Inhomogeneous quantum turbulence in a channel

Makoto Tsubota Osaka City University, Japan

1. Introduction
2. Quantum turbulence in atomic BECs
3. Quantum turbulence in superfluid helium

   3-1. Previous simulation for the homogeneous normal fluid flow (1980's-2010)

   3-2. Recent visualization experiments (2006-)

   3-3. The new simulation for the inhomogeneous normal fluid flow (2013-)

General review articles on quantum turbulence

・ M. Tsubota, K. Kasamatsu and M. Kobayashi, in ***Novel Superfluids***, ed. K. H. Bennemann and J. B. Ketterson (Oxford University Press, 2013), Vol.1, p.156; arXiv:1004.5458

・ M. Tsubota, M. Kobayashi and H. Takeuchi, **Physics Reports** 522, 191 (2013); arXiv:1208.0422

・ C. F. Barenghi, L. Skrbek, K. R. Sreenivasan, **PNAS** 111 (Suppl. 1), 4647 (2014); arXiv: 1404.1909

1. Introduction

What is quantum turbulence?

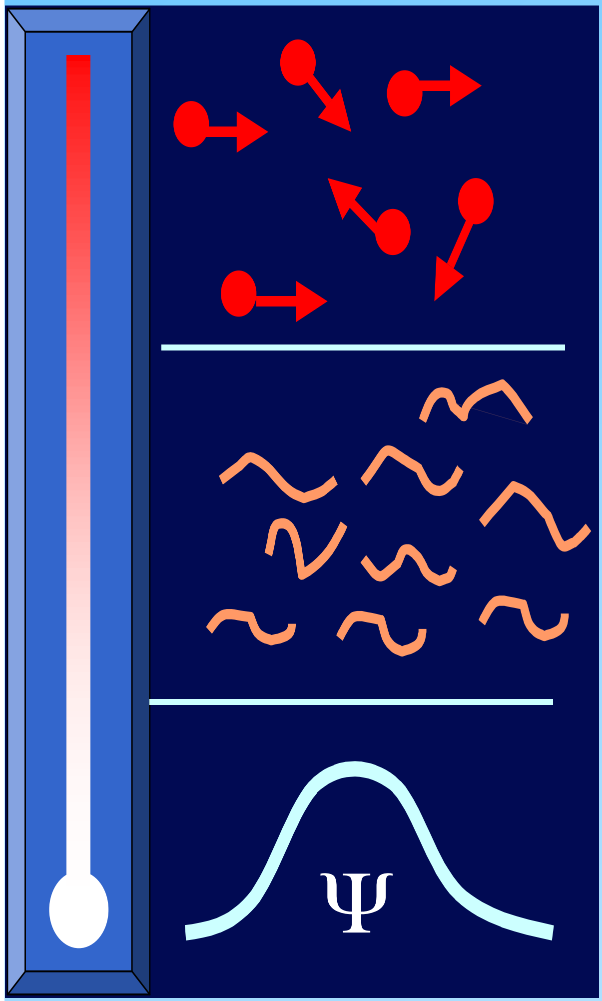
Quantum turbulence(QT) means

Turbulence in quantum condensed fluids .

The main stages of QT are

- Superfluid helium (since 1950’s)
- Atomic Bose-Einstein condensates(BECs) (since 1995)

Bose-Einstein condensation (BEC) and the macroscopic wave function



Physics of scalar BEC at 0K is described by the macroscopic wave function (order parameter).

$$\Psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\exp\left(i\theta(\mathbf{r},t)\right)$$

$n(\mathbf{r},t)$: Density of the Bose condensate

$\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla\theta(\mathbf{r},t)$: Superfluid velocity

In a weakly interacting BEC, $\Psi(\mathbf{r},t)$ obeys the Gross-Pitaevskii (GP) equation.

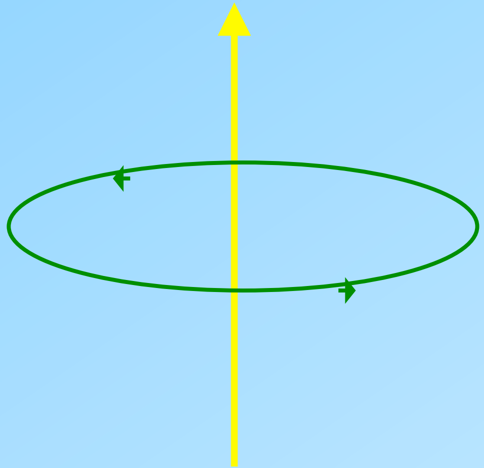
$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2\right]\Psi(\mathbf{r},t)$$

A quantized vortex is a vortex of superflow in a BEC. Any rotational motion in superfluid is sustained by quantized vortices.

(i) The circulation is quantized.

$$\oint \boldsymbol{v}_s \cdot d\boldsymbol{s} = \kappa n \ (n = 0, 1, 2, \cdots)$$
$$\kappa = h/m$$

A vortex with $n \geqq 2$ is unstable.



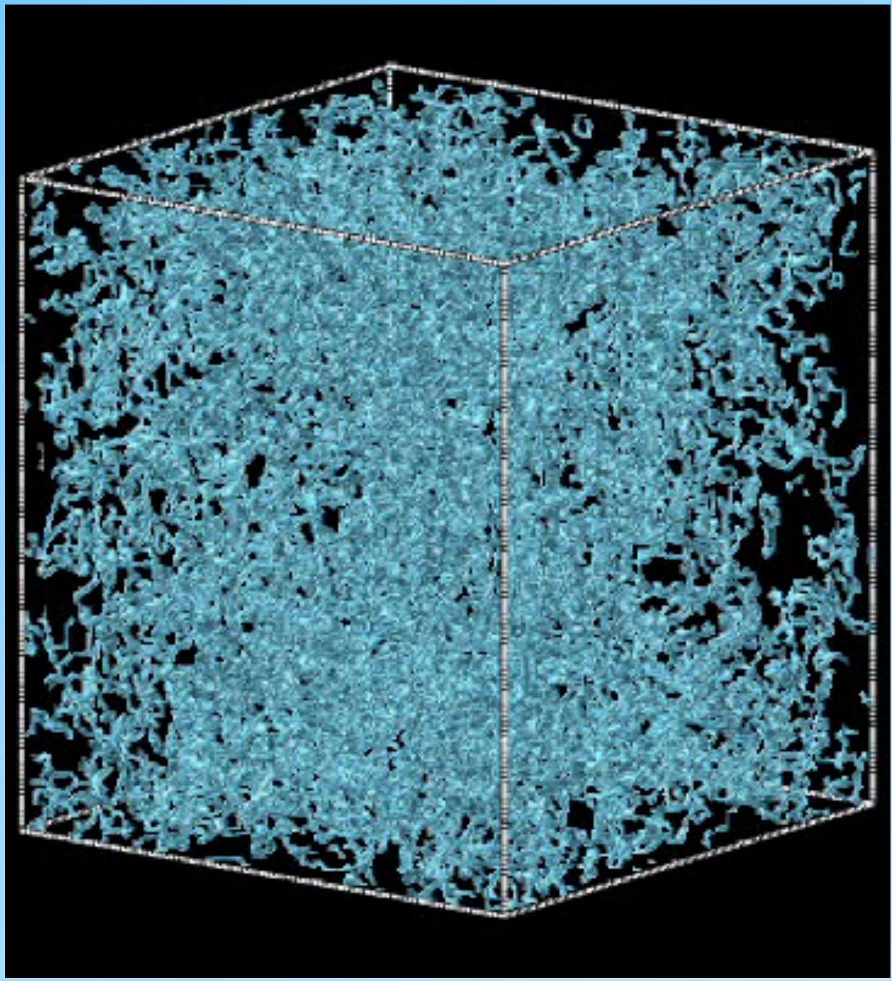
⇨ **Every vortex has the same circulation.**

(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

⇨ **The vortex is stable once it is nucleated.**

A quantized vortex is definite and well-defined!!

Vortex tangle in ***Quantum Turbulence***.



Numerical simulation by the Gross-Pitaevskii model.

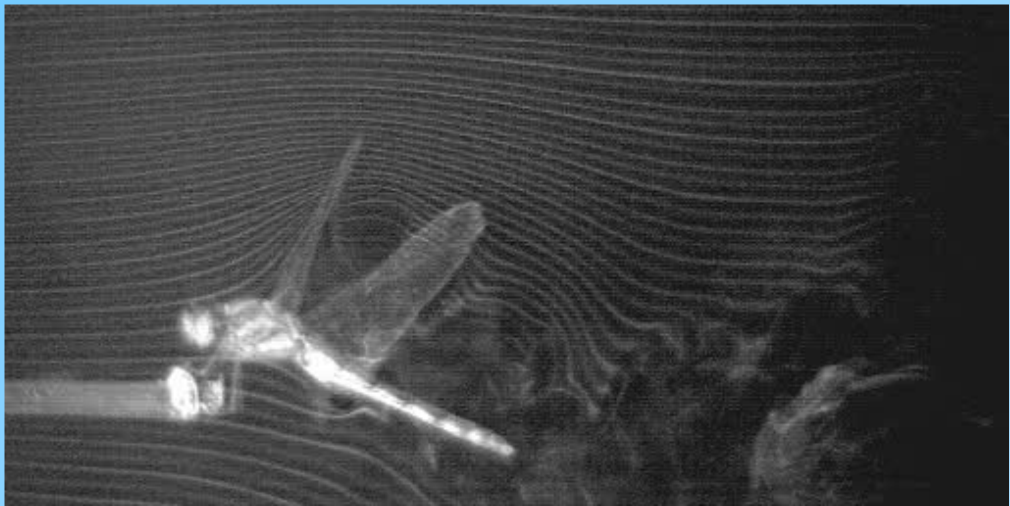
The blue lines show the thin cores of quantized vortices.

Vortices are disordered spatially and temporally.

So are superflow created by the vortices.
→ Superfluid turbulence

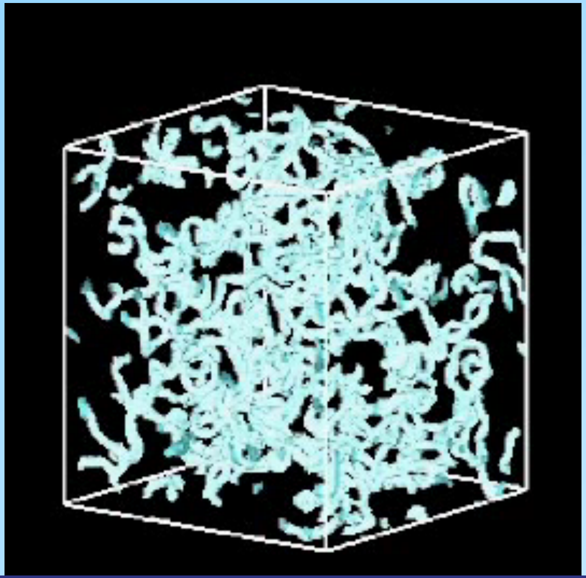
Classical Turbulence (CT) vs. Quantum Turbulence (QT)

Classical turbulence



http://www.nagare.or.jp/mm/2004/gallery/iida/dragonfly.html

Quantum turbulence



Motion of vortex cores

Elements are better-defined in QT than CT.

・The vortices are unstable. Not easy to identify each vortex.

・The circulation differs from one to another, not conserved.

・The quantized vortices are stable topological defects.
・Every vortex has exactly the same circulation.
・Circulation is conserved.

Models available for simulation of QT

Gross-Pitaevskii (GP) model for the macroscopic wave function

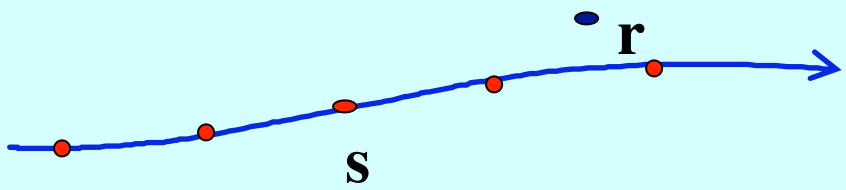
$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2\right]\Psi(\mathbf{r},t)$$

ATOMIC BECS

Vortex filament model(VFM) **Biot-Savart law**

$$\mathbf{v}_s(\mathbf{r}) = \frac{\kappa}{4\pi}\int\frac{(\mathbf{s}-\mathbf{r})\times d\mathbf{s}}{|\mathbf{s}-\mathbf{r}|^3}$$

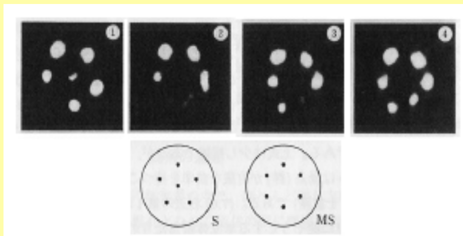
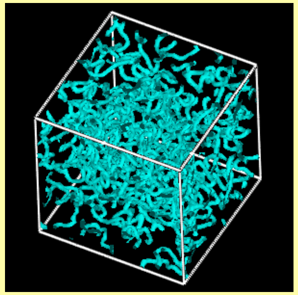
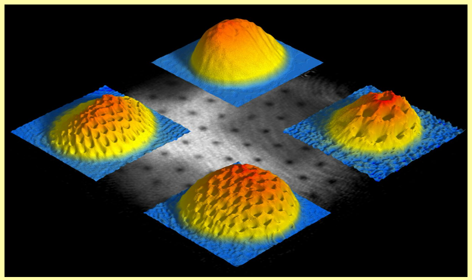


A vortex makes the superflow of the Biot-Savart law, and moves with this local flow.

SUPERFLUID HELIUM

2. Quantum turbulence in atomic BECs

There are two main cooperative phenomena of quantized vortices; Vortex lattice under rotation and Quantum turbulence.

	Vortex lattice	Quantum turbulence
Superfluid He		
Atomic BEC		Few works, but recently active

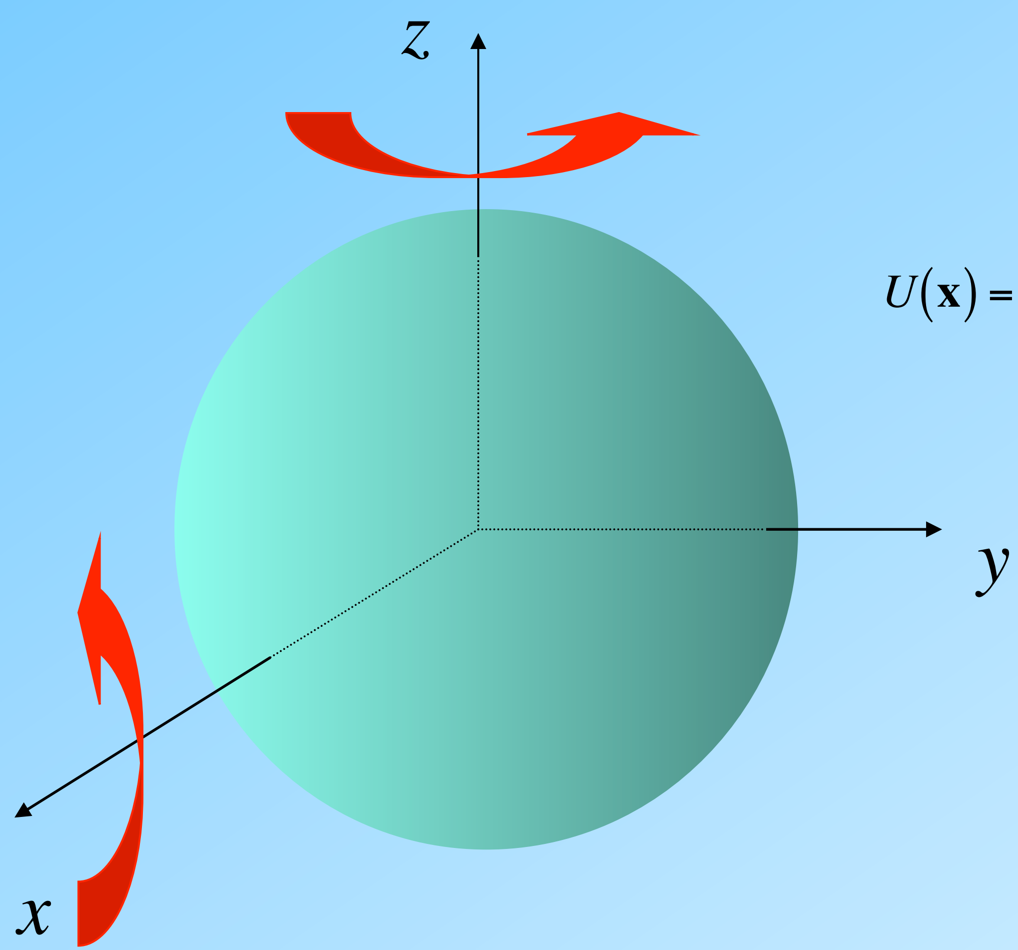
Some methods of how to create QT in a trapped BEC.

- Phase printing: N. G. Berloff, B. V. Svistunov, PRA66, 013603(2002)
- Manipulating the trapping potential: M. Kobayashi, M. Tsubota, PRA76, 045603(2007)
- Stirring the condensate: A. J. Allen *et al*., PRA89, 023602(2014) other several works

QT in a trapped BEC

M. Kobayashi and M. Tsubota, Phys. Rev. A76, 045603 (2007)

Making QT by combining two rotations



1. Trap the BEC in a weakly elliptic potential.

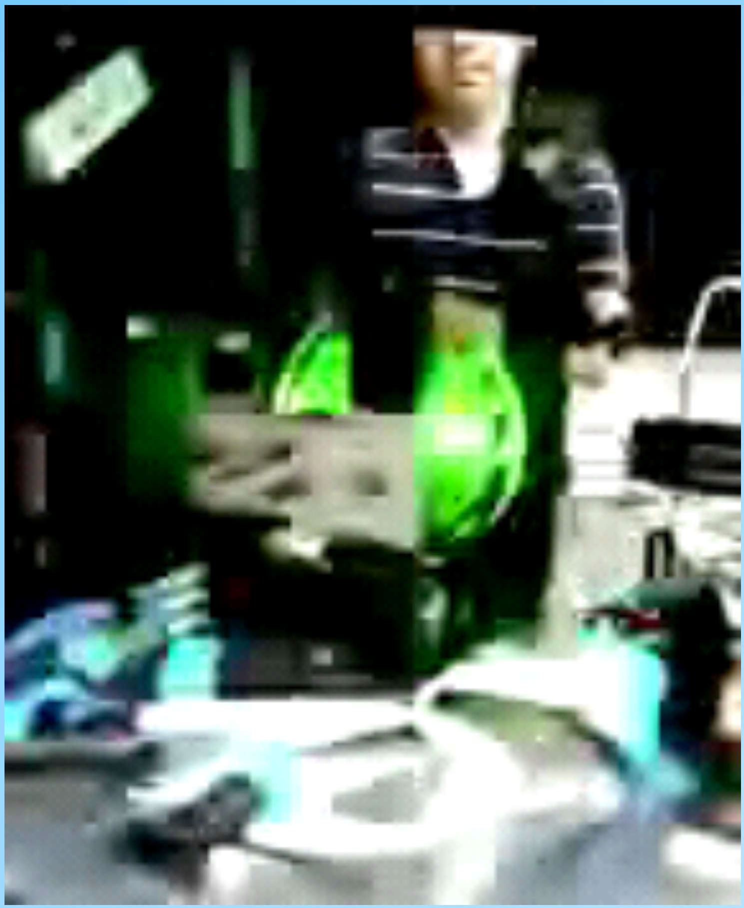
$$U(\mathbf{x}) = \frac{m\omega^2}{2}\left[(1-\varepsilon_1)(1-\varepsilon_2)x^2 + (1+\varepsilon_1)(1-\varepsilon_2)y^2 + (1+\varepsilon_2)z^2\right]$$

2. Rotate the system first around the x-axis, next around the z-axis.

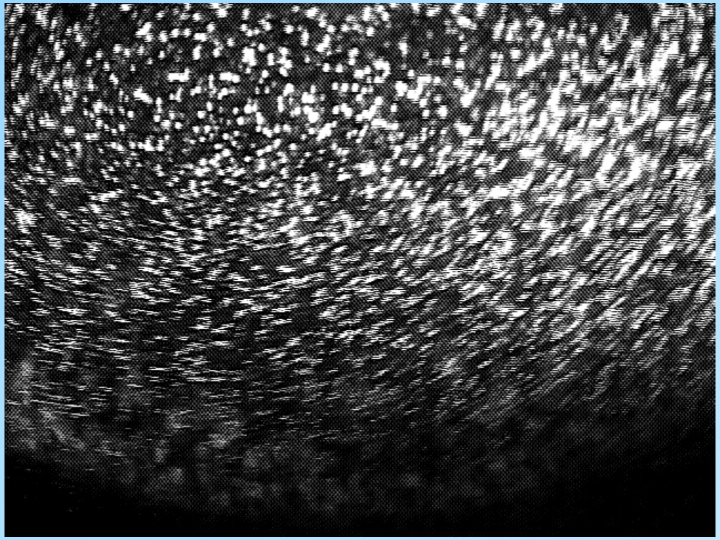
$$\Omega(t) = \left(\Omega_x,\ \Omega_z \sin\Omega_x t, \Omega_z \cos\Omega_x t\right)$$

Actually this idea has been already used in CT.

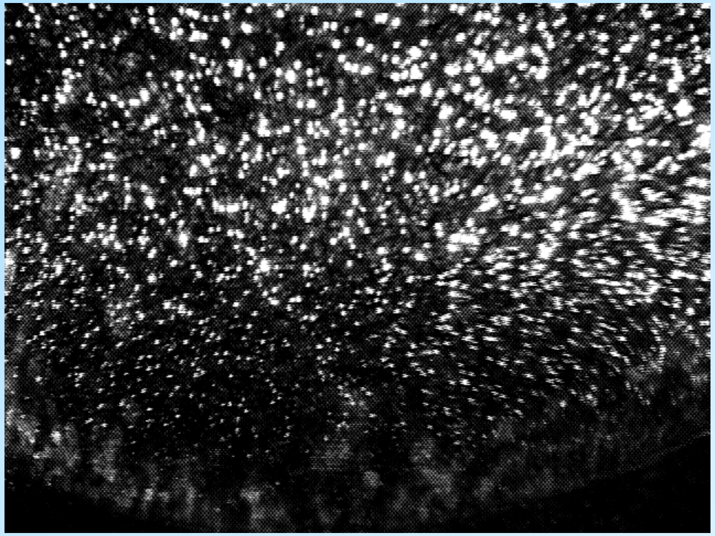
S. Goto, N. Ishii, S. Kida, and M. Nishioka, Phys. Fluids 19, 061705 (2007)



Rotation around one axis



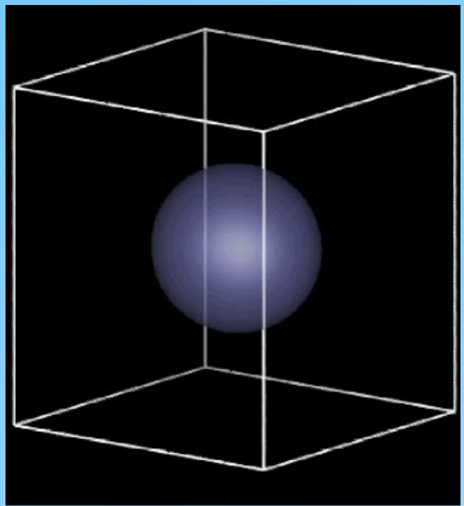
Rotation around two axes



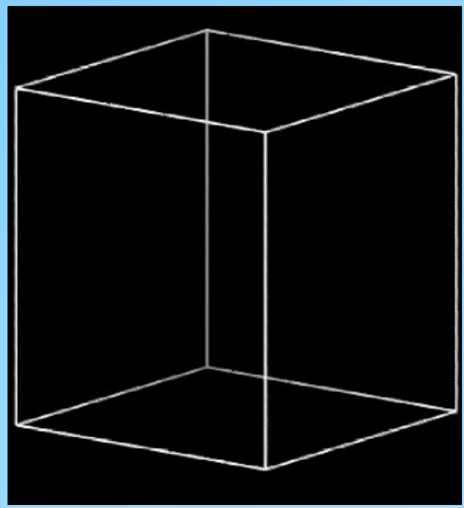
QT made by two precessions in a trapped BEC

Two precessions ($\omega_x \times \omega_z$)

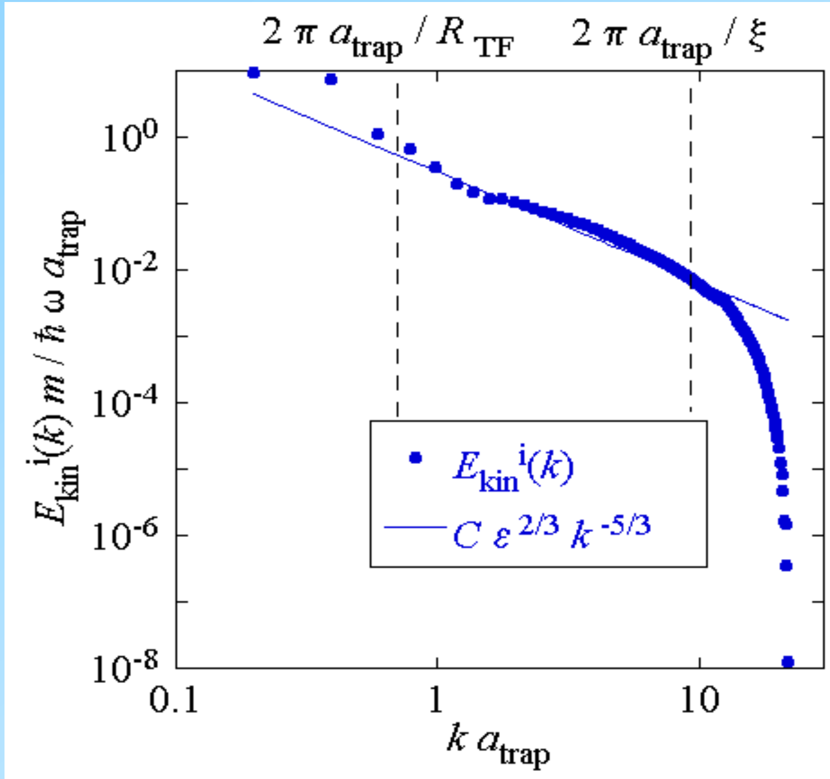
M. Kobayashi and MT, Phys. Rev. A76, 045603 (2007)



Condensate density



Quantized vortices



Simulation of the Gross-Pitaevskii model

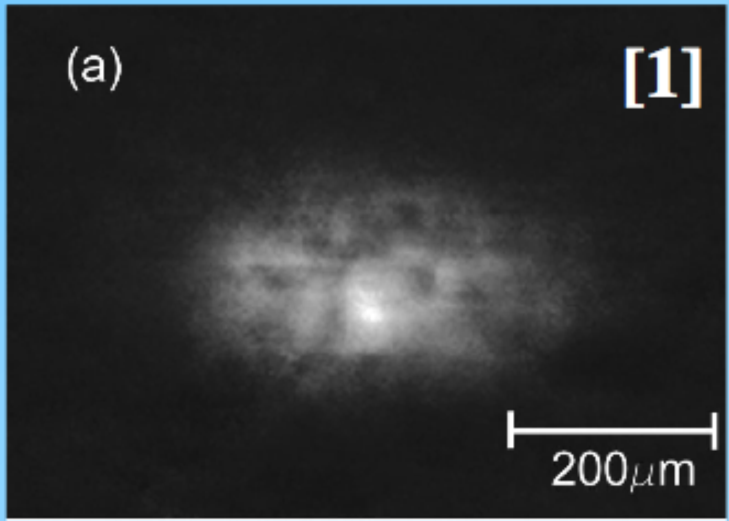
$$n \approx 1.78 \pm 0.194$$

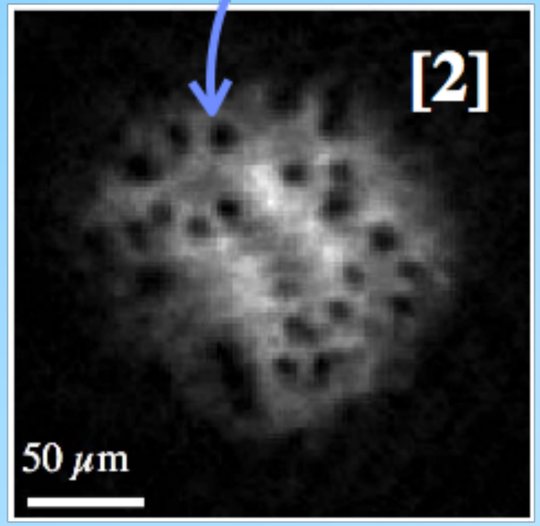
We confirmed a scaling law of the energy spectrum similar to the Kolmogorov -5/3 law.

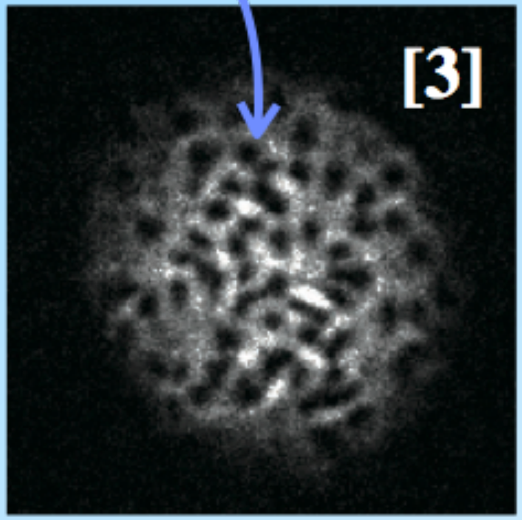
QT is realized experimentally.

- Density distribution in turbulence

Quantized vortex







3D ***2D***

[1] E. A. L. Henn et al. , Phys. Rev. Lett. 103, 045301 (2009).
[2] K. E. Wilson et al., Annu. Rev. Cold At. Mol. 1, 261 (2013).
[3] Woo Jin Kwon et al., Phys. Rev. A 90, 063627 (2014).

2-2. QT in two-component BECs

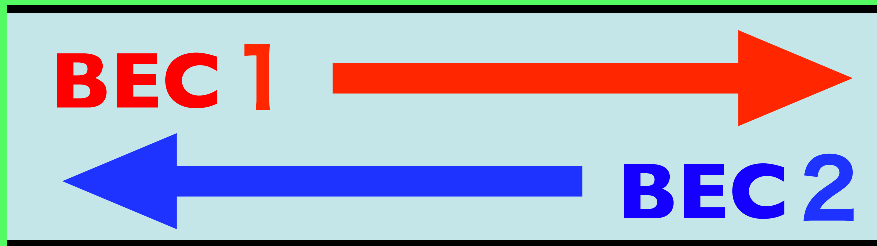
$$i\hbar\frac{\partial}{\partial t}\Psi_1 = -\frac{\hbar^2}{2m_1}\nabla^2\Psi_1 + g_{11}|\Psi_1|^2\Psi_1 + g_{12}|\Psi_2|^2\Psi_1$$
$$i\hbar\frac{\partial}{\partial t}\Psi_2 = -\frac{\hbar^2}{2m_2}\nabla^2\Psi_2 + g_{22}|\Psi_2|^2\Psi_2 + g_{12}|\Psi_1|^2\Psi_2$$

g_{11}, g_{22} : intracomponent interaction

g_{12} : intercomponent interaction

$g_{11}g_{22} > g_{12}^2$ ➡ The mixture is stable.

However,



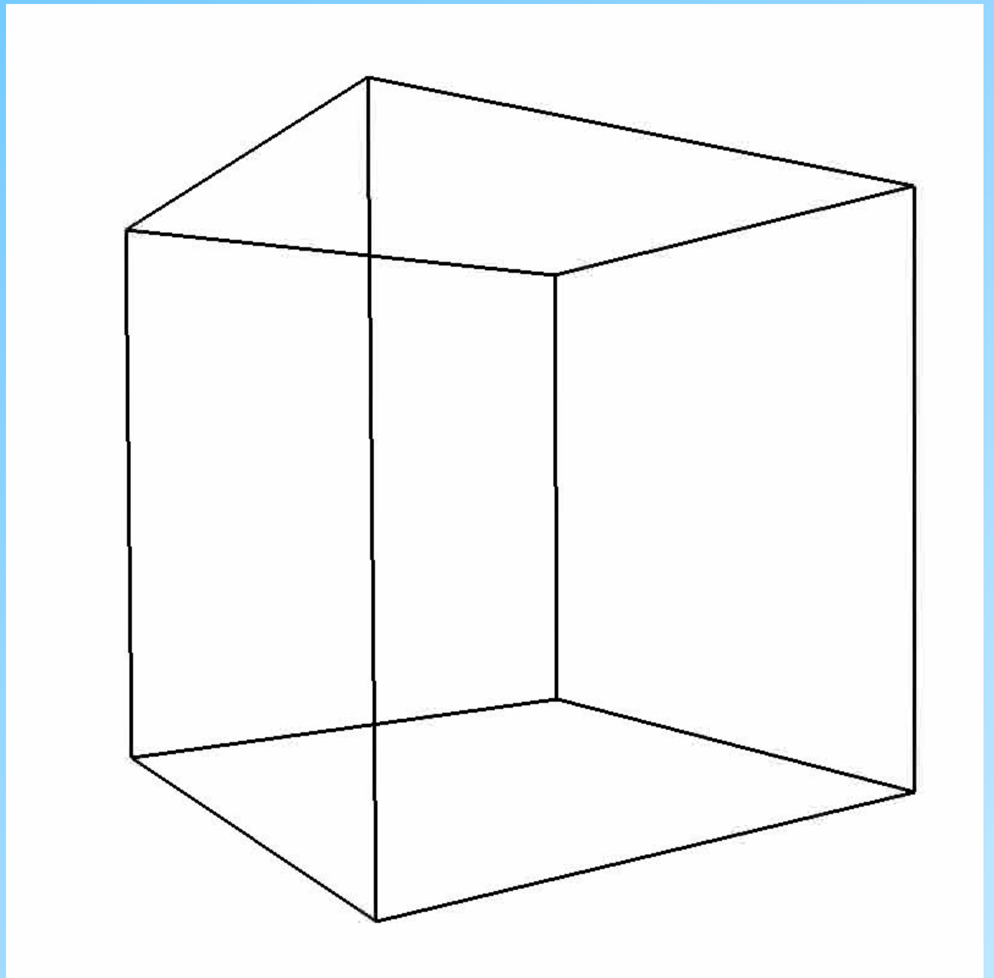
The large relative velocity should make it unstable.

V. I. Yukalov and E. P. Yukalova, Laser Phys. Lett. **1**, 50 (2004).

C. Hamner *et al*., Phys. Rev. Lett. **106**, 065302(2011).

3D 2-component QT

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \quad V'_R = 4.71, \quad \gamma = 0.9$$



Counterflow of two BECs

Solitons

$\rightarrow$ Vortex loops

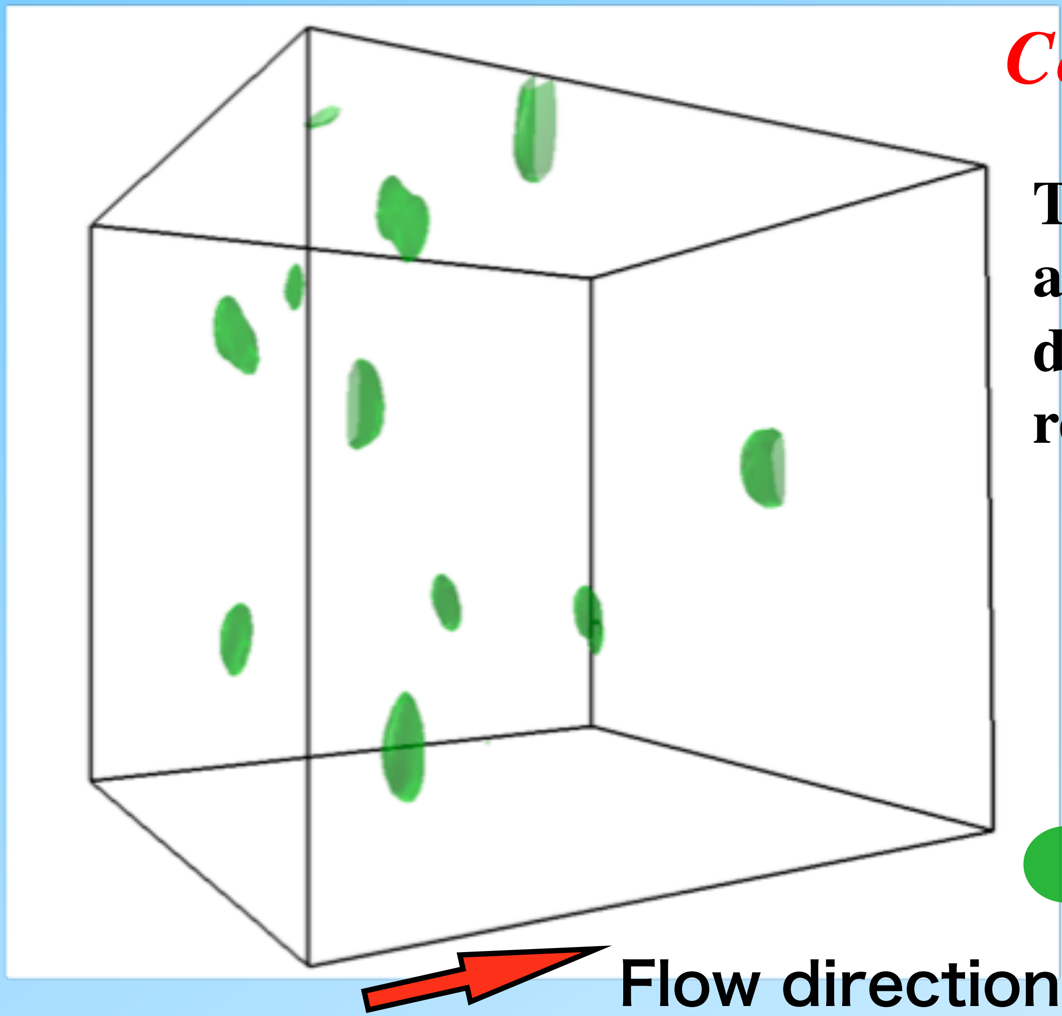
$\rightarrow$ QT

H. Takeuchi, S. Ishino, MT,
PRL105, 205301(2010)
S. Ishino, MT, H. Takeuchi,
PRA83, 063602(2011)

Flow direction

Scenario to turbulence (1)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \; V_R' = 4.71, \; \gamma = 0.9$$



Counterflow of two BECs

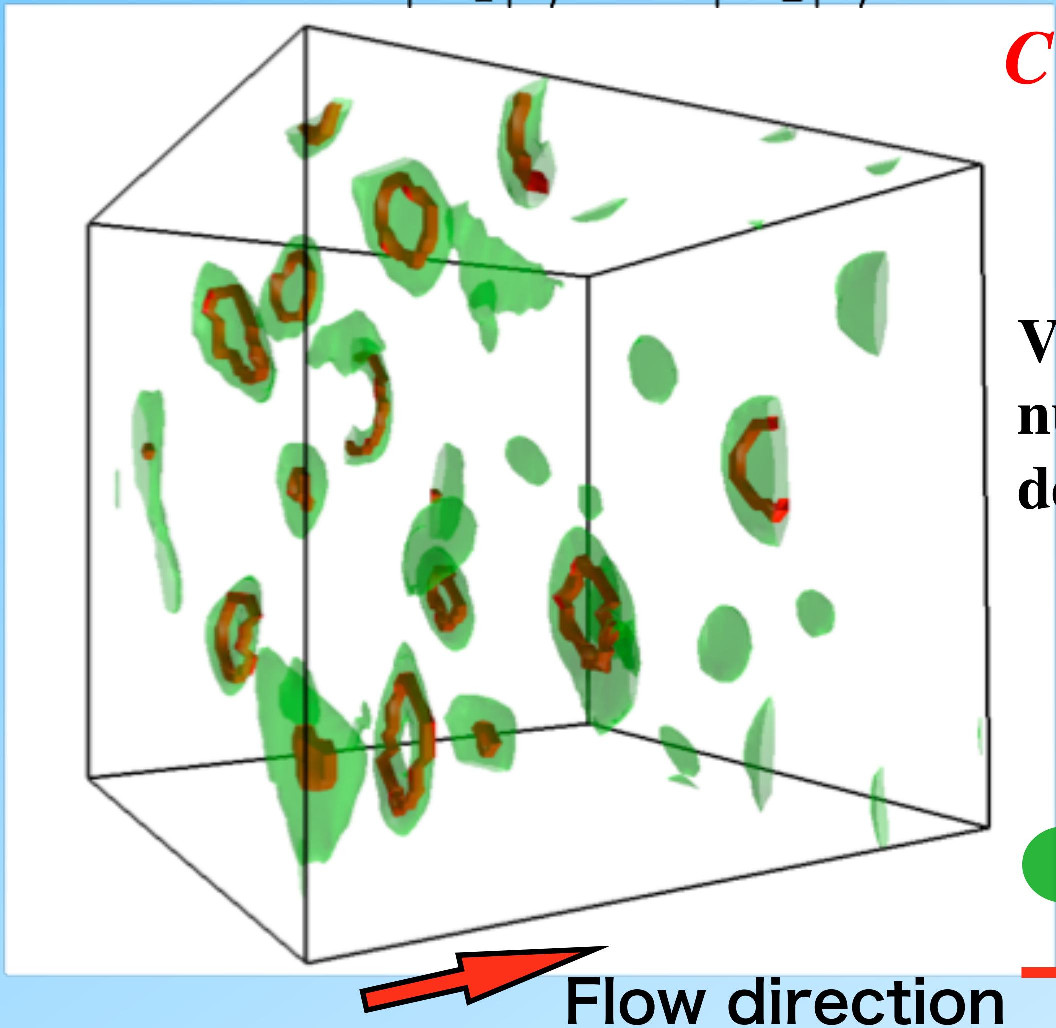
The unstable mode is amplified to lead to the disk-shaped low density regions.

Isosurface of $|\Psi_1|^2/n = 0.1$

Scenario to turbulence (2)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \ \ V_R' = 4.71, \ \ \gamma = 0.9$$

Counterflow of two BECs



Vortex rings are nucleated inside the low density regions.

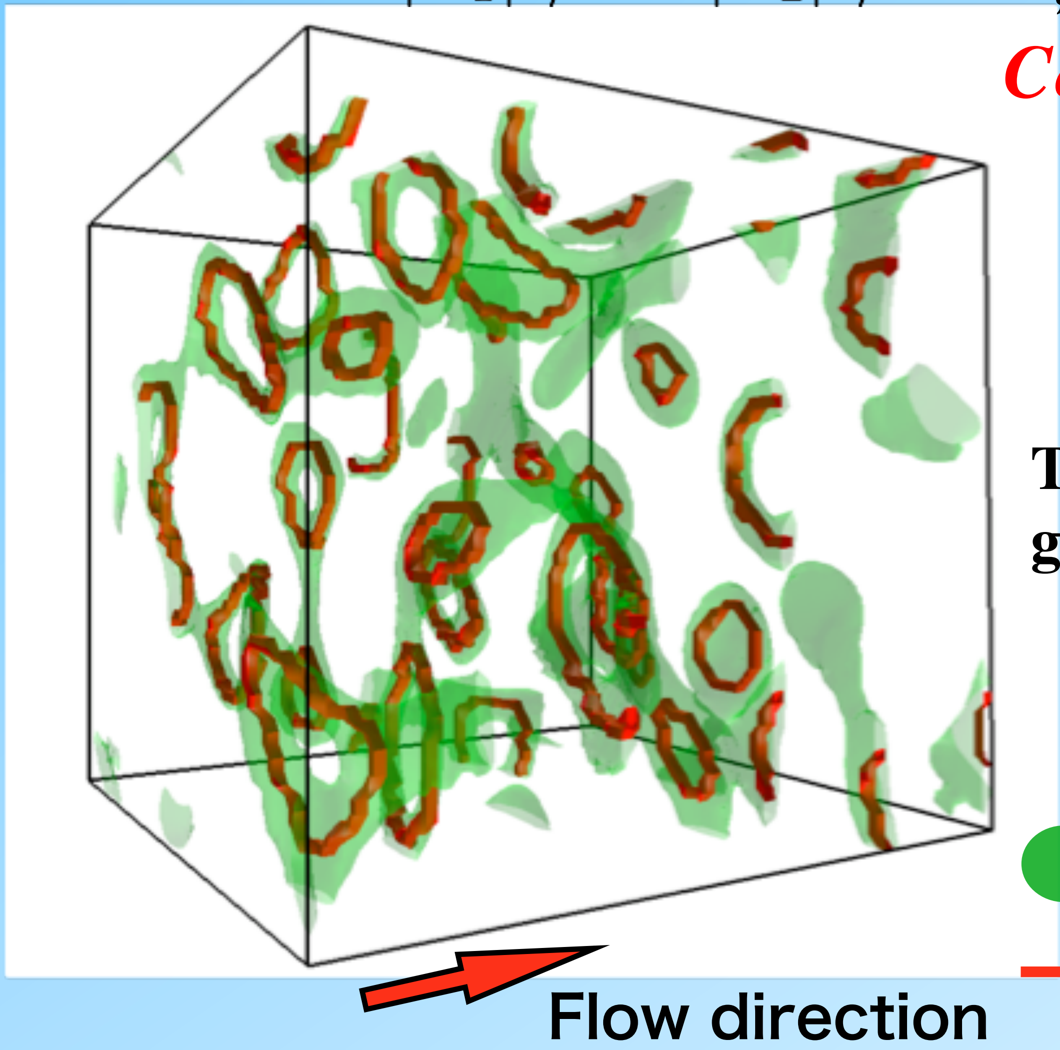
Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (3)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \quad V_R' = 4.71, \quad \gamma = 0.9$$

Counterflow of two BECs



The vortices expand and grow.

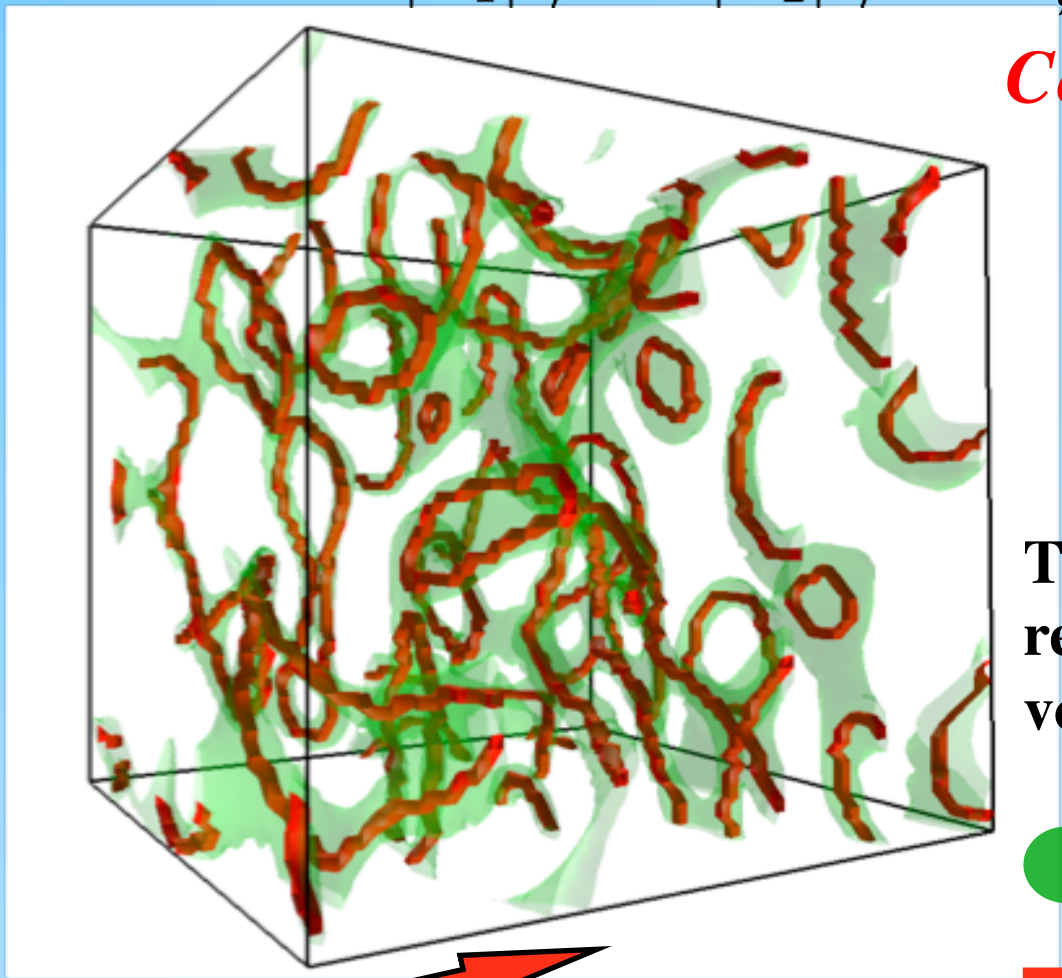
Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (4)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \quad V_R' = 4.71, \quad \gamma = 0.9$$

Counterflow of two BECs



Flow direction

The vortices expand to reconnect with other vortices.

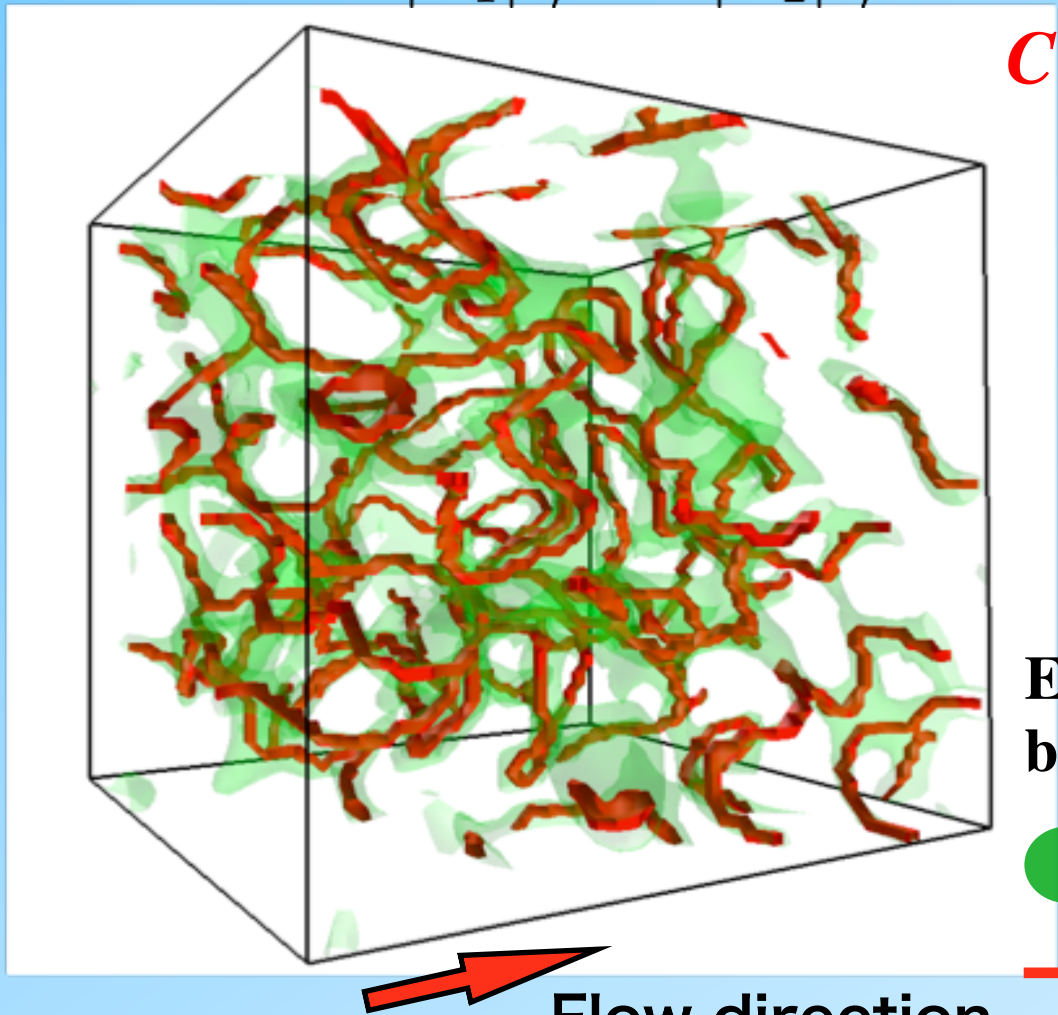
Isosurface of $|\Psi_1|^2/n = 0.1$

Vortex core of component 1

Scenario to turbulence (5)

$$|\Psi_1|^2/n = |\Psi_2|^2/n = 1, \quad V_R' = 4.71, \quad \gamma = 0.9$$

Counterflow of two BECs

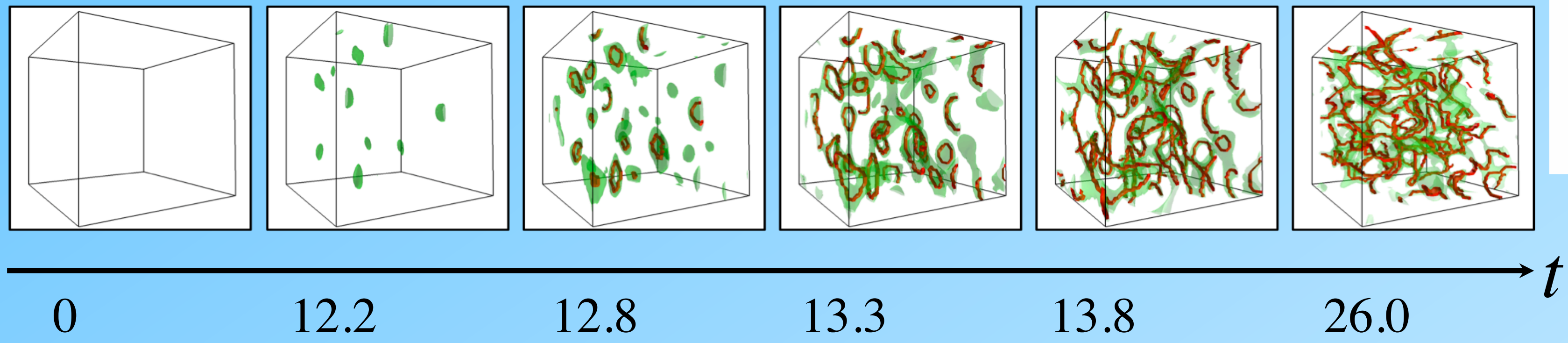


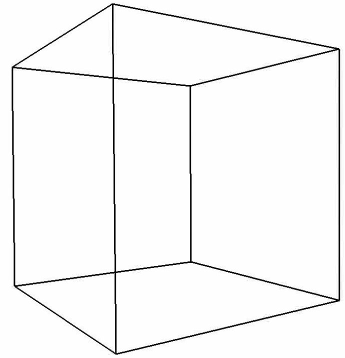
Eventually the vortices become tangled.

Isosurface of $|\Psi_1|^2/n = 0.1$

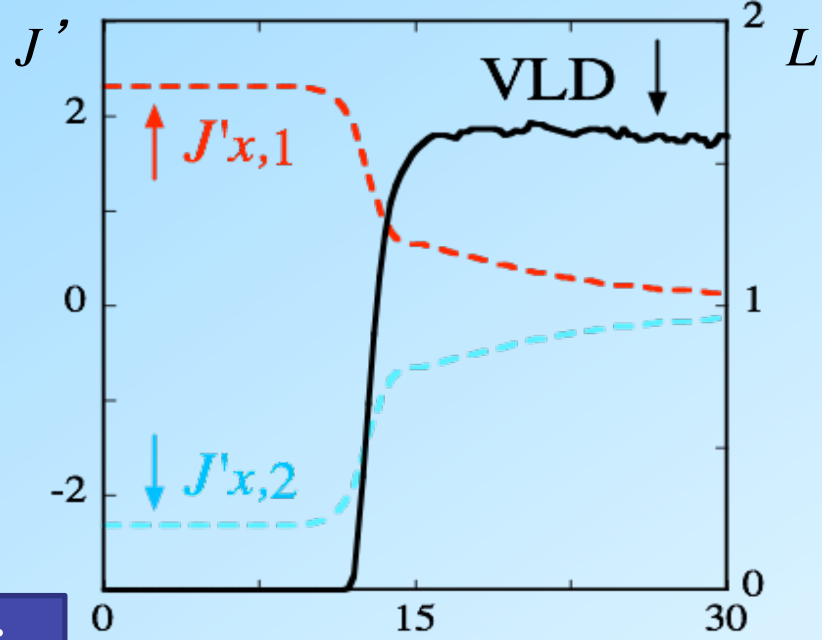
Vortex core of component 1

Scenario to turbulence





momentum exchange



As the vortices grow, the two superfluids start to communicate and exchange the momentum, eventually reducing the relative motion.

"Mutual friction" between superfludis

Various kinds of quantum turbulence in atomic BECs.

Quantized vortices

QT in single-component BECs   QT in two-component BECs

Spins

Spin turbulence in spinor BECs

K. Fujimoto, MT, PRA85, 033642(2012)
MT, Y. Aoki, K. Fujimoto, PRA88, 061601(R) (2013)

Waves

Bogoliubov wave turbulence in BECs

K. Fujimoto, MT, PRA91, 053260(2015)

Spin Waves

Spin wave turbulence in BECs

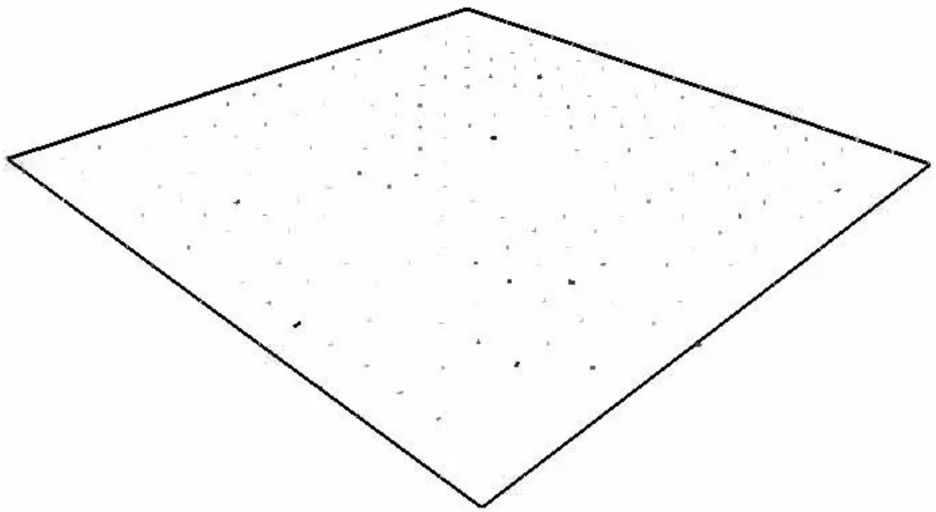
K. Fujimoto, MT, PRA93, 033620(2016)

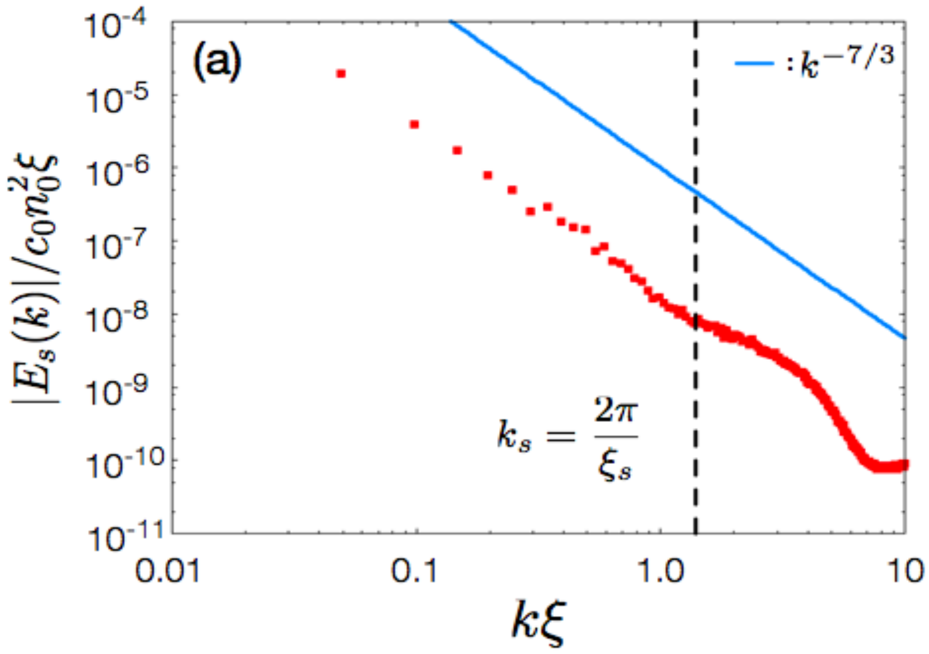
Spin turbulence in spinor BECs

K. Fujimoto, MT: PRA85, 033642(2012), PRA85, 053641(2012)

$$s_i = \sum_{m,n=-1}^{1} \psi_m^* (S_i)_{mn} \psi_n$$

Time-development of the spin density vector ***s***





$$E_s \propto k^{-7/3}$$

3. Quantum turbulence in superfluid helium-

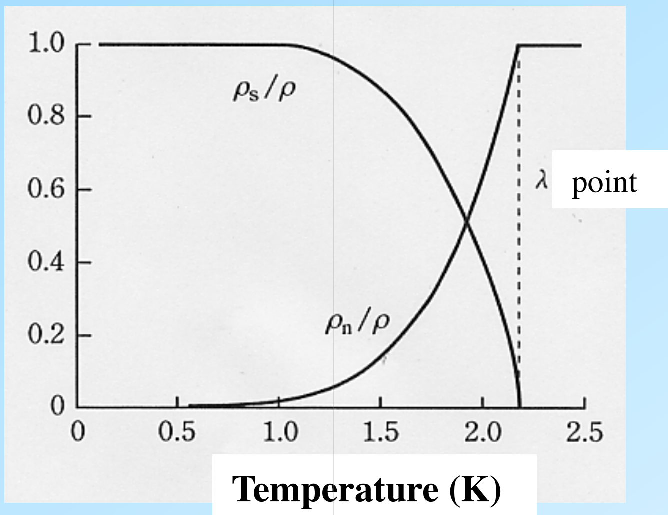
Liquid ^{4}He enters the superfluid state below 2.17 K (λ point) with Bose-Einstein condensation.

Its hydrodynamics are well described by the two-fluid model:

The two-fluid model (Tisza, Landau)

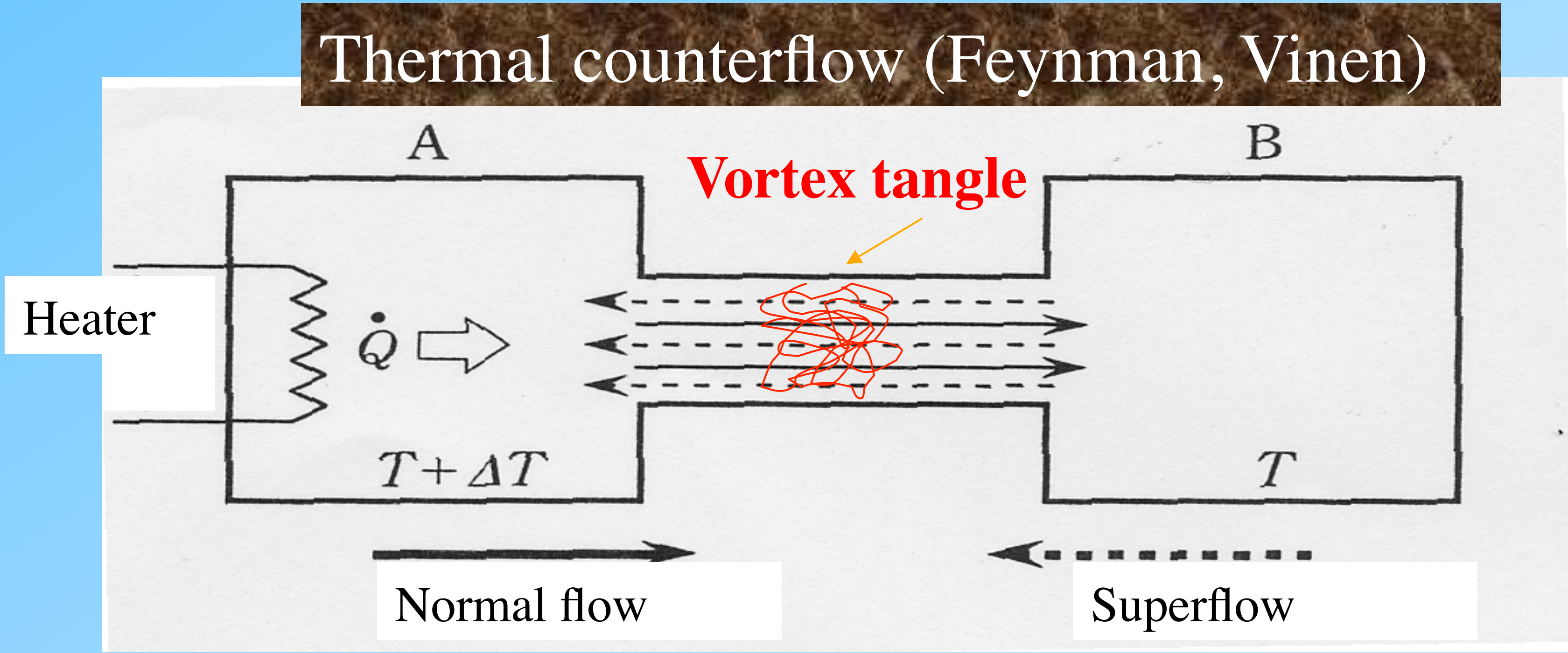
The system is a mixture of inviscid superfluid and viscous normal fluid.

$$\rho = \rho_s + \rho_n \qquad \mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$



	Density	Velocity	Viscosity	Entropy
Superfluid	$\rho_s(T)$	$\mathbf{v}_s(\mathbf{r})$	0	0
Normal fluid	$\rho_n(T)$	$\mathbf{v}_n(\mathbf{r})$	$\eta_n(T)$	$s_n(T)$

3-1. Previous simulation for the homogeneous normal fluid flow (1980's-2010)

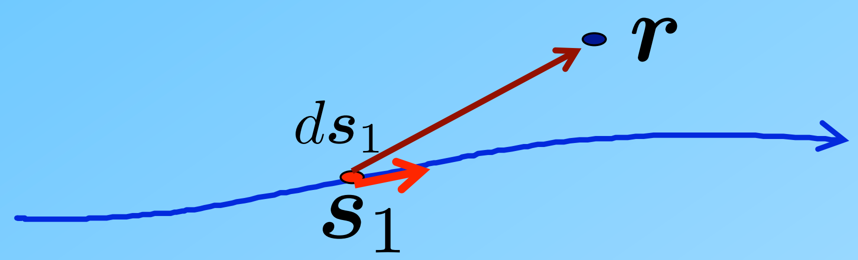


Most simulations were performed for homogeneous systems.

・K. W. Schwarz, Phys. Rev. B38, 2398 (1988) LIA

・H. Adachi, S. Fujiyama, M. Tsubota, Phys. Rev. B81, 104511(2010) full Biot-Savart

Vortex filament model (VFM)



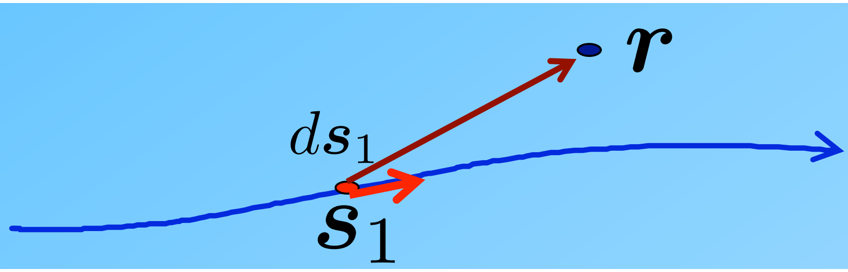
A vortex filament is represented by the parametric form $\boldsymbol{s}(\xi, t)$.

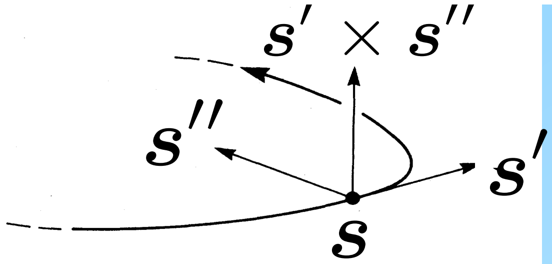
The vortex filament makes the superfluid velocity field by the Biot-Savart law.

$$\boldsymbol{v}_s(\boldsymbol{r}) = \frac{\kappa}{4\pi} \int_{\mathcal{L}} \frac{(\boldsymbol{s}_1 - \boldsymbol{r}) \times d\boldsymbol{s}_1}{|\boldsymbol{s}_1 - \boldsymbol{r}|^3}$$

Every point $\boldsymbol{s}$ on the filament moves with its local superfluid velocity: $\dfrac{d\boldsymbol{s}}{dt} = \boldsymbol{v}_s(\boldsymbol{r} = \boldsymbol{s})$

When we try to obtain $\boldsymbol{v}_s(\boldsymbol{s})$ and let $\boldsymbol{s}_1 \to \boldsymbol{r} = \boldsymbol{s}$, this integral becomes divergent. What to do?





$$\boldsymbol{s}' = \frac{d\boldsymbol{s}}{d\xi}, \boldsymbol{s}'' = \frac{d^2\boldsymbol{s}}{d\xi^2}$$

By introducing some cut-off, the Biot-Savart integral is divided into two terms(Arms & Hama 1965).

$$\frac{d\boldsymbol{s}_0}{dt} = \frac{\beta}{4\pi}\boldsymbol{s}' \times \boldsymbol{s}'' + \cancel{\frac{\kappa}{4\pi}\int_{\mathcal{L}}' \frac{d\boldsymbol{s} \times (\boldsymbol{r} - \boldsymbol{s})}{|\boldsymbol{r} - \boldsymbol{s}|^3}}$$

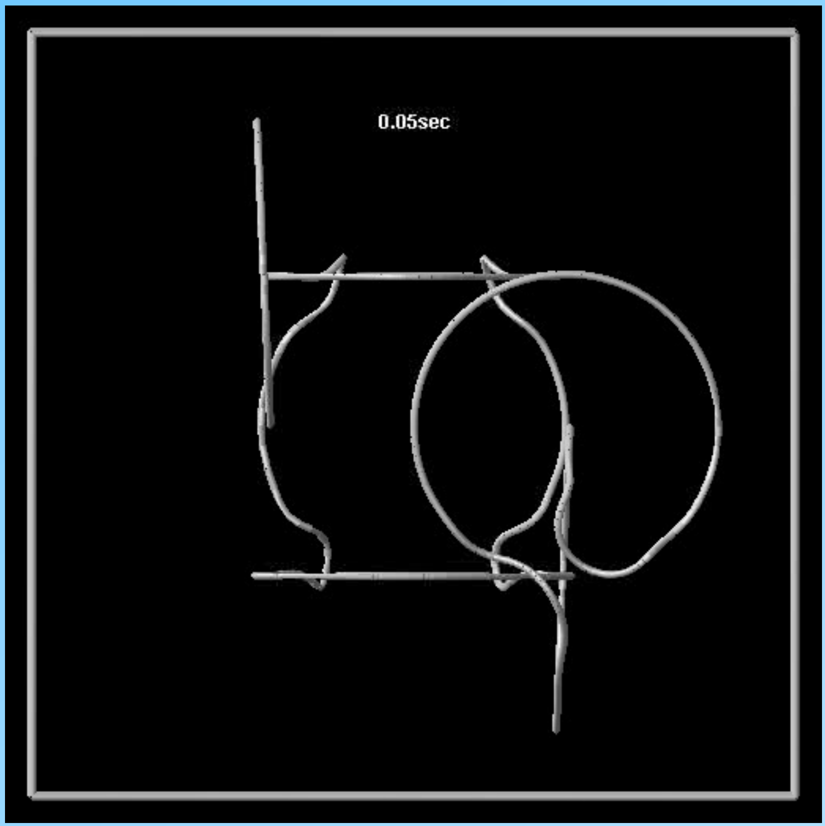
Local term　　　　Nonlocal term

$$\frac{d\boldsymbol{s}}{dt} = \frac{d\boldsymbol{s}_0}{dt} + \alpha \boldsymbol{s}' \times (\boldsymbol{v}_n - \frac{d\boldsymbol{s}_0}{dt}) - \alpha' \boldsymbol{s}' \times \left[\boldsymbol{s}' \times (\boldsymbol{v}_n - \frac{d\boldsymbol{s}_0}{dt})\right]$$

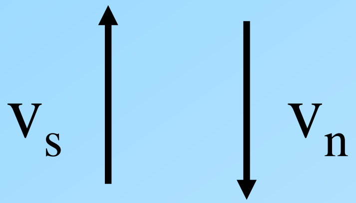
Mutual friction at finite temperatures

The approximation neglecting the nonlocal term is called the LIA(Localized Induction Approximation).

$$\frac{d\boldsymbol{s}_0}{dt} = \frac{\beta}{4\pi}\boldsymbol{s}' \times \boldsymbol{s}''$$



Simulation under LIA



K. W. Schwarz, Phys. Rev. B38, 2398 (1988).
-Obtained a statistically steady state by the vortex filament model (VFM) under the localized induction approximation (LIA).

Periodic boundary conditions for all three directions

Schwarz's simulation(1) PRB38, 2398(1988)

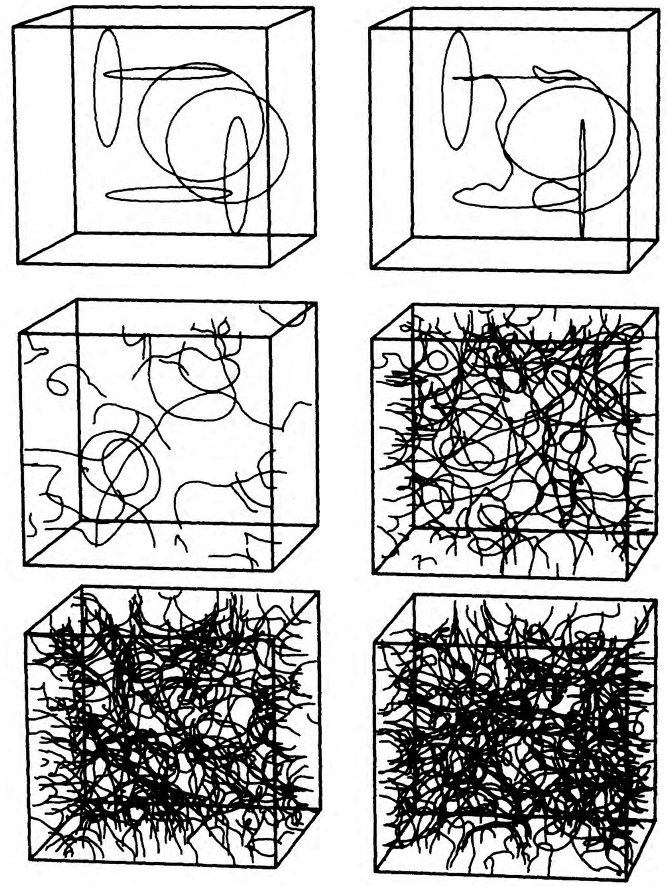


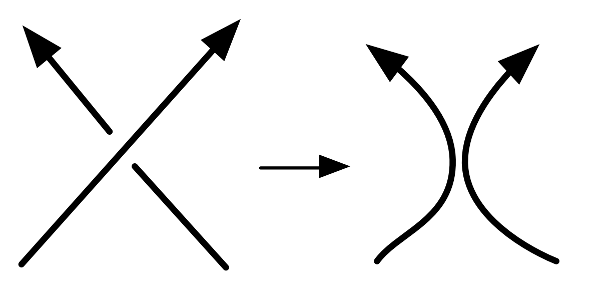
FIG. 4. Case study of the development of a vortex tangle in a real channel. Here, $\alpha=0.10$, corresponding to a temperature of about 1.6 K, and $v_{s,0}=75$ into the front face of the channel section shown. Upper left: $t_0=0$, no reconnections; upper right; $t_0=0.0028$, three reconnections; middle left: $t_0=0.05$, 18 reconnections; middle right: $t_0=0.20$, 844 reconnections; lower left: $t_0=0.55$, 12 128 reconnections; lower right: $t_0=2.75$, 124 781 reconnections.

However, this simulation had nontrivival serious problems.

1. Vortex reconnections were modeled artificially.

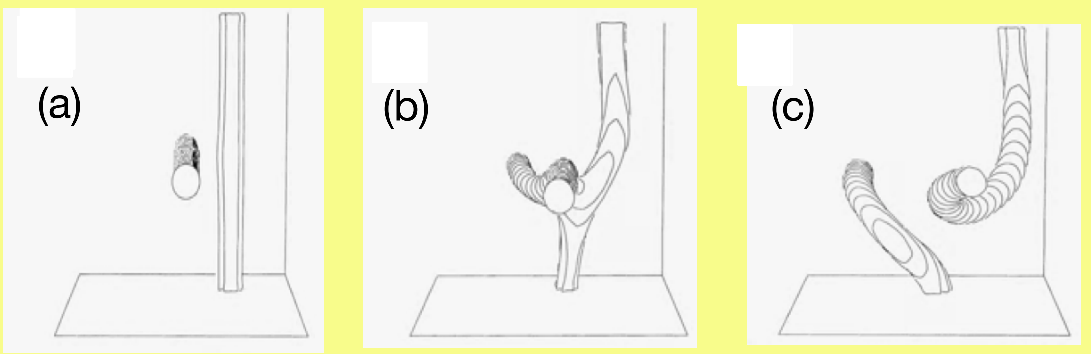
Reconnection of quantized vortices (1)

In the field of classical fluid dynamics, vortex reconnections are believed to occur by the viscous diffusion of vorticity.

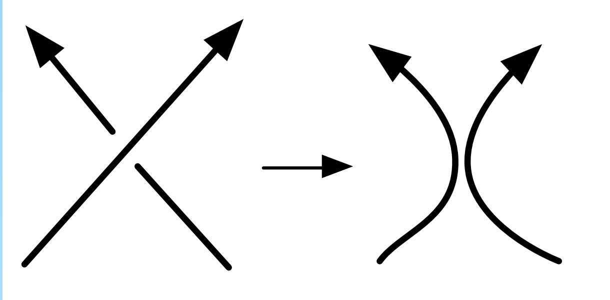


Can quantized vortices reconnect? The vortex filament model (VFM) cannot answer the question.

The simulation of the Gross-Pitaevskii model shows reconnection. J. Koplik and H. Levin, PRL71, 1375 (1993)



Reconnection of quantized vortices (2)



Reconnections in VFM are modeled with an algorithmical procedure. However, this procedure is more or less arbitrary.

Schwarz's simulation(2) PRB38, 2398(1988)

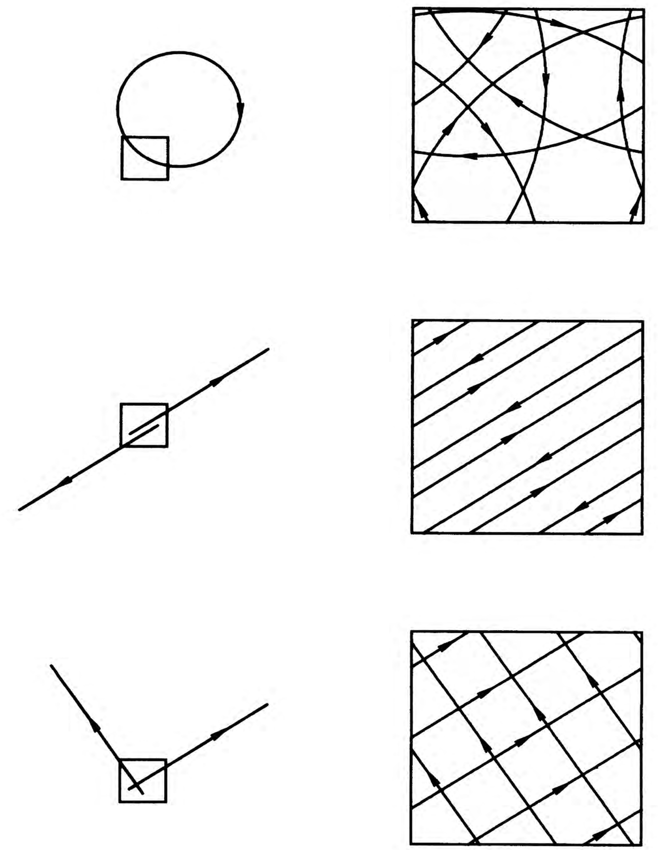


FIG. 8. Mapping of various vortex configurations into the computational volume, showing the appearance of the unit cell when all space is filled by the repetition of these objects. The end points of the lines represent equivalent points in the unit cell. Top row: closed loops; middle row: parallel infinite lines characteristic of a dead-end fluctuation; bottom row: infinite lines after randomizing procedure designed to reestablish three-dimensional behavior. The illustrations are intended to be purely schematic.

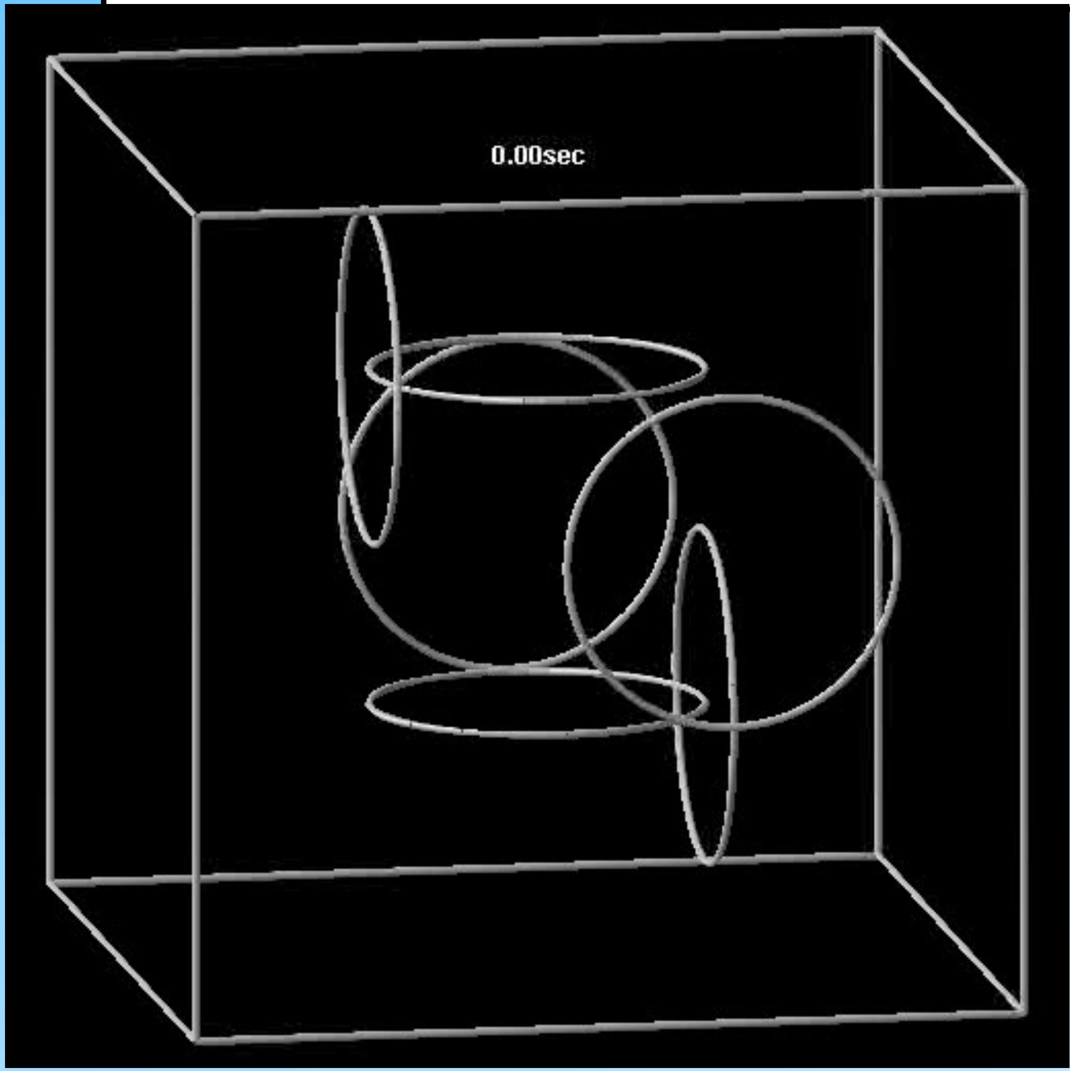
However, this simulation had nontrivial serious problems.

1. Vortex reconnections were modeled artificially.

2. All calculation was performed by the LIA.

→ He used an artificial mixing procedure in order to obtain the steady state.

Simulation by the **full Biot-Savart law**



H. Adachi, S. Fujiyama, M. Tsubota, Phys. Rev. B81, 104511(2010) .

$\boldsymbol{v}_n$

$\boldsymbol{v}_s$

BOX (0.1cm)^3 $T = 1.6$ K

Vns = 0.367cm/s

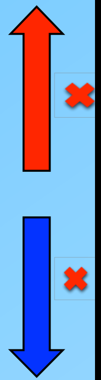
Periodic boundary conditions for all three directions

The statistically steady states were obtained without the artificial mixing procedure.

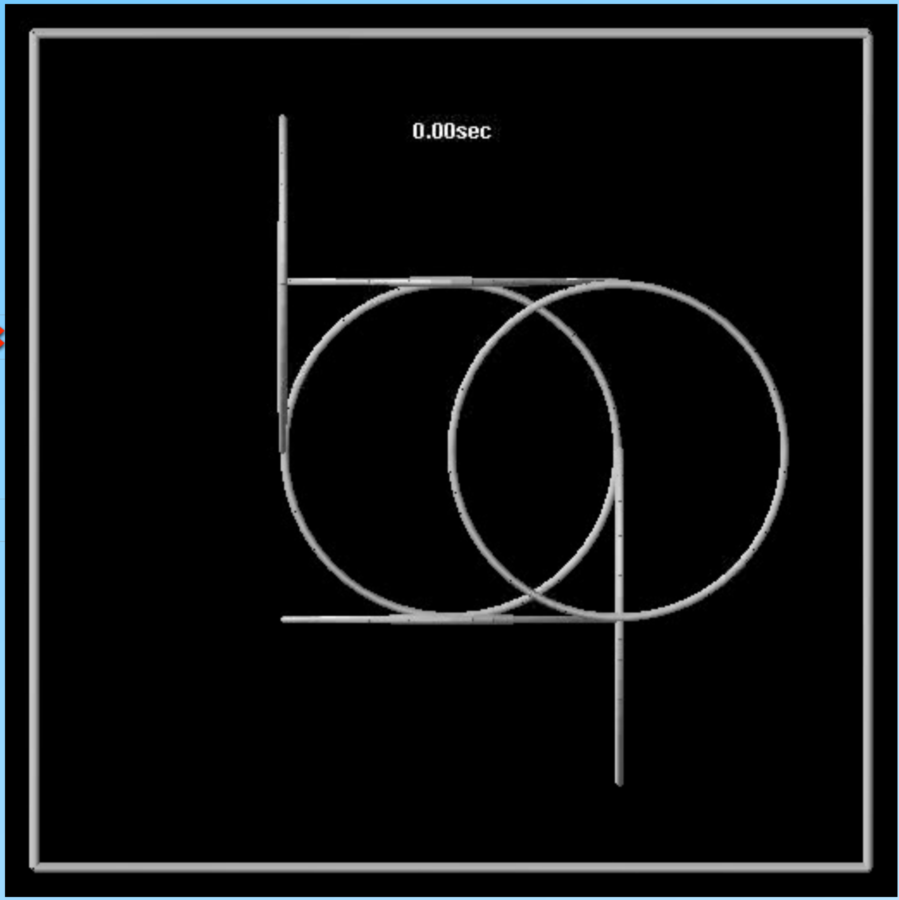
Comparison between LIA and full Biot-Savart

Full Biot-Savart $T = 1.6$ K LIA

v_n



v_s



We need intervortex interaction.

Vortices become anisotropic, forming layer structures.

3-2. Recent visualization experiments (2006-)

Visualizing quantized vortices and the profile of the normal fluid flow

Maryland, Tallahassee, Prague

- G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, Nature 441, 588(2006)

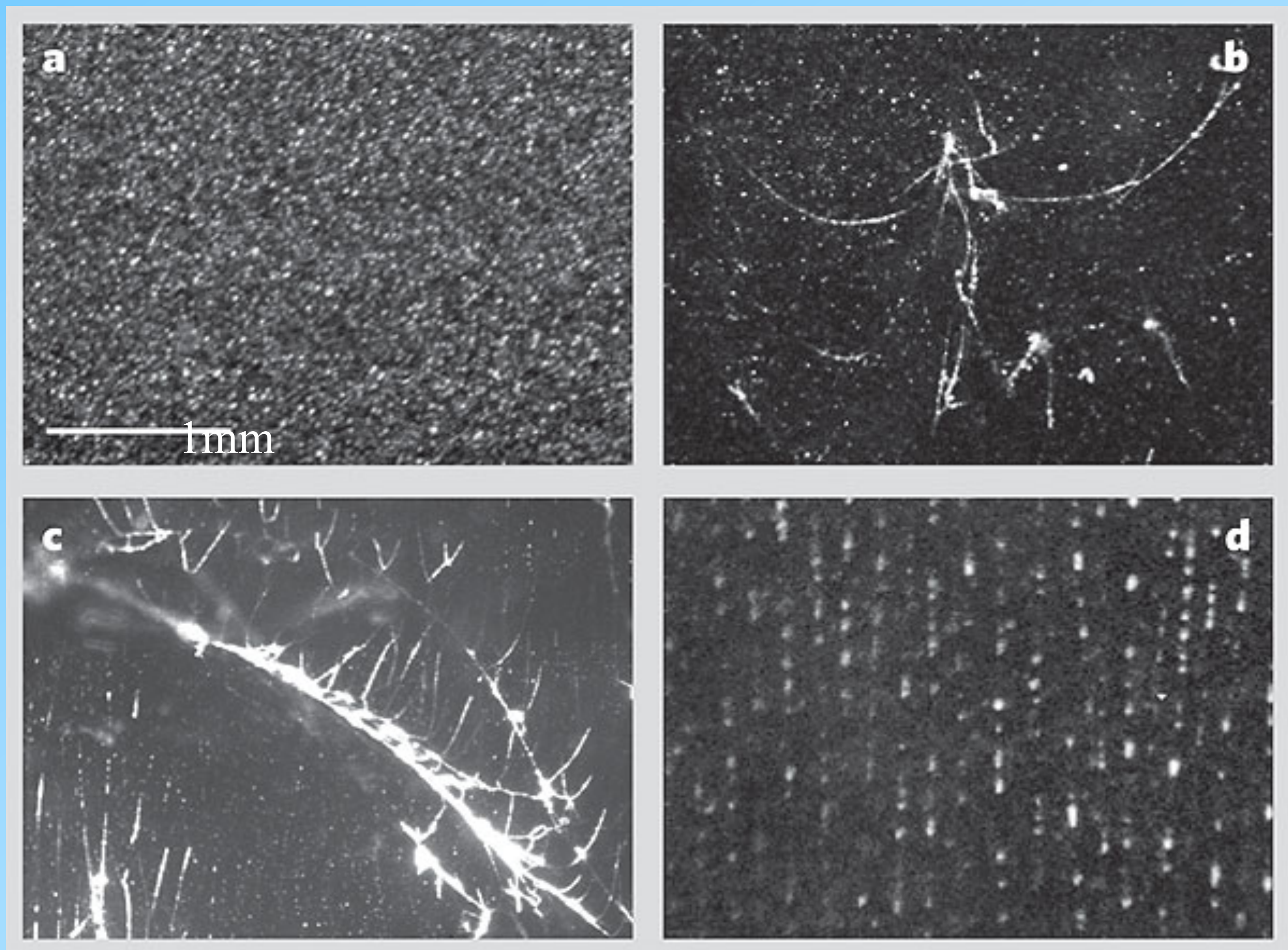
- A. Marakov, J. Gao, W. Guo, S. W. Van Sciver, G. G. Ihas, D. N. McKinsey, W. F. Vinen, Phys. Rev. B 91, 094503(2015).

- M. La Mantia, L. Skrbek, Phys. Rev. B 90, 014519 (2014).

Visualization of quantized vortices

G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, Nature 441, 588(2006)

$T > T_\lambda$



$T < T_\lambda$

Solid hydrogen particles are trapped by quantized vortices.

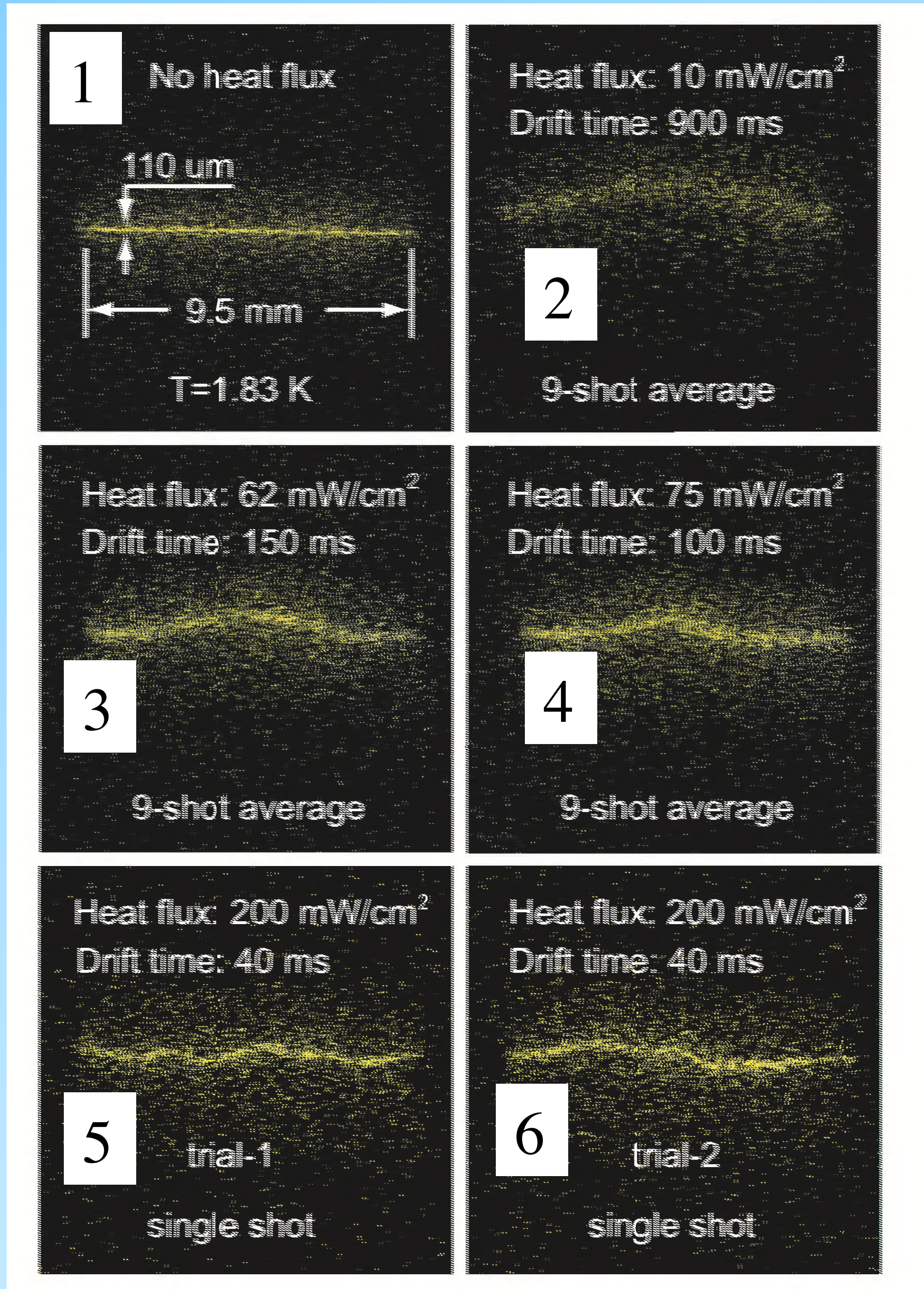
Marakov *et al*. observed a novel profile of the normal fluid flow by using metastable He_2^* molecules.

A. Marakov, J. Gao, W. Guo, S. W. Van Sciver, G. G. Ihas, D. N. McKinsey, W. F. Vinen, Phys. Rev. B 91, 094503(2015).

1 No heat flux → No flow
2 Poiseuille flow (laminar)
3, 4 Tail-flattened flow(laminar)
5, 6 Turbulence

$\mathbf{v}_n$

$\mathbf{v}_s$



Such tail-flattened flow has never been observed even in a classical fluid.

Present status of the numerical simulation in superfluid helium

Almost all simulations

One way

Superfluid (Vortex filament model)

Prescribed normal fluid (Navier-Stokes equation)

Desirable simulations

Both ways

Superfluid (Vortex filament model)

Normal fluid (Navier-Stokes equation)

Lagrangian

Eulerian

3-3. The new simulation for the inhomogeneous normal fluid flow

(1) Counterflow quantum turbulence of He-II in a square channel: Numerical analysis with nonuniform flows of the normal fluid

S. Yui and M. Tsubota, Phys. Rev. B91, 184504 (2015): arXiv: 1502.06683

(2) Logarithmic velocity profile (the log-law) of quantum turbulence of superfluid ^{4}He

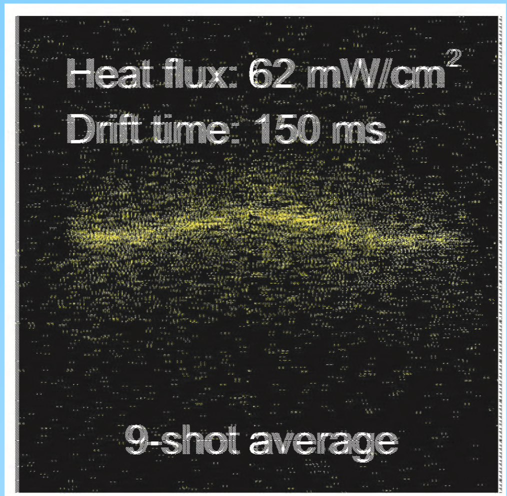
S. Yui, K. Fujimoto and M. Tsubota, Phys. Rev. B 92, 224513 (2015): arXiv:1508.01347

What is lacking in the previous simulations?

Most previous numerical works suppose

• Periodic boundary for all three directions

• Prescribing the homogeneous profile of the normal fluid

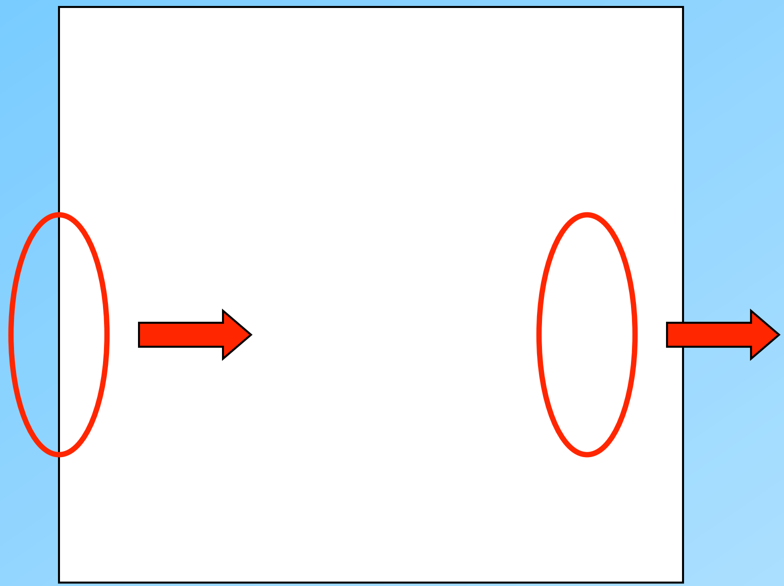


In order to understand these phenomena, we need

* solid boundary condition in a channel
* coupling of superfluid and normal fluid

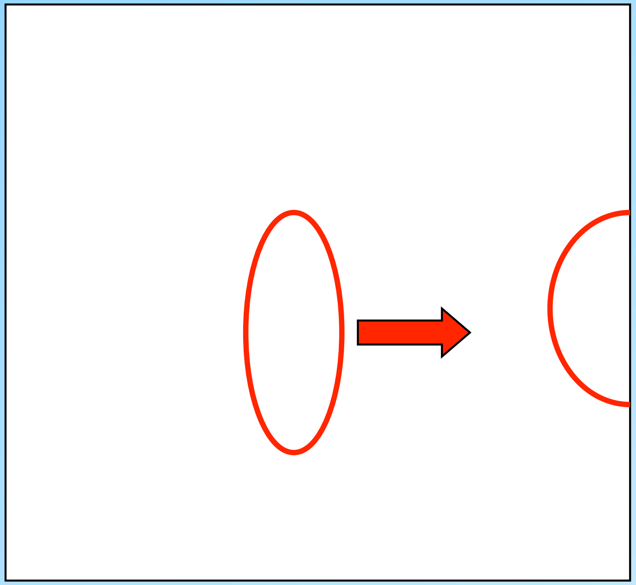
Difference between solid- and periodic boundary conditions

Periodic



A vortex ring that comes out of the right enters the system from the left again.

Solid boundary



A vortex ring moving to the right reconnects with the solid wall.

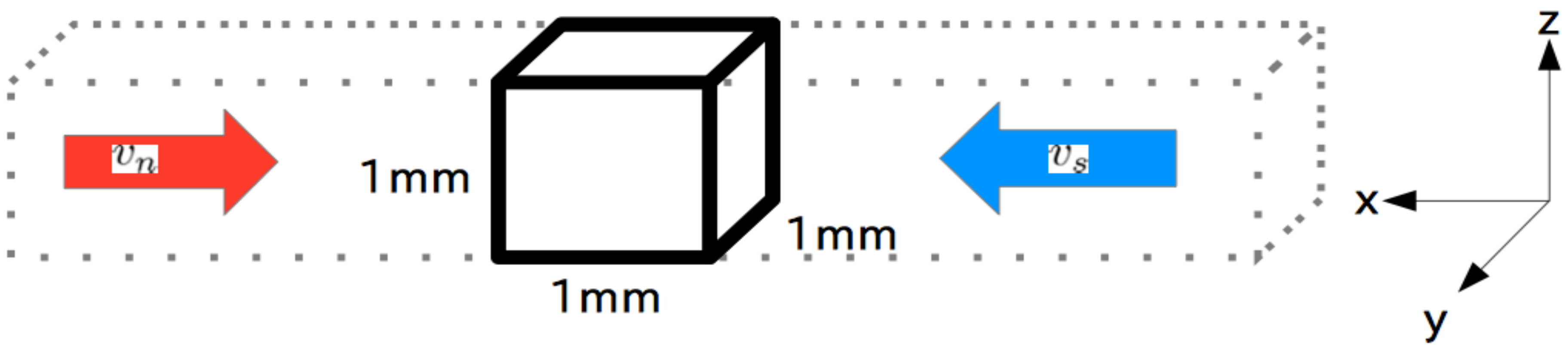
--> Solid walls can work as an absorber for vortices.

(1) Counterflow quantum turbulence of He-II in a square channel: Numerical analysiswith nonuniform flows of the normal fluid

S. Yui and M. Tsubota,
Phys. Rev. B91, 184504 (2015)

- Square cross section 1mm ×1mm
- Computational volume is 1mm ×1mm × 1mm
- Periodic B. C. along the x-axis, and solid smooth B. C. for other walls.
- T= 1.3K, 1.6K and 1.9K

Full Biot-Savart calculation

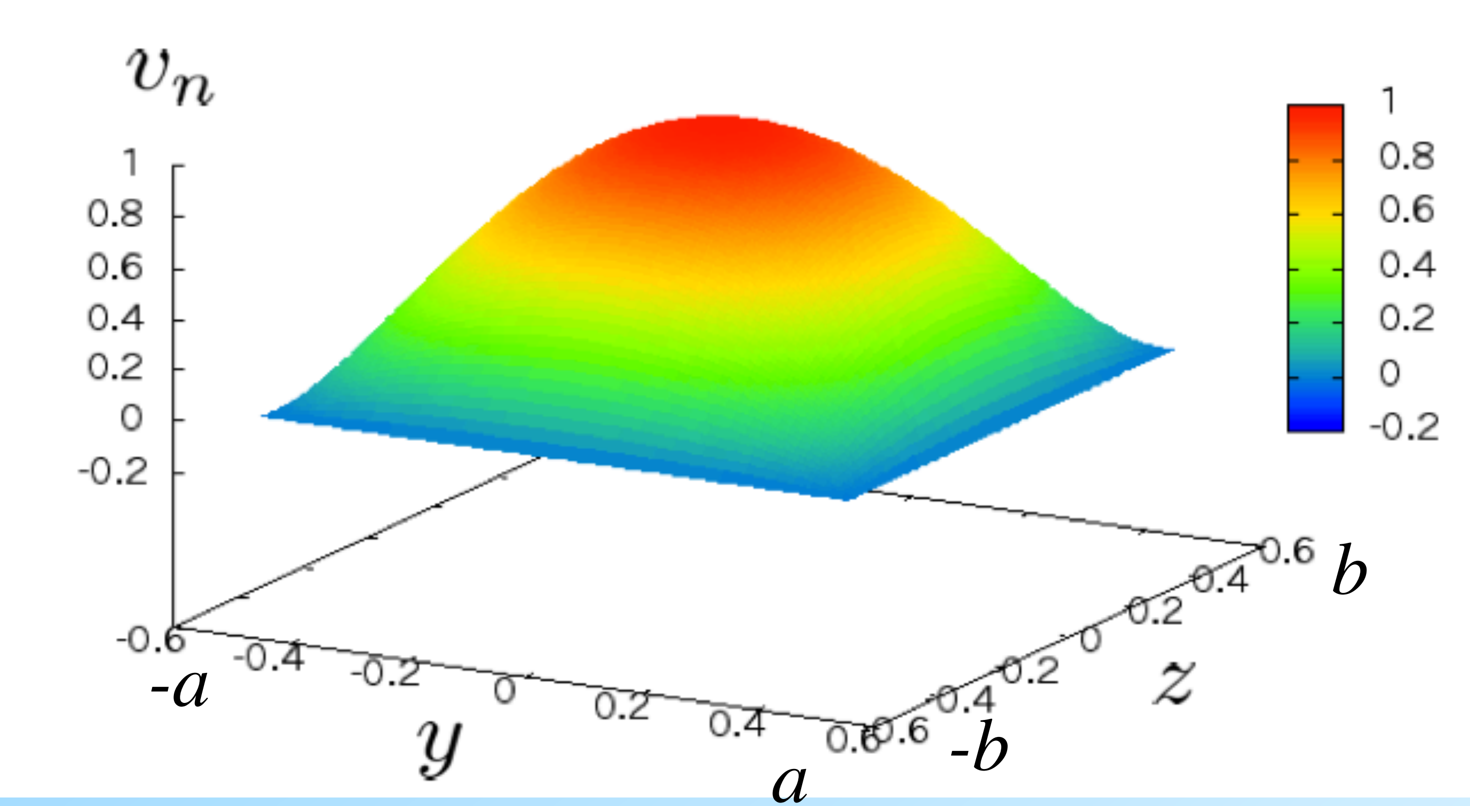


(1) Poiseuille flow in a rectangular channel

For the cross section $(-a < y < a, -b < z < b)$

$$v_n(y,z) = u_0 \sum_{i=1,3,5,\cdots}^{\infty} (-1)^{\frac{i-1}{2}} \left[1 - \frac{\cosh(i\pi z/2a)}{\cosh(i\pi b/2a)} \right] \frac{\cosh(i\pi y/2a)}{i^3}$$

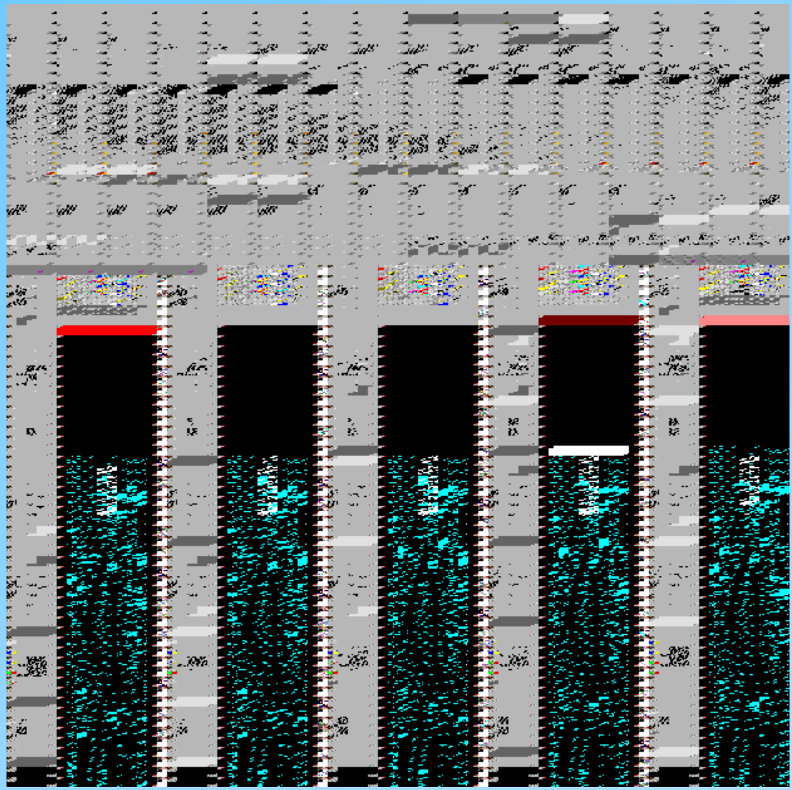
The Handbook of Fluid Dynamics, edited by R. W. Johnson (CRC, Boca Raton,1998)



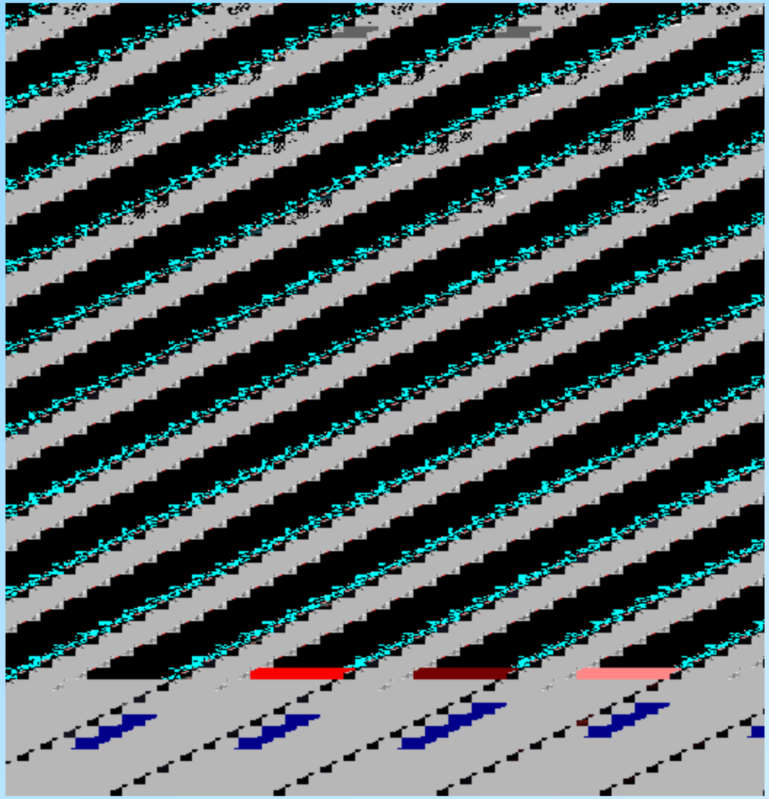
Vortex tangle in a square channel

$\otimes v_{ns}$ 1.4cm/s

T=1.3K

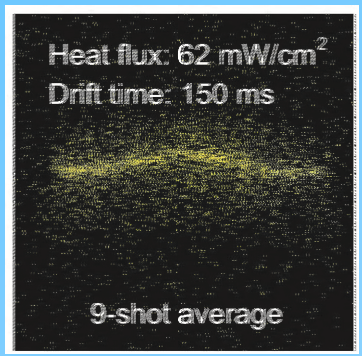


T=1.9K



1. Vortices expand from the center toward the channel walls, trapped by the walls.
2. Vortices are denser near the walls than the center.
3. At higher temperatures, the strong mutual friction grows the vortices rapidly and densely.

(2) Tail-flattened flow in a rectangular channel

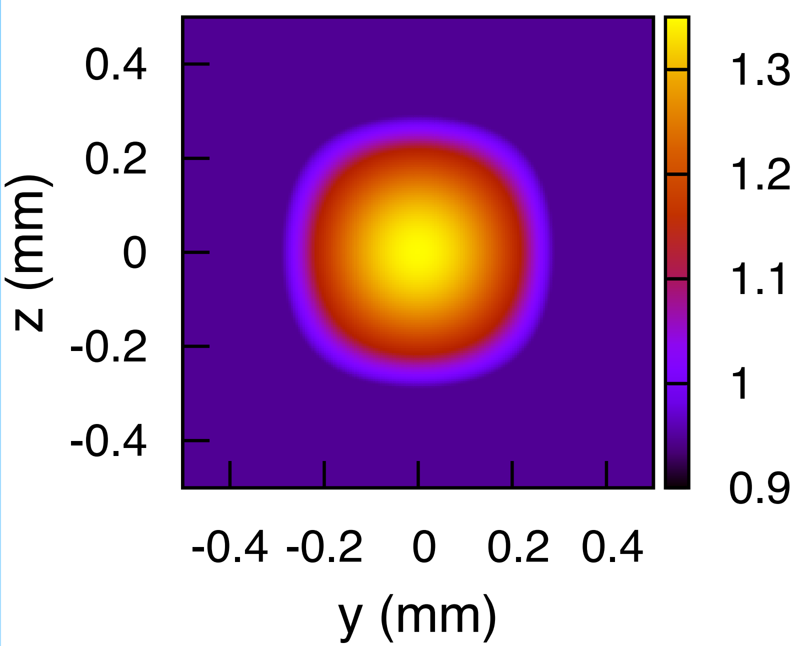


How to make the flow profile?

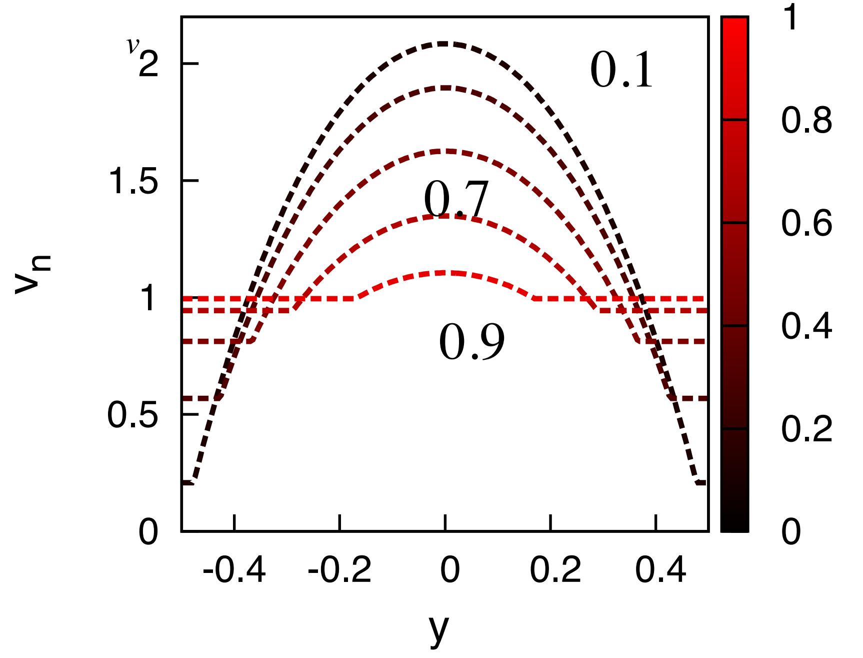
Combining the Poisuille flow $v_n^p(\mathbf{r})$ and the flat flow $v_n^p(0)$

$$v_n(\mathbf{r}) = u_0 \max[v_n^p(\mathbf{r}),\ h v_n^p(0)]$$

h=0.7



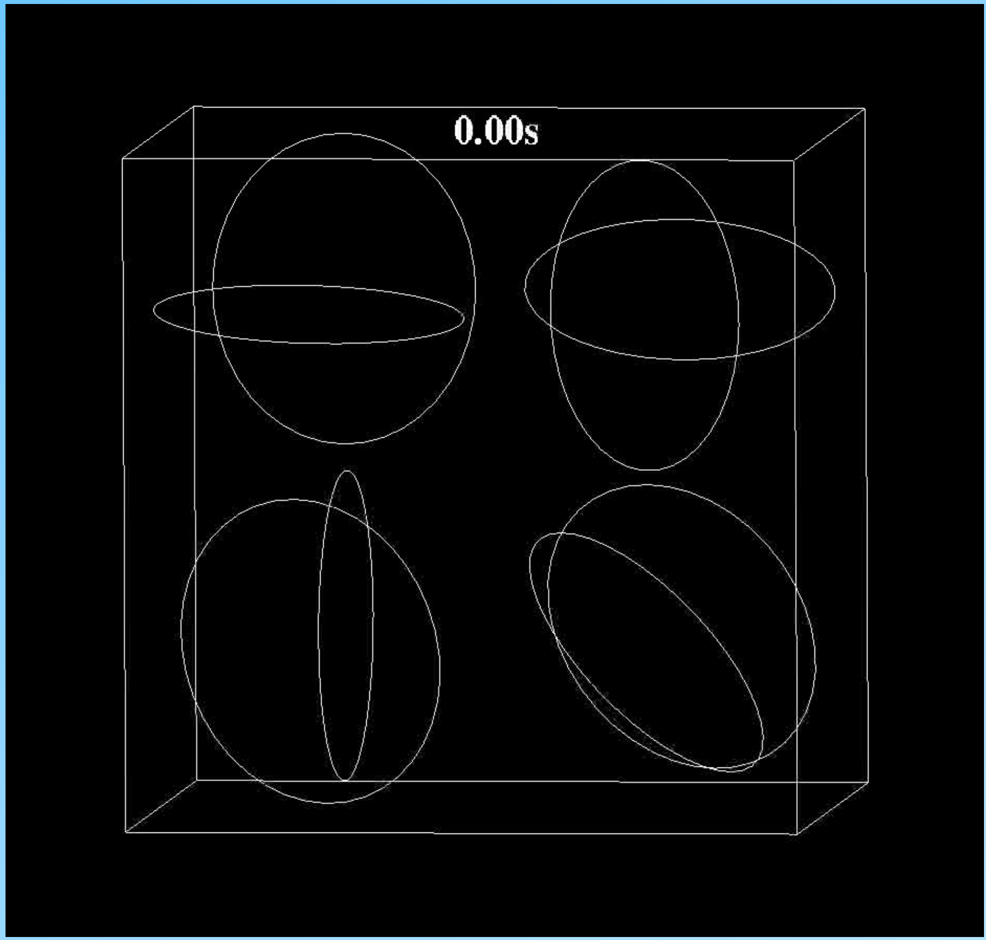
Parameter h



Increasing h makes the flow profile more uniform.

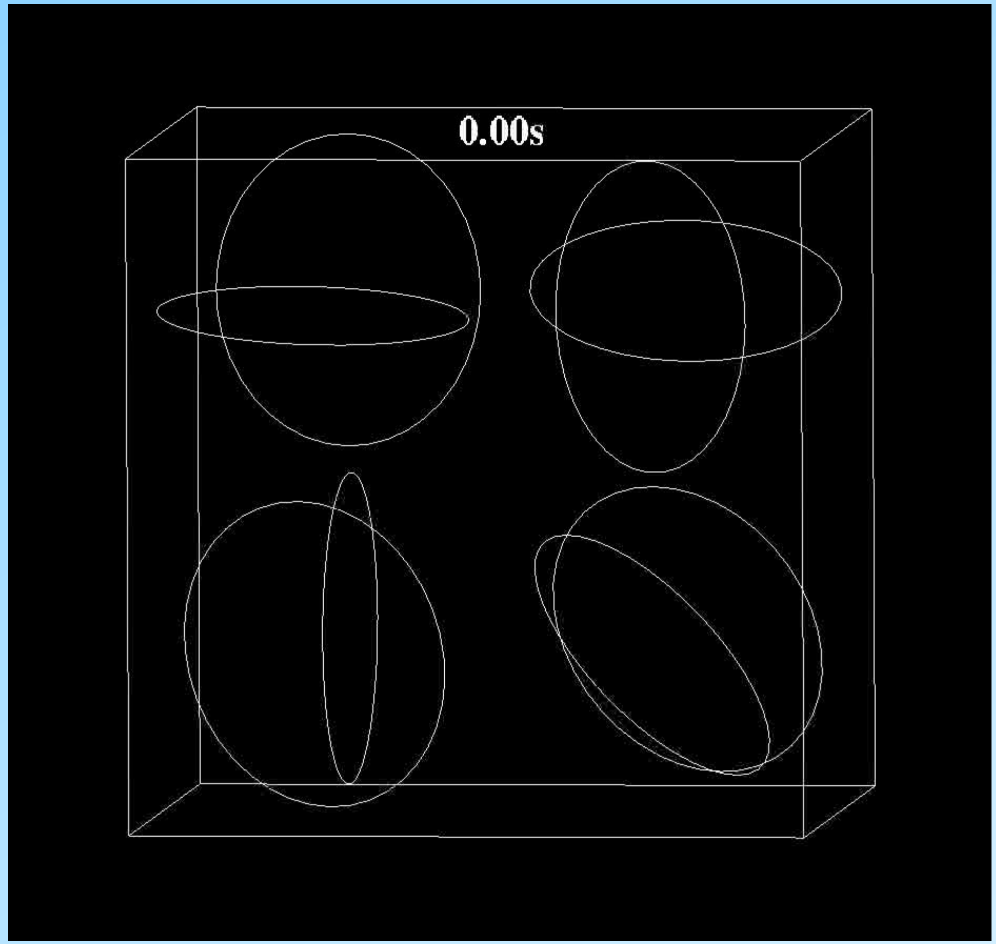
Tail-flattened flow with h=0.7

T=1.9K, v_{ns}=0.5cm/s



Poiseuille flow

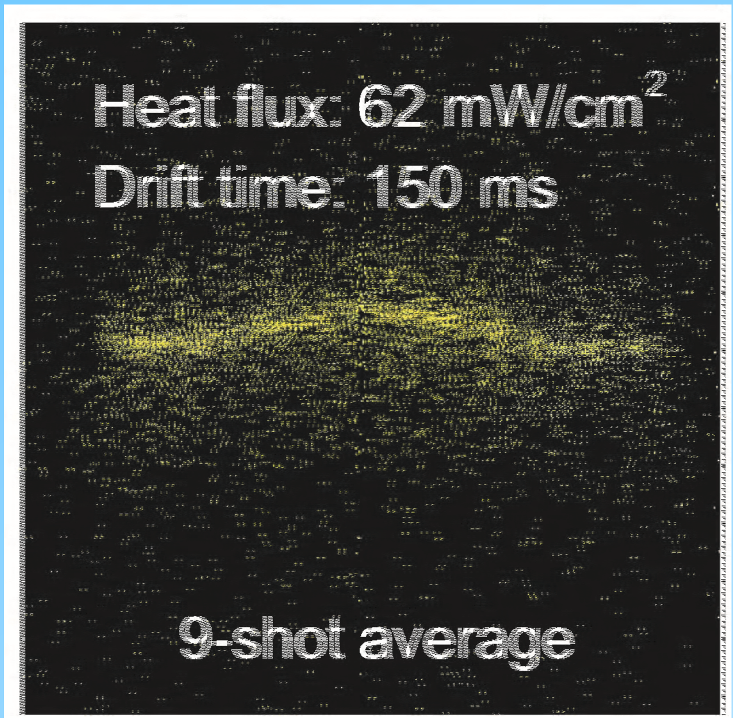
T=1.9K, v_{ns}=0.7cm/s



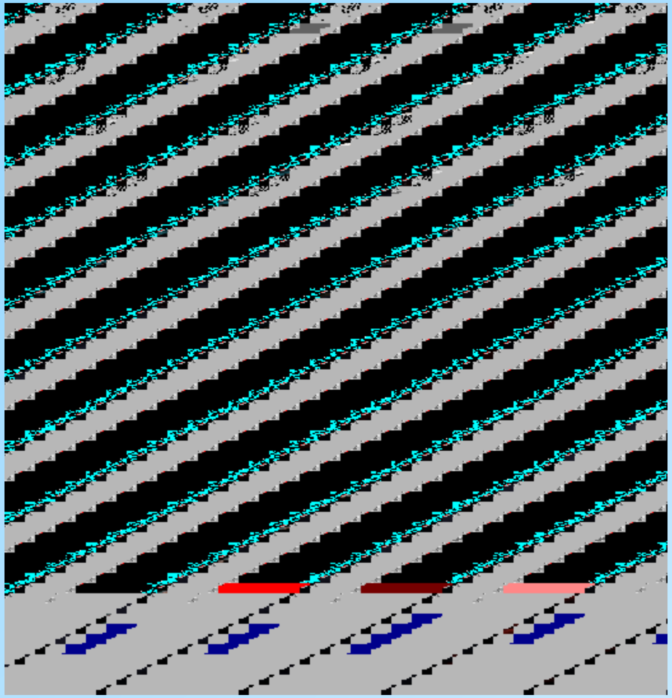
Vortex tangle is more homogeneous in tail-flattened flow than in Poiseuille flow.

What causes the tail-flattened flow?

Vortex tangle made under the prescribed Poiseuille flow



$\boldsymbol{v}_{ns}$



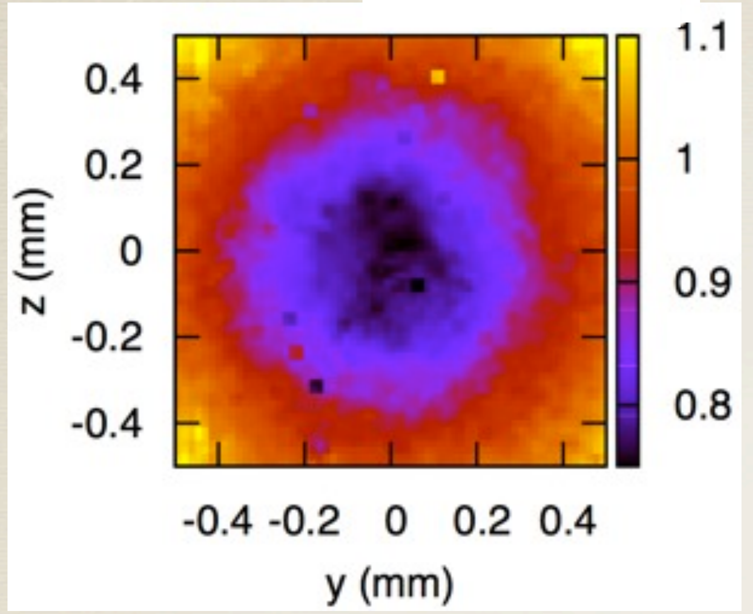
$\otimes v_{ns}$

If we turn on the mutual friction from vortices to normal fluid, the Poiseuille profile may be changed.

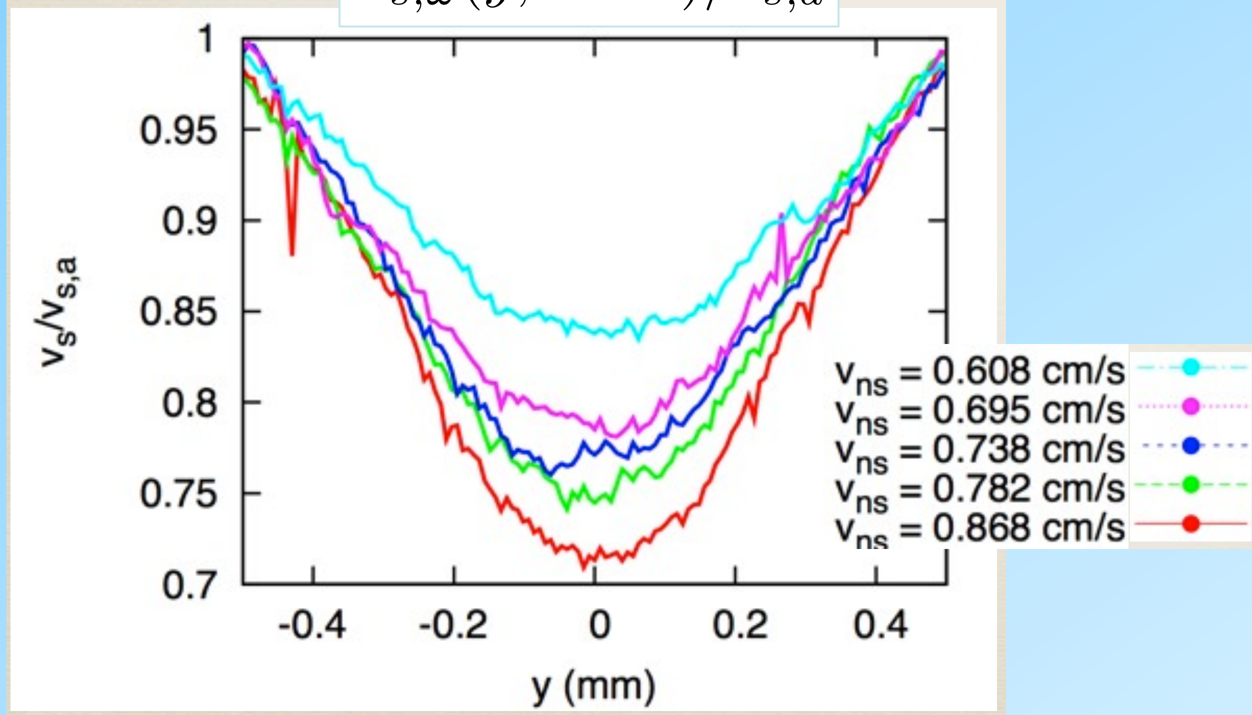
Cf. 2D simulation: L. Galantucci, M. Sciacca, C. F. Barenghi, Phys. Rev. B92, 174530 (2015)

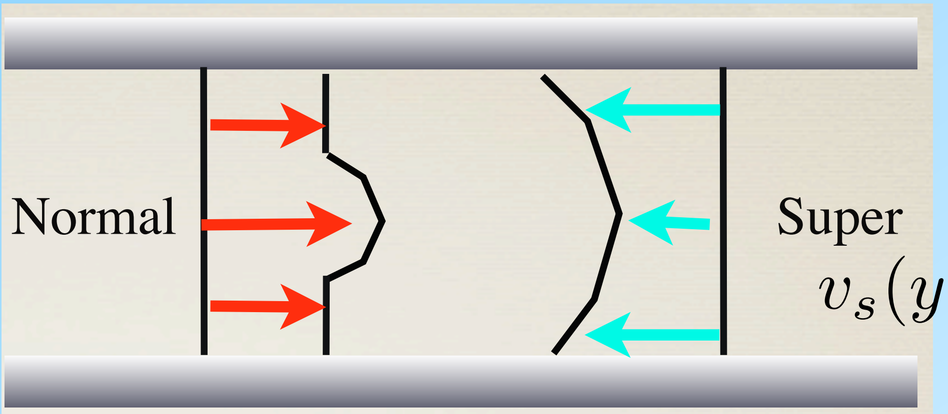
Superfluid velocity field $v_{s,\omega}(y,z)$ created by the vortex tangle

$v_{s,\omega}(y,z)/v_{s,a}$



$v_{s,\omega}(y,z=0)/v_{s,a}$



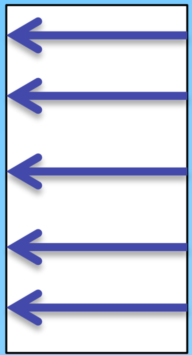


Superfluid velocity by the tangle is along v_n, depressing $v_{s,a}$.

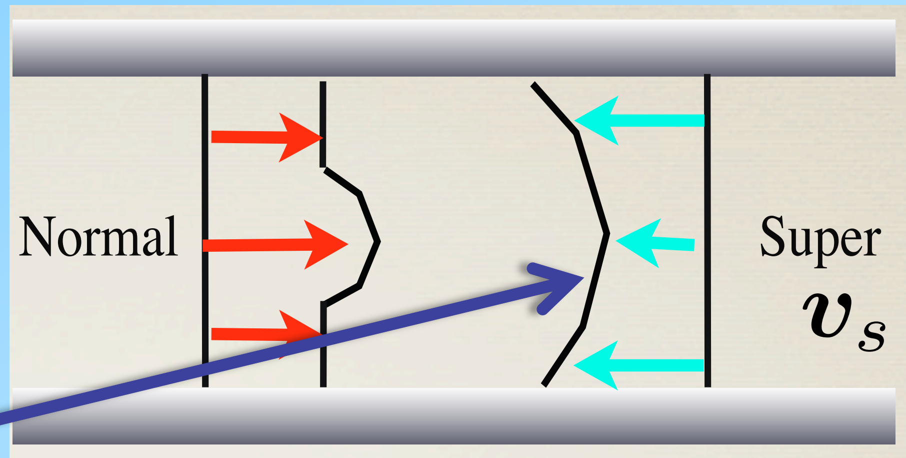
$$v_s(y,z) = v_{s,a} + v_{s,\omega}(y,z)$$

Superfluid velocity

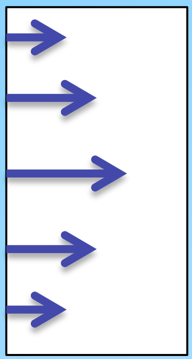
$$\boldsymbol{v}_s = \boldsymbol{v}_{s,a} + \boldsymbol{v}_{s,vortex}$$



Applied velocity $\boldsymbol{v}_{s,a}$

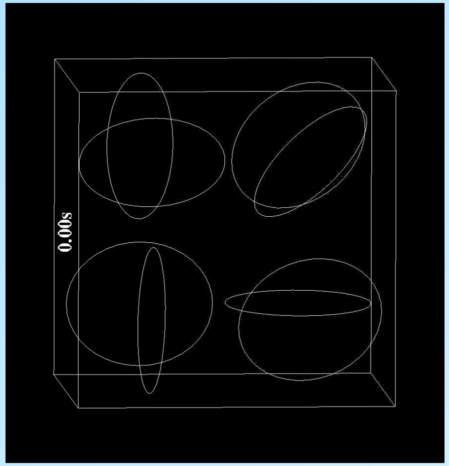


+

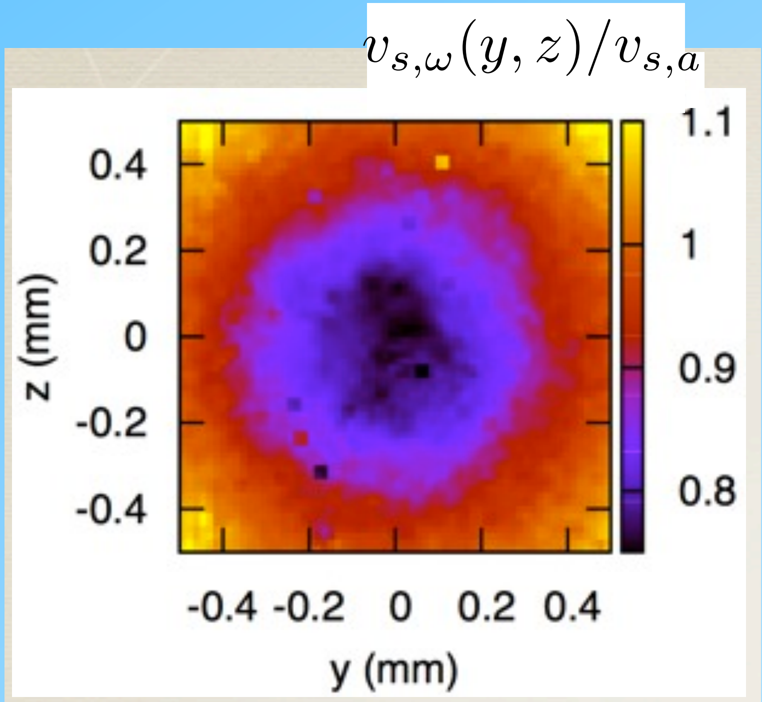


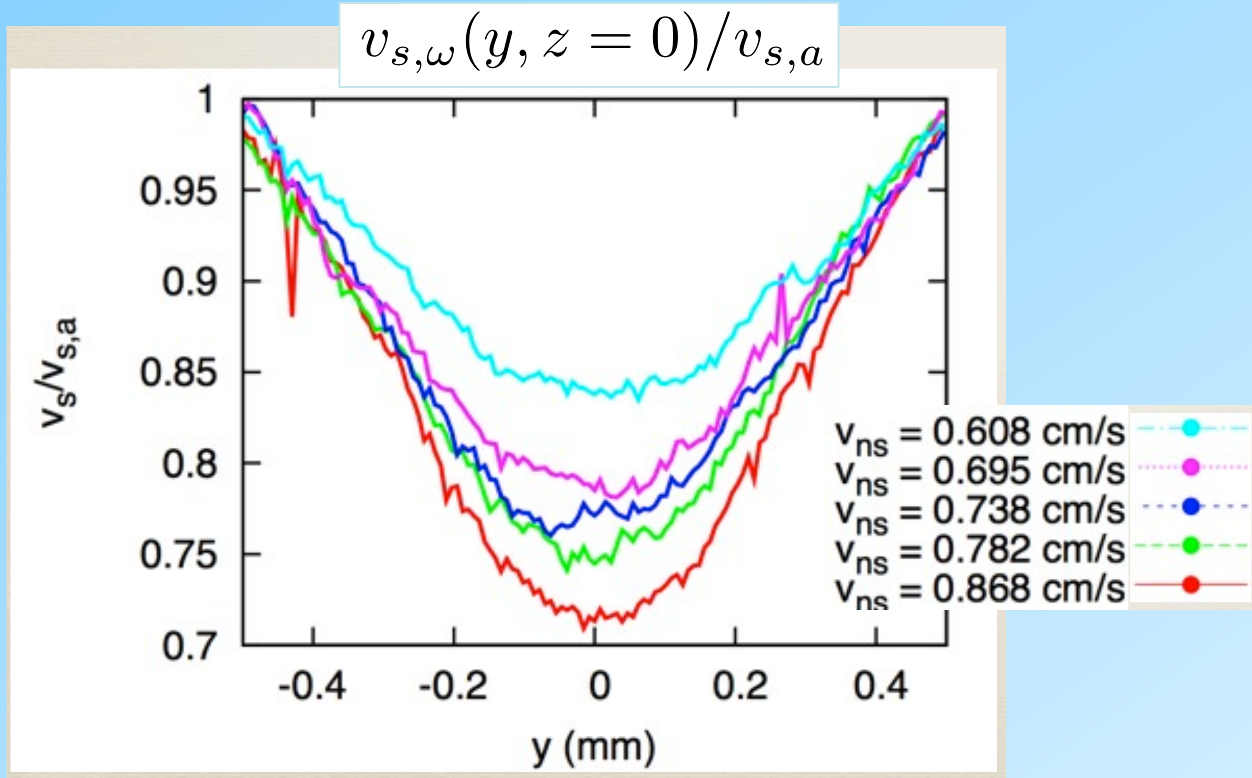
Velocity made by the vortex tangle

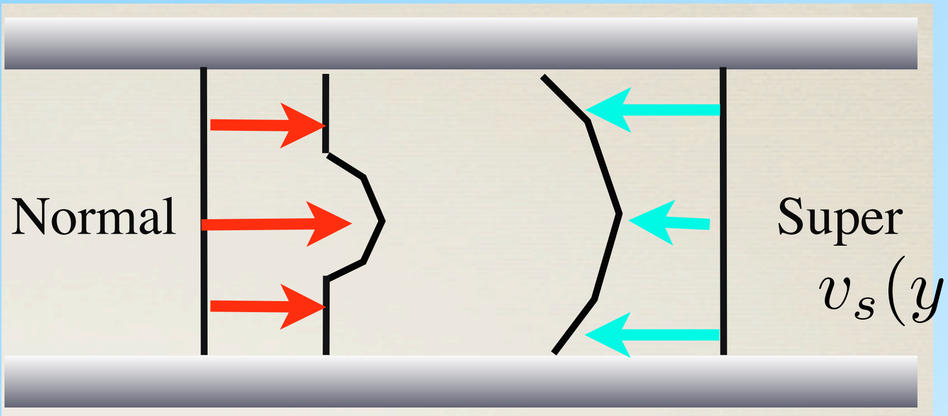
$$\boldsymbol{v}_{s,vortex} = \frac{\kappa}{4\pi} \int_{\mathcal{L}} \frac{(\boldsymbol{s}_1 - \boldsymbol{r}) \times d\boldsymbol{s}_1}{|\boldsymbol{s}_1 - \boldsymbol{r}|^3}$$



Superfluid velocity field $v_{s,\omega}(y,z)$ created by the vortex tangle







Superfluid velocity by the tangle is along v_n, depressing $v_{s,a}$.

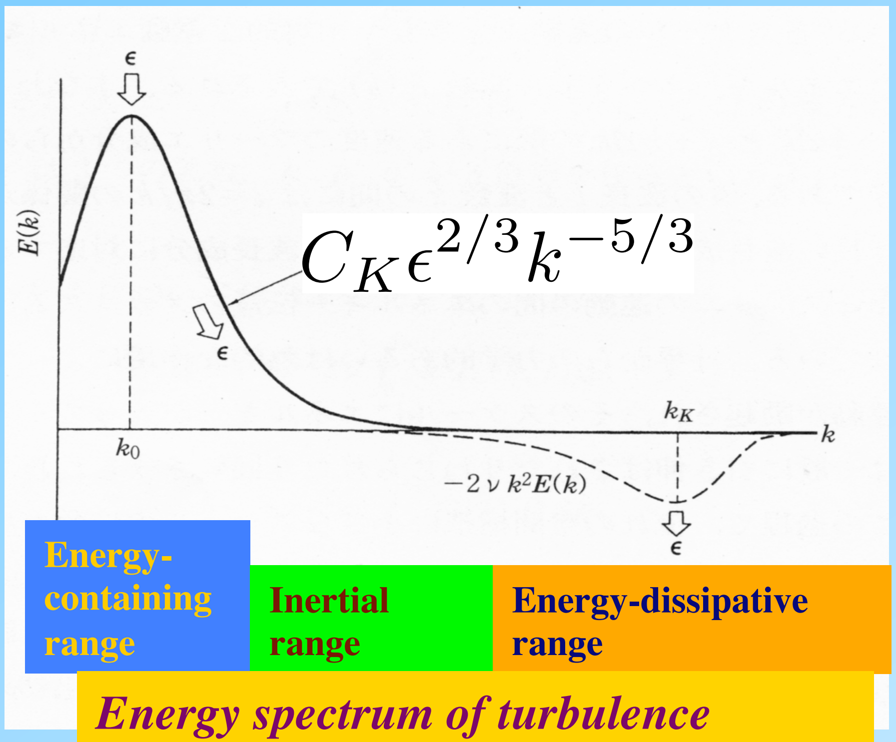
$$v_s(y,z) = v_{s,a} + v_{s,\omega}(y,z)$$

(2) Logarithmic velocity profile of quantum turbulence of superfluid ^{4}He

S. Yui, K. Fujimoto and M. Tsubota, Phys.Rev.B92, 224513 (2015)

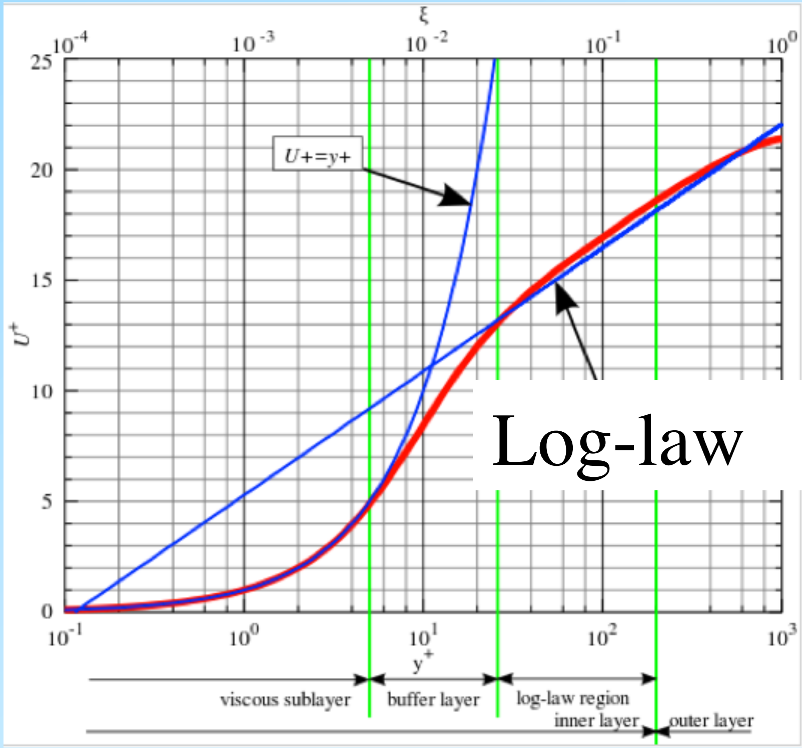
Two well-known statistical laws in classical turbulence

Kolmogorov -5/3 law in the bulk

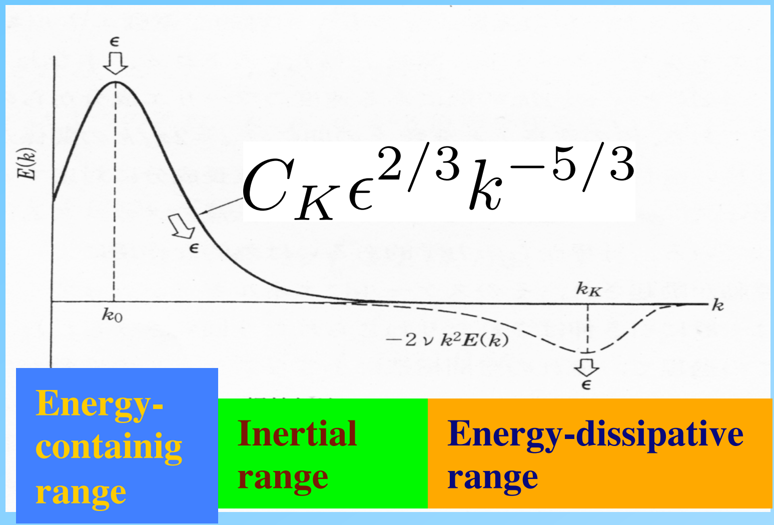


Energy spectrum of turbulence

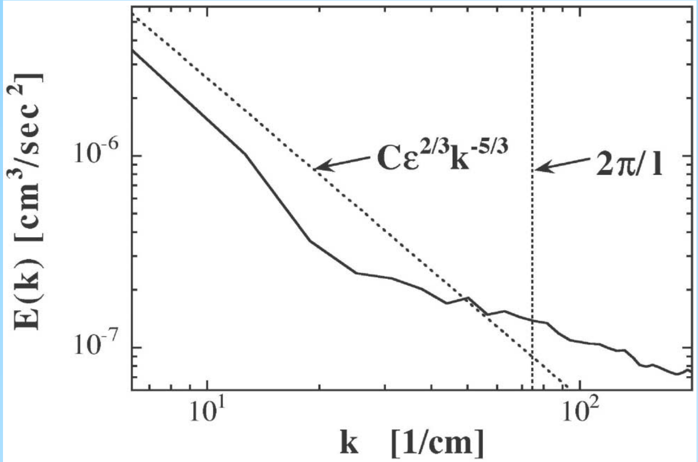
Log- law near walls



Classical turbulence vs Quantum turbulence

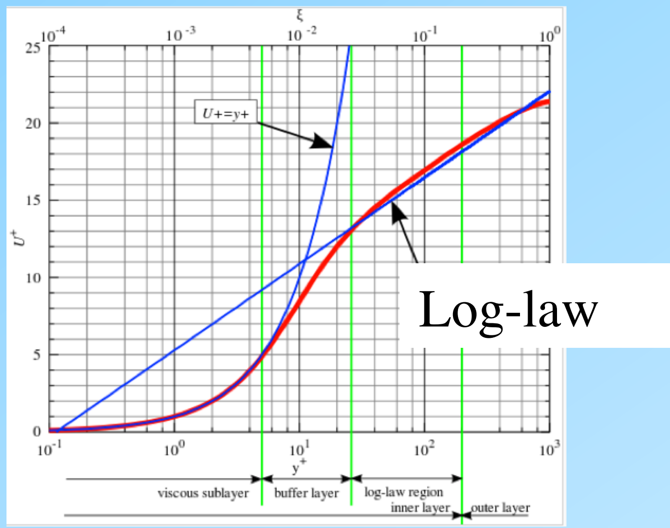


Kolmogorov -5/3 law in the bulk

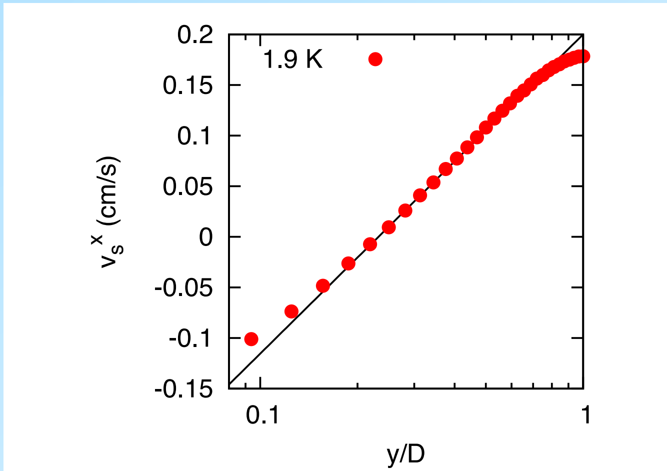


T. Araki, M. Tsubota and S. K. Nemirovskii, PRL89, 145301(2002)

Paris, Osaka, New Castle, …

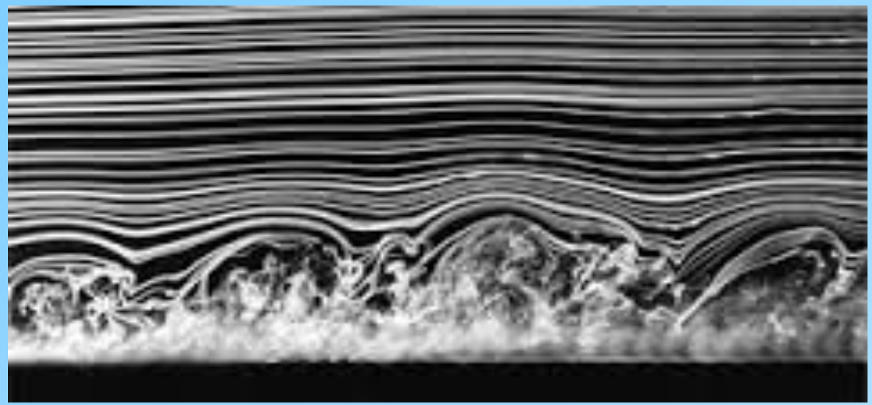


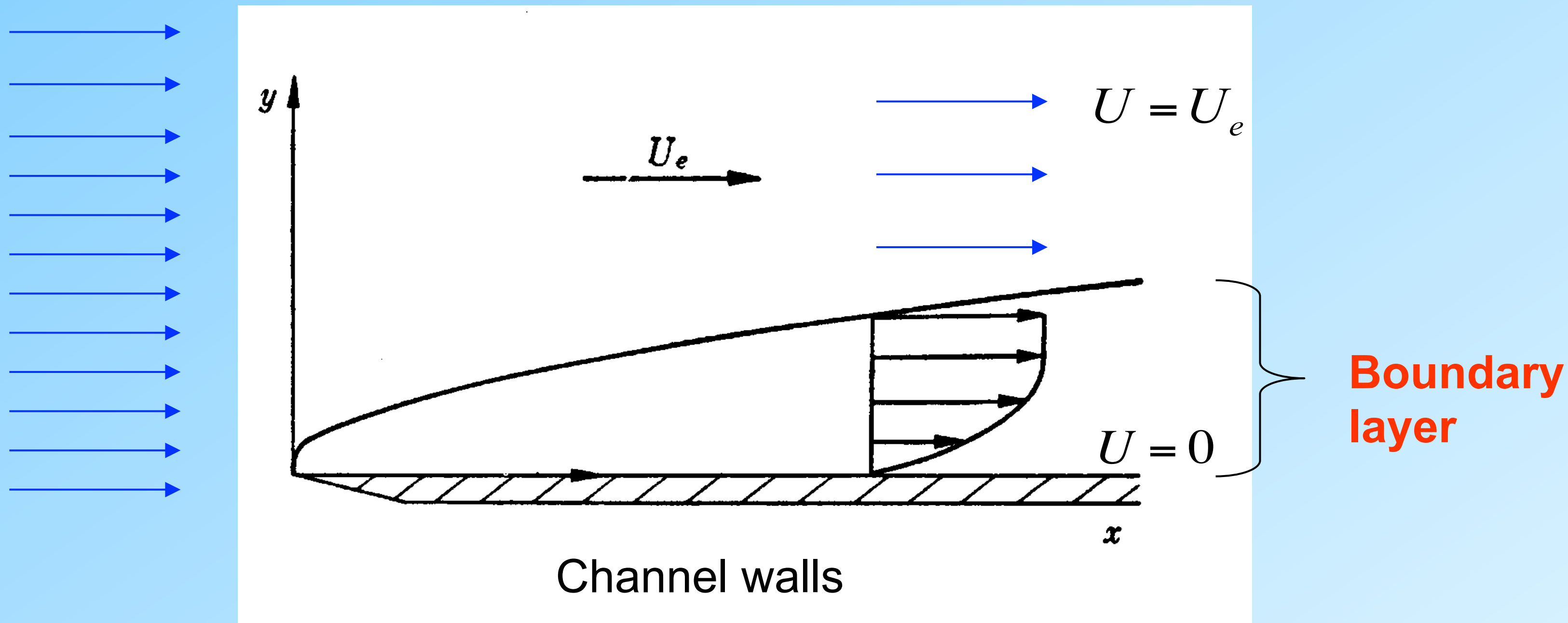
Log- law near walls



S. Yui, K. Fujimoto and M. Tsubota, PRB92, 224513 (2015)

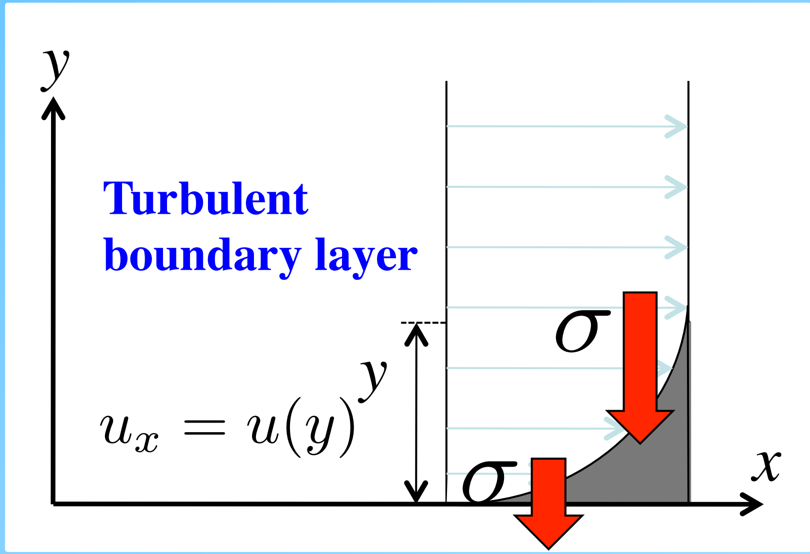
Turbulent boundary layer in a classical fluid





How to derive the log-law (1)

cf. Prandtl: Boundary layer theory
Landau-Lifshitz: Fluid Mechanics



Averaged velocity $u_x = u(y), u_y = u_z = 0$

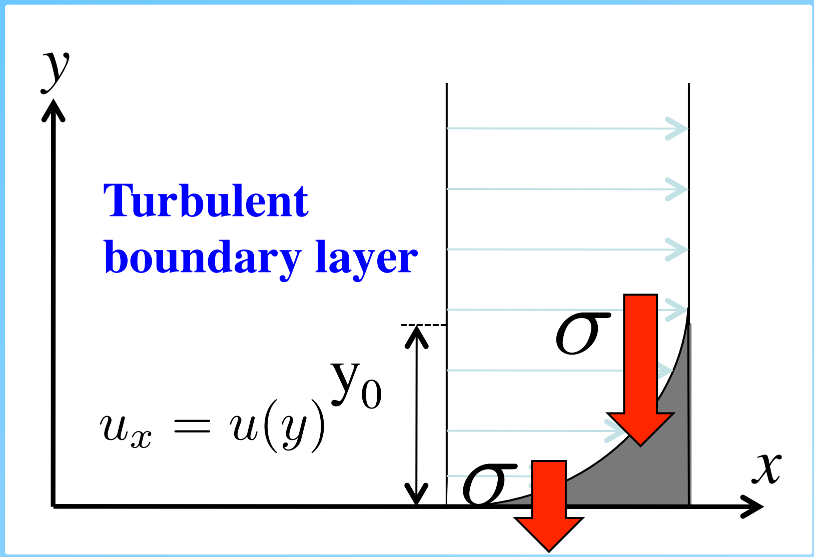
- Viscosity is not available except near the walls..
- Constant momentum flux σ (Reynolds stress) flows from the bulk to the walls.
- σ dissipates by the viscosity near the walls.

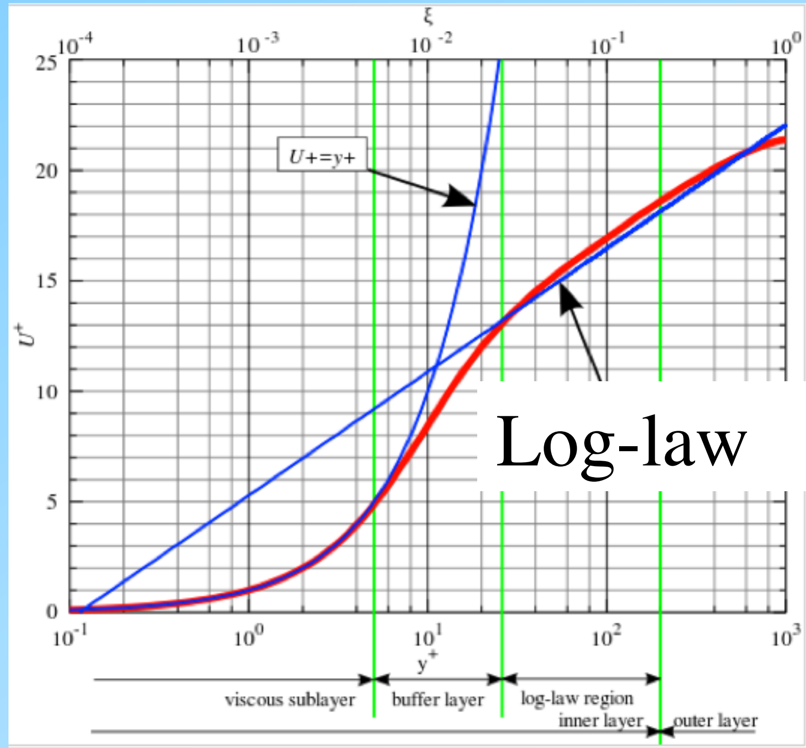
du/dy is determined only by the fluid density ρ, momentum flux σ, distance y.

Dimension $[du/dy] = 1/T, [\rho] = M/L^3, [\sigma] = M/(L \cdot T^2), [y] = L$

$$\frac{du}{dy} = \frac{\sqrt{\sigma/\rho}}{by}, \qquad b = 0.417 \text{ :Karman constant}$$

How to derive the log-law (2)





What determines the width y_0 of the boundary layer ?

$$Re = \frac{v_* y_0}{\nu} \sim 1 \qquad \rightarrow \qquad y_0 \sim \frac{\nu}{v_*}$$

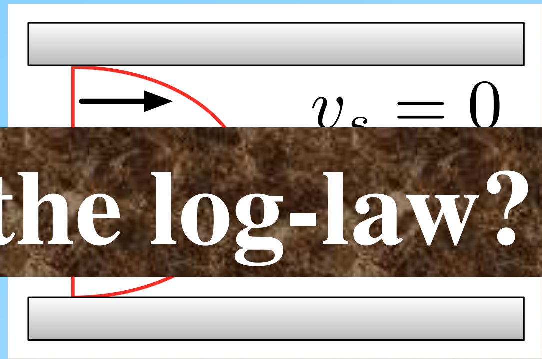
By some considerations,

$$\frac{u}{v_*} = \frac{1}{b} \log\left(\frac{y}{y_0}\right)$$

Quantum-turbulent boundary layer

S. Yui, K. Fujimoto and M. Tsubota,
Phys. Rev. B 92, 224513 (2015)

Pure normal flow between two parallel plates



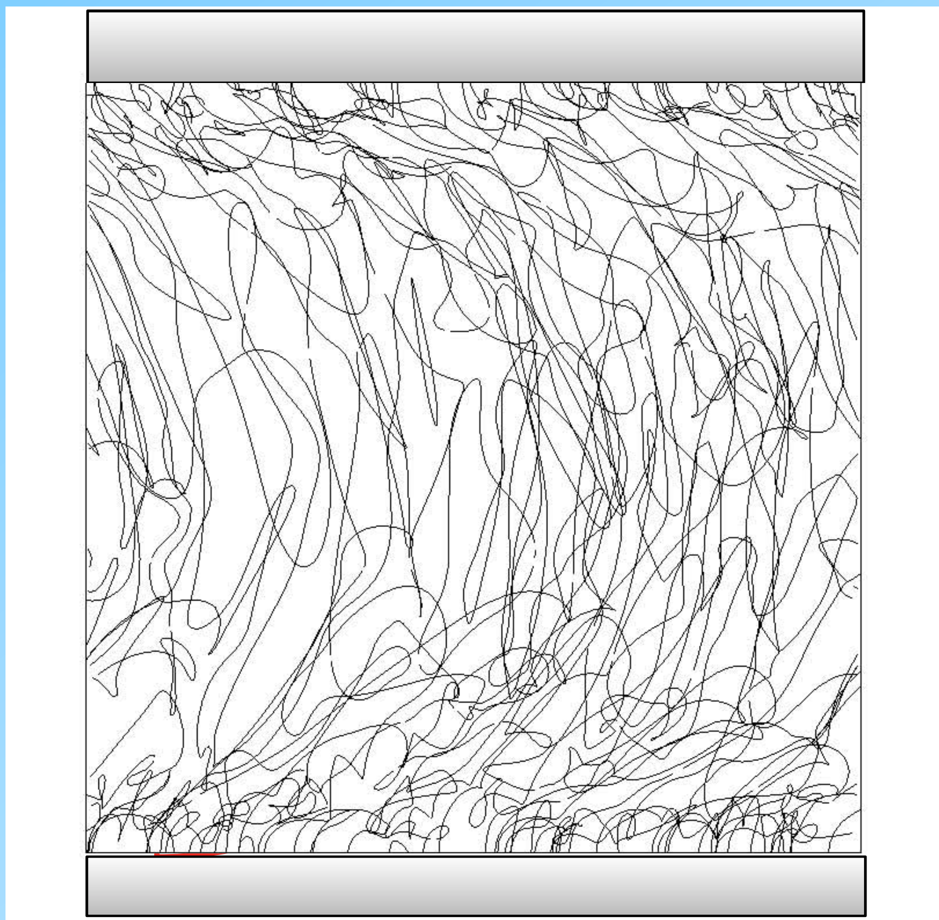
How about the log-law?

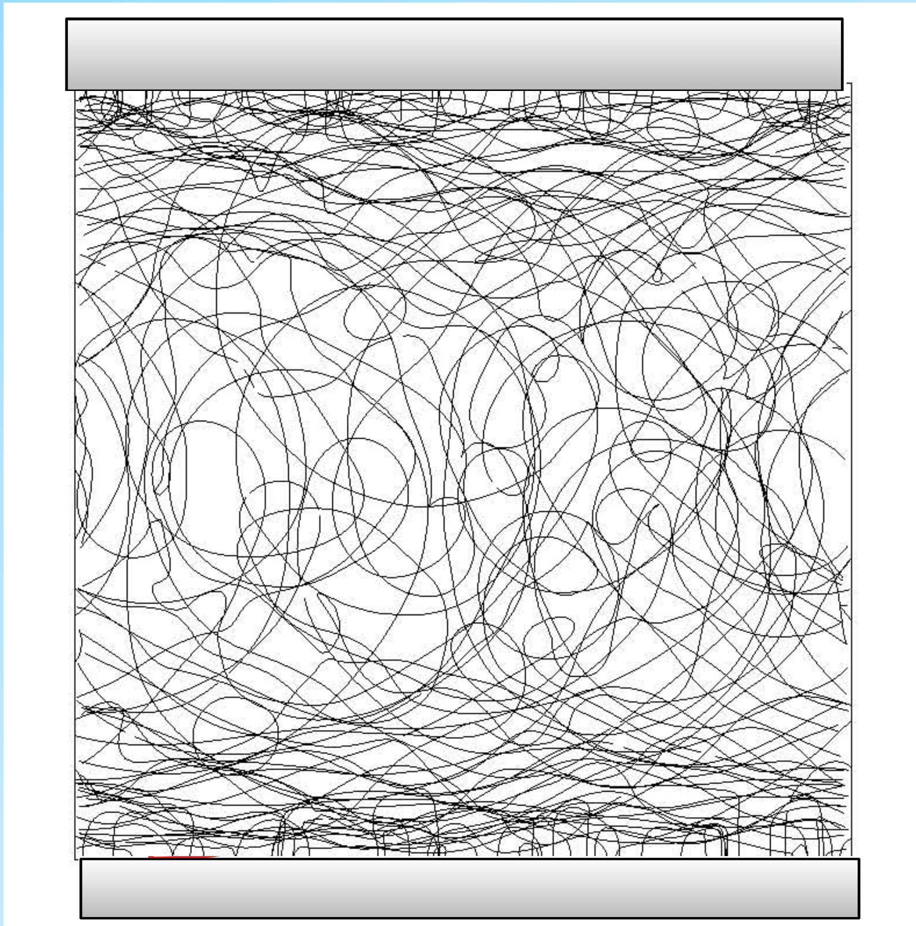
$\longrightarrow v_n$

T=1.9K

$\overline{v_n}$ =0.9cm/s

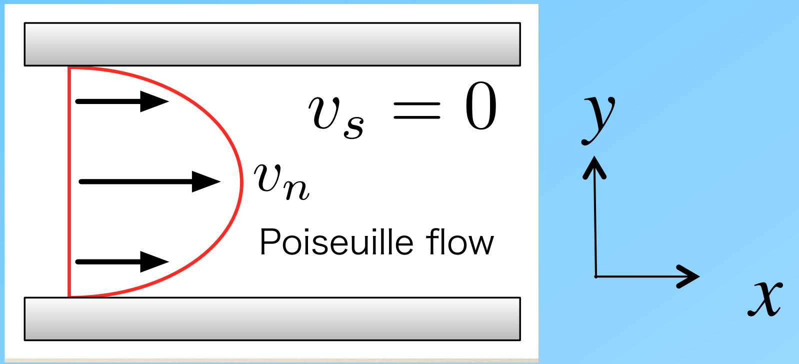
$\otimes\, v_n$



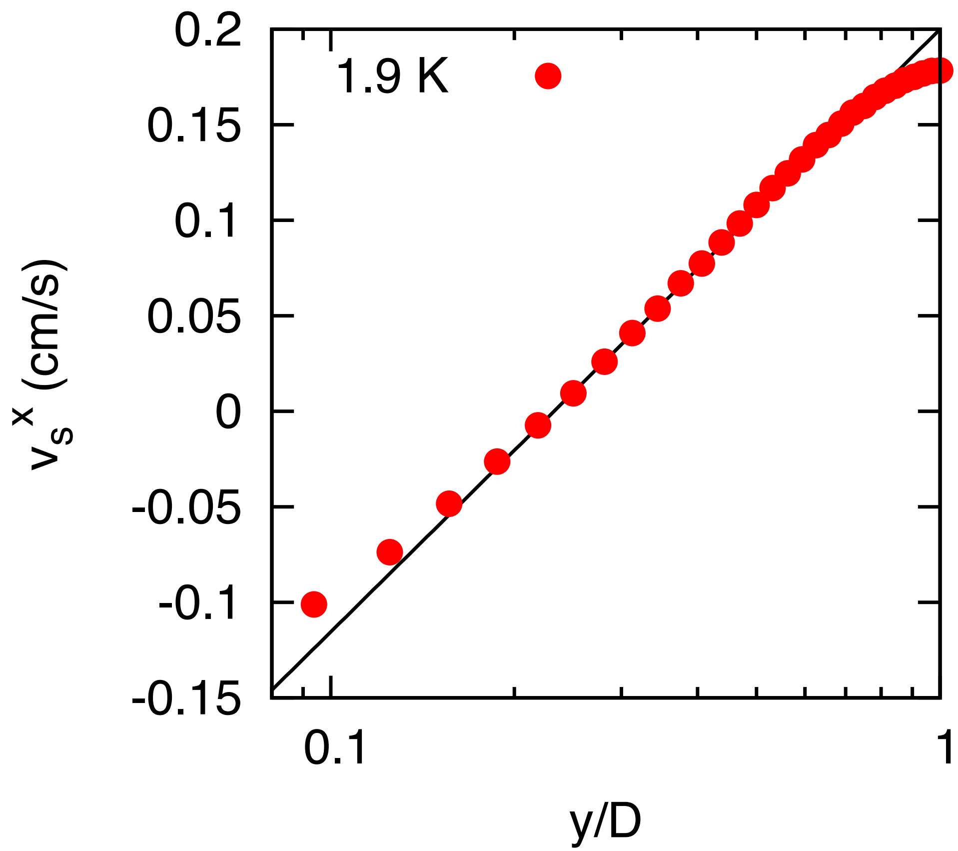


Quantum-turbulent boundary layer

$$\boldsymbol{v}_s = \frac{\kappa}{4\pi}\int_{\mathcal{L}} \frac{(\boldsymbol{s}_1 - \boldsymbol{r}) \times d\boldsymbol{s}_1}{|\boldsymbol{s}_1 - \boldsymbol{r}|^3}$$

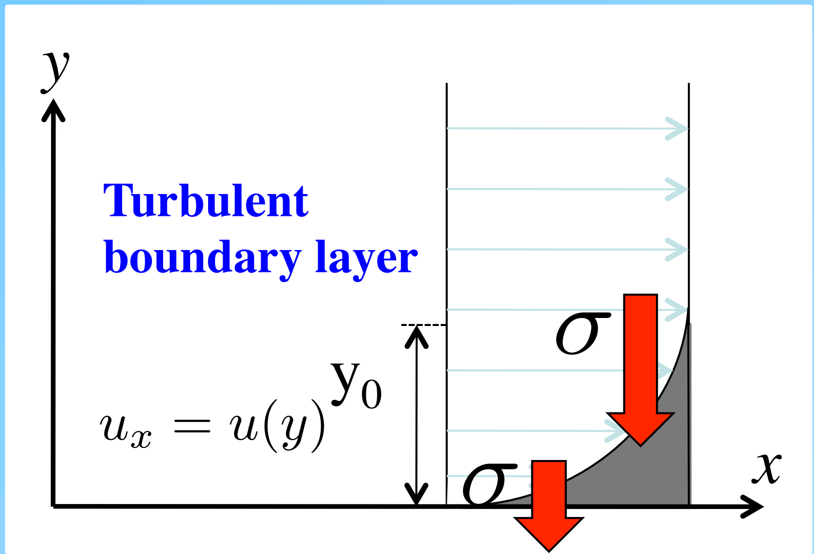


The averaged velocity of turbulent superfluid flow obeys the log-law !

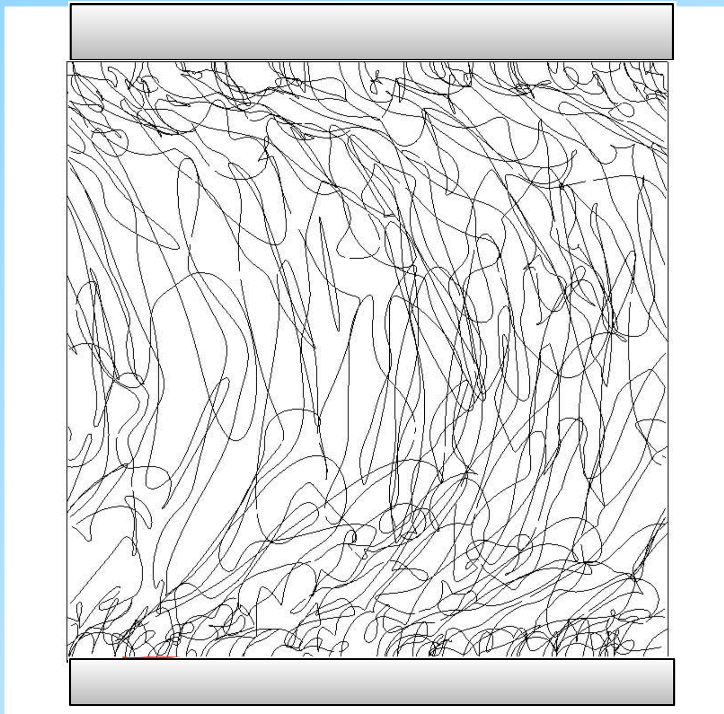


Analogy between classical and quantum cases

Classical



Quantum



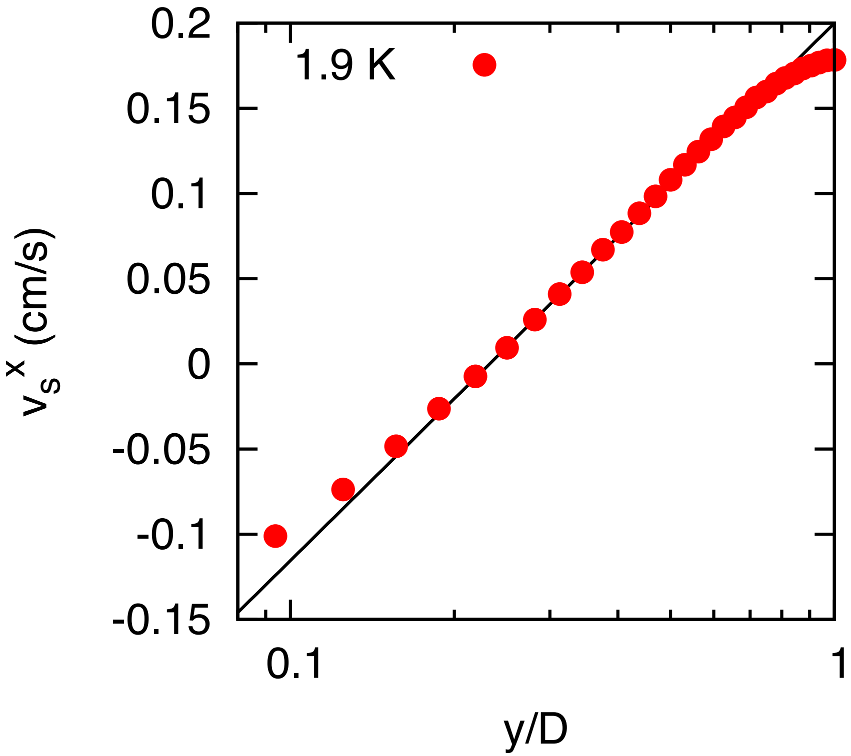
Momentum is transferred toward the walls.

Vortices move toward the walls with the momentum.

How about the Karman constant?

Classical case

$$v_* = \sqrt{\sigma/\rho}$$



$$v_s^x = \frac{v_q^*}{\kappa_q}\left[\log\left(\frac{y}{D}\right) + c\right]$$

T (K)	v_0 (s/cm)	v_q^*/κ_q (s/cm)	c –
1.9	0.184	0.141	1.46
1.6	0.079	0.070	1.40
1.3	0.025	0.028	1.14

We know v_q^*/κ_q from the fitting. Since we have no theory for v_q^*, however, we cannot obtain the Karman constant κ_q.

Summary

1. We review the research of numerical simulation of quantum turbulence in atomic BECs and superfluid helium.

2. The recent visualization experiments open the door for inhomogeneous turbulence in superfluid helium.

3. We discussed the two topics.

 3-1. Inhomogeneous turbulence in a square channel

S. Yui, M. Tsubota, Phys. Rev. B91, 184504(2015)

 3-2. Log-law in turbulent boundary layer

S. Yui, K. Fujimoto, M. Tsubota, Phys. Rev. B 92, 224513(2015)