A stochastic model for speciation by mating preferences

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CMAP

04/07/2016

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Motivations

Pundamilia pundamilia

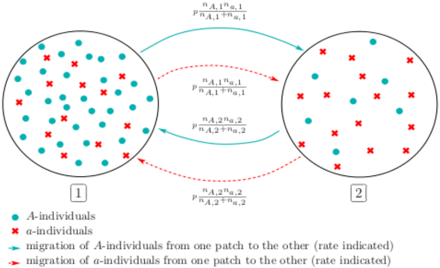
Pundamilia nyererei

What is the effect of a **mating preference** on the **speciation**?

- **Homogamy**: individuals with similar phenotypes have a higher reproductive success,
- **Heterogamy**: individuals with different phenotypes have a higher reproductive sucess.

Model description

- Two patches,
- Haploid population,
- two alleles: a and A,



Dynamics of the population $Z^K_{A,1}$

$$Z^K_{A,1} = \frac{\sharp\{\text{individuals of type } A \text{ in the patch } 1\}}{K}$$

- Birth: $b\frac{\beta Z^K_{A,1}+Z^K_{a,1}}{Z^K_{A,1}+Z^K_{a,1}}Z^K_{A,1}$,

→ **Homogamy** : two mating individuals that carry the same allele have a probability β **times larger** to give birth to a viable offspring.

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- Death: $(d + c(Z^K_{A,1} + Z^K_{a,1}))Z^K_{A,1}$,

Dynamics of the population $Z_{A,1}^K$

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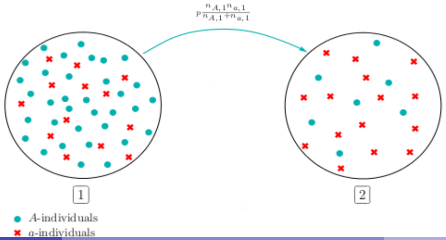
- Death: $(d + c(Z_{A,1}^K + Z_{a,1}^K))Z_{A,1}^K$,
- Migration toward the patch 2: $p\frac{Z_{a,1}^K}{Z_{A,1}^K + Z_{a,1}^K} Z_{A,1}^K$,



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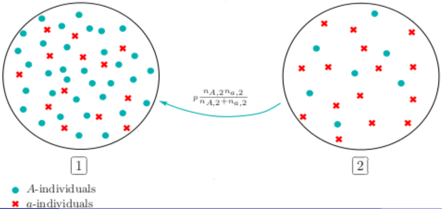

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The total population is represented by

$$\mathbf{Z}^K_t = (Z^K_{A,1}, Z^K_{a,1}, Z^K_{A,2}, Z^K_{a,2})$$

Large population approximation

Assumptions:

- $\boxed{K \to +\infty}$
- $(\mathbf{Z}_0^K)_{K\geq 1}$ converges to $\mathbf{z}^0$.

Theorem

$$\lim_{K\to\infty} \sup_{s\leq T} \|\mathbf{Z}^K(s) - \mathbf{z}^{(\mathbf{z}^0)}(s)\| = 0 \quad \textit{in probability}, \tag{1}$$

where $\|.\|$ *denotes the* L^∞*-Norm on* $\mathbb{R}^4$*, and* $\mathbf{z}^{(\mathbf{z}^0)}$ *is the solution to the dynamical system*

$$\begin{cases} \frac{d}{dt}z_{A,1}(t) = z_{A,1}\left[b\frac{\beta z_{A,1}+z_{a,1}}{z_{A,1}+z_{a,1}} - d - c(z_{A,1}+z_{a,1}) - p\frac{z_{a,1}}{z_{A,1}+z_{a,1}}\right] + p\frac{z_{A,2}z_{a,2}}{z_{A,2}+z_{a,2}} \\ \frac{d}{dt}z_{a,1}(t) = z_{a,1}\left[b\frac{\beta z_{a,1}+z_{A,1}}{z_{A,1}+z_{a,1}} - d - c(z_{A,1}+z_{a,1}) - p\frac{z_{A,1}}{z_{A,1}+z_{a,1}}\right] + p\frac{z_{A,2}z_{a,2}}{z_{A,2}+z_{a,2}} \\ \frac{d}{dt}z_{A,2}(t) = z_{A,2}\left[b\frac{\beta z_{A,2}+z_{a,2}}{z_{A,2}+z_{a,2}} - d - c(z_{A,2}+z_{a,2}) - p\frac{z_{a,2}}{z_{A,2}+z_{a,2}}\right] + p\frac{z_{A,1}z_{a,1}}{z_{A,1}+z_{a,1}} \\ \frac{d}{dt}z_{a,2}(t) = z_{a,2}\left[b\frac{\beta z_{a,2}+z_{A,2}}{z_{A,2}+z_{a,2}} - d - c(z_{A,2}+z_{a,2}) - p\frac{z_{A,2}}{z_{A,2}+z_{a,2}}\right] + p\frac{z_{A,1}z_{a,1}}{z_{A,1}+z_{a,1}}. \end{cases} \tag{2}$$

[Ethier, Kurtz 1986]

Stationary states

There exists 4 stable stationary states:

- 2 equilibria for which each type remains present

$$(\zeta, 0, 0, \zeta), \quad (0, \zeta, \zeta, 0)$$

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where $\zeta = \frac{b\beta - d}{c}$.

Stationary states

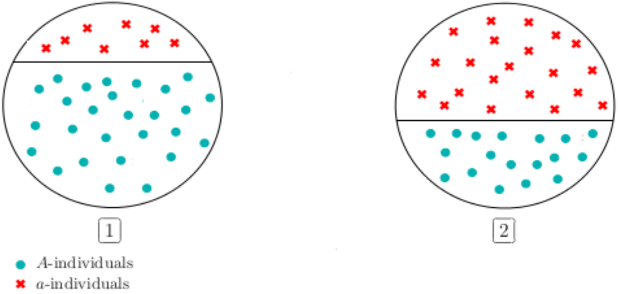
There exists 10 unstable stationary states:

- the extinction state: $(0, 0, 0, 0)$,
- 4 equilibria with only one type in only one patch,
- the equilibrium with both types remaining in both patches with the same proportion of each type,
- 4 equilibria with both types remaining in both patches with different proportions of each type.

Long time behavior of (2)

Let

$$\mathcal{D} := \{\mathbf{z} \in \mathbb{R}_+^4, z_{A,1} - z_{a,1} > 0, z_{a,2} - z_{A,2} > 0\},$$



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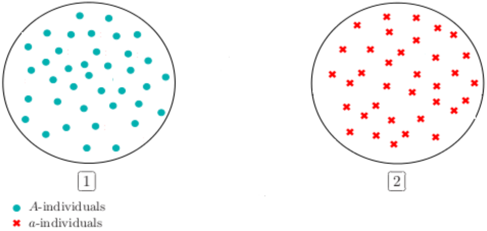
Theorem

Let $p < p_0$. Then any solution to (2) which starts from $\mathcal{D}$ converges to the equilibrium $(\zeta, 0, 0, \zeta)$.



Long time behavior of (2)

Let

$$\mathcal{D} := \{\mathbf{z} \in \mathbb{R}_+^4, z_{A,1} - z_{a,1} > 0, z_{a,2} - z_{A,2} > 0\},$$

Theorem

Let $p < p_0$. Then if the initial condition of (2) lies in

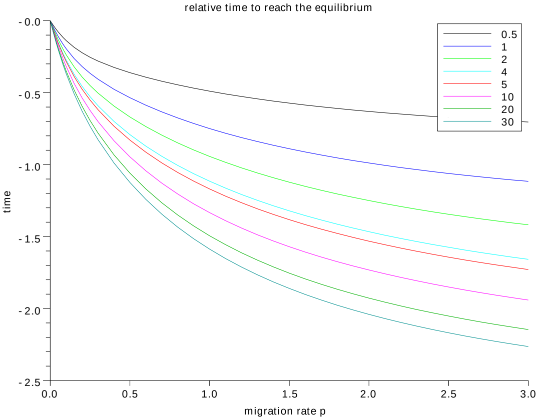
$$\mathcal{K}_p := \left\{\mathbf{z} \in \mathcal{D},\ \{z_{A,1} + z_{a,1},\ z_{A,2} + z_{a,2}\} \in \left[\frac{b(\beta+1) - 2d - p}{2c}, \frac{2b\beta - 2d + p}{2c}\right]\right\},$$

there exist two positive constants k_1 and k_2, depending on the initial condition, such that for every $t \geq 0$,

$$\|\mathbf{z}(t) - (\zeta, 0, 0, \zeta)\| \leq k_1 e^{-k_2 t}.$$

Influence of the migration parameter p

$$p \mapsto T_\varepsilon(p) - T_\varepsilon(0)$$



$$z_{a,1}(0) = z_{A,1}(0) - 0.1, \quad z_{A,2}(0) = 1, \quad z_{a,2}(0) = 30.$$

Influence of the migration parameter p

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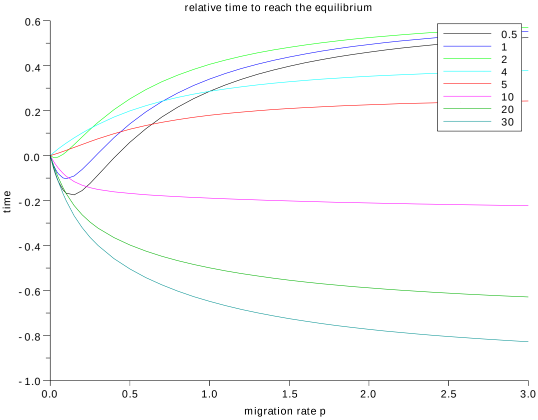
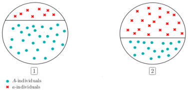


$$z_{a,1}(0) = z_{A,1}(0) - 0.1, \quad z_{A,2}(0) = 15, \quad z_{a,2}(0) = 16.$$

Time before speciation

If $(\mathbf{Z}^K(0))_{K\geq 1}$ converges to $\mathbf{z}^0$



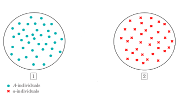
Theorem

Let $\varepsilon > 0$ *and* $p < p_0$*, the time before* $\mathbf{Z}^K$ *reaches the set*

$$\mathcal{B}_\varepsilon := [\zeta - \varepsilon, \zeta + \varepsilon] \times \{0\} \times \{0\} \times [\zeta - \varepsilon, \zeta + \varepsilon],$$

is

$$\frac{1}{b(\beta - 1)} \log(K)$$



when K *tends to* $+\infty$*.*

Conclusion : Without any ecological difference, **mating preferences** are enough to entail **a reproductive isolation**.


Thank you for
your attention