Material-dictated upper bounds for scattering response in linear systems

Owen Miller
Postdoc, MIT Applied Math

Collaborators:

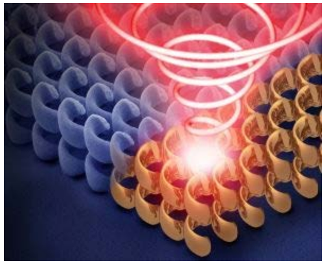
Prof. Steven Johnson, MIT Applied Math
Prof. Eli Yablonovitch, UC Berkeley EECS

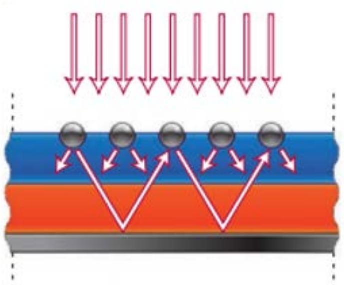
Prof. John Joannopoulos
Prof. Marin Soljacic
Dr. Homer Reid
Emma Anquillare
Dr. Wenjun Qiu

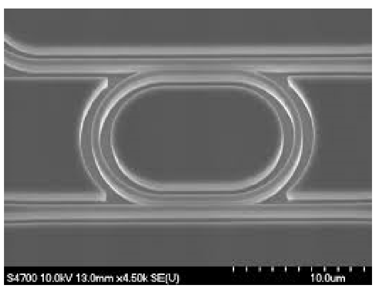
Prof. Alejandro Rodriguez
Brendan DeLacy
Dr. Chia-Wei Hsu
Prof. Thanos Polimeridis

Prof. Xiang Zhang
Samarth Bhargava
Vidya Ganapati
Chris Keraly

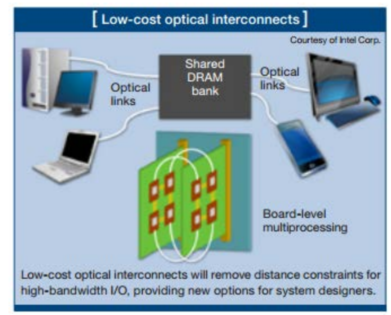
Principles







Technology



← *Design* →

The design challenge:

Lots of degrees of freedom in geometry,
but what structure is best for given device & materials?

And what performance & phenomena are possible?

Limits on what is possible

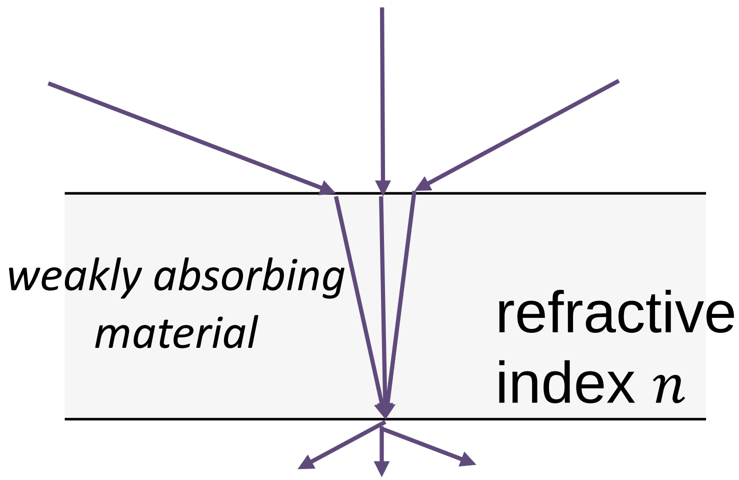
Typically, we know the materials, but don't know the structure, so geometry-independent limits on engineering performance are especially useful in guiding design:

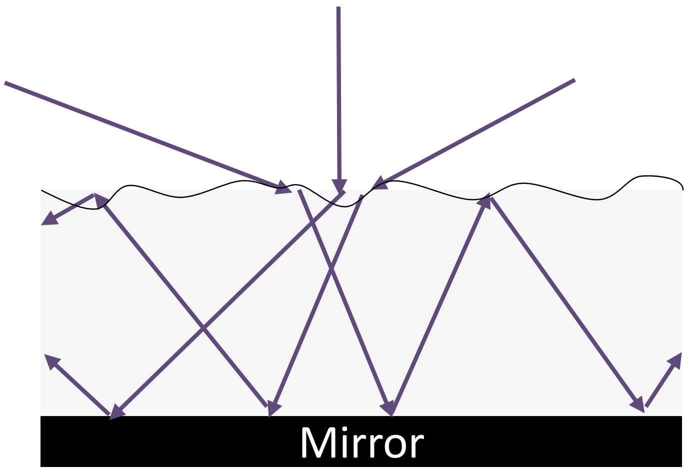
- How strongly/quickly can light be absorbed, scattered, transmitted?
- How efficiently can energy be converted from one form to another?
  — e.g. between different frequencies, radiation from heat, radiation from excited electrons (spontaneous emission)
- How long can light be stored in a given volume?
- What is the interplay with bandwidth, material loss, volume?

Example: Yablonovitch limit

E. Yablonovitch, "Statistical Ray Optics" [*JOSA* 72, 899 (1982)]

Enhancing the absorption efficiency of a thin film (e.g. solar cell):





single-pass absorptivity $A \ll 1$

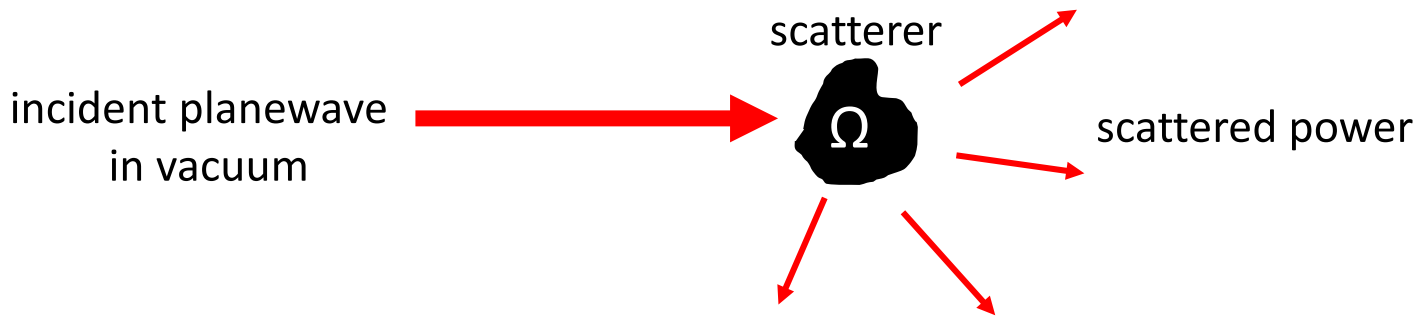
Yablonovitch:
texture+mirror absorptivity $\leq 4n^2A$

Derived under very restrictive assumptions (originally ray optics!), but has been very hard to beat (and nearly tight) in practical structures.

Other examples

- Wheeler–Chu bounds on antenna quality factor Q (per volume)
- Wiener (1912), Hashin–Shtrikman (1963), Bergman (1981), Milton (1981), bounds on homogenized properties of composites
- Black-body limit on thermal radiation (in far field for linear and/or equilibrium surface)
- Manley–Rowe limits to nonlinear frequency conversion
- Speed-of-light (*c*) limit on energy transport
- …

Geometry-*dependent*, *material-independent* limits



Spherical scatterers: Hamam et al. *PRA* 75, 053801 (2007)
Ruan & Fan *APL* 98, 43101 (2011)
Generic scatterers: Kwon & Pozar *IEEE TAP* 57, 3720 (2009)
Liberal et al. *IEEE TAP* 62, 4726 (2014)

scattered power $\lesssim O(N^2)$
where N = # multipole orders that can be excited

… depends non-trivially on shape
(N ~ diameter as size $\longrightarrow \infty$)
but not on the materials!

JP Hugonin et al., *PRB* 91, 180202 (2015)

scattered, absorbed power $\lesssim \langle \mathbf{E}_{inc}, (\mathrm{Im}\, G_0{}^*)^{-1} \mathbf{E}_{inc} \rangle_\Omega$

where G_0 is vacuum Green's function — depends on shape Ω but not materials!

New results

[O. D. Miller et al., *PRL* 115, 204302 (2015) & *Opt. Exp.* 24, 3329 (2016)]

- Limits on scattering & absorption by particles, on the local density of states, and also on near-field thermal radiation
- Very general, simple derivations from energy conservation & optical theorem
- Independent of geometry and bandwidth, depend only on the materials
- ~Tight (within a small constant factor) in many cases … all?

Review: Maxwell & Materials

$$\underbrace{\begin{pmatrix} & \nabla\times \\ -\nabla\times & \end{pmatrix}}_{\text{anti-Hermitian}} \underbrace{\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}}_{\psi} = \frac{\partial}{\partial t}\left[\psi + \underbrace{\chi * \psi}_{\phi = \begin{pmatrix} \mathbf{P} \\ \mathbf{M} \end{pmatrix}} \right] + \underbrace{\begin{pmatrix} \mathbf{J} \\ \mathbf{K} \end{pmatrix}}_{\text{current sources}}$$

polarization

continuum, local, linear materials: 6x6 susceptibility $\chi(x,t)$

(breaks down for metals at < 10nm scales ⇒ nonlocal; or very strong fields ⇒ nonlinear)

frequency domain: $\frac{\partial}{\partial t} \to -i\omega$

passive materials: $\omega \,\mathrm{Im}\, \chi(x,\omega) > 0$

i.e., polarization currents can dissipate but not supply energy

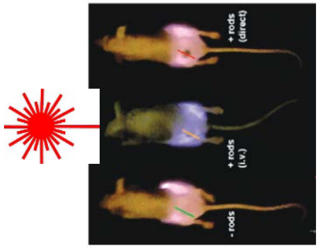
Starting Problem: Obscurant Nanoparticles

Goal: dilute, randomly-arranged particles to absorb or scatter light over a broad bandwidth

"smoke grenades"

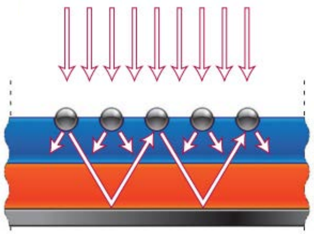
Related applications:

cancer therapy



JACS 128, 2115 (2006)

solar cells



Nat. Phot. 9, 205 (2010)

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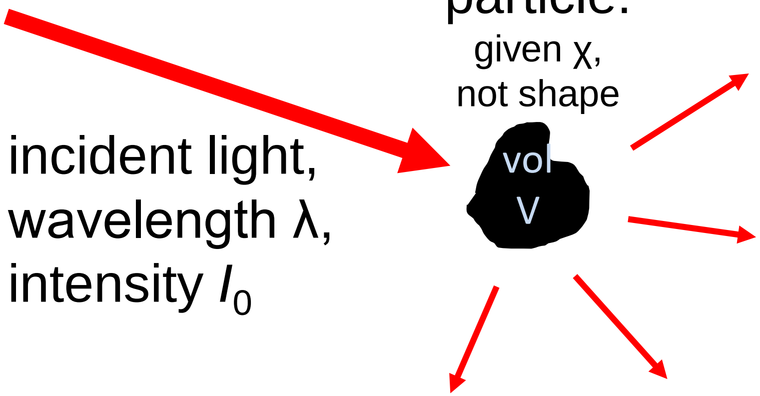
particle:
given χ, not shape

incident light, wavelength λ, intensity I_0



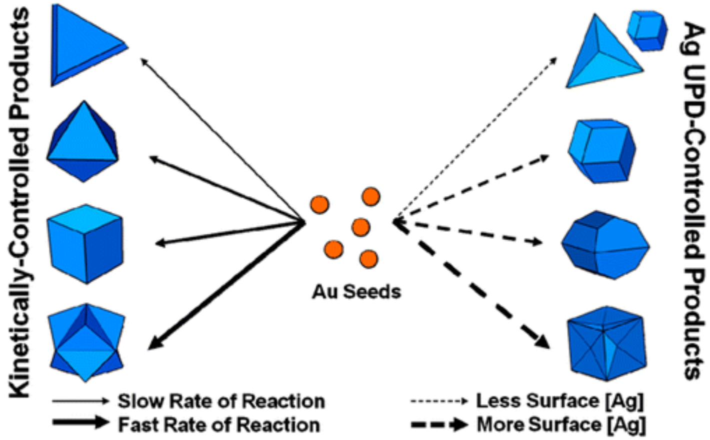
scattered light, power P_{scat}

extinction cross-section $\sigma_{ext} = (P_{abs} + P_{scat}) / I_0$

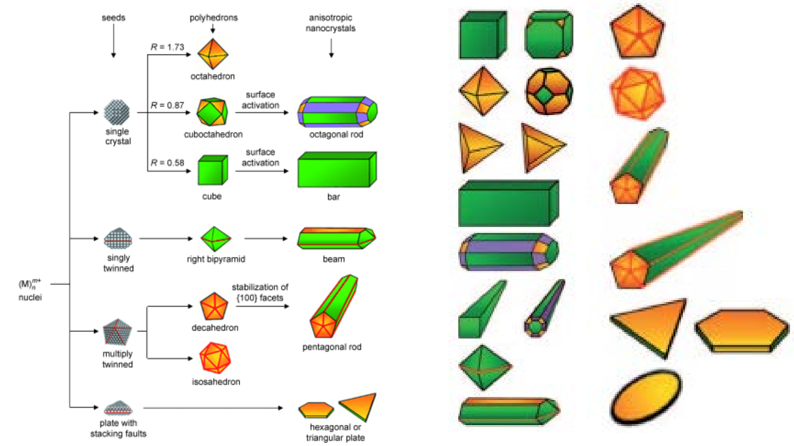
Key question: What is the best σ_{ext} / volume?

… averaged incident angles & polarizations …
(over some bandwidth)

A rapidly growing experimental toolkit…

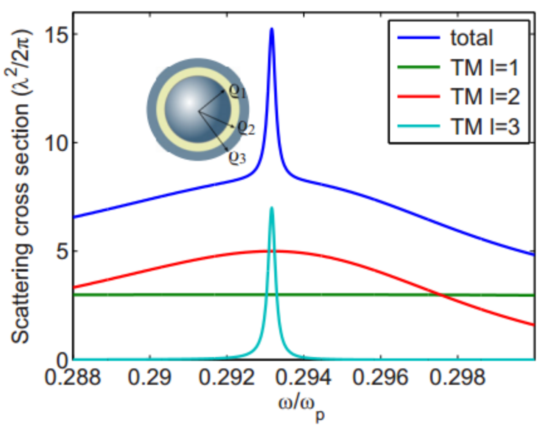


C. Mirkin et. al., JACS *134*, 14542 (2012)



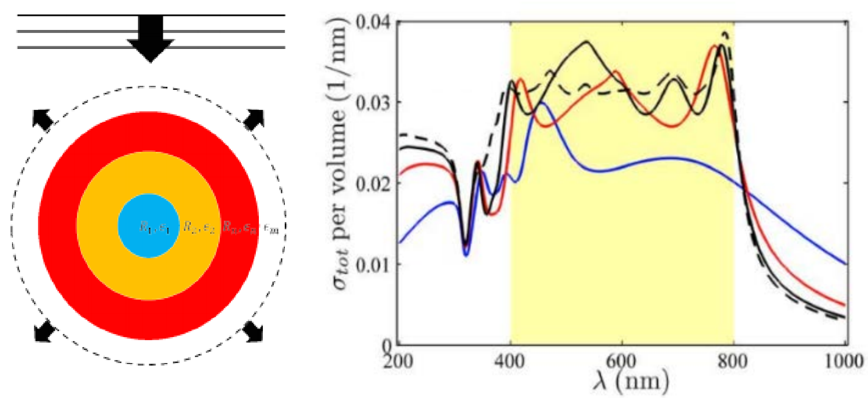
Y. Xia et. al. ACIE *48*, 60 (2009)

…with only limited theoretical designs



size $\approx \lambda/4$

Fan et. al. PRL 105, 13901 (2010)
APL 98, 43101 (2011)

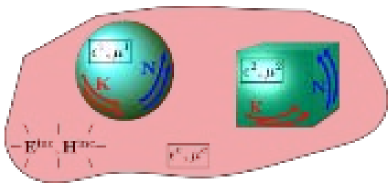


size $\ll \lambda$

Qiu, DeLacy Johnson, Joannopoulos, & Soljacic,
Opt. Exp. 20, 18494 (2012)

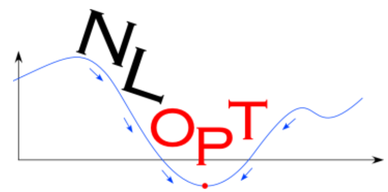
to start with:
computational exploration of non-spherical shapes

“Warmup” problem: Optimizing Ag ellipsoids



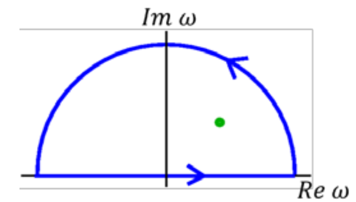
boundary-element method

\+



nonlinear optimization

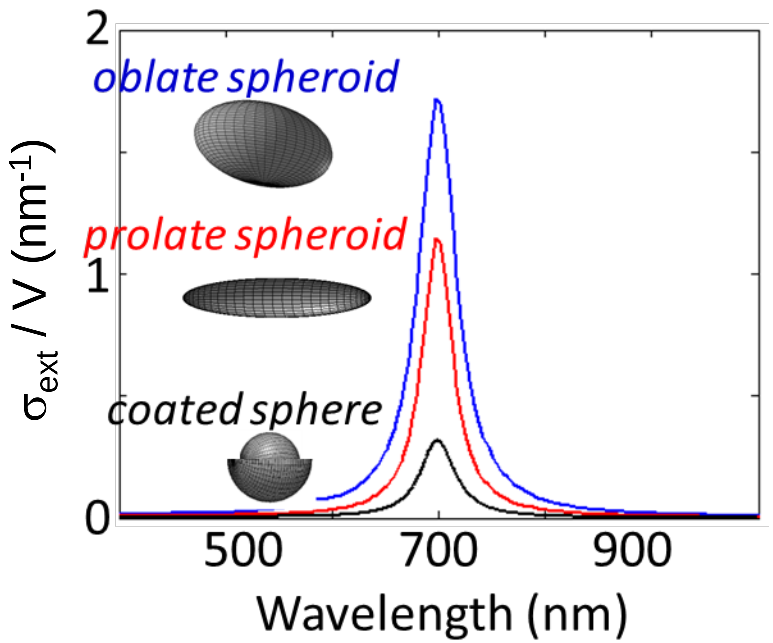
\+



complex-ω transformation

Hashemi et al *PRA* 86, 013804 (2012)
Liang et al *OE* 21, 30812 (2013)

angle- and pol.-averaged



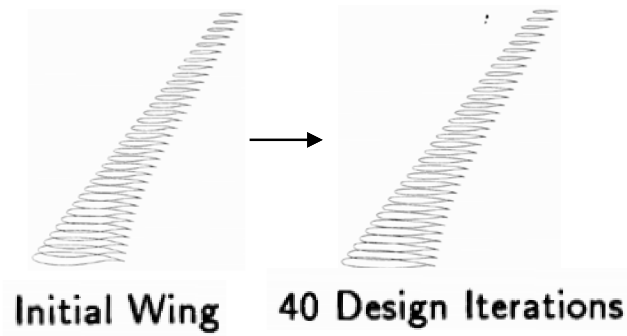
- Disk ≈6x better than coated sphere
- converges to min. thickness, 3nm
- Disk > needle

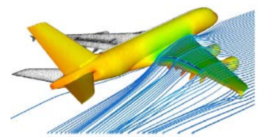
- Tuning shape > adding coatings
- Ellipsoids: 6x improvement

how about other shapes?

“Adjoint”-based optimization: Newly emerging in photonic design

Aerodynamics

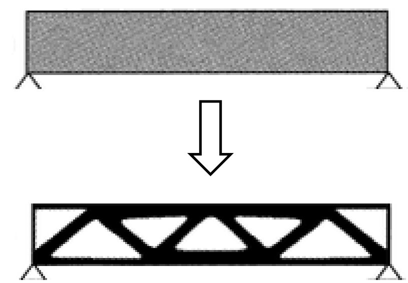




Boeing

A. Jameson JSC 3, 233 (1988)
O. Pironneau JFM 64, 97 (1974)
MB Giles & NA Pierce FTC 65, 393 (2000)

Elasticity



Altair PDG

Bendsoe & Sigmund, “Topology Optimization” (2003)

Deep Learning

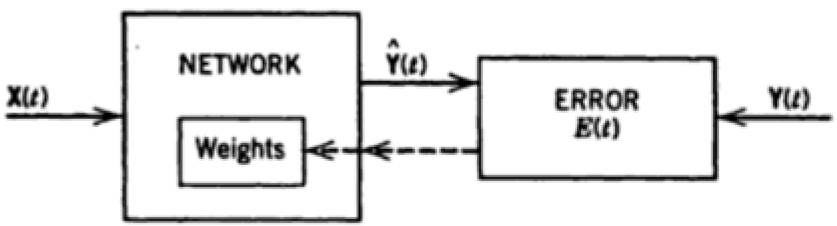
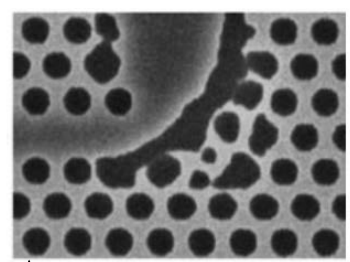
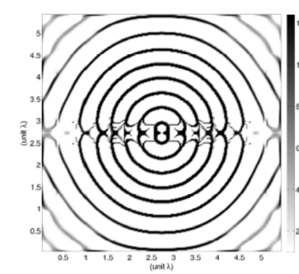


FIGURE 8.3 Basic backpropagation (in pattern learning).

Werbos, “The Roots of Backpropagation” (1994)
Rumelhart et al. Nature 323, 533 (1986)

Photonics

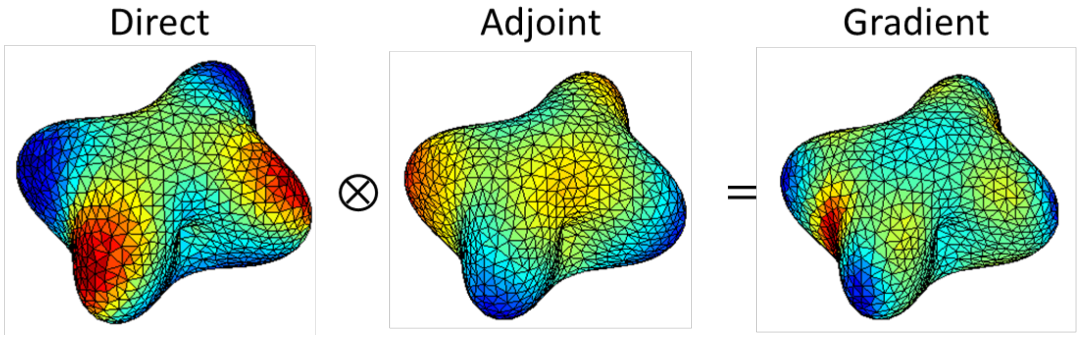




Sigmund et. al. LPR 5, 308 (2011)
X. Liang & SG Johnson OE 21, 30812 (2013)

Fast computation of N derivatives, for any N!

Arbitrary-shape optimization: Ag nanoparticles

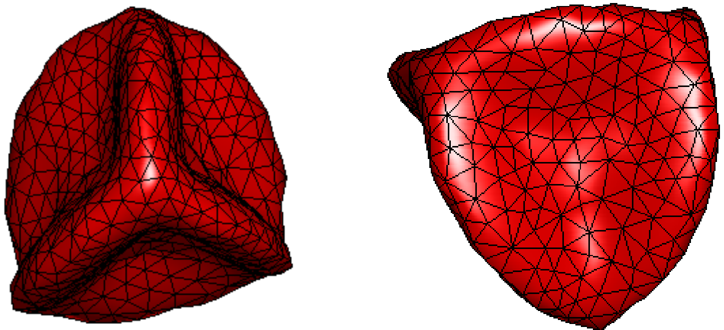


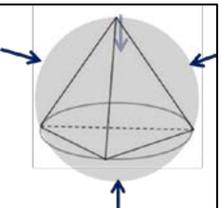
1000 parameters, c_{lm}

$$r(\theta, \phi) = \sum c_{lm} Y_l^m(\theta, \phi)$$

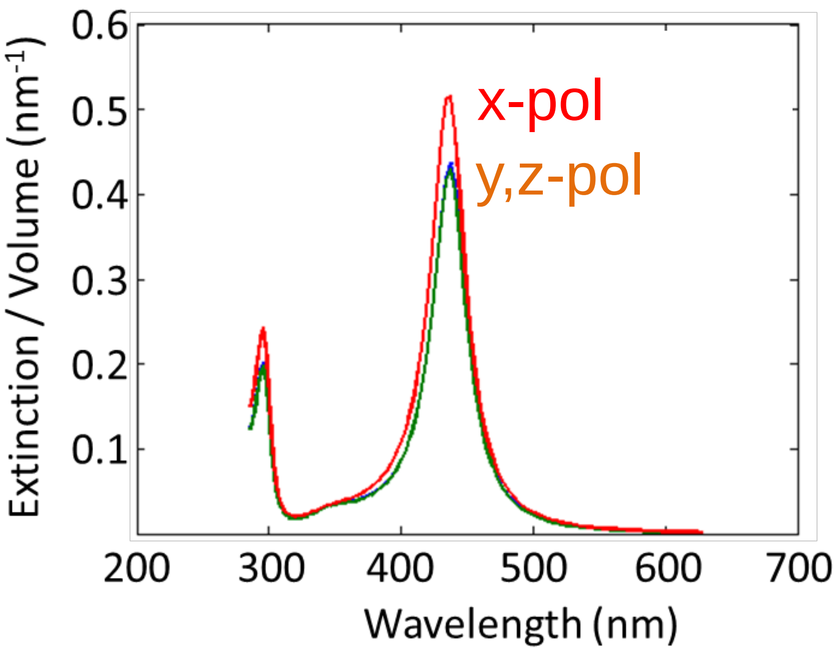
Y_l^m = spherical harmonic

Optimal Structure:
"Deflated Tetrahedron"



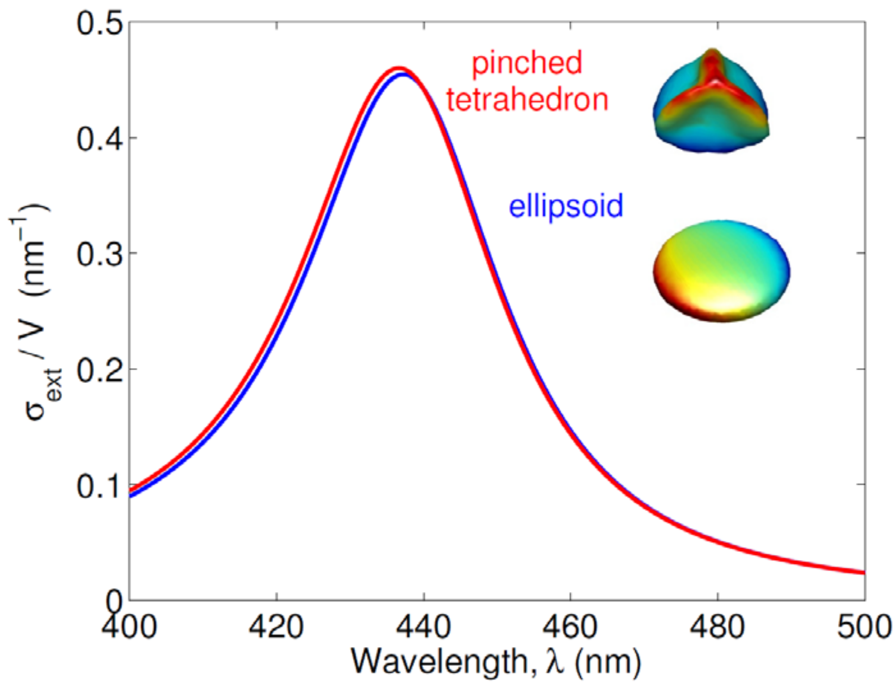


Dimensions ≈10nm



≈ equal for all 3 polarizations!

Angle- and polarization-averaged:



Surprise: Almost exactly the same?!

General shape optimum < 3% better than ellipsoidal optimum

… hitting an upper bound?

Empirical observation:

optimum metal structure always far subwavelength

(≈ quasistatic, absorption-dominated)

Surface-integral equation (SIE) version of Poisson:

$$\underbrace{-\int_S n(x)\cdot G^E(x,x')\sigma(x')dx'}_{\mathcal{K}_S^*[\sigma]} + \left(\frac{1}{2}+\frac{1}{\chi}\right)\sigma(x) = E^{\text{inc}}(x)\cdot n(x)$$

bounded self-adjoint operator
(for right inner product)

[O.D. Kellogg (1929), *Foundations of Potential Theory*]
[Ammari, …, Milton (2012)]

Eigenvalues: $\hat{K}\sigma = (L_i - 0.5)\sigma$, where $L_i \in [0, 1]$

⇒ *angle-averaged response:*

$$\sigma_{\text{ext}} = \frac{2\pi}{3\lambda}\sum_i p_i \,\text{Im}\left(\frac{1}{L_i + 1/\chi(\omega)}\right) \qquad p_i = \frac{1}{3}\sum_\alpha \langle x_\alpha, u_i\rangle\langle v_i, n_\alpha\rangle$$

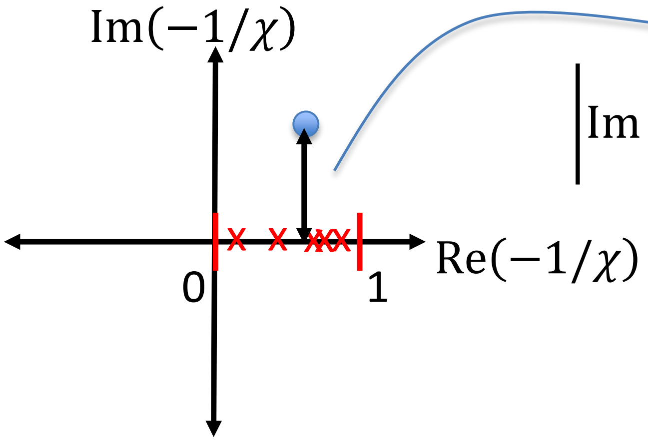
left/right eigenvectors

Resonances (poles) at certain materials (real χ < –1), for fixed ω!

Sum rules for the cross-section

$$\frac{\sigma_{ext}}{V} = \frac{2\pi}{3\lambda}\sum_i p_i \,\mathrm{Im}\left(\frac{1}{L_i + 1/\chi(\omega)}\right)$$

lossy materials:



$$\left|\mathrm{Im}\,\frac{1}{L_i + 1/\chi}\right| \leq \frac{1}{-\mathrm{Im}\,\chi^{-1}} = \frac{|\chi|^2}{\mathrm{Im}\,\chi}$$

sum rule #1:

Fuchs *PRB* 11, 1732 (1975)

$$\sum p_i = \frac{1}{3}\sum_\alpha \sum_n \langle x_\alpha, \sigma_n\rangle\langle \tau_n, s_\alpha\rangle = \int_S \hat{n}\cdot x dx = V$$

sum rule #2:

Fuchs *PRB* 14, 5521 (1976)

$$\frac{\sum \ p_i L_i}{\sum \ p_i} = \sum_\alpha \sum_n L_n\langle x_\alpha, \sigma_n\rangle\langle \tau_n, s_\alpha\rangle = \frac{1}{3}$$

An interesting connection

Single-particle quasistatic surface-integral equations

Composite Bounds (Milton & Bergman)

$$\sigma_{\text{ext}} = \frac{2\pi}{3\lambda}\sum_i p_i \operatorname{Im}\left(\frac{1}{L_i + 1/\chi(\omega)}\right)$$

$$\varepsilon_* = \varepsilon_2 \prod_i \frac{\tau - \tau_i{}'}{\tau - \tau_i}$$

sum rule #1:
Fuchs (1975)

$$\sum p_i = \int_S \hat{n} \cdot x dx = V$$

$$\frac{\partial \varepsilon_*}{\partial \varepsilon_1}(\varepsilon_1, 1) = p_1$$

sum rule #2:
Fuchs (1976)

$$\frac{\sum \; p_i L_i}{\sum \; p_i} = \frac{1}{3}$$

$$\frac{\partial^2 \varepsilon_*}{\partial^2 \; \varepsilon_1}(\varepsilon_1, 1) = -\frac{2}{3} p_1 p_2$$

In the dilute limit, these formulations are equivalent!

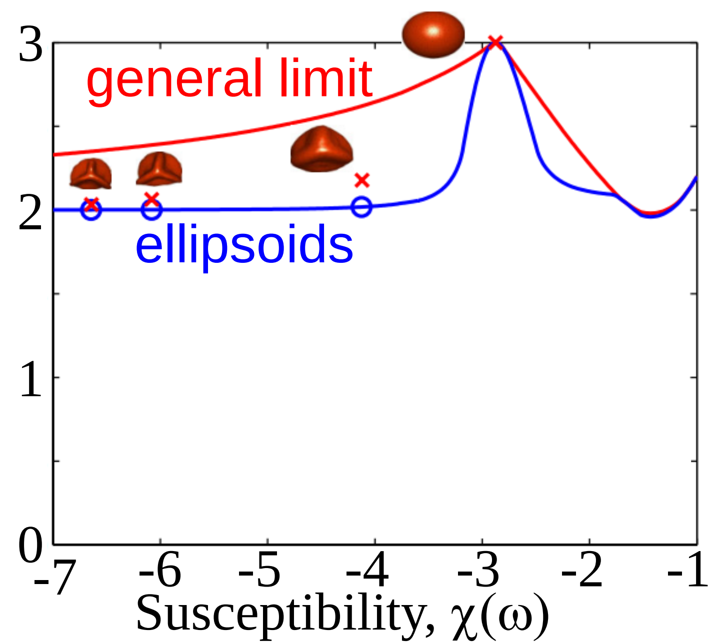
A fundamental limit

$$\frac{\sigma_{\text{ext}}}{V} \leq \frac{2\pi}{3\lambda} \begin{cases} \frac{2\chi_r^3(1+\chi_r)+\chi_i^2(3+2\chi_r+4\chi_r^2)+2\chi_i^4}{\chi_i(\chi_i^2+(1+\chi_r)^2)} & 0 < -\frac{\chi_r}{|\chi|^2} < \frac{1}{3} \\ 3\chi_i - \frac{\chi_r}{\chi_i}|\chi|^2 & \frac{1}{3} < -\frac{\chi_r}{|\chi|^2} < 1 \\ \chi_i\left(2 + \frac{1}{\chi_i^2+(1+\chi_r)^2}\right) & \text{else,} \end{cases}$$

for typical metals
(Im χ << |1+Re χ|):

$$\boxed{\frac{\sigma_{\text{ext}}(\omega)}{V} \leq \frac{2}{3}\frac{\omega}{c}\frac{|\chi(\omega)|^2}{\text{Im}\,\chi(\omega)}}$$

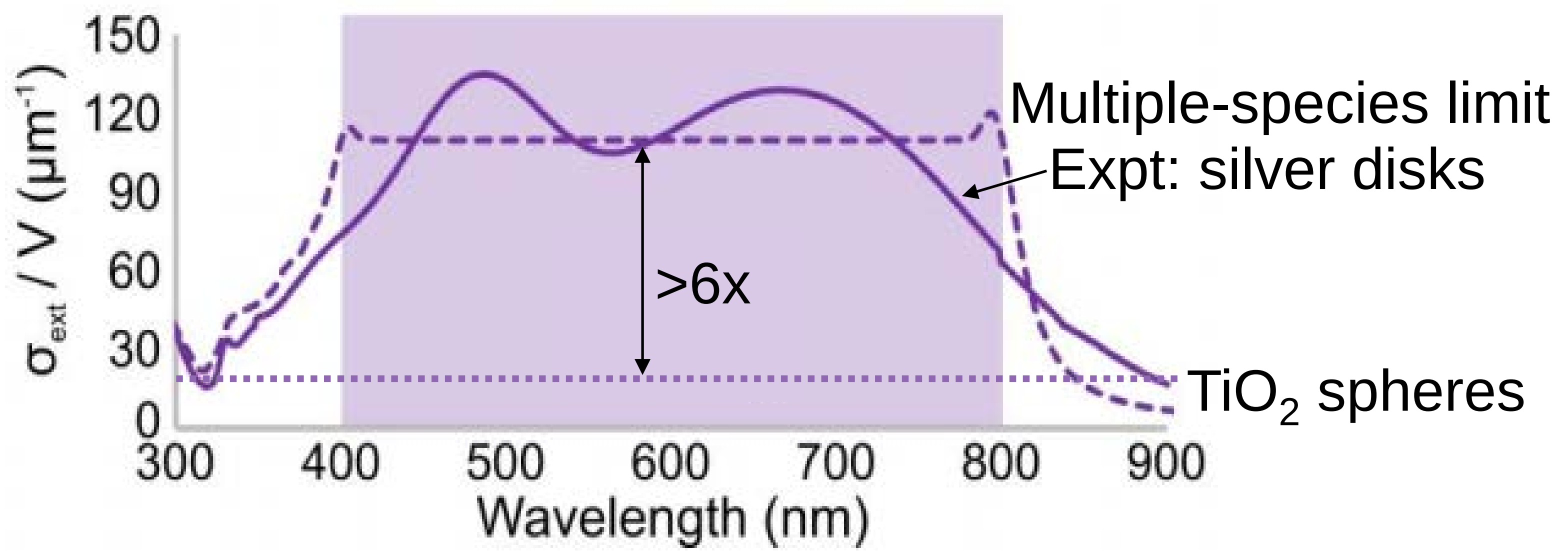
independent of shape!



[Owen Miller et. al. *Phys. Rev. Lett. 112, 123903* (2014)]

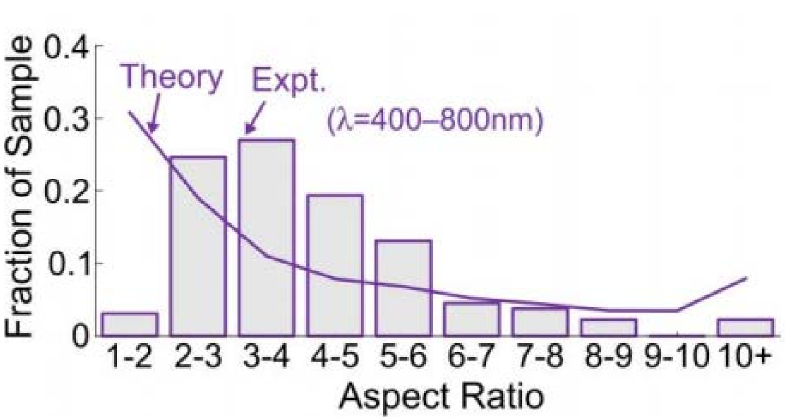
Experimental demonstration: Tailored aspect-ratio silver nanoparticles

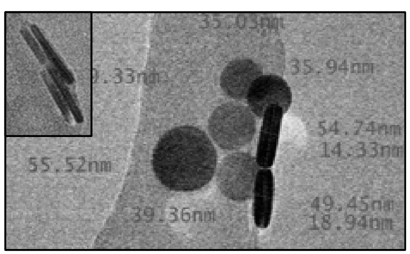
[E. Anquillaire, Owen Miller et. al., Submitted, arXiv: 1510.01768]



Experimental Synthesis:

thermal conversion of colloidal thin-disk particles





Aspect ratios:
1.5:1 – 10:1

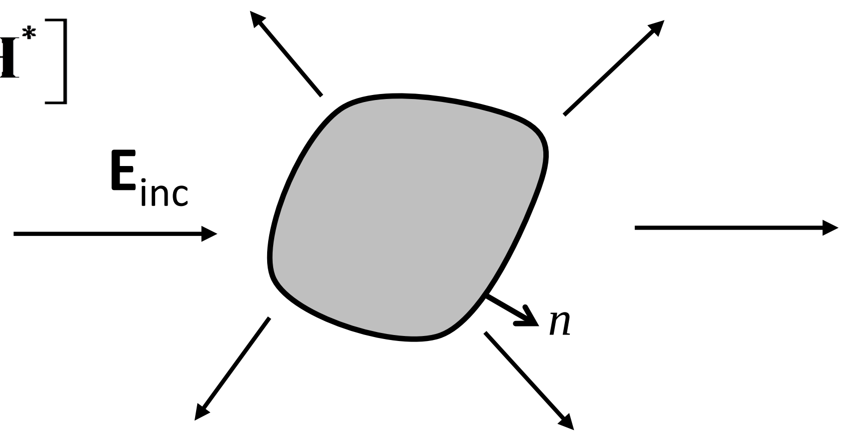
Emma Anquillare
Soljacic group (MIT)

Now, generalize to
full electrodynamics (not quasistatic)

… and other objectives besides σ_{ext}

Key component: Optical theorem

Power/area = Poynting $\frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]$



Absorbed power = incoming flux:

$$= -\frac{1}{2}\,\mathrm{Re}\int_S E \times H^* \cdot \hat{n}$$

Scattered power:

$$= \frac{1}{2}\,\mathrm{Re}\int_S E_{\mathrm{scat}} \times H^*_{\mathrm{scat}} \cdot \hat{n}$$

Extinction = absorption + scattering:

$$= -\frac{1}{2}\,\mathrm{Re}\int_S \hat{n}\cdot(E_{\mathrm{inc}} \times H^*_{\mathrm{scat}} + E_{\mathrm{scat}} \times H^*_{\mathrm{inc}})$$

$$= -\frac{1}{2}\,\mathrm{Re}\int_V \nabla\cdot(E_{\mathrm{inc}} \times H^*_{\mathrm{scat}} + E_{\mathrm{scat}} \times H^*_{\mathrm{inc}})$$

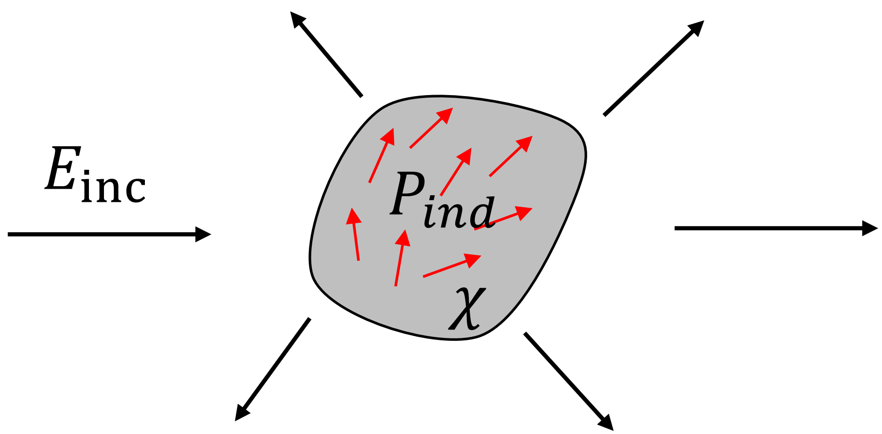
$$\boxed{= \frac{\omega}{2}\,\mathrm{Im}\int_V E^*_{\mathrm{inc}} \cdot \chi E}$$

Lytle et al *PRE* 71, 056610 (2005)
Hashemi et al *PRA* 86, 013804 (2012)

the extinction is proportional to (the imaginary part of) the overlap of the incident field and the induced polarization currents, $\mathbf{P}_{ind} = \chi\mathbf{E}$

Bounding the induced polarization

Optical theorem: the extinguished power is proportional to the (imaginary part of) a linear functional of $\mathbf{P}_{\text{ind}} = \chi\mathbf{E}$



$$P_{\text{ext}} \propto \text{Im} \int_V E^*_{\text{inc}} \cdot P_{\text{ind}}$$

linear in P_{ind}

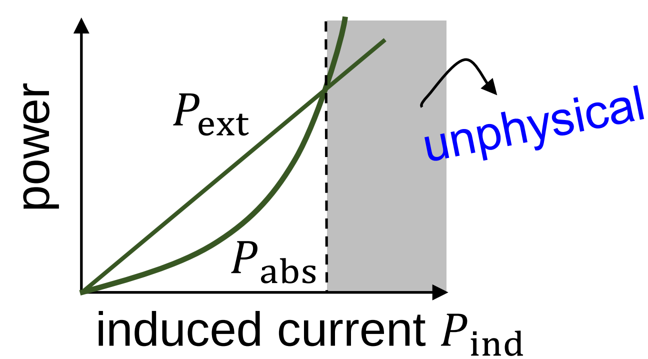
meanwhile…

$$P_{\text{abs}} \propto (\text{Im}\,\chi) \int_V E^* \cdot E$$

$$= \left(\text{Im}\frac{1}{\chi}\right) \int_V P^*_{\text{ind}} \cdot P_{\text{ind}}$$

absorption is *quadratic* in P_{ind}

extinction (abs. + scat.) > absorption



the rest is easy:

Optimize desired objective
subject to absorption ≤ extinction

(typically a convex optimization problem,
can be solved analytically)

General limits to optical response

[Owen Miller et. al, *Opt. Exp.* 24, 3329 (2016)]

By energy conservation, variational calculus ($\partial P_{abs}/\partial P_{ind} = 0$, etc.) and standard optimization theory (optimality conditions)…

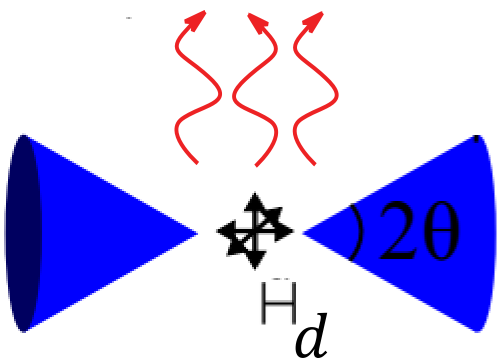
$$\frac{\sigma_{\text{abs}}}{V}, \frac{\sigma_{\text{scat}}}{V} \leq \beta \frac{\omega}{c} \frac{|\chi|^2}{\text{Im}\,\chi} \qquad \beta = \begin{cases} 1 & \text{absorption} \\ 1/4 & \text{scattering} \end{cases}$$

for more general sources and media: (magnetic / anisotropic / chiral / inhomogeneous $\bar{\bar{\chi}}$)

$$P_{abs}, P_{scat} \leq \beta\omega(\text{incident energy inside } V)\left\|\bar{\bar{\chi}}^{\dagger}(\text{Im}\,\bar{\bar{\chi}})^{-1}\bar{\bar{\chi}}\right\|$$

Similar limit to power radiated by dipole at distance *d*, i.e. the local density of states (LDOS)

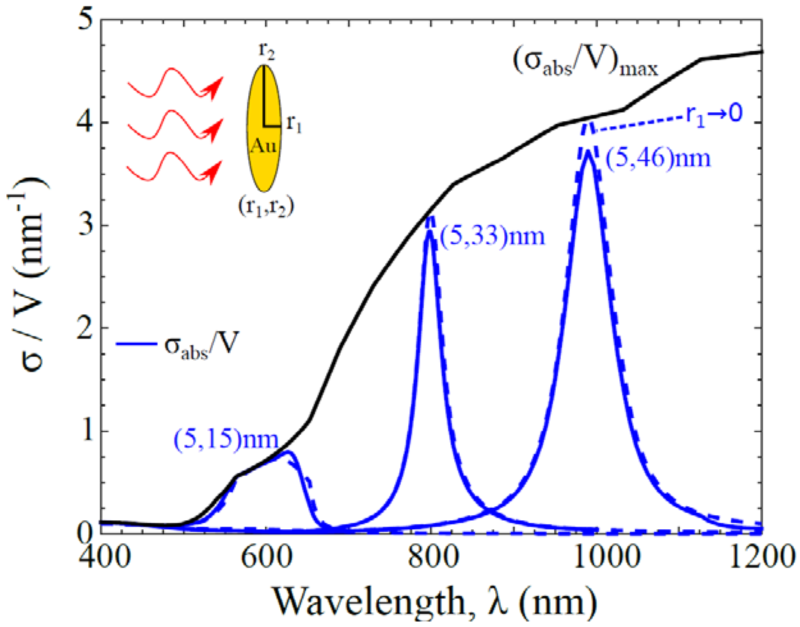
$$\frac{\rho_{rad}}{\rho_0}, \frac{\rho_{nr}}{\rho_0} \leq \frac{\beta}{8(kd)^3} \frac{|\chi|^2}{\text{Im}\,\chi}$$

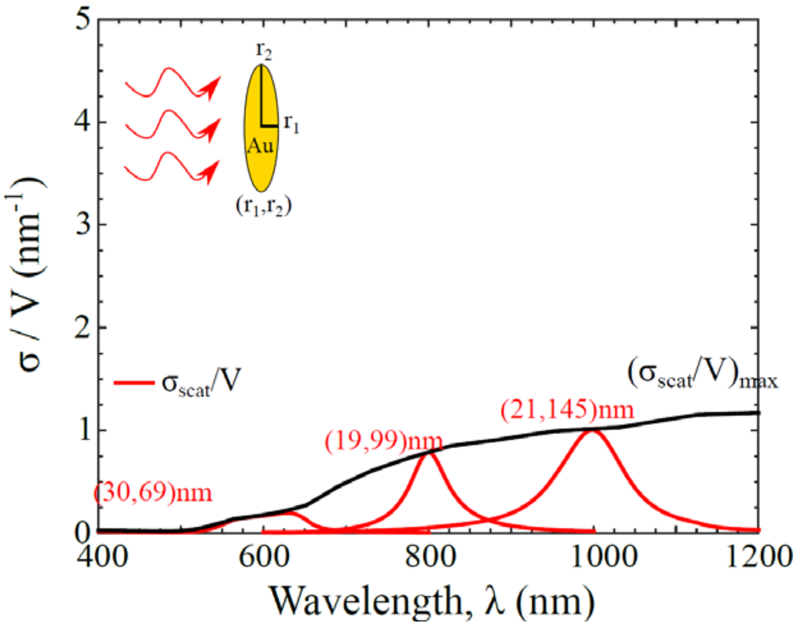


How tight are these bounds?

$$\frac{\sigma_{\text{abs}}}{V} \leq \frac{\omega}{c} \frac{|\chi|^2}{\text{Im}\,\chi}$$

$$\frac{\sigma_{\text{scat}}}{V} \leq \frac{1}{4} \frac{\omega}{c} \frac{|\chi|^2}{\text{Im}\,\chi}$$

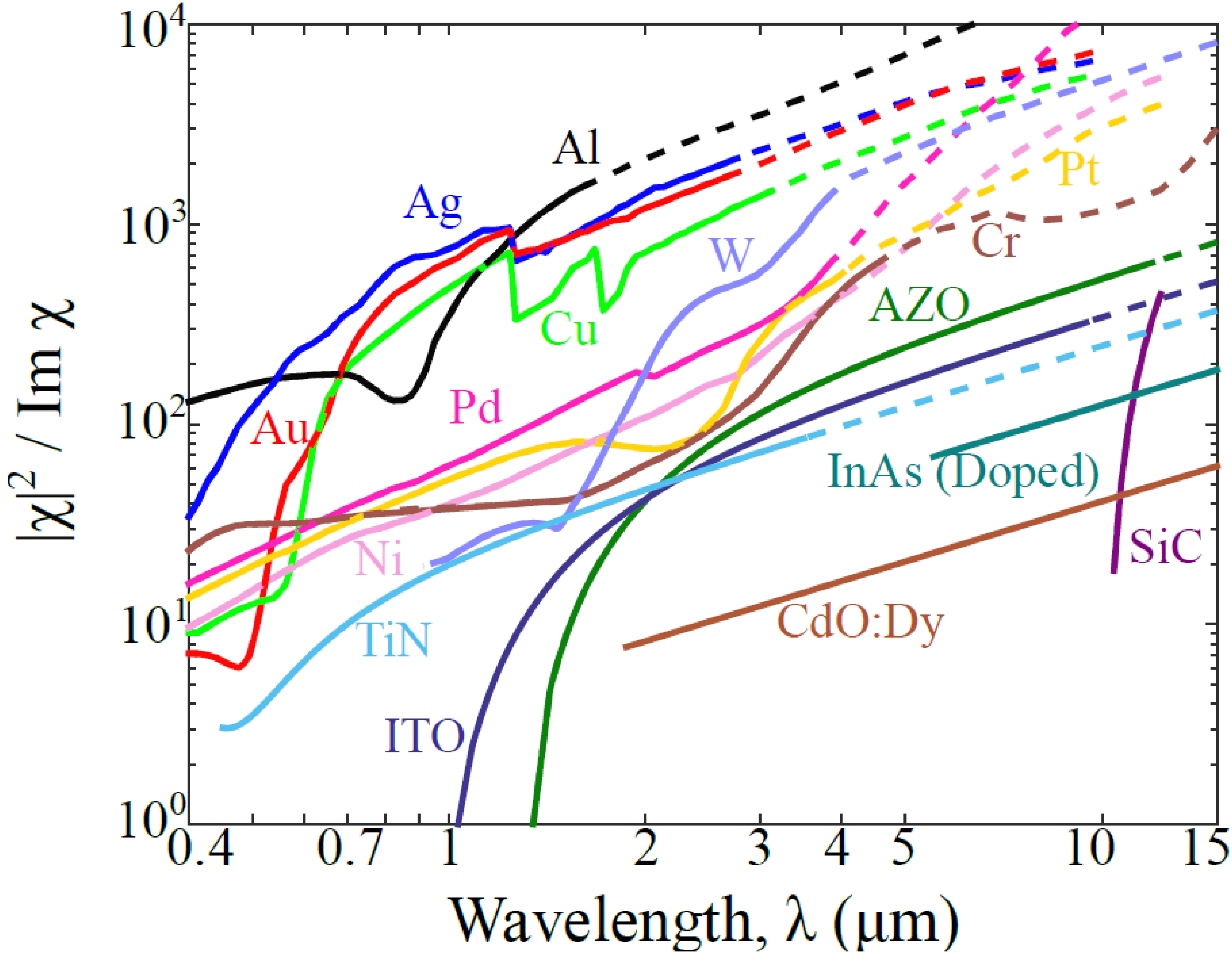




plane-wave excitation:
the limits are "tight" across many frequencies

for absorption and scattering

“Best” materials vs. wavelength



dashed lines: optimal ellipsoids require aspect ratios > 30:1

Resonances in Physics

Inherent to the concept of resonance is the resonant frequency

Mathematical eigen-equations:

electromagnetism $$\frac{1}{\varepsilon(x)} \nabla \times \nabla \times E_n = \omega_n^2 E_n$$

linear elasticity $$(\lambda + \mu)\nabla\nabla \cdot u_n + \mu \nabla^2 u_n = -\rho \omega_n^2 u_n$$

quantum mechanics $$-\frac{\hbar^2}{2m} \nabla^2 \Psi_n + U \Psi_n = \hbar \omega_n \Psi_n$$

⋮

(subject to appropriate boundary conditions)

Differential vs. integral equation formulations

Differential Equation

$$-\nabla \times \nabla \times E + \varepsilon(x)\frac{\omega^2}{c^2}E = i\mu_0\omega J$$

Volume Integral Equation (VIE)

$$E(x) - \int_V G^0(x - x')\chi(x')E(x') = E_{\text{inc}}(x)$$

Eigen-equations:

$$\frac{1}{\varepsilon(x)}\nabla \times \nabla \times E_n = \omega_n^2 E_n$$

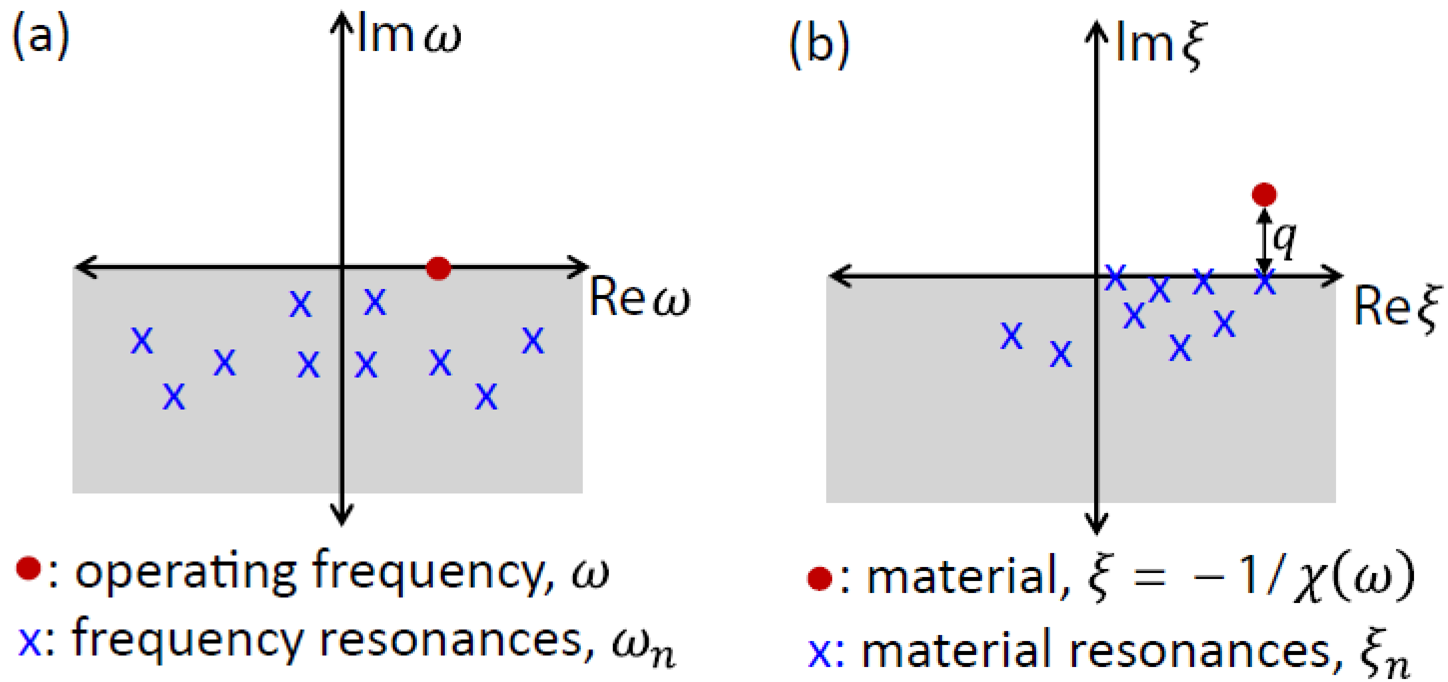
$$\int_V G^0(x - x'; \omega)E_n(x')dx' = -\frac{1}{\chi_n}E_n(x)$$

fixed structure
+ fixed permittivity
→ resonant frequency

fixed structure
+ fixed frequency
→ resonant susceptibility

(assumes piecewise-homogeneous media)

frequency vs. material resonances



Lossy materials (e.g. metals)
→ $\mathrm{Im}\,\chi > 0$, $\mathrm{Im}\,\xi > 0$

$$P_{\text{ext}} = \frac{\omega}{2}\,\mathrm{Im}\sum_i \frac{p_i}{\xi_i - \xi(\omega)}$$

$$\mathrm{Im}\left(\frac{1}{\xi_i - \xi(\omega)}\right) > \frac{1}{\mathrm{Im}\xi(\omega)} = \frac{|\chi(\omega)|^2}{\mathrm{Im}\chi(\omega)}$$

Back to energy-conservation: Generalizations

[O. D. Miller et al, unpublished]

Similar limits can be derived

(1) for other linear wave equations, such as the Lamé–Navier equations of elastic media:

incident shear wave in isotropic medium:
$$\frac{\sigma^S}{V} \le \beta k_0 \frac{|\Delta\mu|^2}{\mu_0 \mathrm{Im}\, \Delta\mu}$$

μ_0 = ambient shear modulus

$\Delta\mu$ = difference of scatterer – μ_0

(2) for local two-dimensional materials (e.g. graphene)

$$\frac{\sigma_{\text{scat,abs}}}{A} \le \beta k_0 \left(\mathrm{Re}\, \sigma_\delta^{-1}\right)^{-1}$$

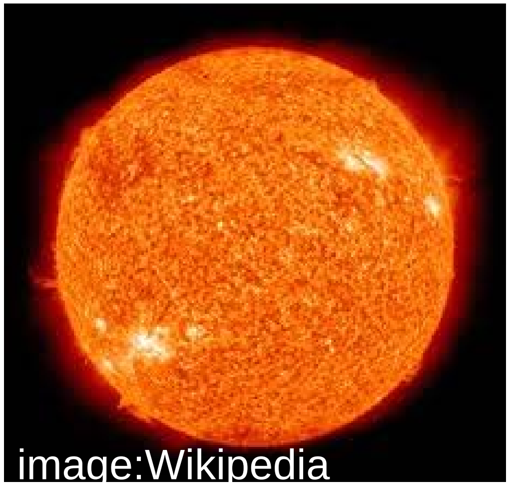
σ_δ = surface conductivity

(3) for nonlocal (e.g. hydrodynamic) constitutive equations

$$\frac{1}{\chi} P = E \longrightarrow \alpha \nabla (\nabla \cdot P) + \beta P = E$$

Radiative Heat Transfer

Far-field



Kirchoff's Law:

absorptivity = emissivity

= 1 "blackbody" definition (ray-optical)

Stefan-Boltzmann

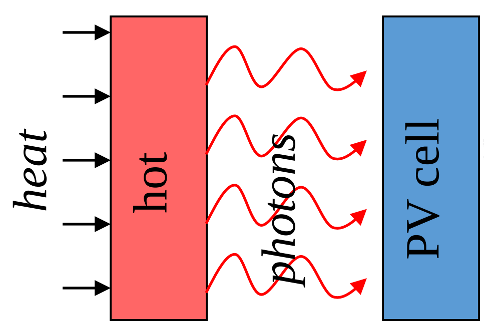
$$H/A \leq \int 1 \cdot \Theta(\omega,T)/\lambda^2 = \sigma T^4$$

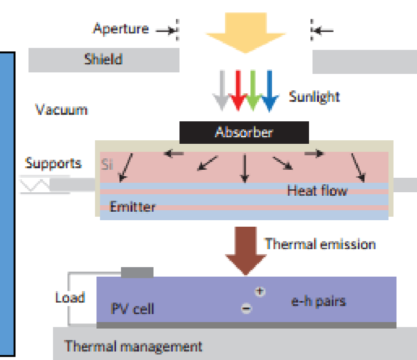
Near field

In the near field, (evanescent) thermal transport can exceed "black-body" limit

Difficulty: comp/expt progress is very recent

e.g. for future thermo-photovoltaic systems?

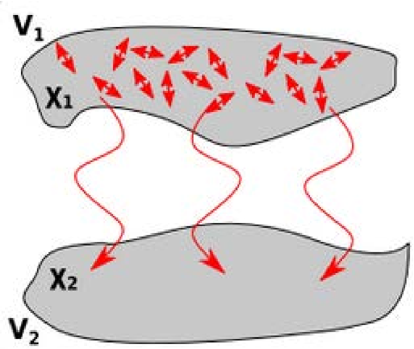




Wang et. al.
Nat Nano 9, 126 (2014)

Near-field radiative heat transfer: Milestones

$T_1 > 0$



$T_2 = 0$

$$\langle J_i(x,\omega) J_j(x',\omega)^* \rangle = \frac{4\varepsilon_0 \omega}{\pi} \Theta(\omega, T_1) \chi''(\omega) \delta(x - x') \delta_{ij}$$

Rytov / Polder / Van Hove
(1950–1980's)

stochastic theory, plate–plate heat transfer, possibility of greater-than-blackbody transfer

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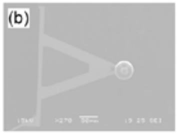
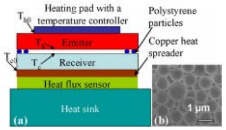
J Pendry (1999): (unrealistic) theoretical bounds to plane–plane transfer

G Chen et. al. (2008):
first expt. measurements > blackbody transfer

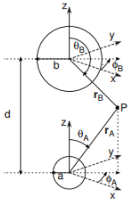


PRB 78, 115303

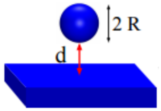
APL 92, 133106



Multiple groups (2008–2011): first rigorous
sphere–sphere and sphere–plate theory

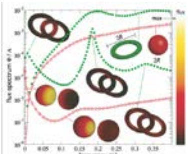


G. Chen et. al.
PRB 77, 75125 (2008)

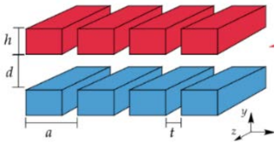


S. Fan et. al.
PRB 84, 245431 (2011)

SG Johnson et. al. (2011+): generic
generic full-Maxwell solvers

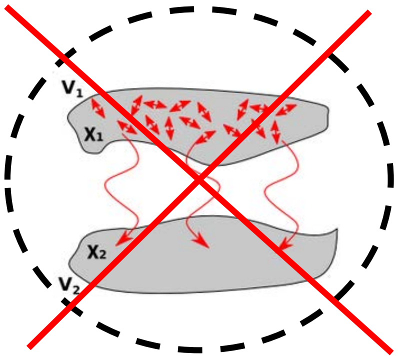


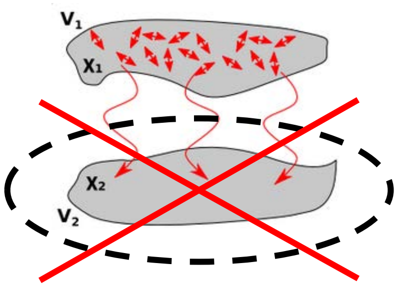
PRB 86, 220302 (2012)



PRL 107, 114302 (2011)

Straightforward extension of limits to near-field heat transfer?





dotted line = bounding surface

Difficulty: sources are embedded within (arbitrary-shape) scattering body

→ no conventional optical theorem

limits: two scattering problems + reciprocity

(1) redefine "incident" and "scattered" fields:

$$E^1_{\mathrm{inc}}(x) = \int_{V_1} G^1(x,x')J(x')$$

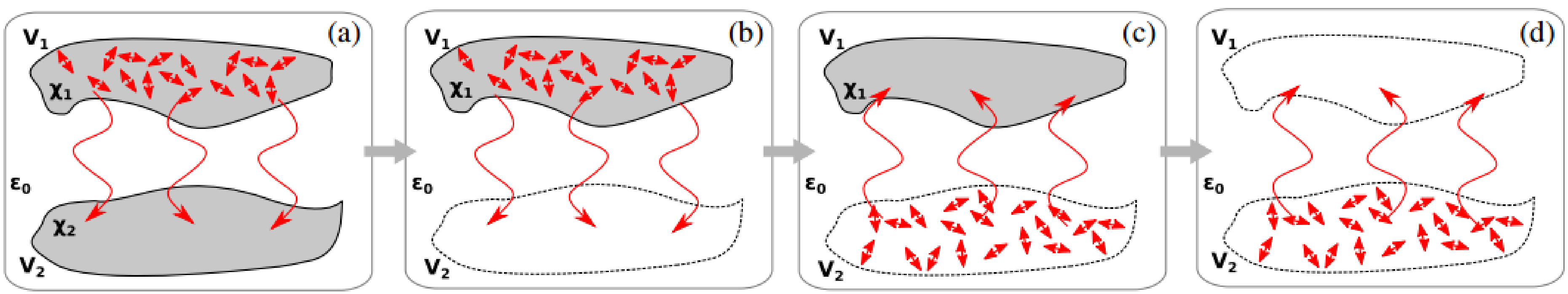
$$E^1_{\mathrm{scat}}(x) = \int_{V_2} G^1(x,x')P_{\mathrm{ind}}(x')$$

G^1 = Green's function *in the presence* of body 1

(2) Bound absorption in body 2, from *unknown* field

(3) Reciprocity: switch source + measurement points

(4) Bound the energy transmitted back to V_1



$P_{\mathrm{abs}} \propto (\mathrm{Im}\chi_2)\int_{V_2}|\mathbf{E}|^2$

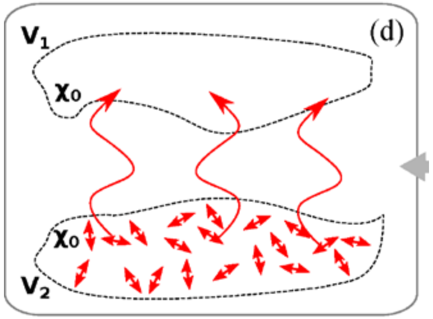
energy conservation $\leq \frac{|\chi_2|^2}{\mathrm{Im}\chi_2}\int_{V_2}|\mathbf{E}_{\mathrm{inc},1}|^2$

reciprocity $\propto \frac{|\chi_2|^2}{\mathrm{Im}\chi_2}(\mathrm{Im}\chi_1)\int_{V_1,V_2}\|\boldsymbol{G}_1\|_F^2$

energy conservation $\leq \frac{|\chi_1|^2}{\mathrm{Im}\chi_1}\frac{|\chi_2|^2}{\mathrm{Im}\chi_2}\int_{V_1,V_2}\|\boldsymbol{G}_0\|_F^2$

Upper limits to near-field heat transfer

[Owen Miller et al, Phys. Rev. Lett. 115, 204302 (2015)]



energy conservation

$\leq \frac{|\chi_1|^2}{\text{Im}\chi_1} \frac{|\chi_2|^2}{\text{Im}\chi_2} \int_{V_1,V_2} \|G_0\|_F^2$

Heat flux at a given frequency ω is bounded by:

$$\Phi(\omega) \leq \frac{2k^4}{\pi} \frac{|\chi_1(\omega)|^2}{\text{Im}\,\chi_1(\omega)} \frac{|\chi_2(\omega)|^2}{\text{Im}\,\chi_2(\omega)} \int_{V_1}\int_{V_2} \left\|G^0(\mathbf{x}_1,\mathbf{x}_2)\right\|_F^2$$

G^0 = vacuum Green's function

$\sim \frac{1}{r^3}$ in the near field

for a minimum separation *d*:

$$\frac{\Phi(\omega)}{A} \leq \frac{1}{16\pi^2 d^2} \frac{|\chi_1(\omega)|^2}{\text{Im}\,\chi_1(\omega)} \frac{|\chi_2(\omega)|^2}{\text{Im}\,\chi_2(\omega)}$$

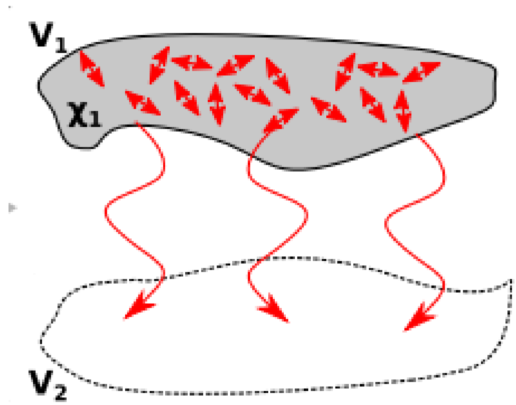
near-field enhancement $\sim 1/d^2$

emission and absorption equally enhanced by $|\chi|^2 / \text{Im}\,\chi$

Design rules in the near field

(1) In the absence of the absorber, V_2, the fields emitted by V_1 into V_2 should be amplified by material enhancement ratio:

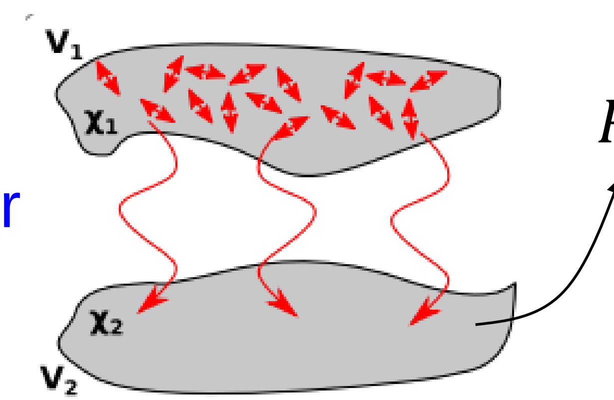
Optimal-emitter condition



$$E_{\text{inc}} \sim \frac{|\chi_1|^2}{\operatorname{Im} \chi_1} E_{\text{dipole}}$$

(2) With both bodies present, the currents induced in the absorber should be further enhanced by the second material enhancement ratio

Optimal-absorber condition



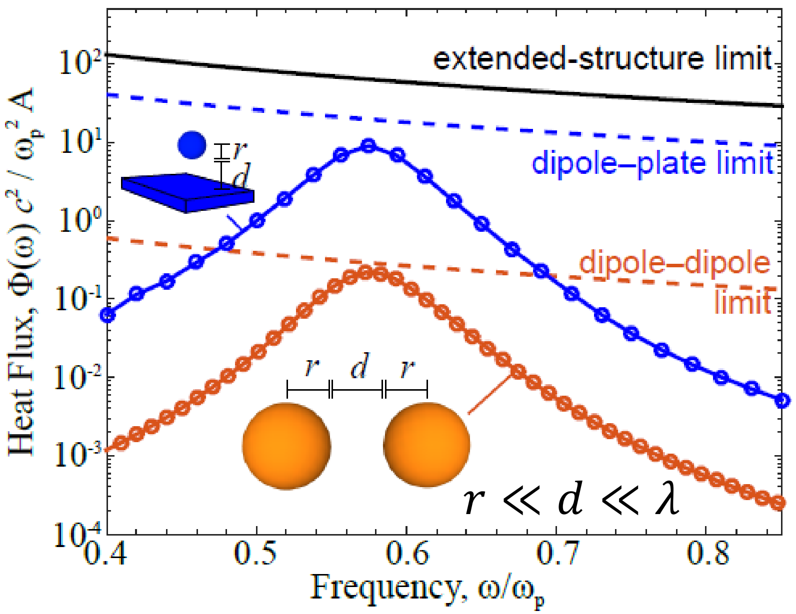
$$P_{\text{ind}} \sim \frac{|\chi_2|^2}{\operatorname{Im} \chi_2} E_{\text{inc}}$$

$$\sim \frac{|\chi_1|^2}{\operatorname{Im} \chi_1} \frac{|\chi_2|^2}{\operatorname{Im} \chi_2} E_{\text{dipole}}$$

Are the limits achievable in known structures?

First consider simple structures and the generic limit:

$$\Phi(\omega) \le \frac{2k^4}{\pi} \frac{|\chi_1(\omega)|^2}{\operatorname{Im}\chi_1(\omega)} \frac{|\chi_2(\omega)|^2}{\operatorname{Im}\chi_2(\omega)} \int_{V_1}\int_{V_2} \left\| \boldsymbol{G}^0(\mathbf{x}_1,\mathbf{x}_2)\right\|_F^2 .$$



Drude metal,
plasma frequency ω_p
and dissipation $\gamma = 0.1\omega_p$

$$\Phi(\omega)]_{\text{dipole–dipole}} \le \frac{3}{4\pi^3} \frac{|\chi_1(\omega)|^2}{\operatorname{Im}\chi_1(\omega)} \frac{|\chi_2(\omega)|^2}{\operatorname{Im}\chi_2(\omega)} \frac{V_1 V_2}{(r_1+r_2+d)^6}$$

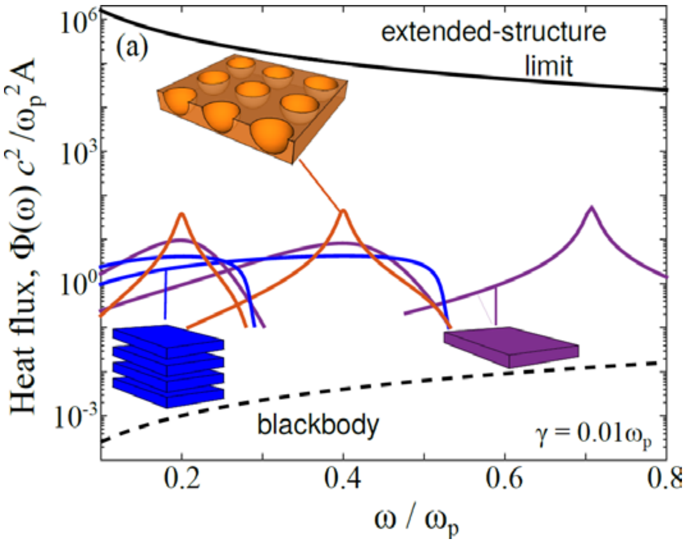
sphere-sphere reaches the limit

$$[\Phi(\omega)]_{\text{dipole-to-ext}} \le \frac{1}{8\pi^2} \frac{|\chi_1(\omega)|^2}{\operatorname{Im}\chi_1(\omega)} \frac{|\chi_2(\omega)|^2}{\operatorname{Im}\chi_2(\omega)} \frac{V}{(r+d)^3}$$

sphere-plate off by 2x (pol. mismatch), correct scaling

Are the limits achievable in known structures?

What about extended (planar) structures?

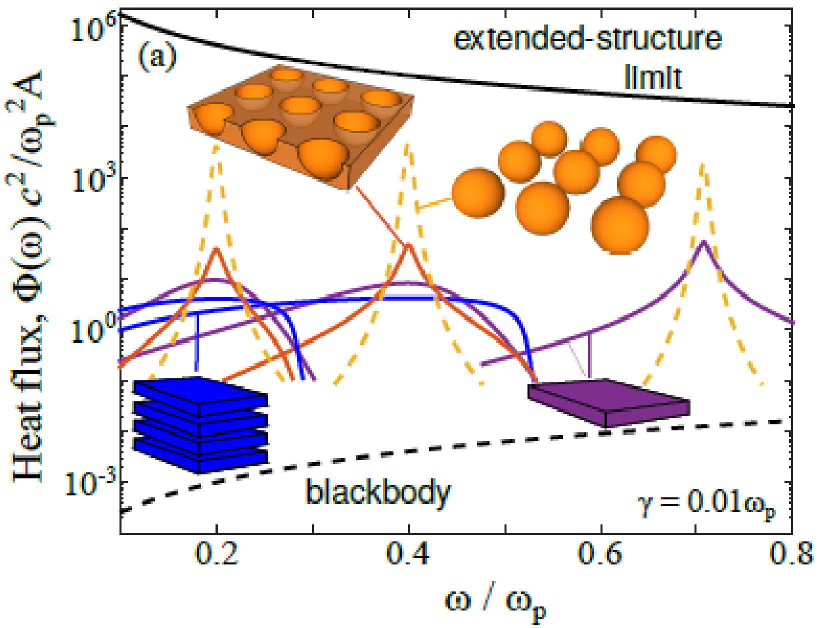


$$\left[\frac{\Phi(\omega)}{A}\right]_{\text{plate-plate}} \sim \frac{1}{d^2} \ln\left[\frac{|\chi|^4}{(\operatorname{Im}\chi)^2}\right]$$

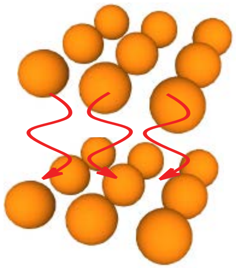
$$\left[\frac{\Phi(\omega_{\text{res}})}{A}\right]_{\text{HMM-to-HMM}} = \frac{\ln 2}{4\pi^2 d^2}$$

Are the limits achievable in known structures?

What about extended (planar) structures?



Promising avenue: periodic nanostructure interactions

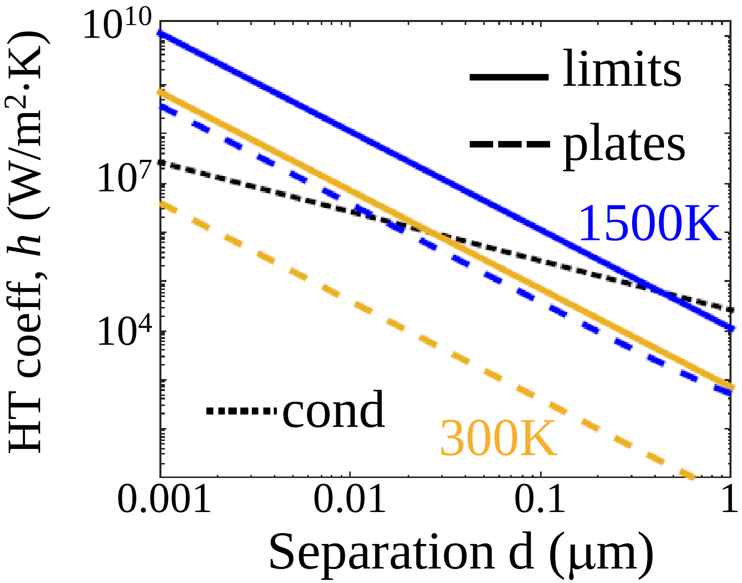


arrays of dipolar spheres interacting additively (overly idealized)

to simultaneously achieve $|\chi|^2/\mathrm{Im}\,\chi$ (via particles) and $1/d^2$ (via array) enhancements

Reaching the limits: new possibilities in heat transfer

Given optimal flux (and smallest bandwidth, $\Delta\omega/\omega_{res}$, for a metal):



radiative > conductive transport (in air)

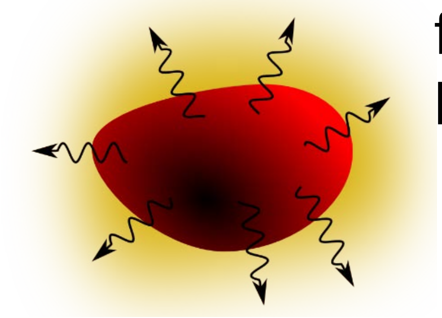
possible at:

T=300K, d=30nm

or

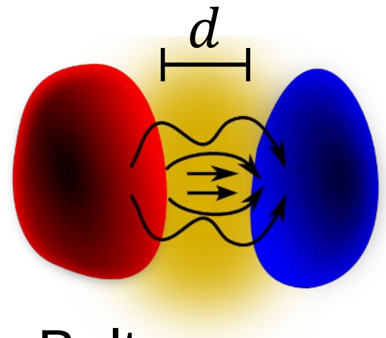
T=1500K, d=0.5μm

air, $\kappa_{cond} = 0.026 \frac{\mathrm{W}}{\mathrm{m \cdot K}}$



far-field heat transfer

$\leq \sigma T^4 A$



near-field heat transfer

$\leq \sigma T^4 A \left(\frac{\lambda_T}{d}\right)^2 \left(\frac{|\chi|^3}{\mathrm{Im}\,\chi}\right)$

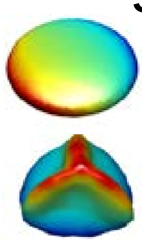
σ = Stefan–Boltzmann constant

Progress, and New Questions

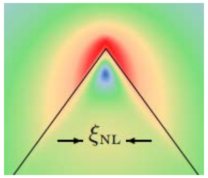
New upper bounds to optical responses
proportional to $|\chi|^2 / \mathrm{Im}\, \chi$

Linear elasticity? QM?

Absorbing & Scattering Nanoparticles

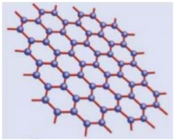


Comp. optimized structures
to reach highest-possible
absorption/scattering rates



Nonlocal interactions?

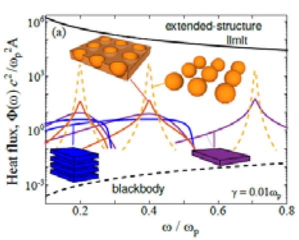
[Mortensen, *PNFA* 11, 303 (2013)]



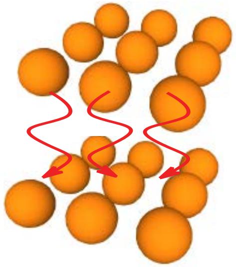
Single-layer absorbers?

[Geim et al. *RMP* 81, 109 (2009)]

Radiative Heat Transfer



New limits to
near-field transport



What are the optimal structures?