SINGLE/MULTI-COMPONENT QUANTUM GASES:



NON-EQUILIBRUM MODELS



& APPLICATIONS TO EXPERIMENTS



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<u>GENERAL CASE: QUANTUM GAS AT 0 < T < Tc</u>

A typical experiment, features both condensed and thermal atoms

 $T \sim 100nK$ $N \sim 10^3 - 10^8$ atoms $n_{3D} \sim 5*10^{12} - 10^{14} cm^{-3}$



Need to go beyond the (simple) Gross-Pitaevskii Equation!

Require a self-consisent non-equilibrium finite Temperature theory to describe both "subsystems" & their couplings



There are (at least) 2 fundamentally different, yet complementary, approaches to partially condensed (T > 0) Systems





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Kinetic Approaches (explicit BEC separation) **Stochastic Approaches** (no explicit BEC separation)

BEC + Dynamical Thermal Cloud with full self-consistent coupling

NON-BEC

BEC



Modes up to a cut-off described in a unified manner (classical field) coupled to a Heat Bath



Collective Modes/Dynamics Full BEC – Thermal Coupling

(far from critical region)

Random (shot-to-shot) Fluctuations Quenches / Low-D & Universality (high-lying modes "unaffected")

MODELLING COLD ATOMS AT T > 0

There are (at least) 2 fundamentally different, yet complementary, approache

Kinetic Ap (explicit BEC

BEC + Dynamica with full self-con



Collective Mod Full BEC – The

(far from critica,,

e.g. Additional Scheme

'Classical Field' Evolution

(e.g. Berloff – Barenghi – Davis)



Only consider modes populated 'clasically' (up to some cutoff) such that N(E) >> 1

[Microcanonical Ensemble]

> 0) Systems

ic Approaches **BEC** separation)

a cut-off described nner (classical field) to a Heat Bath



to-shot) Fluctuations _ow-D & Universality ...g modes "unaffected"



MODELLING REVIEWS:

Quantum Gases: Finite Temperature and Non-Equilibrium Dynamics Proukakis et al. (Eds), Imperial College Press/World Scientific (2013)

Berloff, Brachet & Proukakis, PNAS 111 (Suppl. 1) 4675 (2014) Proukakis & Jackson, J Phys B 41, 203002 (2008) Blakie, Bradley, Davis, Ballagh & Gardiner, Adv. Phys. 57, 363 (2008)





Kinetic Approaches (explicit BEC separation)

KINETIC MODEL ("ZNG" or Gross-Pitaevskii-Boltzmann)



Based on Symmetry-breaking; System artificially (conveniently) separated into 2 parts



$$\hat{\Psi} \rightarrow \left\langle \hat{\Psi} \right\rangle + \hat{\delta} = \phi + \hat{\delta}$$

& follow truncated hierarchy of coupled equations of motion

KINETIC MODEL ("ZNG" or Gross-Pitaevskii-Boltzmann)

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APPLICATION I: EXCITATION FREQUENCIES



"Scissor's Mode" Excitation

Expt: Marago et al, PRL 86, 3938 (2001) Theory: Jackson-Zaremba, PRL 87, 100404 (2001)



Quadrupolar Excitations

П

0.9

0.7

T'

Expt: Jin et al, PRL 78, 764 (1997) Theory: Jackson-Zaremba PRL 88, 180402 (2002)

→ Excellent Quantitative Agreement with Experiments

APPLICATION II: EXPERIMENTAL DARK SOLITON DECAY

B

Jackson, Proukakis & Barenghi, Phys. Rev. A 75, R051601 (2007)

EXPERIMENT S Burger *et al.* Phys. Rev. Lett. **83**, 5198 (1999)

GROSS-PITAEVSKII THEORY **T = O**

> GENERALIZED MEAN FIELD (ZNG) **T > O**



→ Soliton Disappears on Experimental Timescales

Soliton Oscillations at extremely low T, subsequently observed in Hamburg; Heidelberg

APPLICATION III: EXPERIMENTAL VORTEX DECAY



Jackson, Proukakis, Barenghi & Zaremba, Phys. Rev. A 79, 053615 (2009)



Off-centered Vortex spirals out from centre and decays gradually with sound emission





→ Findings are Qualitatively Consistent with Vortex Experiments
 See e.g. Freilich et al., Science 329 (2010)

APPLICATION III: EXPERIMENTAL VORTEX DECAY



Jackson, Proukakis, Barenghi & Zaremba, Phys. Rev. A 79, 053615 (2009)



Off-centered Vortex spirals out from centre and decays gradually with sound emission

Vortex position grows exponentially

$$\frac{dr_{v}}{dt} = \omega \left(\frac{\gamma}{\omega}\right) r_{v} = \alpha \omega r_{v}$$







Indirect Evidence from Experimental co-rotating vortex dipole decay study

APPLICATION IV: SURFACE EVAPORATIVE COOLING

Move Thermal Cloud onto a Room-Temperature Surface & Study Atom Loss & BEC Growth at Different Distances from Surface

Effective Potential near Surface

surface

tota

Study Atom Loss & BEC Growth at Different Distances from Surface



Theory-Experiment Comparison (different "hold" positions)

J. Maerkle, A.J. Allen, P. Federsel, B. Jetter, A. Günther, J. Fortágh, N. P. Proukakis, and T. E. Judd PRA 90, 023614 (2014)





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SGPE describes the entire multi-mode system describing the low-lying modes





STOCHASTIC GROSS-PITAEVSKII (SGPE) MODEL



SGPE describes the entire multi-mode system describing the low-lying modes

$$i\hbar \frac{\partial \Phi(x,t)}{\partial t} = \left(1 - i\gamma\right) \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{TRAP} - \mu + g\left|\Phi(x,t)\right|^2\right] \Phi(x,t) + \eta(x,t)$$

 \rightarrow Results obtained by averaging over noise realizations $\eta(x,t)$

$$\langle \eta^*(x,t)\eta(x',t')\rangle = 2\hbar\gamma k_B T\delta(x-x')\delta(t-t')$$

so supposed to be interpreted after suitable 'trajectory' averaging Stoof-Bijlsma J Low Temp Phys 124, 431 (2001); Gardiner-Davis J Phys B 36, 4731 (2003)



- * Contain element of stochasticity
 * Qualitatively reproduces single experimental realisations
- * Wash out density fluctuations to produce smooth profiles
- * Suitable for extracting global features (densities, correlation functions, etc.)



Properties Characterised by Densities & Lowest Order Correlation Functions

Quasi-1D: Ab Initio Prediction of densities & coherences



Quasi-2D: Scale-invariance & Universality



Experiments: Paris & Amsterdam

PRL 97, 250403 (2006) PRL 100, 090402 (2008) PRL 105, 230402 (2010) PRL 91, 010405 (2003) EPJD 35, 155 (2005)

Ab Initio SGPE Modelling: NPP et al., PRA 84, 023613 (2011) PRA 86, 013627 (2012)

> Experiment: Chicago

Nature 470, 236 (2011)

Ab Initio SGPE Modelling: Cockburn & NPP PRA 86, 033610 (2012)

Detailed Theoretical Benchmarking: Cockburn et al. PRA 83, 043619 (2011)

APPLICATION II: DARK SOLITON EVOLUTION



Resort to 'statistical analysis' of soliton decay times Soliton is 'lost' within each run, when it becomes comparable to the noise



- → Significant spread between different soliton trajectories/lifetimes
- → Predictions of few very long-lived soliton trajectories

Consistent with Experimental Findings .(but in different regime!)

Cockburn, Nistazakis, Horikis, Kevrekidis, Proukakis & Frantzeskakis PRL 104, 174101 (2010)

Consider rapidly quenching a system through the phase transition



Hadzibabic et al, Science 347 (2015)

→ System Grows Locally-Coherent Patches with Uncorrelated Phases Separated by Phase Defects (e.g. Dark Solitons / Vortices)

Kibble-Zurek Model:

Number of Defects:

$$N \propto (\tau_Q)^{-\alpha}$$

(Defect "Type" Depends on Dimensionality)

Review: del Campo & Zurek, Int J Mod Phys A 29, 1430018 (2014)



Critical Exponents Experimentally Characterised in a Box-like Trap Hadzibabic et al, Science 347 (2015) Dalibard et al., Nat. Comm. (2015)

Model Trento Experimental Quench Sequence: Quench from T > Tc to T < Tc over tens-hundreds of ms





Consider 3 different experimentally-relevant quench rates



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Random (shot-to-shot) Fluctuations Quenches / Low-D & Universality (high-lying modes "unaffected") COMPARISON OF METHODS IN MUTUAL VALIDITY REGIME:



Rooney, Allen, Zülicke, Proukakis & Bradley PRA **93**, 063603 (2016) Allen, Allen, Zaremba, Barenghi & Proukakis, PRA **87**, 013630 (2013) What About

2-Component

Bose-Einstein Condensates ?



Phase Profiles Controlled by Inter- / Intra- Atomic Interactions





Phase Profiles Controlled by Inter- / Intra- Atomic Interactions



Feshbach Resonances

position (µm)

PRL 101, 040402 (2008)

PRL 101, 040402 (2008)

position (µm)

2-COMPONENT BEC EXPERIMENTAL GALLERY:





IMMISCIBLE 2-COMPONENT TEMPERATURE QUENCHES

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Schematic of Possible Evolutionary Dynamics

[assuming here Rb grows faster than Cs, at least initially]



→ Profiles Observed in Experiments may only be *Metastable* Profiles

IK Liu ... NPP., Phys. Rev. A 93, 023628 (2016)

IMMISCIBLE 2-COMPONENT TEMPERATURE QUENCHES

Perform Temperature & Chemical Potential Quench on an Immiscible Quasi-1D Equilibrium Binary Mixture (here Rb-Cs)

 $T_i = 80nK \rightarrow T_i' = 20nK \qquad \mu_i \rightarrow \mu_i' \ge \mu_i$

Observe 3 Characteristic Evolutionary Stages:







T > 0 TWO-COMPONENT CONDENSATE THEORY

2



<u>T > 0 TWO-COMPONENT CONDENSATE THEORY</u>





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New Relevant (Im)Miscibiity Parameter $\Delta n_{\rm nc}$ for Symmetric Trapped BEC Mixtures:

Rb (Species 1)
 K (Species 2)

$$\begin{array}{c} \text{norm} -\mathbf{max} \ n_{c,1}(\mathbf{r}) & \text{max} \ n_{c,2}(\mathbf{r}) \\ \hline -1.0 & -0.5 & 0.0 & 0.5 & 1.0 \\ \hline (i) \ \Delta = 0 & \Delta n_{\text{norm}} \\ \hline \Delta >> 0 & \Delta n_{\text{norm}} \\ \hline -1 & -1 & +1 \end{array}$$

 $n_{c,1}(\mathbf{0})$ $n_{c,2}(\mathbf{0})$

Lee, Jorgensen, Liu, Wacker, Arlt & NPP, arXiv:1604.08063 (2016)









Phase separation Boundary: Green → Blue Criterion in Trap can Deviate Significantly from Homogeneous Condition ! This depends critically also on atom numbers Lee, Jorgensen, Liu, Wacker, Arlt & NPP, arXiv:1604.08063 (2016)



Equilibrium Profiles for T > 0

(choosing to have approximately constant BEC atom number)



Miscibility / Immiscibility Essentially Fixed by BEC Atom Numbers / Interactions

... but ... Thermal Effects are Important for Dynamics !



Consider Dipole Oscillations of Perturbed Co-Trapped Quantum Gases



s
$$\Delta n_{\text{norm}} = \frac{n_{c,1}(\mathbf{0})}{\max n_{c,1}(\mathbf{r})} - \frac{n_{c,2}(\mathbf{0})}{\max n_{c,2}(\mathbf{0})}$$

Change in Behaviour Is associated with Changing Values of

 $\Delta n_{
m norm}$

Temperature significantly affects Damping Rate and Frequency

Behavioural "Crossover"

 $0 \le \Delta \le 0.5$

in Probed Parameter Range:



a Good Parameter to Map Out



Consider Dipole Oscillations of Perturbed Co-Trapped Quantum Gases



→ Temperature Enhances Damping in Both Cases ! Effect of Collisions minimal in both cases (noticeable over long timescales)

Multi-component Atomic Condensates and ROtational Dynamics









IOP Institute of Physics Quantum Optics, Quantum

New Journal of Physics

Control Group

NewcastleGateshead

Information and Quantum

https://conferences.ncl.ac.uk/jqcmacro/ Abstract Deadline: Thursday 30 June

JQC Colloquium: Jean Dalibard (College de France)

MACRO Keynote:

Vanderlei Bagnato (Sao Paulo) Carlo Barenghi (JQC Newcastle) Natalia Berloff (Cambridge/Skoltech) Halina Rubinsztein-Dunlop (Brisbane) Ian Spielman (JQI/NIST) (tentative) Masahito Ueda (Tokyo)

JQC Symposium Plenary: <u>Charles Adams (JQC Durham)</u> <u>Clive Emary (JQC Newcastle)</u> <u>Boris Malomed (Tel Aviv)(tentative)</u> <u>Rob Nyman (Imperial)</u> Short-Term Postdoctoral Position Available between Sep'16 – Feb'17 (Multi-Component BECs in Ring Traps T > 0 Numerical Simulations) INFO: nikolaos.proukakis@ncl.ac.uk



