Math-Cancer, Marseille, 5-10 December 2015

How the dynamics between the populations of cells affects tumor growth

Christophe Letellier





Louise Viger



Clément Draghi

A short introduction to chaos



AND A. SHRIER¹

A short introduction to chaos



A short introduction to chaos

Chaotic Behavior in Simple Reaction Systems

Otto E. Rössler

Institut für Physikalische und Theoretische Chemie der Universität Tübingen

(Z. Naturforsch. 31 a, 259-264 [1976]; received January 5, 1976)

Chemical system theory, exotic kinetic



Poincaré section

> From a chaotic attractor...









Cancer mortality by age and organs



THE MORTALITY FROM CANCER THROUGHOUT THE WORLD

Frederick Hoffman (1865-1945)

First mathematical model in cancer problem

N. S.

BULLETIN OF MATHEMATICAL BIOPHYSICS VOLUME 7, 1945

Loe W (t)

OUTLINE OF A MATHEMATICAL APPROACH TO THE CANCER PROBLEM

N. RASHEVSKY The University of Chicago



Cancer incidence W(t) for 100,000 males for all organs $\log W(t) = \log A_3 + \alpha t - \frac{1}{2} \alpha ct^2$ $Log A_3 = -4.2$ α = 0.236 year⁻¹ $c = 0.01 \text{ year}^{-1}$ Normal longevity $\frac{1}{c} = 88$ ans « we thus can correctly compute the humar normal longevity from data on cancer incidence.»

Survival curves



Number of cancers in US during 2014



Factors affecting the lifetime risk



Lifetime risk: "bad luck" or not?



Peto's paradox

Hypothesis: the probability for presenting malignant cells depends on the number of cell divisions during the lifetime

Corollary 1: longer the lifetime, larger the lifetime risk for a content of the lifeti



> The probability does not linearly increase with the age...

Peto's paradox

Hypothesis: the probability to observe a malignant cell depends on the cell division during the lifetime

Corollary: Bowhead whale should present much more cancers than humans





- > Most likely live up to 250 years without cancer!
- Immune system 4 times more efficient!



Role of the microenvironment

Hypothesis: the probability for presenting malignant cells depends on the number of cell divisions during the lifetime but diagnozing a tumor depends more on the barriers in the microenvironment

Corollary 3: the probability to initiate a cancer depends more on the status of the microenvironment than on the presence of malignant cells



A simple cancer model

Journal of

A

Interactions between the populations of cells in a single tumoral site

$$\begin{cases} \dot{x} = \rho_1 x(1-x) - \alpha_{13} xz \\ \dot{y} = \frac{\rho_2 yz}{1+z} - \alpha_{23} yz - \delta_2 y \\ \dot{z} = z(1-z) - xz - \alpha_{32} yz + \nabla . (K \nabla z) \\ \text{Most cells} \\$$

Realistic parameter values?

			F String	
Cancer origin	ID	Average doubling time (days)	Average doubling time (days)	Average doubling time (days)
Colons	SW480 CaCO-2	5.4	3.4 X	100 391
	SW620 Colo205 HCT-116	3.2 1.0	No.	N2
Prostate	DU145	1.2	3.4 X	65 ≤ 219
Broact	PC-3 HCC1954	1.3	5.6	152
Diedsi	MDA-MB-46	3 1.3	0.0	102
	MCF7	1.2		
	BT474	1.2		
	MDA-MB-23	1 1.2		
Skin	MM8.1	0.8	5.4	147
•••••	A375	0.5		\bigvee L
Lung	A375	Л 1		26 - 114
Lung	SNU-371	4.1	4.4 X	20 2 114
	H2126	1.0	7	\mathbf{N}
	SINU-1330	1.0		
	A549	0.9		
		Citation: Cell Deal © 2012 Macmillan www.nature.com/od	th and Disease (2012) 3, e411; doi:10.1038/cddis Publishers Limited All rights reserved 2041-4889/12 Idis	22012.148
	Review			
	Mitosis-targeted a stand	nti-cancer therapie	es: where they	

Chaotic oscillations in this cancer model



... but induces aggressive cancer

A clinical example





Growth rate $\rho_1 = 1.45$



Role of immune cells

Influence of the inhibition of tumor cells by immune



The action of immune cells does not affect the dynamics

> Sometimes from a limited interest for a therapy

Anti PD-1 immunotherapy (Nivolumab)

Woman, 64 years Smoker Adenocarcinoma with a bone metastasis





Tumor-induced angiogenesis



A cancer model for the angiogenic switch



A cancer model for the angiogenic switch



A cancer model for the angiogenic switch



> Different values of the endothelial cell growth rate ρ_4



> Explain why a cancer evolution cannot be predicted

Spatial simulations of tumor growth

> Depends on the inhibition rate of immune cells by tumor cells

 $\alpha_{yz} = 1.9$ $\varnothing_{\mathsf{T}} \approx 1.3 \,\mathrm{mm}$ $\alpha_{yz} = 3.0$ $\varnothing_{\rm T} \approx 2.9 \,\rm{mm}$

Duration 6000 a.u.t.





in the lung of a women



64 years, smoker, adenocarcinoma with a bone metastasis



Spatial simulations of tumor growth

The tumor growth depends on the inhibition rate of immune cells by tumor cells



- > Chaotic dynamics in each site
- > Very few sites have tumor cells
- Very slow tumor growth

Spatial simulations of tumor growth

- > When there is a local chaotic dynamics
 - *t* = 74 800 a.u.t.





- > Heterogeneous
- > Thick proliferation zone
- Very slow tumor growth

> Woman, 80 years, smoker up to 60 years, weakly evolutive nodule





Adenocarcinoma



Dynamics more important than cell interactions

> Equivalent dynamics but different parameter values



For equivalent dynamics, the host cell growth rate is the most influent parameter...

> Relevance of the micro-environment

Why is observability so important?

- The full description of the states of a system requires a priori the knowledge of the full set of variables spanning the state space
 - Experimentally, only a few variables are measured...
 - > Are we ensured by our ability to correctly define the states of the system?

Conditions for a full observability



This is the case of the Rössler system observed from variable y(t)

 \Rightarrow The system is thus **fully observable**

Observability from different variables



Singular observability manifold

PHYSICAL REVIEW E 86, 026205 (2012)

Influence of the singular manifold of nonobservable states in reconstructing chaotic attractors

Madalin Frunzete,¹ Jean-Pierre Barbot,¹ and Christophe Letellier²

- > Definition of a neighborhood to the singular observability manifold $\mathcal{U}_{\mathcal{M}_s^{obs}} = \{(x, y, z) \in \mathbb{R}^3 | \text{Det } \mathcal{J}_{\Phi_s} | < \epsilon\}$
 - > Assessing the probability to have $x \in \mathcal{A} \cap \mathcal{M}_s^{obs}$





Observability of a cancer model



It should be more efficient to track the environment of the tumor rather than the tumor itself!





Support Care Cancer DOI 10.1007/s00520-013-1954-9

ORIGINAL ARTICLE

Detecting lung cancer relapse using self-evaluation forms weekly filled at home: the sentinel follow-up

Fabrice Denis • Louise Viger • Alexandre Charron • Eric Voog · Christophe Letellier

> > Rationale: since host cells provide the better observability of the tumor dynamics, could we track the environment of the tumor rather than the tumor itself?

Environment = global state of the patient weekly self-evaluated

- 1. Weight
- 2. Lack of appetite

Louise Viger

- 3. Fatigue
- 4. Pain

Fabrice Denis

- 5. Cough
- 6. Breathlessness

> A cohort of 43 patients treated for a lung cancer

- 1. Pulmonary carcinoma (grade 3-4)
- 2 Internet access

> Patient with no relapse

Man, 63 years Smoker, 90 kg, non sporty Treated by radiotherapy Relapse probability = 75 %

jj mm	23 11	30/11	07 12	15 12	21 12	28 12	04 01	11 01	18 01	25 01	02 02	08 02	15 02	22 02	01)03	08 03	15 03	22 03	29 03	05 04	12 04	19 04	26 04	03 05	10 05	17 05	24 05	31 05	07 06	14 06	21 06	28 06	05 07	13 07	19 07	26 07	02 08	09 08
aa	12	12	12	12	12	12	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
POIDS	89	89.5	90	91	91	92	92	91.5	91.5	91.5	91.5	91.5	91.5	91.5	92	92	92	93	93.5	93.5	94	95	95	95.5	96	95	95	95.5	95	95	95	96.5	97	97	96	96	97	95
DELTA POIDS	0	-0.5	4	-2	-2	-3	-3	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2	-2	-1	-1	-1	-1.5	-2	-2.5	-3.5	-3.5	-4	-4.5	-3.5	-3	-3.5	-3	-2	-1.5	-3	-3	-2	4	-0.5	$\left 4 \right $	0
APPETIT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FAIBLESSE	1	1	1	1	1	1	1	1	1	4	1	4	1	1	4	1	4	1	1	1	1	4	1	1	1	1	1	1	4	1	1	4	1	4	1	4	1	1
DOULEUR	1	1	1	1	1	1	1	1	1	4	1	4	1	1	1	1	1	1	1	1	1	4	1	1	1	1	1	1	4	1	1	4	1	4	1	1	1	1
TOUX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ESSOUFFLEMENT	1	1	1	1	1	1	1	1	1	4	1	4	1	1	1	1	1	1	1	1	1	4	1	1	1	1	1	1	4	1	1	4	1	4	1	1	1	1
DEPRIME	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					_		_																															

Before radiotherapy







> Patient with relapse

Man, 65 years Smoker, 86 kg, non sporty Treated by chemiotherapy Relapse probability = 75 %

jj mm	19)08	26 08	02 09	celeo	19 09	23 09	30 09	07 10	14 10	21 10	28 10	04 11	11]11	18 11	25 11	16		12	06 01
aa	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	1 (ریک	• د	14
POIDS	86	86.9	87.2	86.9	87.2	87.4	87.8	87.5	87.9	88.1	88.1	87.7	87.5	88.3	87.5	86.2	86.6	85.6	83
DELTA POIDS	0	-0.9	-1.2	-0.9	-1.2	-1.4	-1.8	-1.5	-1.9	-2.1	-2.1	-1.7	-1.5	-1.4	-0.3	0.7	0.6	1.8	1.8
APPETIT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FAIBLESSE	2	1	1	4	1	4	1	1	1	1	1	1	1	1	1	1	1	1	2
DOULEUR	0	1	1	0	0	4	1	0	0	1	0	1	0	0	2	3	3	3	а
TOUX	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1	0
ESSOUFFLEMENT	3	3	3	3	3	3	3	3	3	3	з	3	- 3	3	3	3	3	3	а.
DEPRIME	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1



2 months before the routine scan





- Detection of cancer relapse advanced by five weeks with respect to the routine scan...
- > Reliability equivalent to a routine follow-up!



- > More than 20% gained at 18 months...
- Cancer relapse treated earlier (5 weeks), better life quality (less stressed, less pain, ...)



3. Avoid using PET or PET-CT scanning as part of routine follow-up care to monitor for a cancer recurrence in patients who have finished initial treatment to eleminate the cancer unless there is high-level evidence that such imaging will change the outcome.

Conclusion

- Based on a deterministic model describing cell interactions, we showed that the micro-environment is relevant for understanding tumor dynamics
- > We are already able to reproduce qualitatively some clinical facts

- Chaotic local dynamics provide strong barriers against tumor growth
- Tracking the environment rather than the tumor is as reliable (if not more) as a routine imaging
 - > Randomized study (phase III) in progress (and very promising)



World Scientific