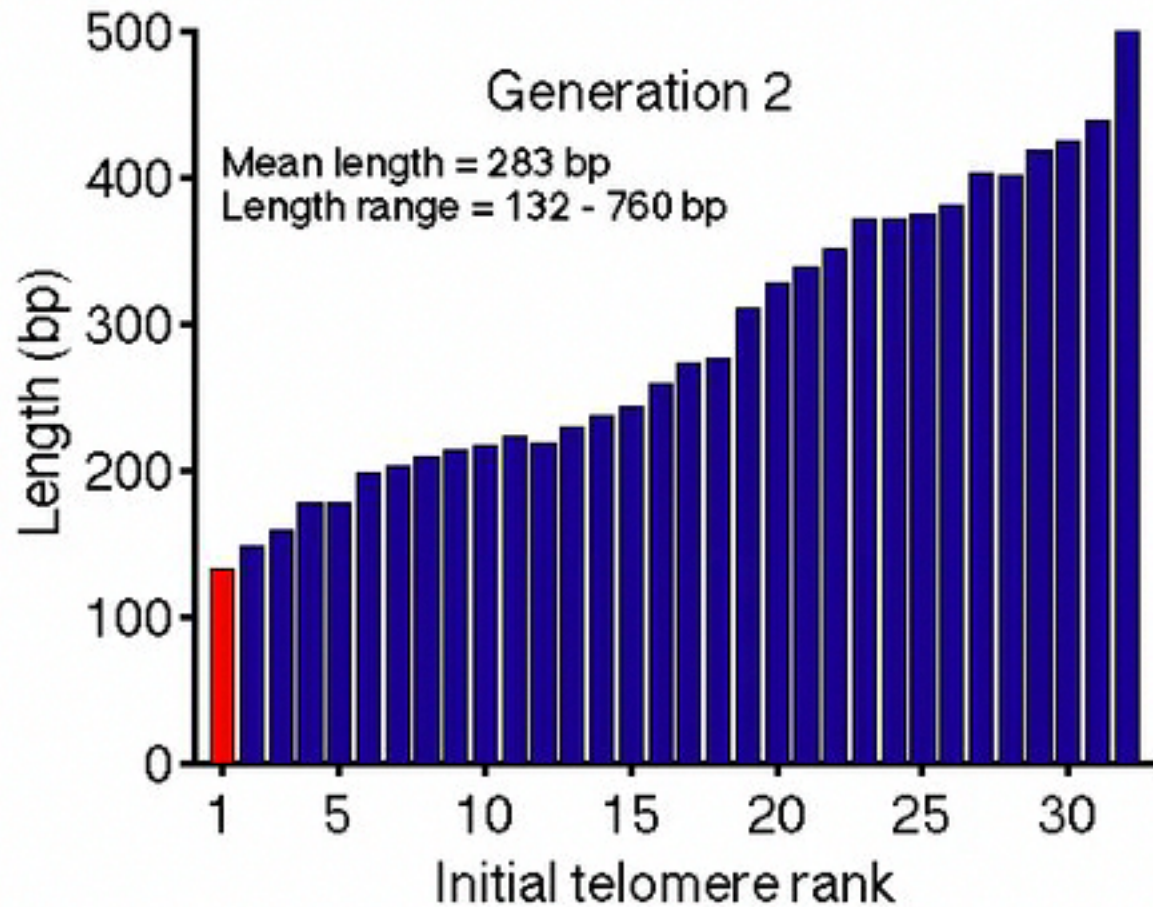
Collaboration with Thibault Bourgeron and Marie Doumic (INRIA)



$$L_k^{n+1} = \begin{cases} L_k^n & \text{if } k \in E \\ L_k^n - s_k & \text{if } k \in E^c \end{cases} \text{ where } E = \sigma(\{1, \dots, 16\})$$

- Stochastic model of telomere shortening, followed in individual lineages

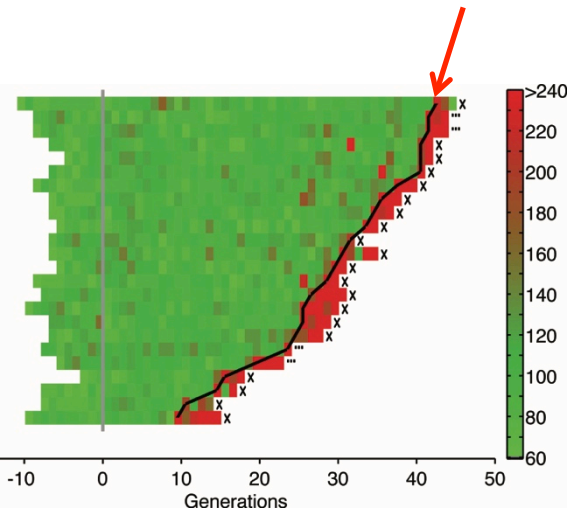
# Simulation of telomere shortening in a cell lineage



# Approach

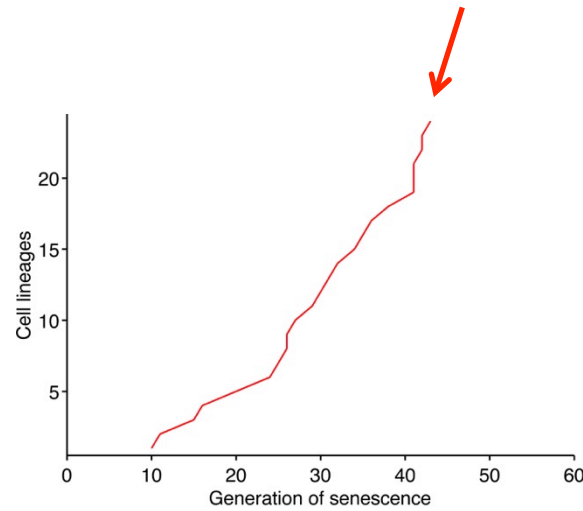
Experimental data:  
Type A lineages

Transition into  
senescence



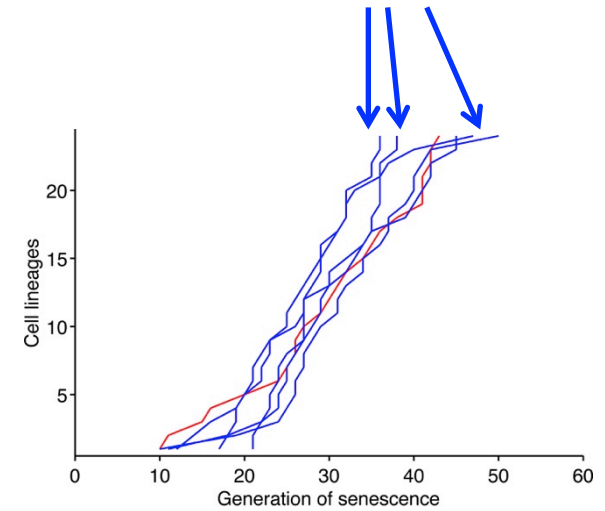
Extract transition data

Transition into  
senescence



Run the model and fit  
the transition profile

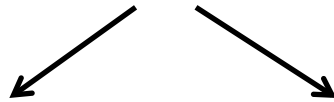
Simulations



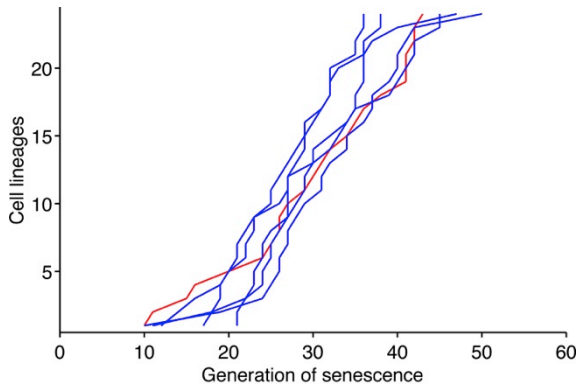
➤ The model was able to reproduce the experimental heterogeneity.

# Sources of senescence heterogeneity

Interclonal variations in telomere distributions?

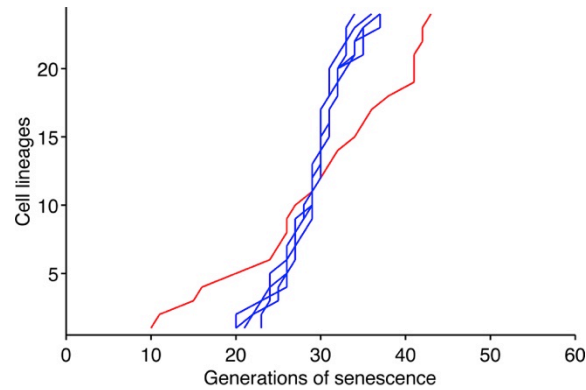
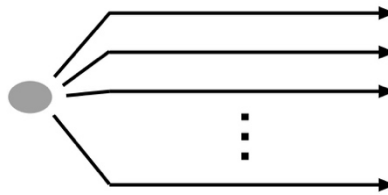


Different initial cells



Coefficient of variations (CV) = 0.29

One initial cell

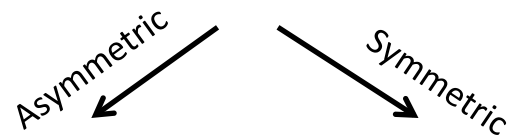
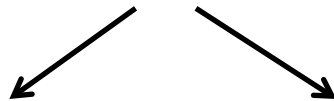


CV = 0.14

# Sources of senescence heterogeneity

Interclonal variations in telomere distributions?

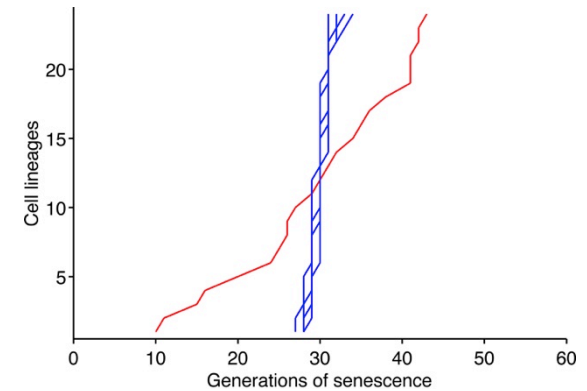
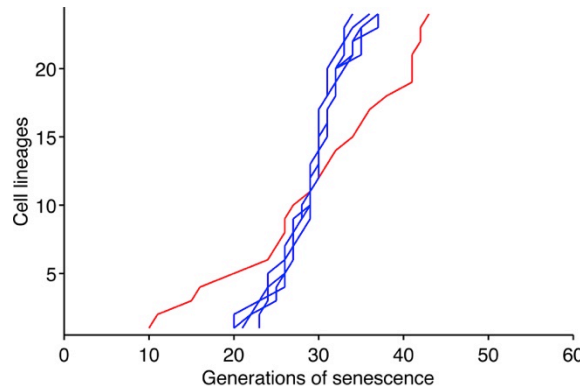
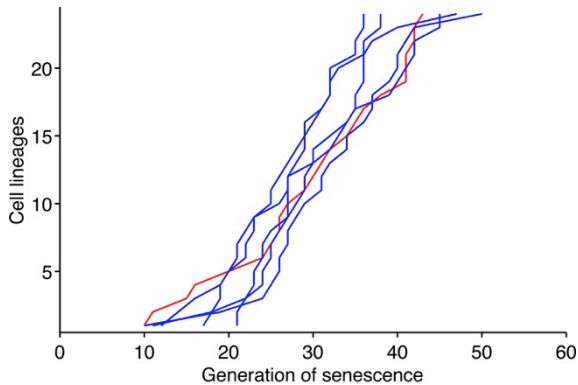
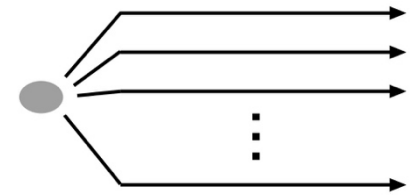
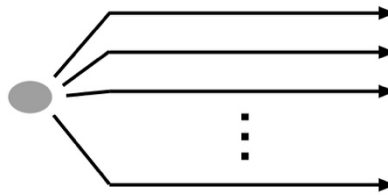
Asymmetry of telomere replication?



Different initial cells

One initial cell

One initial cell (symmetric replication)



Coefficient of variations (CV) = 0.29

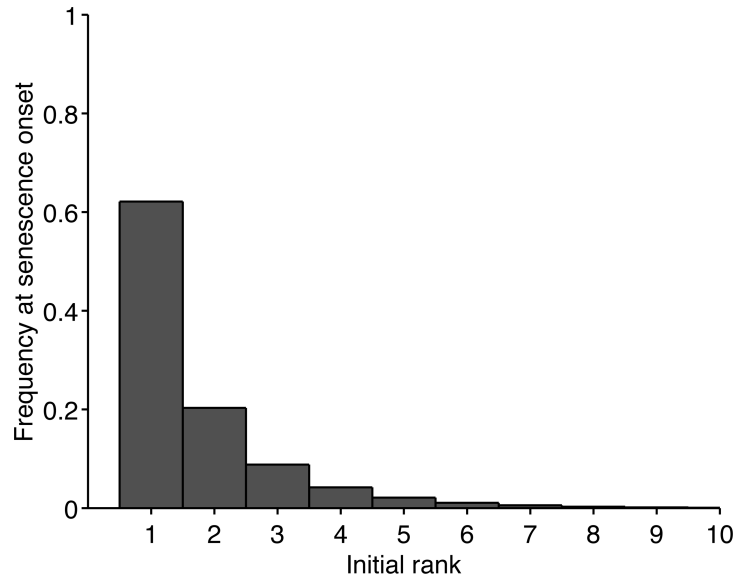
CV = 0.14

CV = 0.04

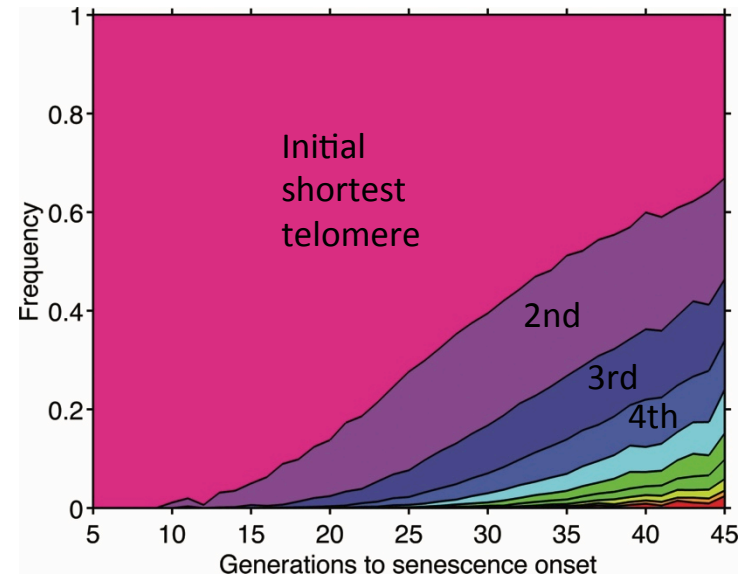
- Different sources of heterogeneity.
- The asymmetry of telomere replication is an unexpected source.

# Consequences of the asymmetry of telomere replication

Initial rank of the final shortest telomere



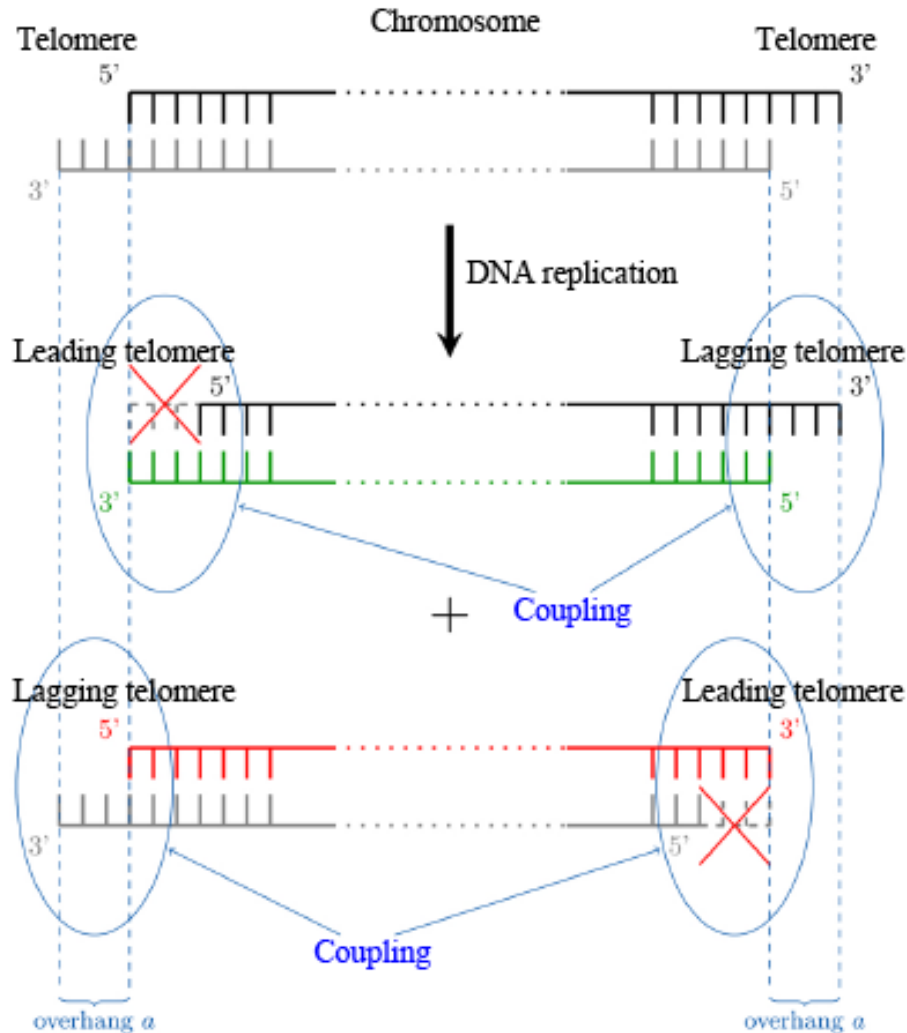
As a function of senescence onset timing



- Short-lived and long-lived lineages experience different telomere dynamics.

# A new layer of complexity: The coupling effect

Collaboration with Sarah Eugène (INRIA) and Thibault Bourgeron (ENS Lyon),  
with help from Marie Doumic, Philippe Robert and Teresa Teixeira



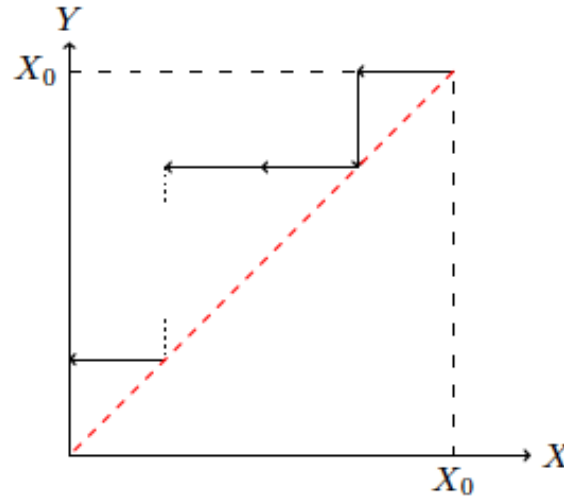
With or without telomerase

$$\begin{pmatrix} L_{n+1}^1 \\ L_{n+1}^2 \end{pmatrix} = \begin{pmatrix} (L_n^1 - a \cdot B_n)^+ + C_n^1 \cdot \mathcal{G}_n^1 \\ (L_n^1 - a \cdot (1 - B_n))^+ + C_n^1 \cdot \mathcal{G}_n^1 \end{pmatrix}$$

➤ The coupling effect adds a new constraint.



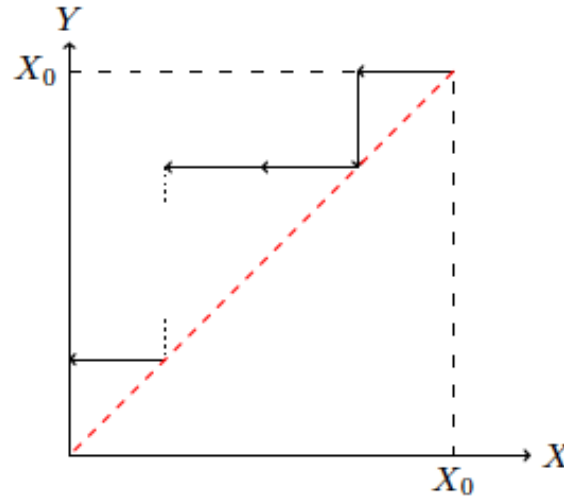
Case of a single chromosome with two equal telomeres  $(x_0, x_0)$



Asymptotic expansion of the time of senescence:

$$\mathbb{E}(T_{x_0}^1) \sim 2x_0 \quad \text{and} \quad \frac{2x_0 - T_{x_0}^1}{\sqrt{x_0}} \xrightarrow[x_0 \rightarrow +\infty]{dist.} |N|, \quad \text{with } N \stackrel{dist.}{\sim} \mathcal{N}(0, 2).$$

# Case of a single chromosome with two equal telomeres $(x_0, x_0)$ and general case



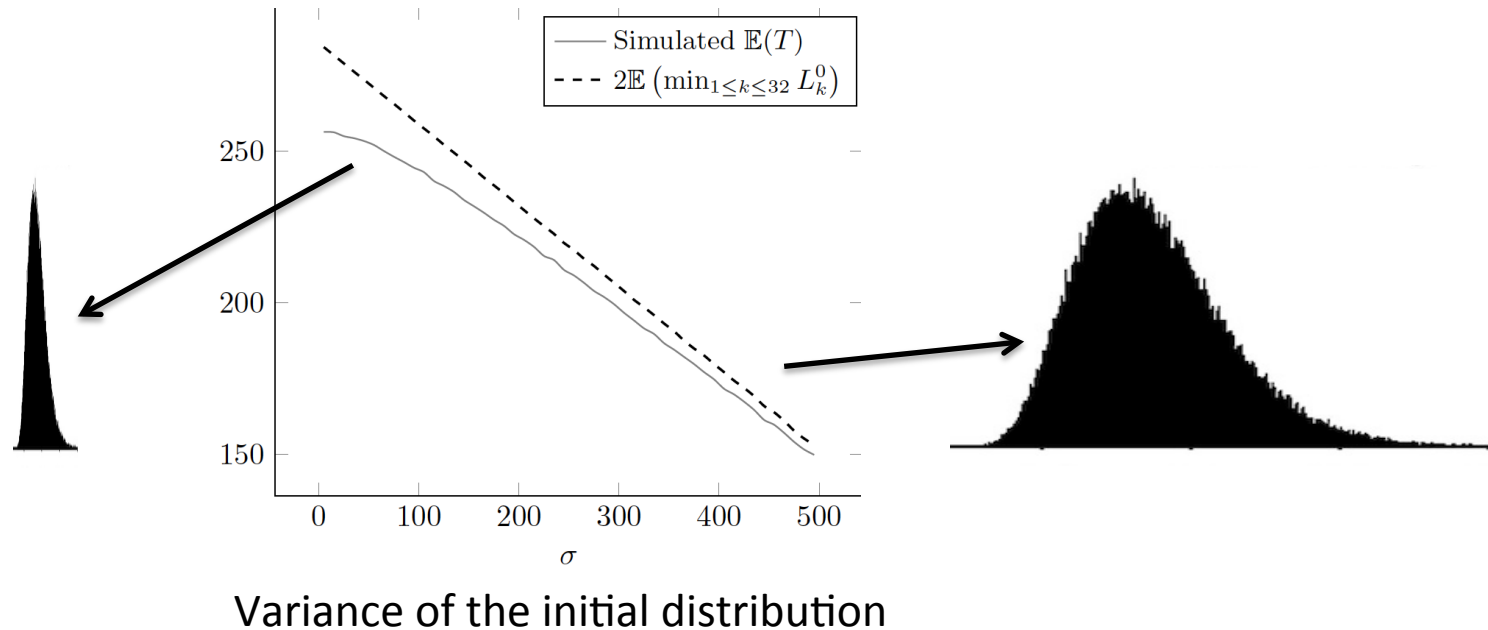
Asymptotic expansion of the time of senescence:

$$\mathbb{E}(T_{x_0}^1) \sim 2x_0 \quad \text{and} \quad \frac{2x_0 - T_{x_0}^1}{\sqrt{x_0}} \xrightarrow[x_0 \rightarrow +\infty]{dist.} |N|, \quad \text{with } N \stackrel{dist.}{\sim} \mathcal{N}(0, 2).$$

And for 16 pairs of telomeres:

$$\mathbb{E}(T_{x_0}) \approx x_0 + \sum_{n=x_0}^{2x_0-1} [\mathbb{P}(2x_0 - \sqrt{x_0} |N| > n)]^{16} = x_0 + \sum_{k=0}^{x_0-1} \left[ \operatorname{erf} \left( \frac{k}{2\sqrt{x_0}} \right) \right]^{16}$$

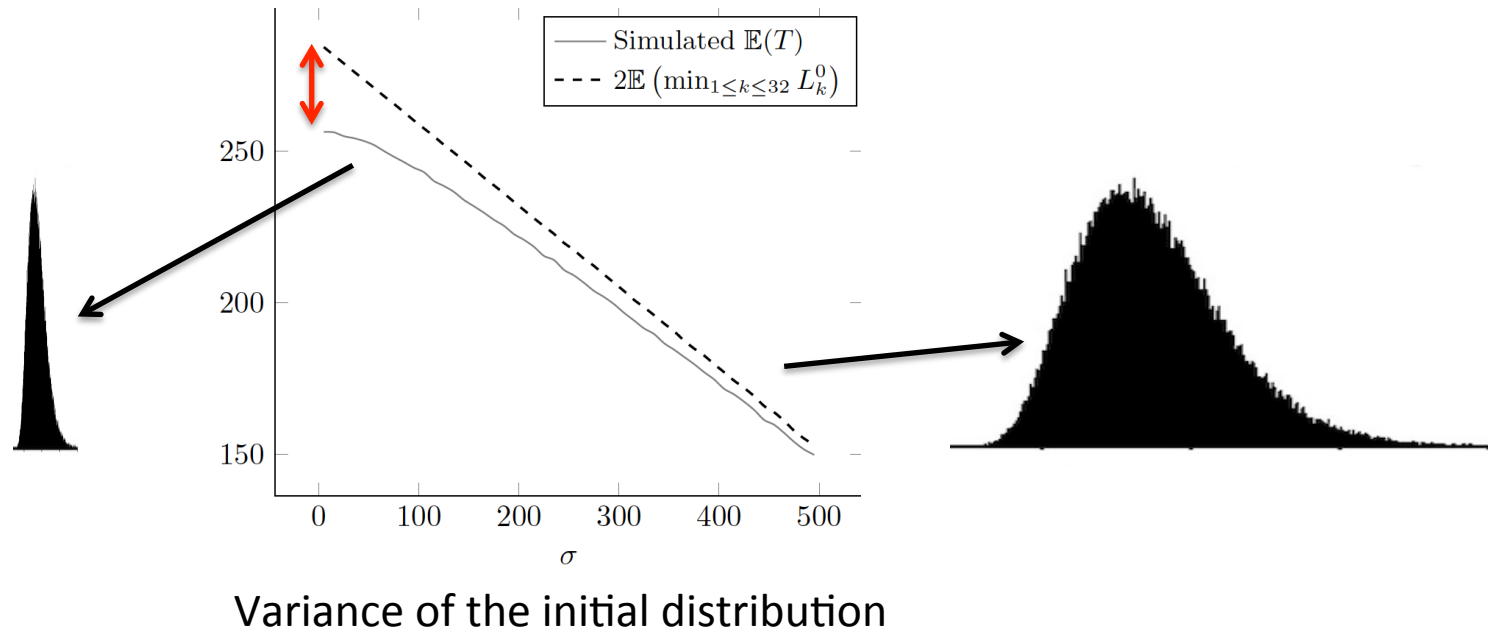
# Biological relevance of the expansion



And for 16 pairs of telomeres:

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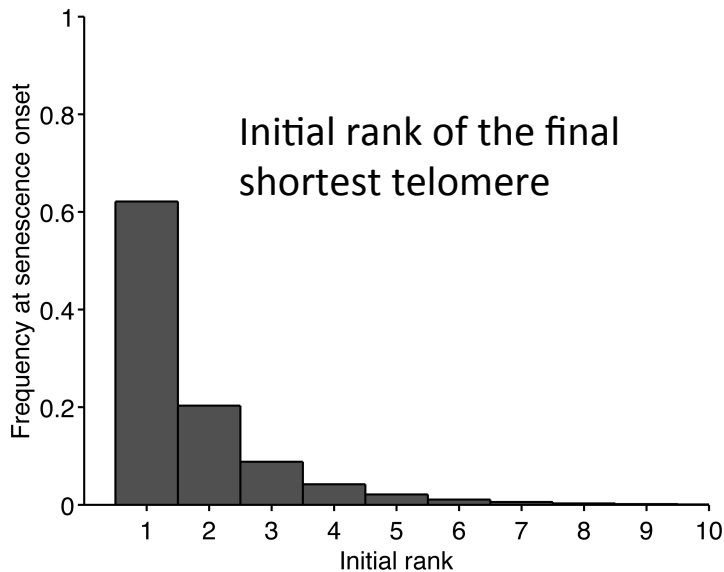
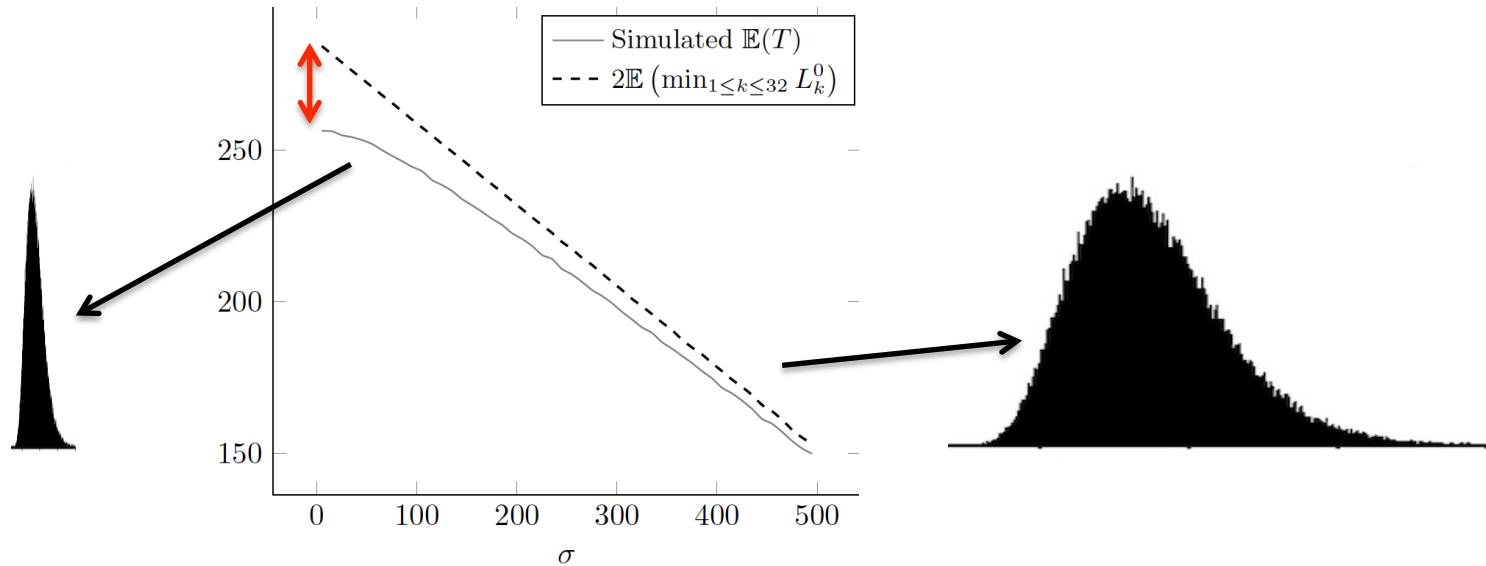
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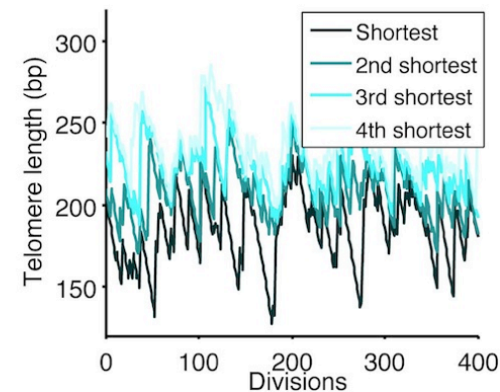
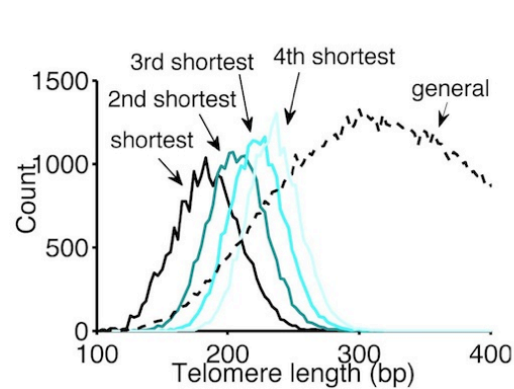
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# Shortening dynamics depends on the initial variance



## Dynamics of the shortest telomeres



# Summary of the mathematical models

Publication	Shortening without telomerase	Shortening and elongation with telomerase	Elongation probability	Comment
Xu <i>et al.</i> 2013 Genetics	N/A	$L_{n+1} = \begin{cases} L_n - a \\ L_n - a + b \end{cases}$	$P(L_n) = \begin{cases} (1 + \beta(L_n - L_0))^{-1} \\ 1 \end{cases}$	Deterministic shortening. <b>Simulations.</b>
Bourgeron <i>et al.</i> 2015 Sci Rep	$L_k^{n+1} = \begin{cases} L_k^n, & \text{if } k \in E \\ L_k^n - s_k, & \text{if } k \in E^c \end{cases}$ where $E = \sigma(\{1, \dots, 16\})$	$L_k^{n+1} = \begin{cases} L_k^n - s \\ L_k^n - s + B \end{cases}$	$P(L_k^n) = \begin{cases} (1 + \beta(L_k^n - L_0))^{-1} \\ 1 \end{cases}$	Stochastic shortening but not for steady-state telomere distribution. <b>Simulations.</b>
Eugène <i>et al.</i> 2016 Submitted (and arxiv)	$\begin{pmatrix} L_{n+1}^1 \\ L_{n+1}^2 \end{pmatrix} = \begin{pmatrix} (L_n^1 - a \cdot B_n)^+ + C_n^1 \cdot G_n^1 \\ (L_n^2 - a \cdot (1 - B_n))^+ + C_n^1 \cdot G_n^1 \end{pmatrix}$		$P(L_n) = \begin{cases} (1 + \beta(L_n - L_0))^{-1} \\ 1 \end{cases}$ OR $P(L_n) = 1_{\{L_n \leq i_s\}}$	Stochastic shortening (also for steady-state telomere distribution) and coupling. <b>Theoretical.</b>

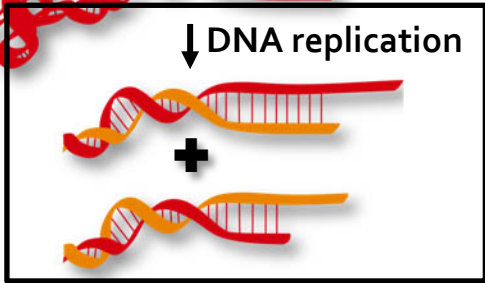
- Each model provided new insights
- Successive layers of refinements

# Replicative senescence

Type A route

Intrinsic source of heterogeneity

↓ DNA replication



↓ DNA replication

↓ DNA replication

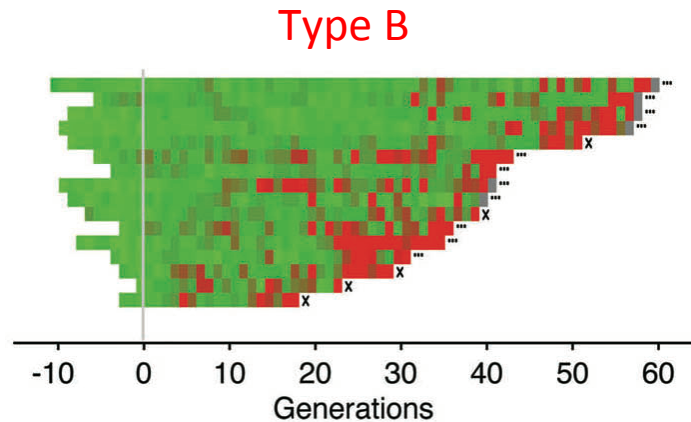
↓ DNA replication



Sharp transition to arrest,  
e.g. single event

Progressive telomere  
shortening

What are **Type B** lineages? Are they important?

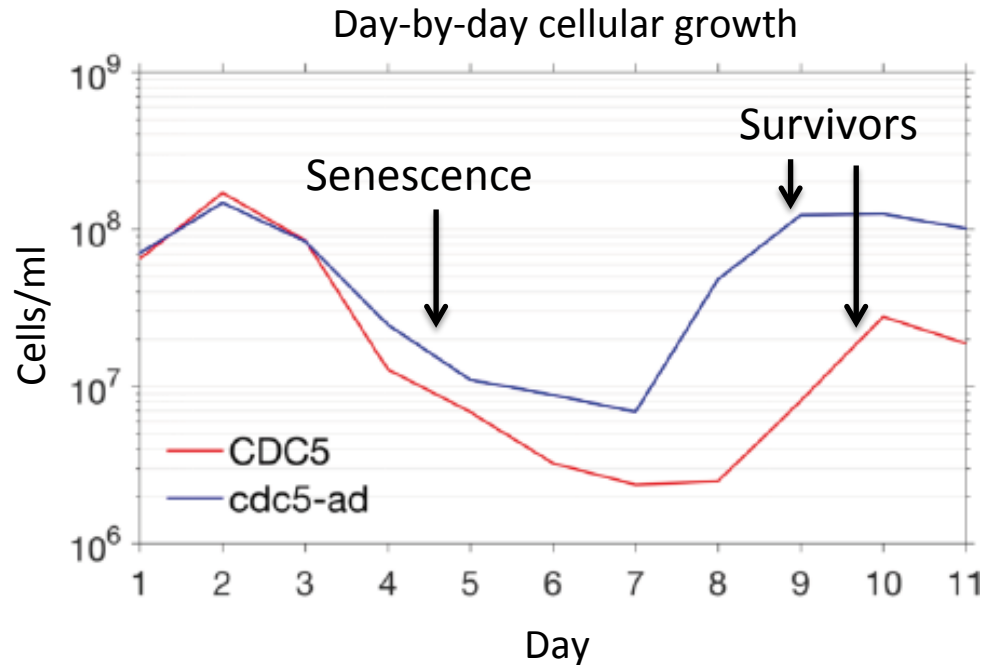


**Our hypothesis:**

**Type B** lineages contributes to many aspects of senescence, for instance the emergence of **post-senescence survivors** (analogous to **cancer** cells).



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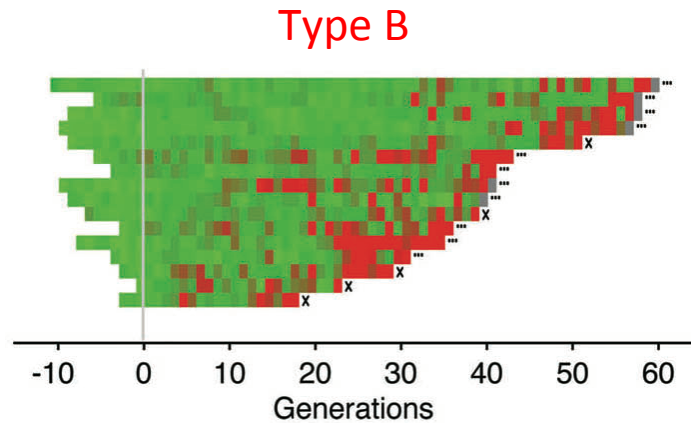


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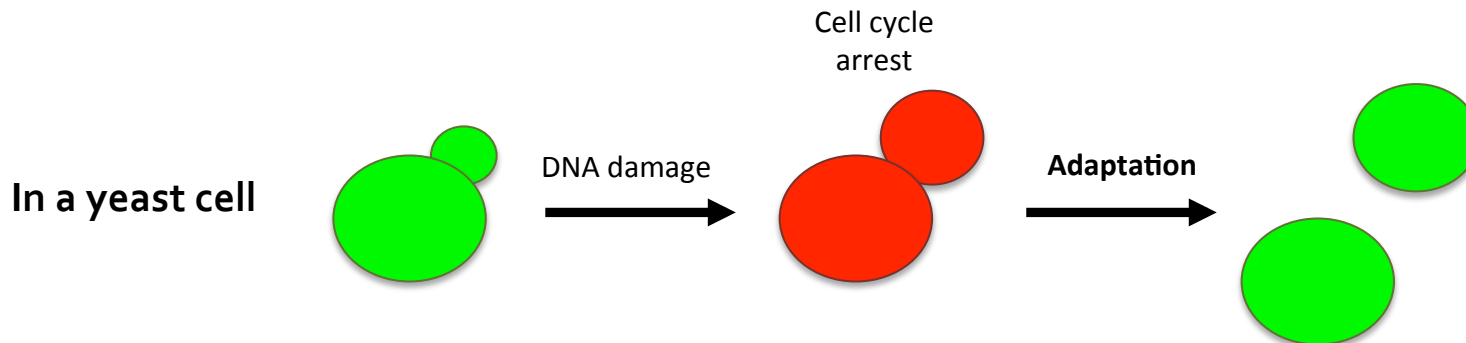
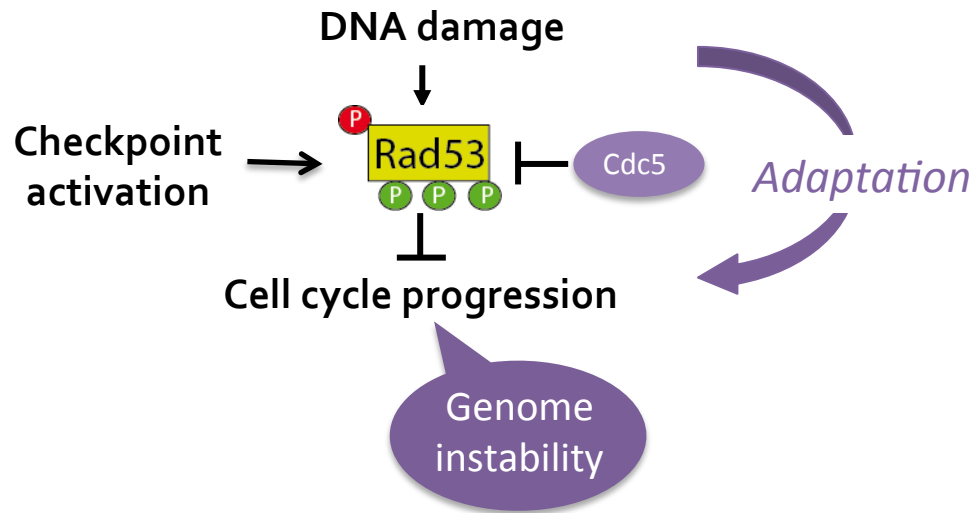
- Several lines of experimental evidence suggesting this is true.

## Most recent evidence



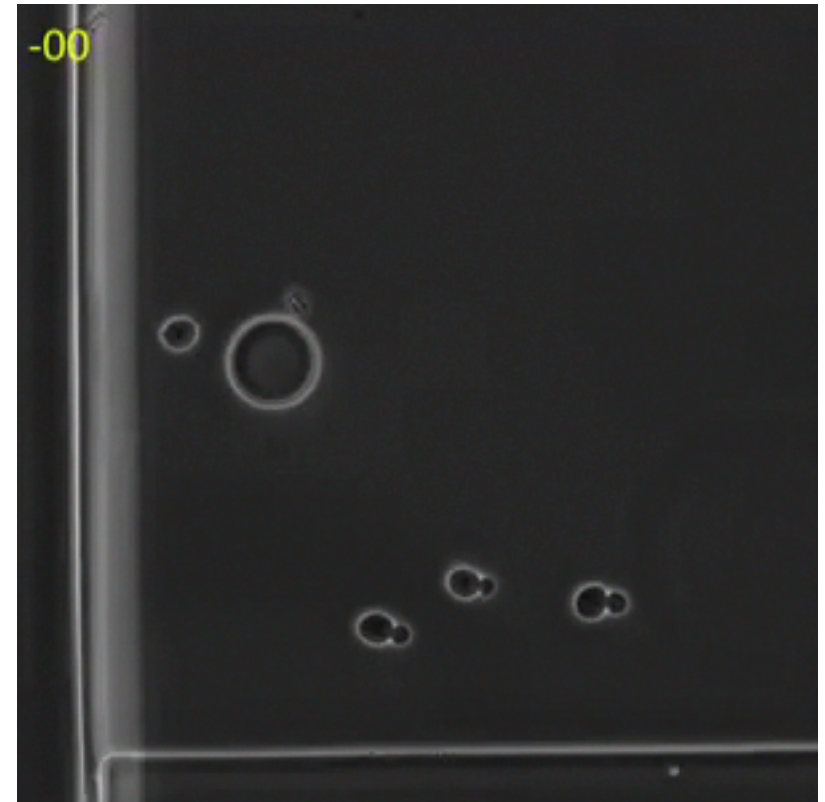
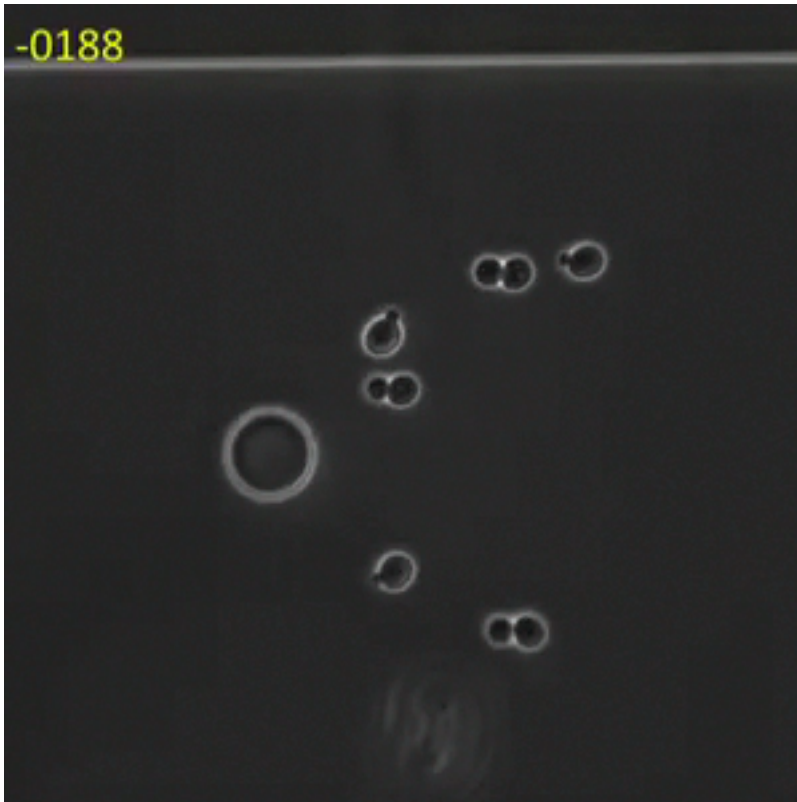
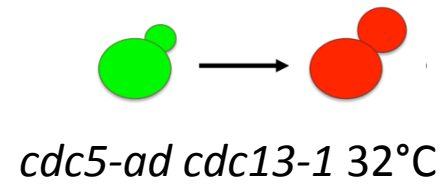
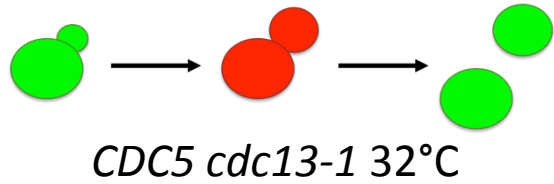
The early delays in **type B** lineages generate genome instability

# Adaptation to DNA damage



# Live cell imaging of adaptation

## Central gene: *CDC5*

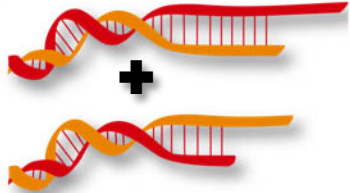


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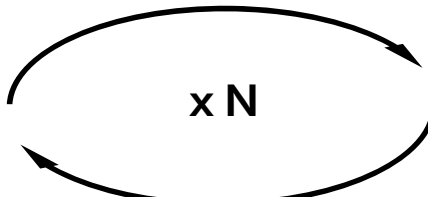
Sharp transition to arrest, e.g. single event

Type B route

Stochastic telomeric damage

Checkpoint activation & cell cycle arrest

Genome instability



Repair (homologous recombination) and/or adaptation



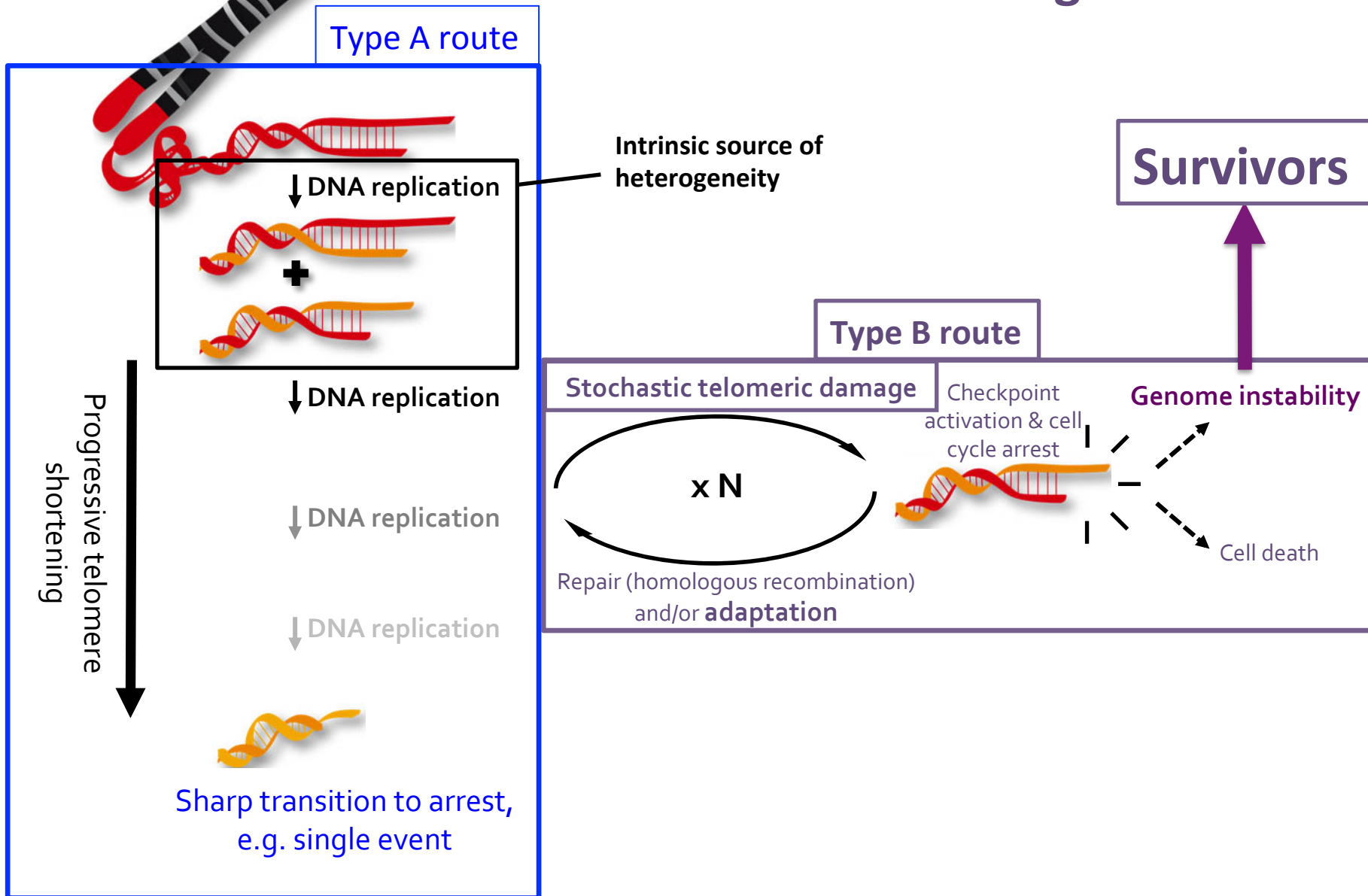
Cell death

Cancer

Progressive telomere shortening



# Perspective in mathematical modelling



# 70 years of progress in the study of the Luria-Delbrück distribution

Assumptions in the Luria-Delbrück model:

- The process starts with one wild-type cell and no mutant,
- Deterministic growth for wild-type and mutant cells,
- Exponential growth (to infinity),
- Mutations occur randomly at a rate proportional to the population size.

Luria and Delbrück

Mean and variance

Lea and Coulson

Approximate pgf, stochastic model

Haldane

Combinatorial approach

Armitage

Exact pgf

Kendall

Arbitrary distribution for cell cycle

Bartlett

Exact pgf, fully stochastic model

Crump and Hoel

Filtered Poisson process theory

Ma and Sarkar

Algorithm for probability function

Many others...

But no closed expression!

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Exact pgf, fully stochastic model

Filtered Poisson process theory

Algorithm for probability function

**Anyone?**

Distribution of **post-senescence survivors!**



# 70 years of progress in the study of the Luria-Delbrück distribution

Assumptions in the Luria-Delbrück model:

- The process starts with one wild-type cell and no mutant,
- ~~Deterministic growth for wild type and mutant cells,~~
- ~~Exponential growth (to infinity),~~
- ~~Mutations~~ occur randomly at a rate proportional to the population size.

Assumptions in the **SURVIVOR** model:

- Stochastic and heterogeneous growth,
- Growth slows down in wild-type cells (senescence),
- Recombination events occur stochastically as a function of telomere lengths,
- No easy way to identify survivors.

Goals:

- To understand how survivors (~cancer cells) emerge,
- To develop a method to estimate the rate of survivor emergence.

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