Abstracts

Matthew Blair : *L^p bounds on eigenfunctions of the Laplacian on polygonal domains.*
Abstract : Given a compact Riemannian manifold (M, g), we consider the problem of establishing upper bounds on the L^p norm of eigenfunctions of the Laplacian $\Delta_g \phi_\lambda = \lambda^2 \phi_\lambda$ in the high frequency limit. When the boundary of the manifold is nonempty, homogeneous Dirichlet or Neumann boundary conditions are imposed. For boundaryless manifolds, a classical result of Sogge establishes bounds of the form

$$\|\phi_\lambda\|_{L^p(M)} \leq C\lambda^{\delta(p,d)} \|\phi_\lambda\|_{L^2(M)}, \qquad 2 < p \leq \infty,$$

$$\delta(p, d) = \max\left(\frac{d-1}{2} - \frac{d}{p}, \frac{d-1}{2}\left(\frac{1}{2} - \frac{1}{p}\right)\right), \qquad d = \dim(M),$$

for some C independent of λ. The bounds apply more generally to "spectral clusters", which are thick quasimodes spectrally concentrated in a window $[\lambda, \lambda + 1]$ and for such modes, the exponent $\delta(p, d)$ is sharp.

Much less is known when the boundary is nonempty as the geometry of generalized bicharacteristic rays for the wave equation complicates the matter of recovering this bound considerably. We survey recent progress on this problem, with emphasis on a joint work with Ford and Marzuola which proves Sogge's bounds for 2 dimensional polygonal domains and flat surfaces with conic singularities.

Virginie Bonnaillie-Noël : *Magnetic Laplacian in singular domains.*
Abstract : In this talk, we are interested in the bottom of the spectrum of Magnetic Laplacian on general three-dimensional corner domains. The main term of the asymptotic can be expressed according to some tangent operators which are Schrödinger operators with constant magnetic field in three-dimensional cones. Then we focus on sharp three-dimensional cones. In the case of circular cones, we prove a complete asymptotic expansion of the bottom of the spectrum as the angle tends to zero and highlight the influence of the magnetic field. For general cone, we prove an upper bound for the ground state energy of the magnetic Laplacian with constant magnetic field on cones that are contained in a half-space. This bound involves a weighted norm of the magnetic field related to moments on a plane section of the cone. When the cone is sharp, i.e. when its section is small, this upper bound tends to 0. A lower bound on the essential spectrum is proved for families of sharp cones, implying that if the section is small enough the ground state energy is an eigenvalue. This circumstance produces corner concentration in the semi-classical limit for the magnetic Schrödinger operator when such sharp cones are involved.
This is a joint work with M. Dauge, N. Popoff and N. Raymond.

Nicolas Burq : *Decay for the damped wave equation in unbounded domains.*
Abstract : We study the decay of the damped wave equation in an unbounded domain. We first prove under the natural *geometric control condition* the exponential decay of the semigroup. Then we prove under a weaker but also very natural condition the logarithmic decay of the solutions (assuming that the initial data are smoother). This is a joint work with R. Joly.

Simon Chandler-Wilde : *Hybrid Asymptotic-Numerical Integral Equation Methods for High Frequency Scattering : Part 1 : High Frequency Solution Behaviour and Algorithm Design.*
Abstract : There has been much interest in recent years in designing and analysing so-called "hybrid asymptotic-numerical methods" for solving high-frequency scattering problems. These methods use knowledge of the high-frequency behaviour of the solution to design basis functions suitable for representing the oscillatory solutions (with this idea most effective in the context of integral-equation methods).

This first talk will discuss how knowledge of high-frequency asymptotics allows one to design appropriate basis functions for the numerical method. In particular, we highlight how the requirements coming from the numerical analysis demand stronger results about the high-frequency asymptotics of the solution than are usually established classically.

Jeffrey Galkowski : *A Quantum Sabine Law for Resonances in Transmission Problems.*
Abstract : We prove a quantum Sabine law for the location of resonances in transmission problems. In this talk, our main applications are to scattering by strictly convex, smooth, transparent obstacles and highly frequency dependent delta potentials. In each case, we give a sharp characterization of the resonance free regions in terms of dynamical quantities. In particular, we relate the imaginary part of resonances to the chord lengths and reflectivity coefficients for the ray dynamics and hence give a quantum version of the Sabine law from acoustics.

Jesse Gell-Redman : *The Feynman Propagator on Asymptotically Minkowski Spaces.*
Abstract : We show that the Feynman propagator for the wave equation on perturbations of Minkowski space is well-defined as an isomorphism between certain weighted b-Sobolev spaces, the target space (i.e. the domain of the d'Alembertian) having additional apriori regularity. Joint with Vasy.

Colin Guillarmou : *Correspondence between Ruelle resonances and quantum resonances for non-compact Riemann surfaces.*

Peter Hintz : *Regularity of waves at the Cauchy horizon of black hole spacetimes.*
Abstract : The Cauchy horizon of a charged or rotating black hole marks the boundary of the region of the spacetime in which solutions to the Cauchy problem for the wave equation are unique. Using microlocal techniques, we analyze regularity and decay of scalar waves along the Cauchy horizon. This complements recent work of Luk and Oh on the blow-up of local energy, and is related to Penrose's Strong Cosmic Censorship conjecture. Joint work with András Vasy.

Oana Ivanovici : *Dispersion estimates for the wave and the Schrodinger equations outside strictly convex obstacle.*
Abstract : We consider a (general) strictly convex domain in R^d of dimension $d > 1$ and we describe dispersion for both wave and Schrödinger equations with Dirichlet boundary conditions. If $d = 2$ or $d = 3$ we show that dispersion does hold like in the flat case, while for $d > 3$, we show that there exist strictly convex obstacles for which a loss occur with respect to the boundary less case (such an optimal loss is obtained by explicit computations). This is a joint work with Gilles Lebeau.

Chris Judge : *Triangles in the hyperbolic plane with no positive Neumann eigenvalues.*
Abstract : In joint work with Luc Hillairet, we show that the Laplacian associated with the generic finite area triangle in hyperbolic plane with one vertex of angle zero has no positive Neumann eigenvalues. This is the first evidence for the Phillips-Sarnak philosophy that does not depend on a multiplicity hypothesis. The proof is based an a method that we call asymptotic separation of variables.

Gilles Lebeau : *On the holomorphic extension of the Poisson Kernel.*
Abstract : Let Ω be an open subset of $\mathbb{R}^d$ with analytic boundary. The Poisson kernel K(x,y), with $x \in \Omega, y \in \partial\Omega$, is the solution of the following elliptic boundary value problem, where $\triangle$ denotes the usual Laplace operator

$$\triangle_x K(x,y) = 0 \quad \text{in} \quad \Omega, \qquad K(x,y)|_{\partial\Omega} = \delta_{x=y}.$$

In this lecture, we are interested in the holomorphic extension in $x \in \mathbb{C}^d$ of $K(x,y)$ near a given point $y \in \partial\Omega$. Very few is known about this problem.

We will first recall old and classical results on the regularity of the Dirichlet problem. Then we will state "conjectures" on the location of the singularities of the holomorphic extension of K and will describe some particular cases where it holds true. Finally, we will explain how this problem is related to propagation of singularities and complex billiard dynamics.

Anna Mazzucato : *Mixed-boundary-value and transmission problems on generalized polyhedral domains.*
Abstract : I will discuss well-posedness of mixed-boundary-value and transmission problems for elliptic equations on curvilinear polyhedral domains, using weighted Sobolev spaces. The domains may have interior faces.The weight is the distance to the singular set. The motivation is the analysis of the Finite Element and Geleralized Finite Element Method for these problems.
This is joint work with Victor Nistor.

Richard Melrose : *The wave equation on Weil-Petersson spaces.*
Abstract : In this somewhat speculative talk I will briefly describe recent results with Xuwen Zhu on the boundary behaviour of the Weil-Petersson metric (on the moduli space of Riemann surfaces) and ongoing work with Jesse Gell-Redman on the associated Laplacian. I will then describe what I think happens for the wave equation in this context and what needs to be done to prove it.

Nicolas Popoff : *Ground state energy of the Robin Laplacian in corner domains.*
Abstract : I will consider the problem of the asymptotics of the first eigenvalue for the Laplacian with Robin boundary condition, when the Dirichlet parameter gets large. I will focus on the case where the domain belongs to a general class of corner domains. I will introduce singular chains associate with tangent geometries of a corner domain, and show that the asymptotics is given at first order by the minimization of a function, called "local energy", defined on singular chains. Using a multiscale analysis, we give an estimate of the remainder. I will also provide a more precise asymptotics when the domain is regular, using an effective Hamiltonian defined on the boundary and involving the mean curvature.

Julie Rowlett : *A Polyakov formula for angular variations.*
Abstract : Polyakov's formula expresses a difference of zeta-regularized determinants of Laplace operators, an anomaly of global quantities, in terms of simple local quantities. Such a formula is well known in the case of closed surfaces (Osgood, Philips, & Sarnak 1988) and surfaces with smooth boundary (Alvarez 1983). Due to the abstract nature of the definition of the zeta-regularized determinant of the Laplacian, it is typically impossible to compute an explicit formula. Nonetheless, Kokotov (genus one Kokotov & Klochko 2007, arbitrary genus Kokotov 2013) demonstrated such a formula for polyhedral surfaces ! I will discuss joint work with Clara Aldana concerning the zeta regularized determinant of the Laplacian on Euclidean domains with corners. We determine a Polyakov formula which expresses the dependence of the determinant on the opening angle at a corner. Our ultimate goal is to determine an explicit formula, in the spirit of Kokotov's results, for the determinant on polygonal domains.

Elmar Schrohe : *The porous medium equation on manifolds with conical singularities.*
Abstract : We study the porous medium equation

$$\begin{aligned} u'(t) - \Delta(u^m(t)) &= f(u,t), \quad t \in (0, T_0], \\ u(0) &= u_0 \end{aligned}$$

on a manifold with conical singularities. We assume that $m > 0$ and $f = f(\lambda, t)$ is a holomorphic function of λ on a neighborhood of $\mathrm{Ran}(u_0)$ with values in Lipschitz functions in t on $[0, T_0]$.

We model the manifold with conical singularities by a compact manifold $\mathbb{B}$ with boundary of dimension ≥ 2, endowed with a degenerate Riemannian metric g, which, in a collar neighborhood $[0,1) \times \partial\mathbb{B}$ of the boundary $\partial\mathbb{B}$, is of the form $g(x,y) = dx^2 + x^2 h(y)$ for a (non-degenerate) Riemannian metric h on $\partial\mathbb{B}$. The Laplacian Δ associated with this metric naturally acts on scales of weighted Mellin-Sobolev spaces.

Given a strictly positive initial value u_0 we show existence, uniqueness and maximal L^p-regularity of a short time solution. In particular, we obtain information on the short time asymptotics of the solution near the conical point. Our method is based on bounded imaginary powers results and R-sectoriality perturbation techniques.

Actually, these solutions turn out to be instantaneously smooth for positive time outside the conic singularity. Moreover, we obtain long time existence of maximal regularity solutions when the initial data are positive and the forcing term f is zero and show how to establish the existence of weak solutions for the case of non-negative initial data and zero forcing term.

(Joint work with Nikolaos Roidos)

Euan Spence :*Hybrid Asymptotic-Numerical Integral Equation Methods for High Frequency Scattering : Part 2 : Norms, Coercivity, Conditioning of Operators.*
Abstract : There has been much interest in recent years in designing and analysing so-called "hybrid asymptotic-numerical methods" for solving high-frequency scattering problems. These methods use knowledge of the high-frequency behaviour of the solution to design basis functions suitable for representing the oscillatory solutions (with this idea most effective in the context of integral-equation methods).

This second talk will discuss how, once new basis functions are designed, new results about boundary-integral operators are needed in order to provide a rigorous error analysis of the hybrid method.

András Vasy : *Asymptotics of radiation fields on long-range asymptotically Minkowski spaces.*
Abstract : I will describe recent work with Dean Baskin and Jared Wunsch on the asymptotics of solutions of the wave equation on long-range asymptotically Minkowski spaces. Asymptotically Minkowski metrics generalize Minkowski space in the same sense that 'scattering metrics' introduced by Melrose generalize Euclidean space, though more stringent assumptions are required in some respects due to the non-elliptic nature of the operator. This project extends (and simplifies !) the earlier joint work in the short-range setting ; the important change is that the long-range assumptions allow Schwarzschild-like behavior at null-infinity.

Boris Vertman : *Ricci Flow on singular edge manifolds.*
Abstract : We discuss recent existence results for a Ricci flow starting at an incomplete manifold with edge singularities and bounded Ricci curvature, flowing for a short time within a class of incomplete edge manifolds. We explain regularity properties for the corresponding family of Riemannian metrics and point out boundedness of the Riemannian curvature tensor along the flow. For Riemannian metrics that are sufficiently close to a Ricci flat incomplete edge metric, such a Ricci flow exists for all times. Our results in particular include the case of Kaehler metrics with isolated conical singularities. The proof works by a careful analysis of the Lichnerowicz Laplacian and the Ricci de Turck flow equation.

Georgi Vodev : *Asymptotic behavior of the interior transmission eigenvalues.*
Abstract : The interior transmission eigenvalues are the eigenvalues of a linear non-symmetric operator whose resolvent may not be compact. I will discuss the question of localization of the transmission eigenvalues on the complex plane as well as the asymptotic behavior of their counting function. We will see that both questions are connected each other. One of the results states that the counting function has a Weyl asymptotics and closer the transmission eigenvalues are to the real axis, better the remainder term is. Some of the results are obtained in collaboration with Vesselin Petkov.