

Coherent optical manipulation of plasma waves

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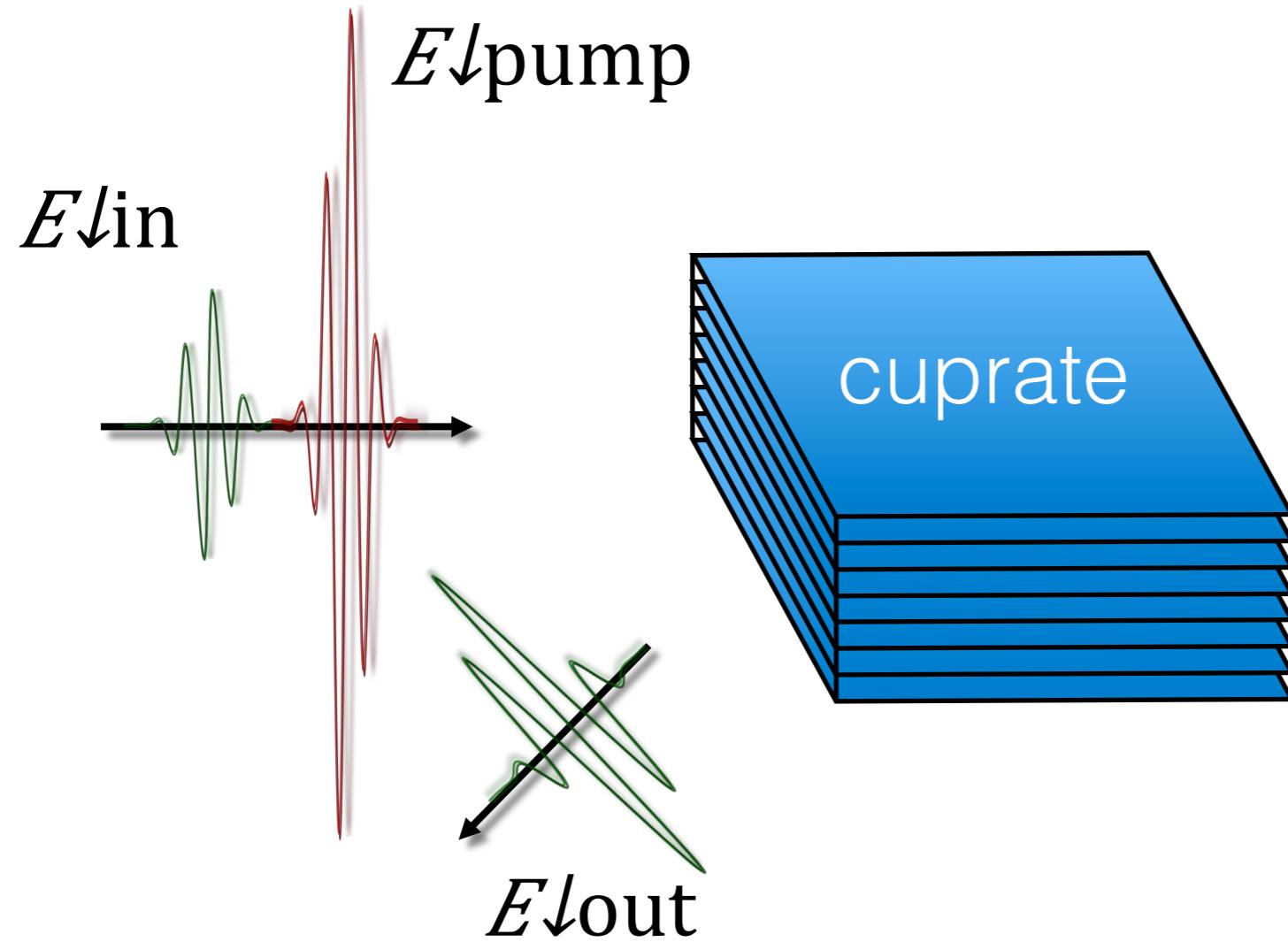


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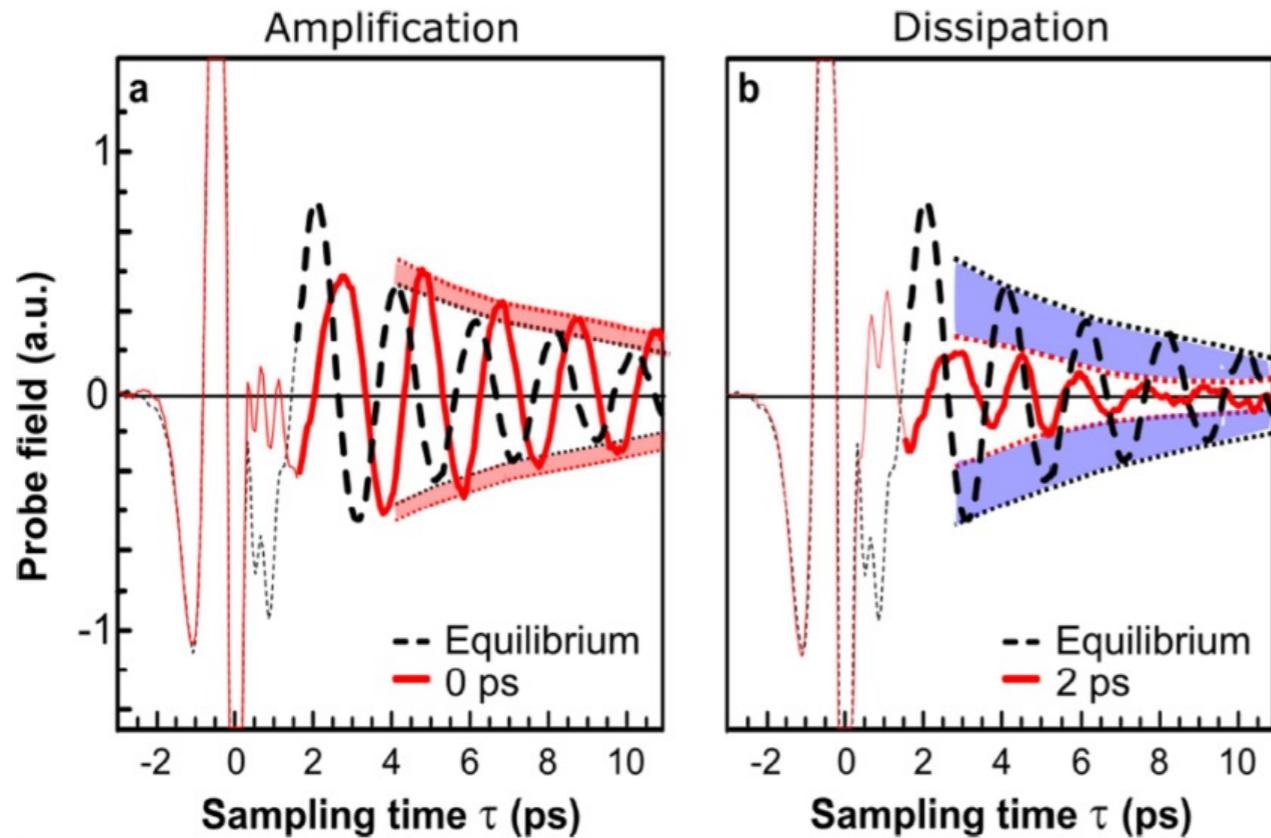
Motivation



- Large nonlinearity:
- manipulate probe optical field
 - drive macroscopic state of cuprate

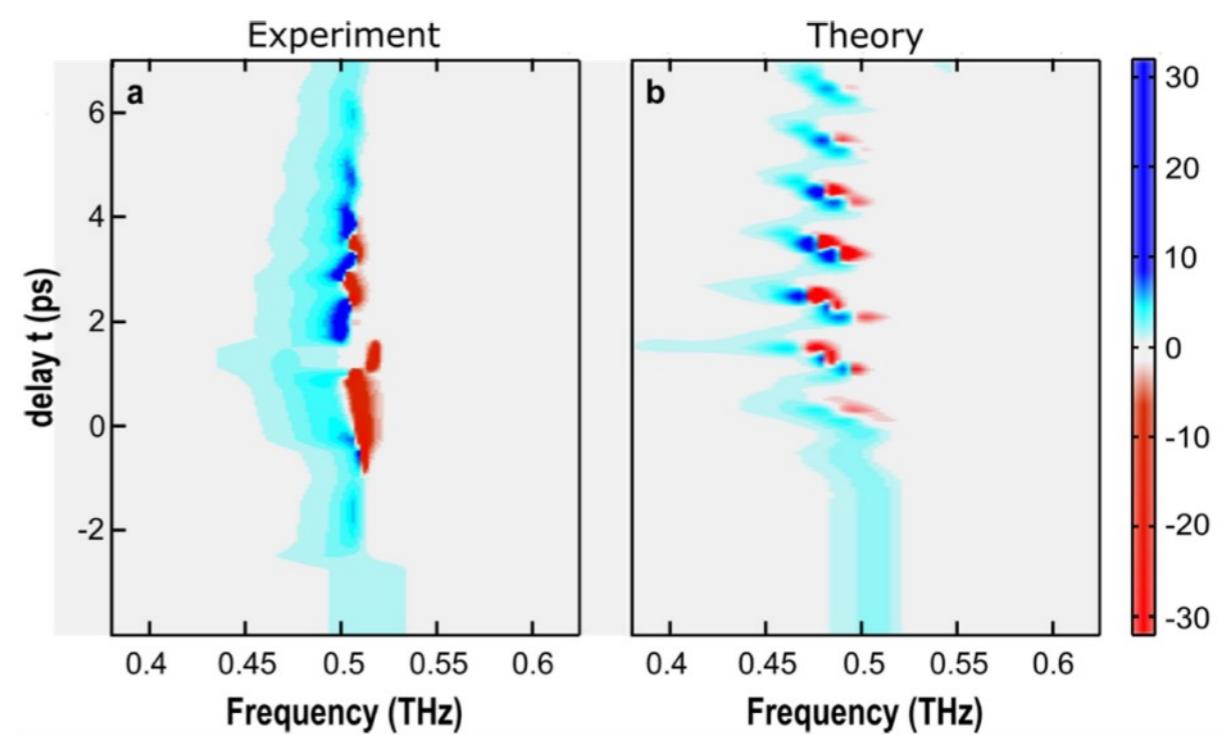
Parametric Amplification of a Terahertz Quantum Plasma Wave, Srivats Rajasekaran, Eliza Casandru, Yannis Laplace, Daniele Nicoletti, Genda D. Gu, Stephen R. Clark, Dieter Jaksch, Andrea Cavalleri, [arXiv:1511.08378](https://arxiv.org/abs/1511.08378) (to appear in Nature Physics 2016).

Experimental results



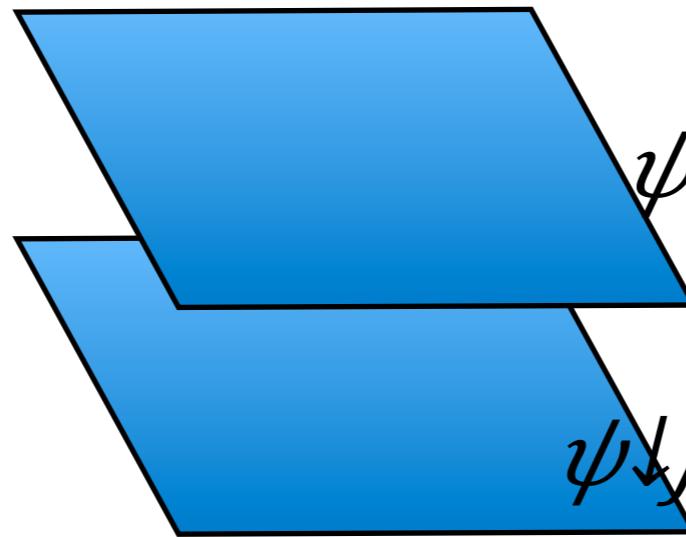
Probe field
amplification (a)
and suppression
(b) due to strong
driving field

Parameter regions
of suppression
(red) and
amplification
(blue)



Josephson physics

$$I = I_{\downarrow 0} \sin(\theta_{\downarrow j} + 1, j)$$



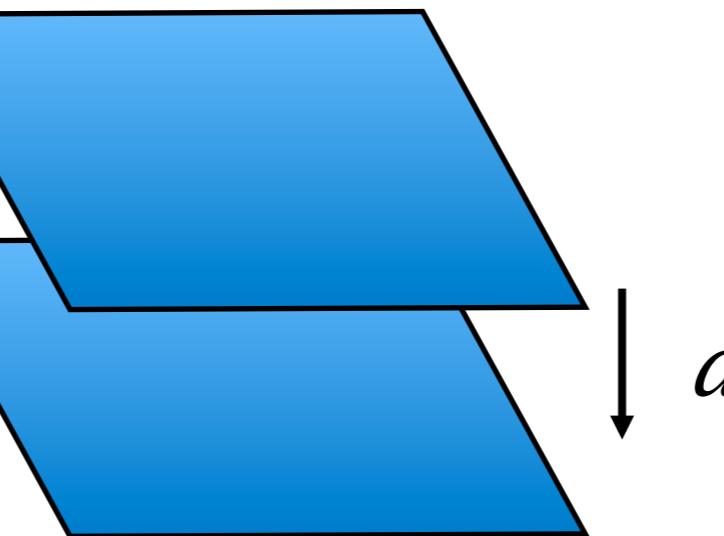
$$\psi_{\downarrow j} = |\psi_{\downarrow j} | e^{\uparrow i \theta_{\downarrow j}}$$

$$\psi_{\downarrow j+1} = |\psi_{\downarrow j+1} | e^{\uparrow i \theta_{\downarrow j+1}}$$

$$\theta_{\downarrow j+1, j} = \theta_{\downarrow j+1} - \theta_{\downarrow j}$$

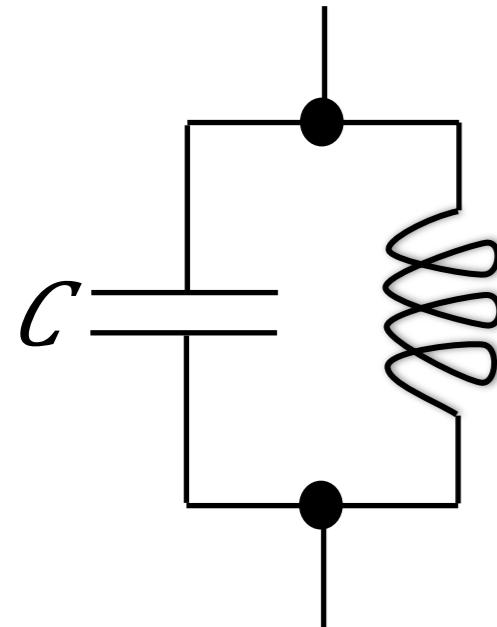
External optical field $E = E_{\downarrow 0} \cos(\omega_{\downarrow j} P t)$ induces a phase difference

$$V = Ed = \hbar/2e \theta_{\downarrow j+1, j}$$



Josephson physics

Assume no excitations above the superconducting gap $|\psi_{\downarrow j}| \approx \text{const}$



$$L \omega_{JP} = 1/LC$$

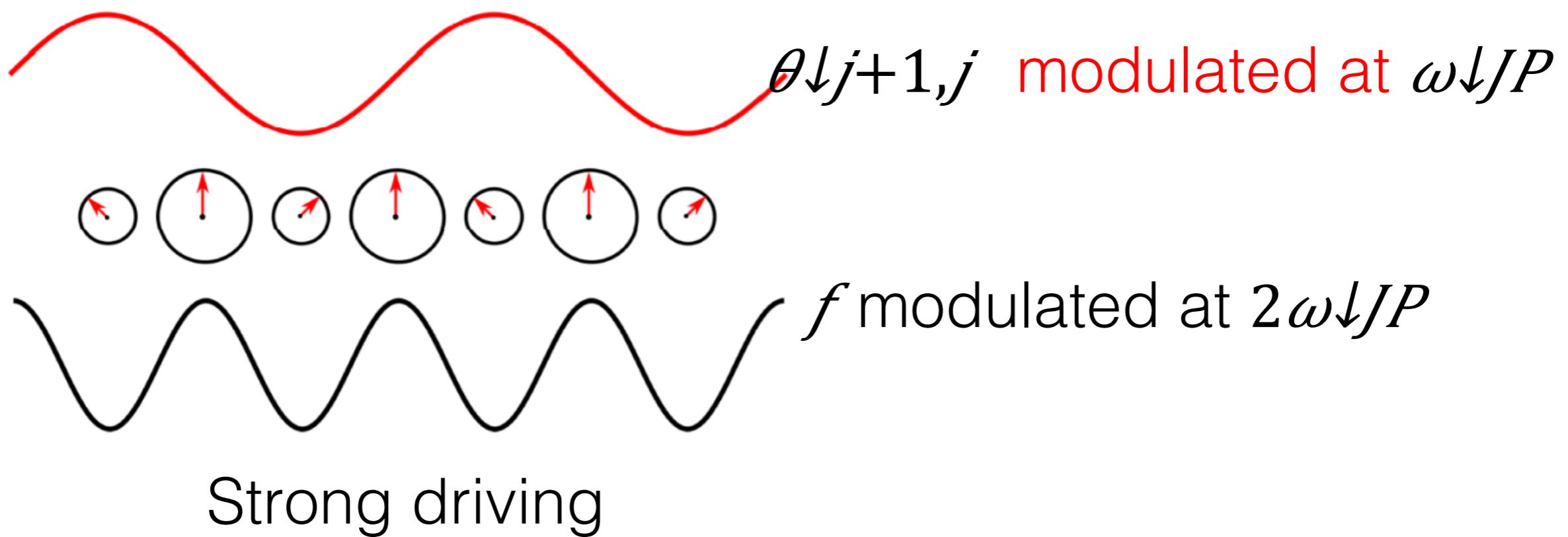
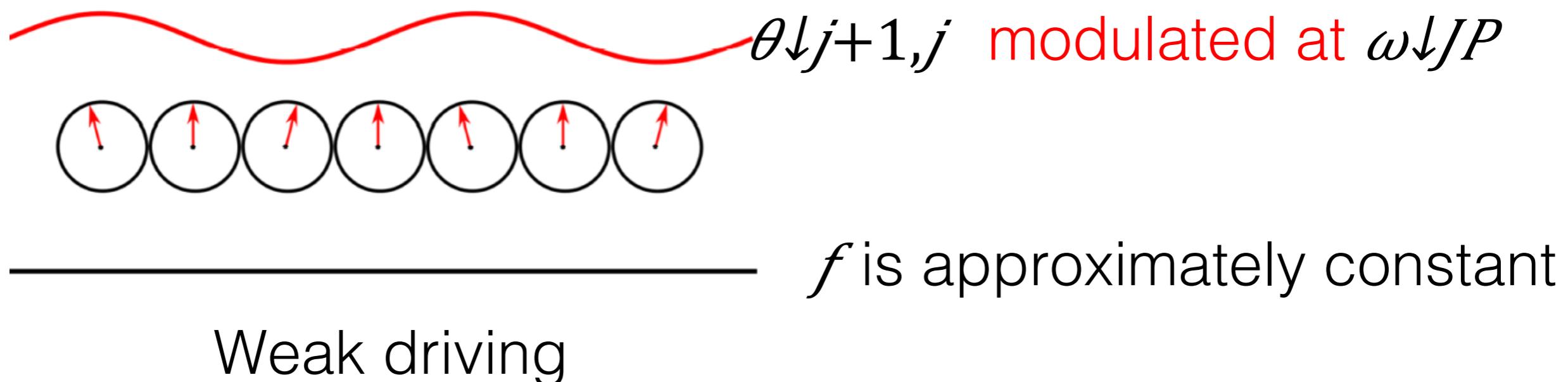
$$L = \hbar/2eI_0 \cos \theta_{j+1,j}$$

$$\theta_{j+1,j} = \theta_{j+1} - \theta_j$$

A strong driving field will change the oscillator strength

$$f_{j+1,j} = f_0 \cos(\theta_0 \cos(\omega_{JP} t)) \theta_0 = 2ed/\hbar\omega_{JP} E_0$$

Linear vs non-linear regime



Pump and probe field

From the Josephson equations we get

$$-\frac{1}{\gamma} \partial \theta_{j+1,j} / \partial t - \epsilon_{jr} / c^{12} \partial^{12} \theta_{j+1,j} / \partial t^{12} = \omega_{jp}^{12} \epsilon_{jr} / c^{12} \theta_{j+1,j}$$

The oscillator strength is modified by the driving field

$$f(t) = \omega_{JP}^{12} (1 - \theta_{j0}^{12} (1 + \cos(2\omega_{JP} t)) / 4)$$

The probe field is described by

$$\begin{aligned} & \frac{1}{\gamma} \partial \theta_{j\text{probe}} / \partial t + \epsilon_{jr} / c^{12} \partial^{12} \theta_{j\text{probe}} / \partial t^{12} + \\ & \omega_{jp}^{12} \epsilon_{jr} / c^{12} (1 - \theta_{j0}^{12} (1 + \cos(2\omega_{JP} t)) / 4) \\ & \theta_{j\text{probe}} = 0 \end{aligned}$$

This is a damped Matthieu equation

Including spatial variations and BCs.

Junction described by a Sine-Gordon equation

$$\frac{\partial^2 \theta_{j+1,j}}{\partial x^2} - 1/\gamma \frac{\partial \theta_{j+1,j}}{\partial t} - \epsilon r/c^2 \frac{\partial^2 \theta_{j+1,j}}{\partial t^2} = \omega_p^2 \epsilon r / c^2 \theta_{j+1,j}$$

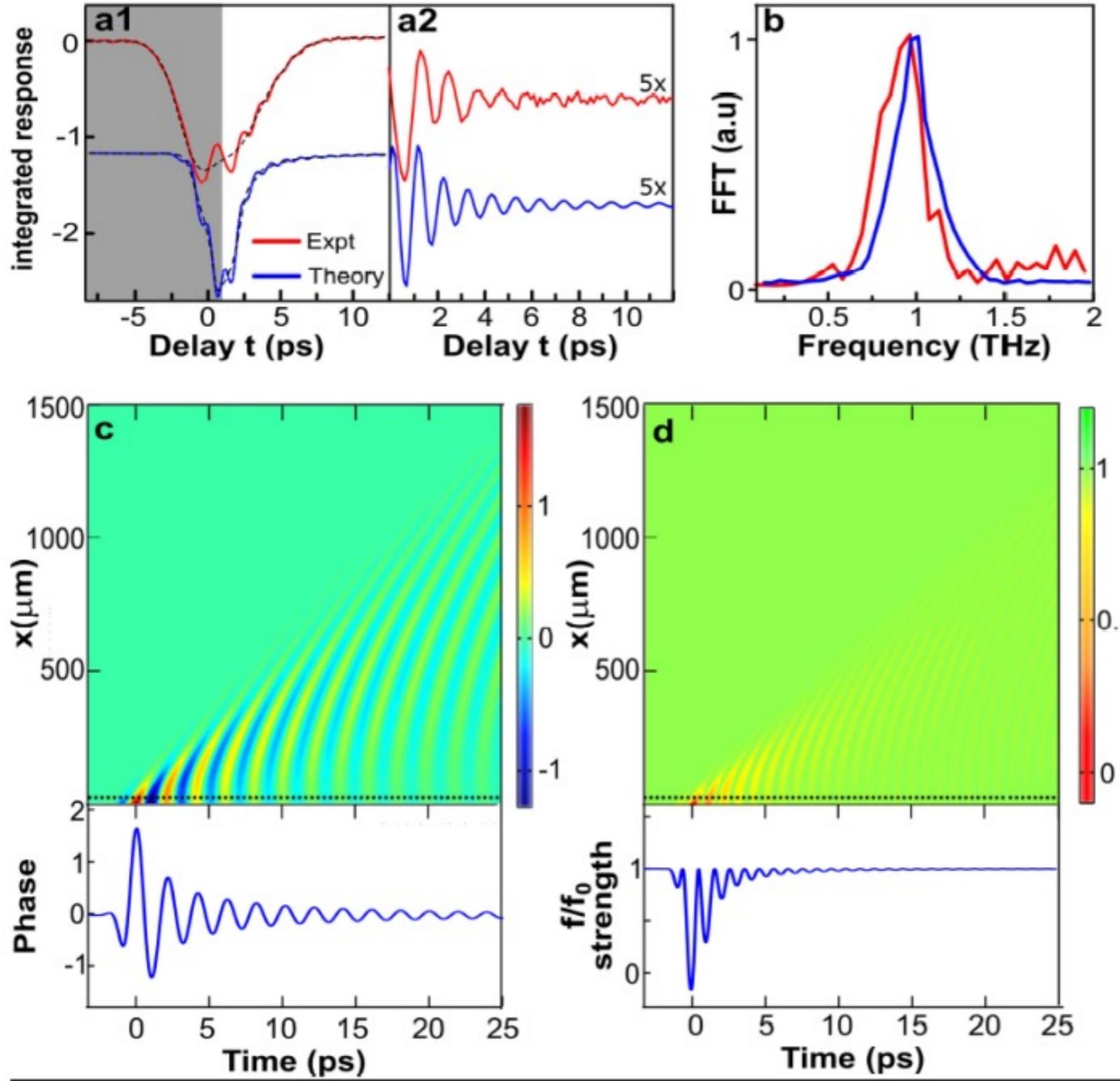
and boundary conditions for the electric E and magnetic field H determined by the Maxwell equations

$$[E_i(t) + E_r(t)]|_{x=-0} = E_c(t)|_{x=+0} = H_0 / \omega_J P \sqrt{\epsilon r} \quad (\partial \theta_{j+1,j} / \partial t)|_{x=+0}$$

$$[H_i(t) + H_r(t)]|_{x=-0} = H_c(t)|_{x=+0} = -H_0 \lambda_j (\partial \theta_{j+1,j} / \partial x)|_{x=+0}$$

with $H_0 = \phi_0 / 2\pi d \lambda_j$ and $\phi_0 = hc/2e$ the flux quantum.

Results

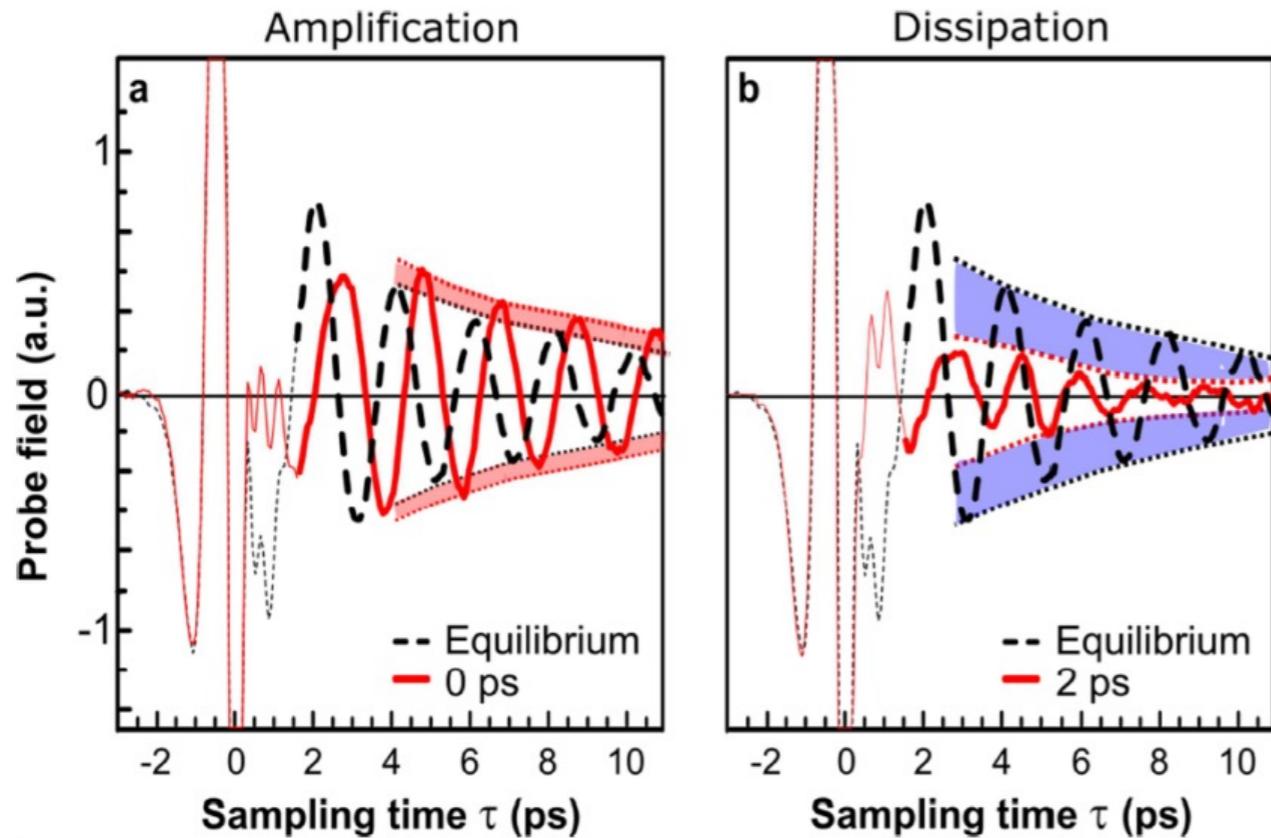


Strong response
at $2\omega \downarrow JP$

Plasma waves moving
into the SC

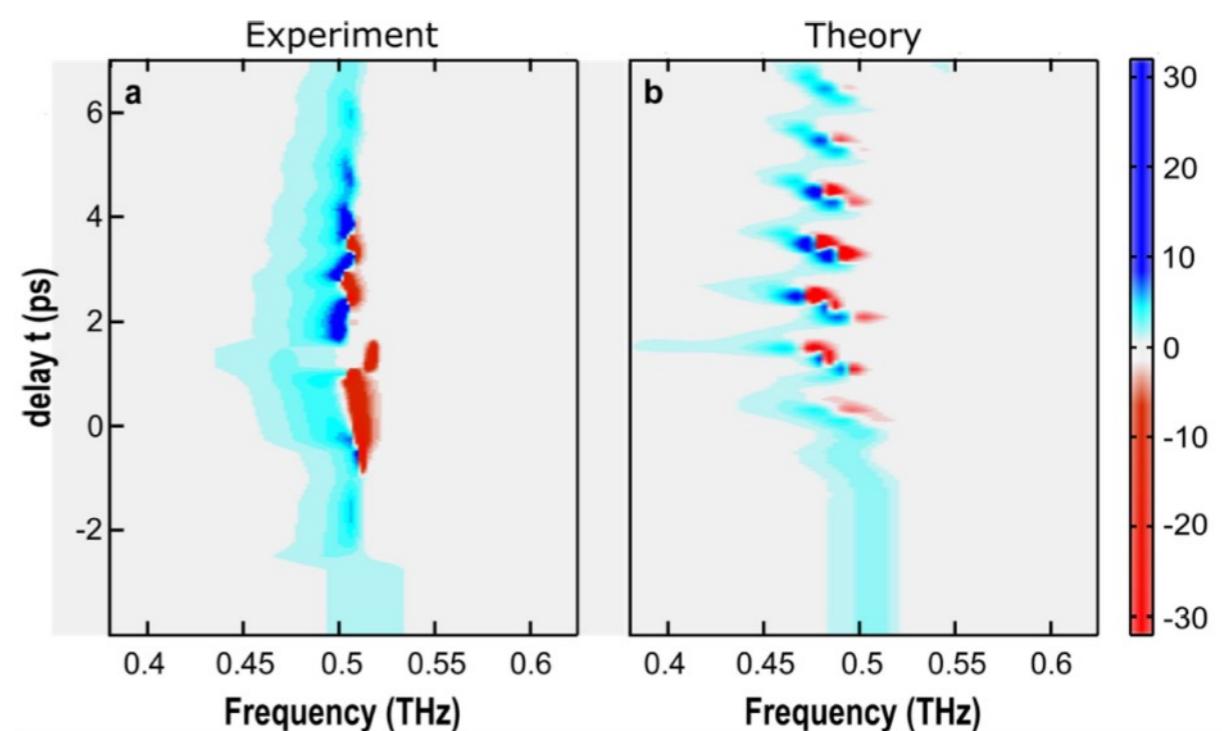
Negative oscillator
strength

Amplification of the probe field

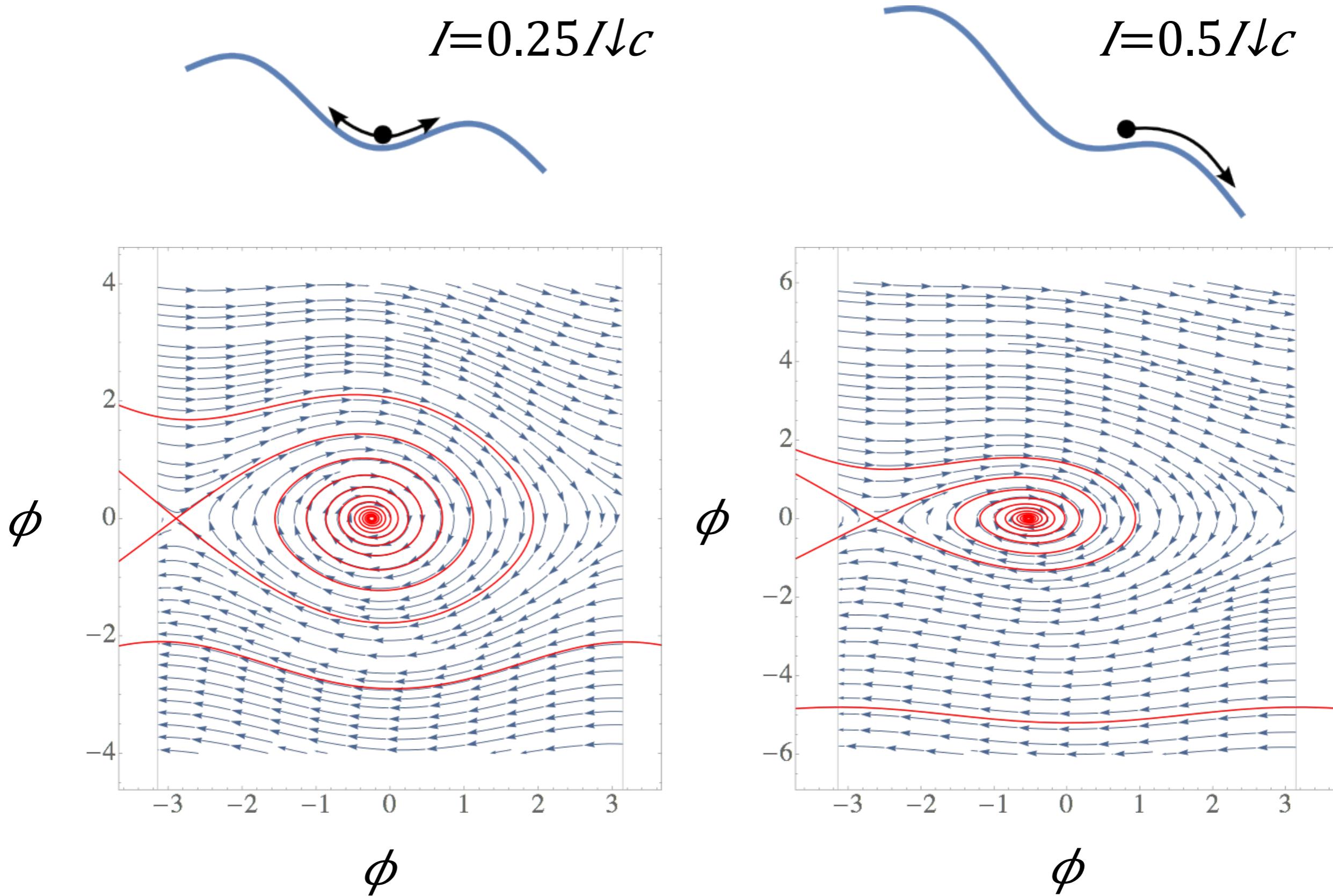


Probe field amplification (a) and suppression (b) due to strong driving field

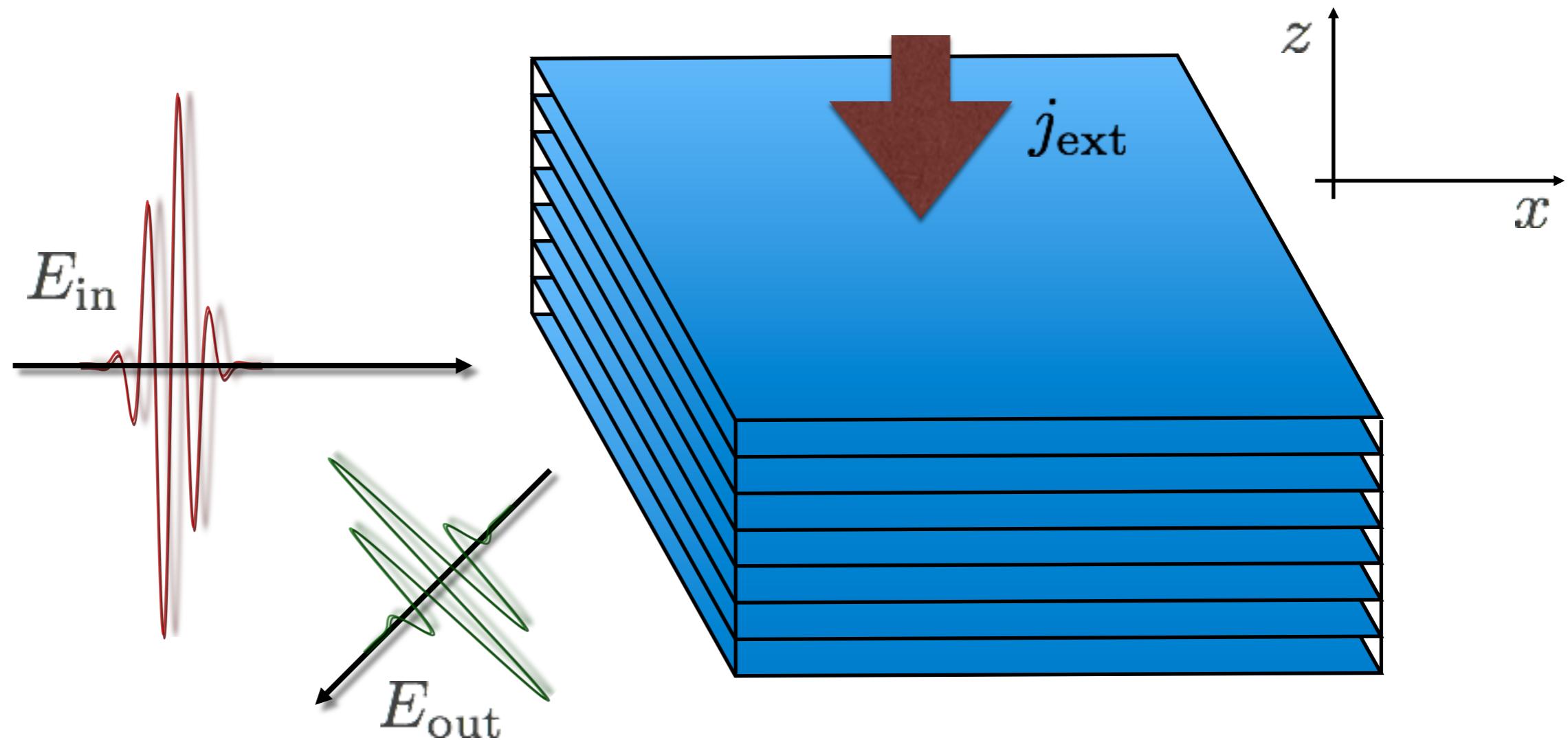
Parameter regions of suppression (red) and amplification (blue)



External currents and phase space



External Currents: Optical Nonlinearity



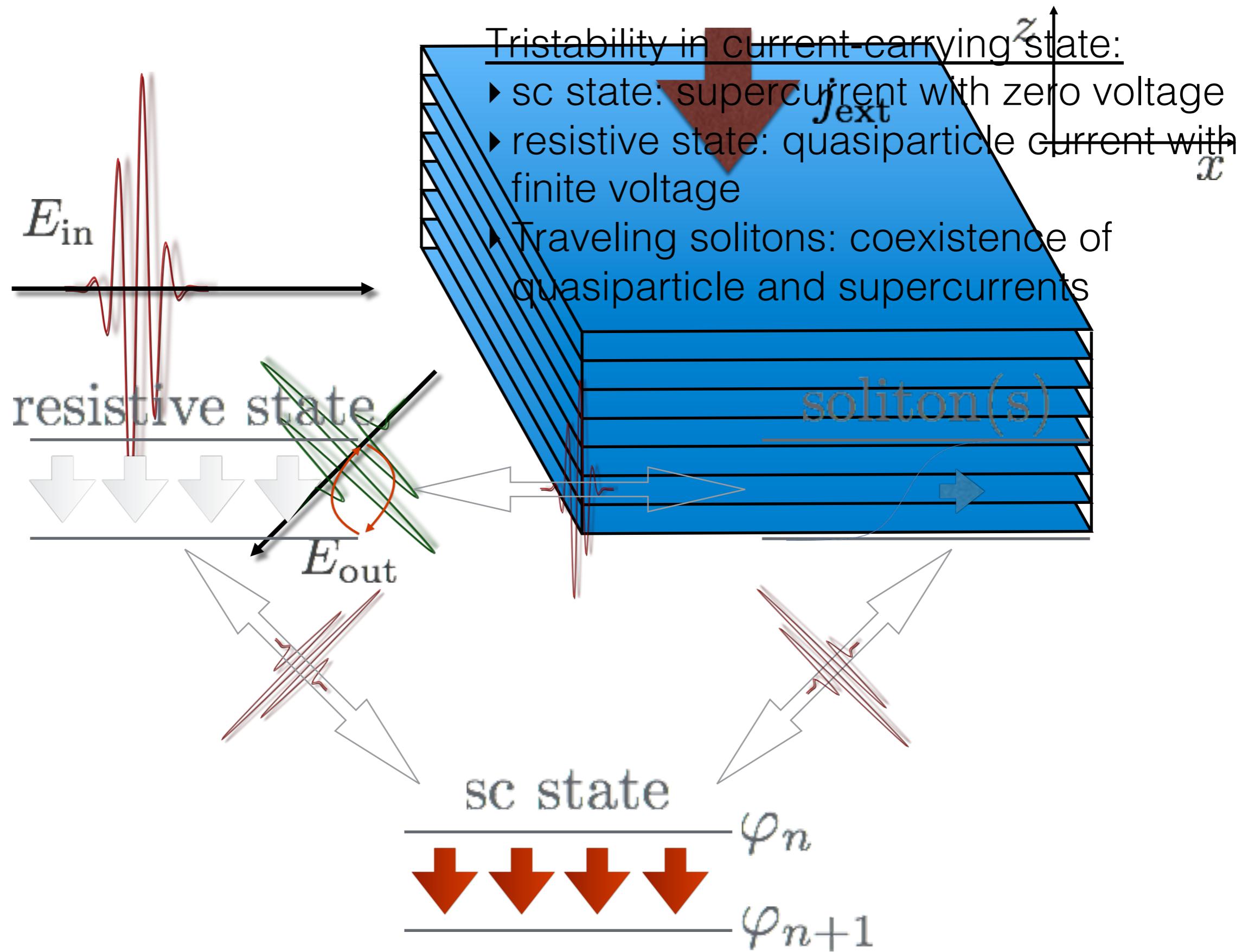
External current: $\sin(\phi_0) = j_{\text{ext}}$

vanishes
without current

Fluctuations: $\sin(\phi_0 + \phi_\epsilon) = \sin(\phi_0) + \cos(\phi_0)\phi_\epsilon - \frac{1}{2}\sin(\phi_0)\phi_\epsilon^2 + \dots$

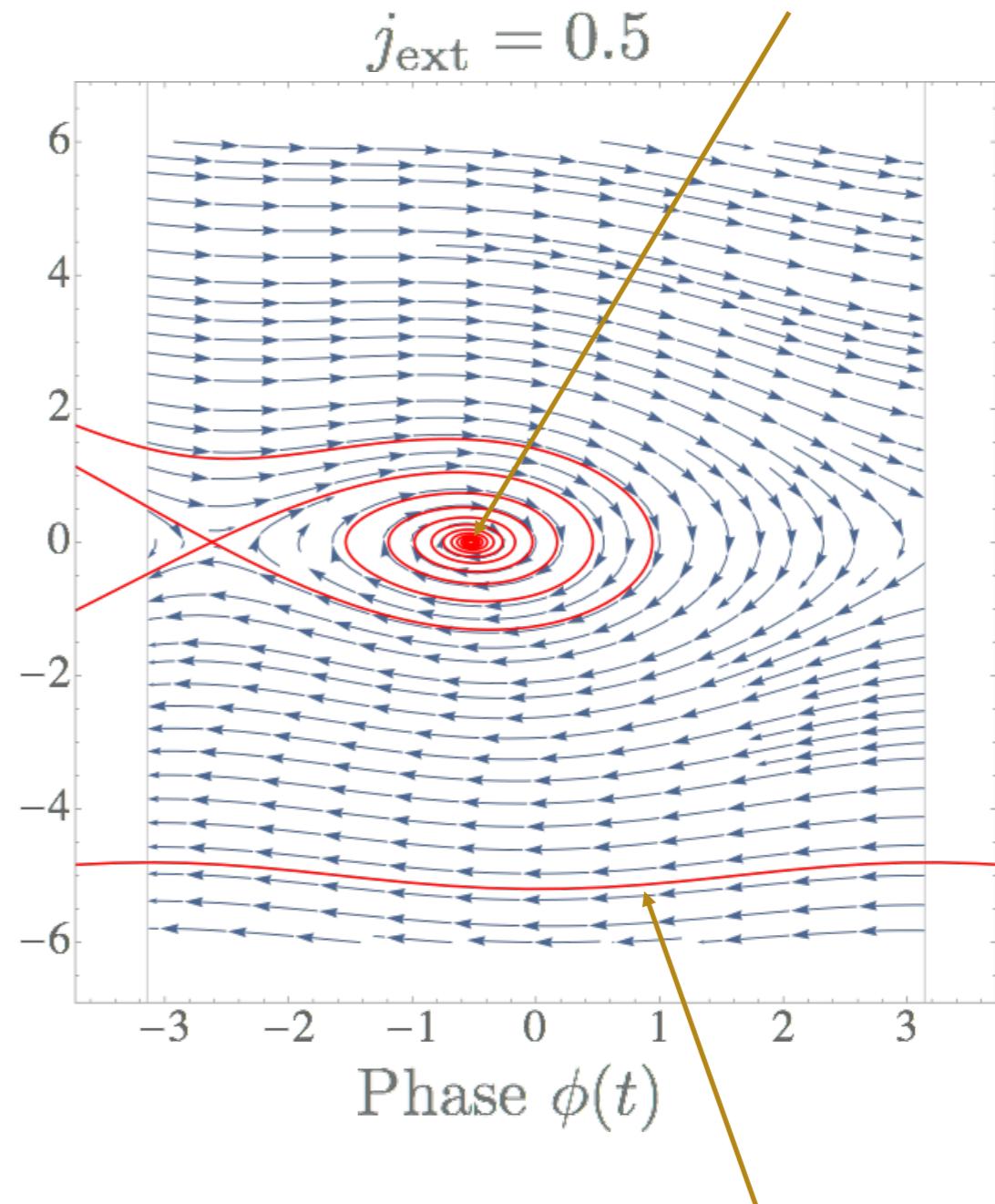
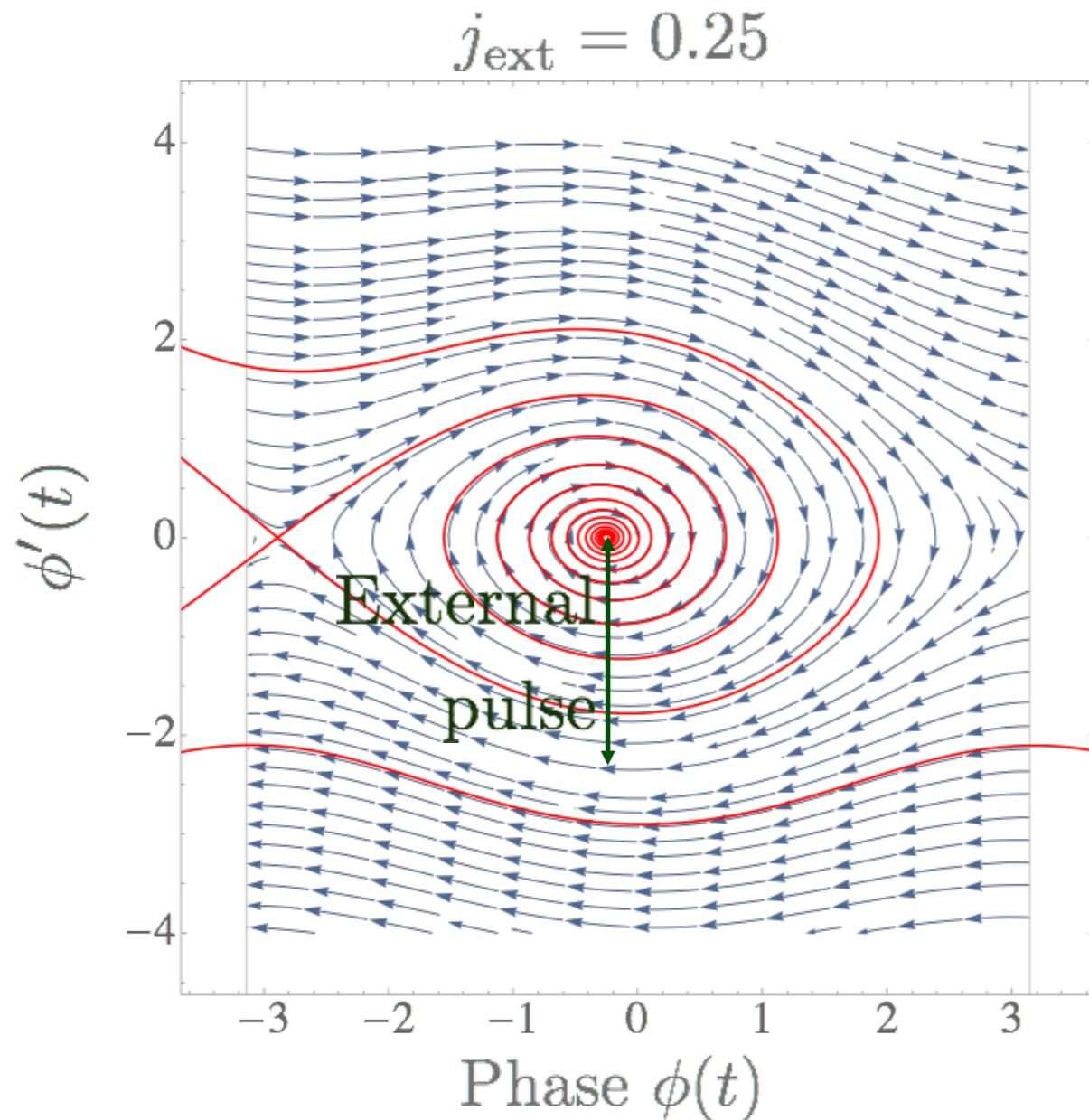
External currents change (enhance) nonlinear response

Currents: Steady states



Phase space diagrams

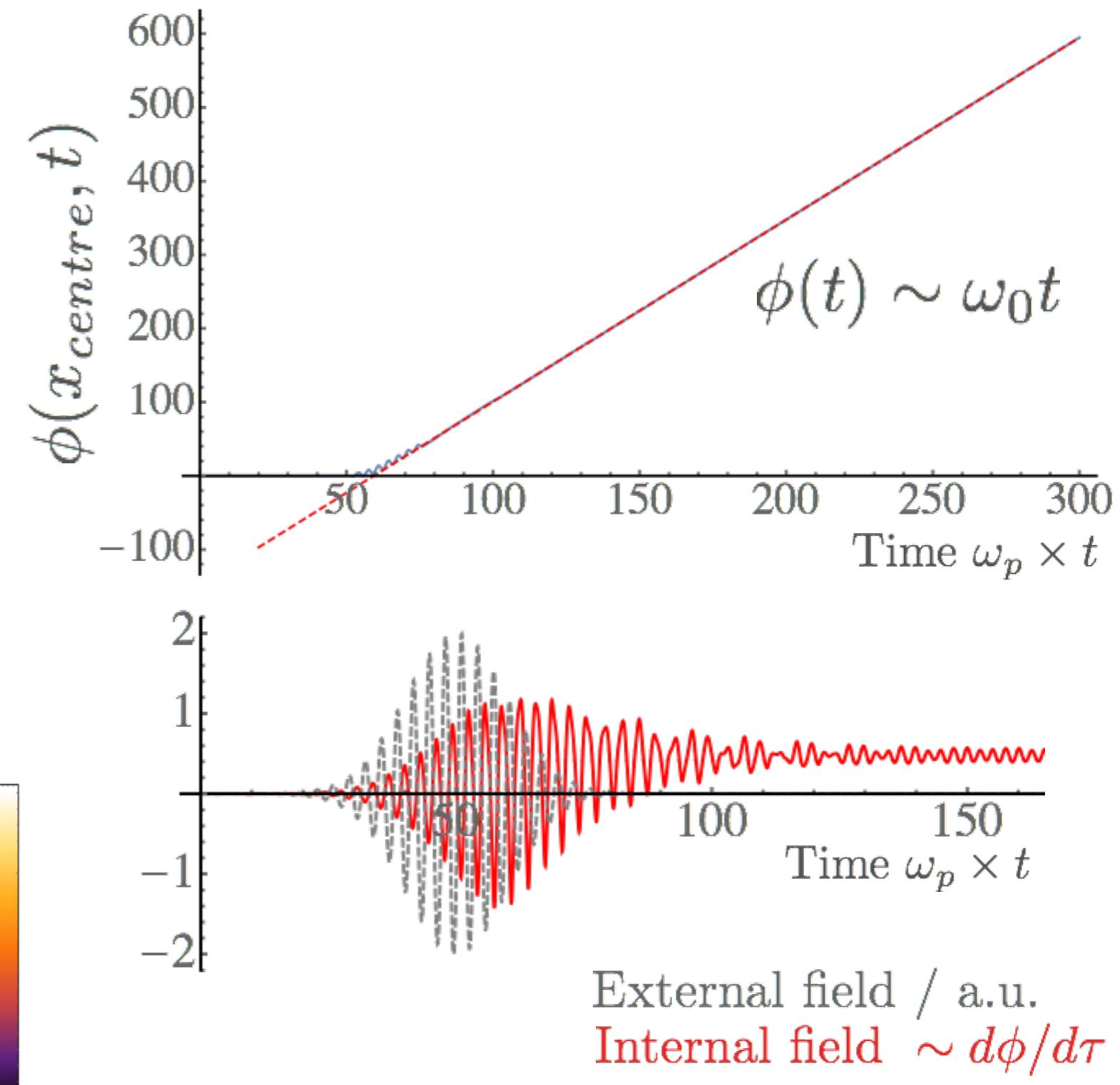
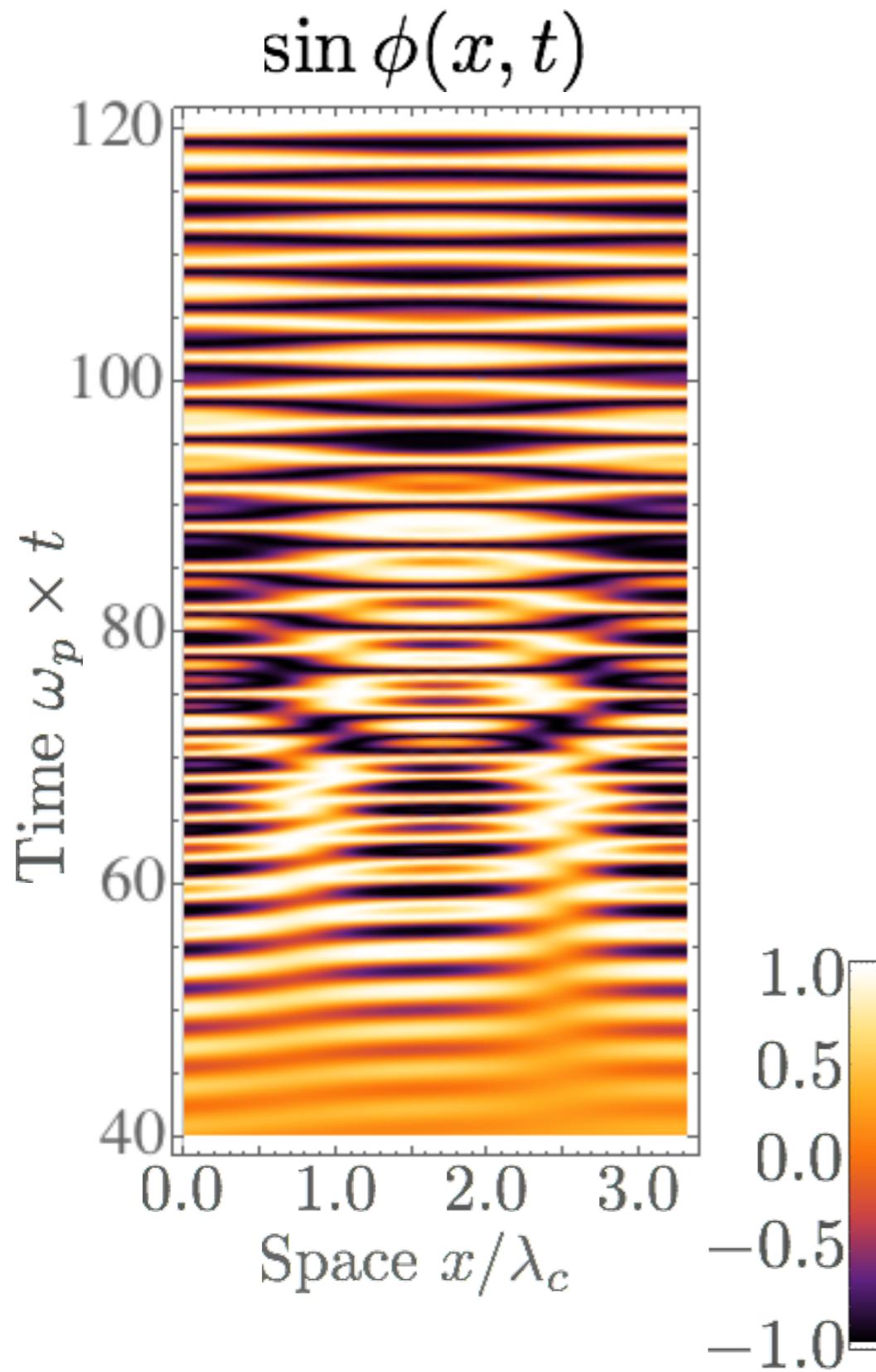
Uniform plasma oscillations: $\phi(x, t) = \phi(t)$ sc state



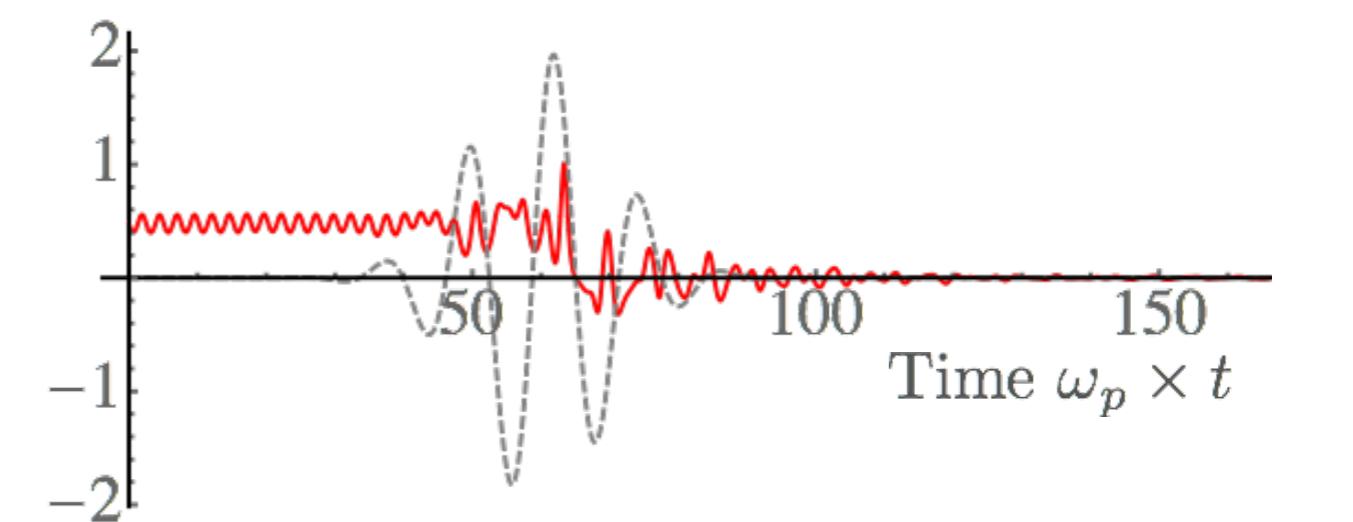
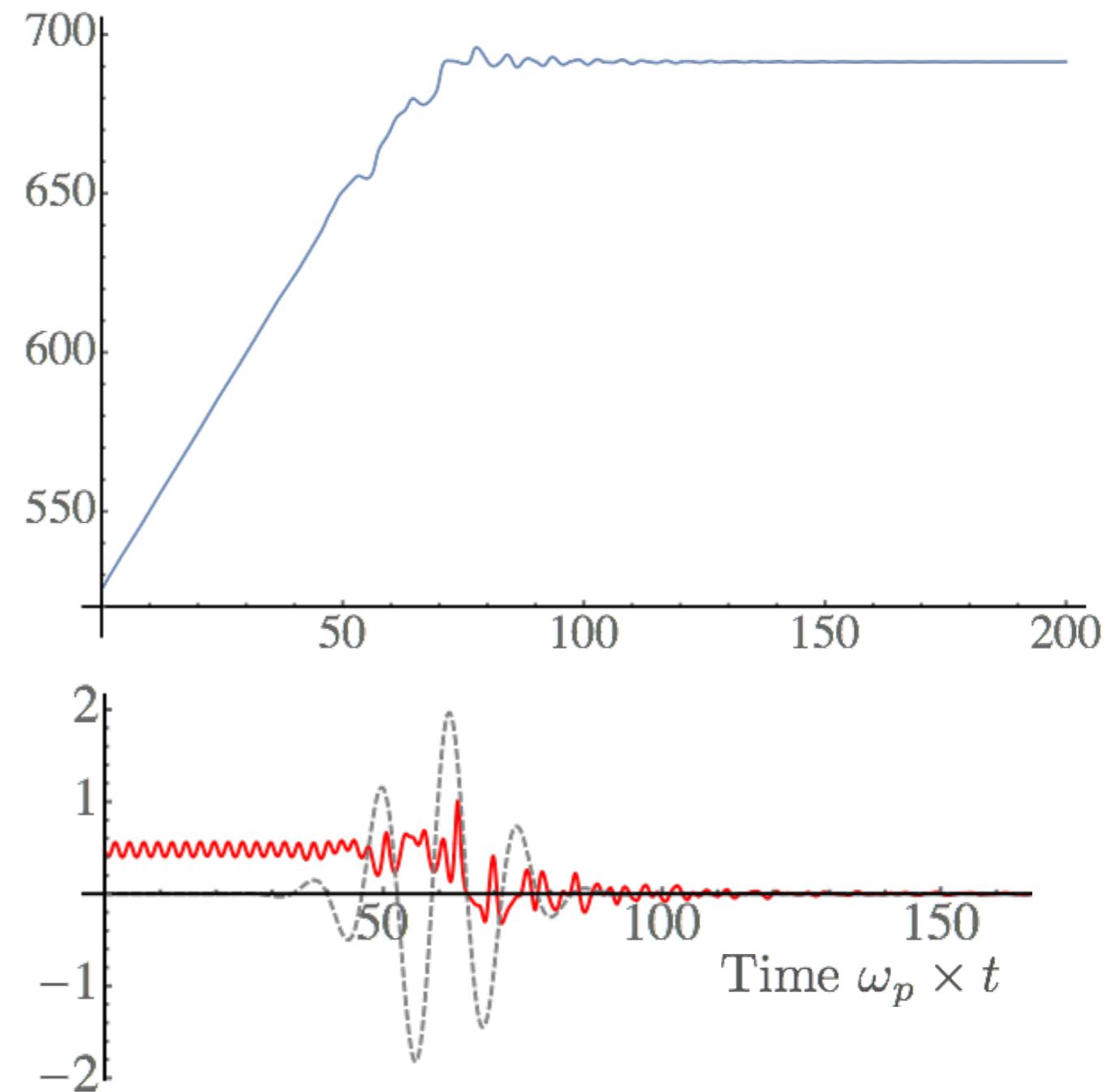
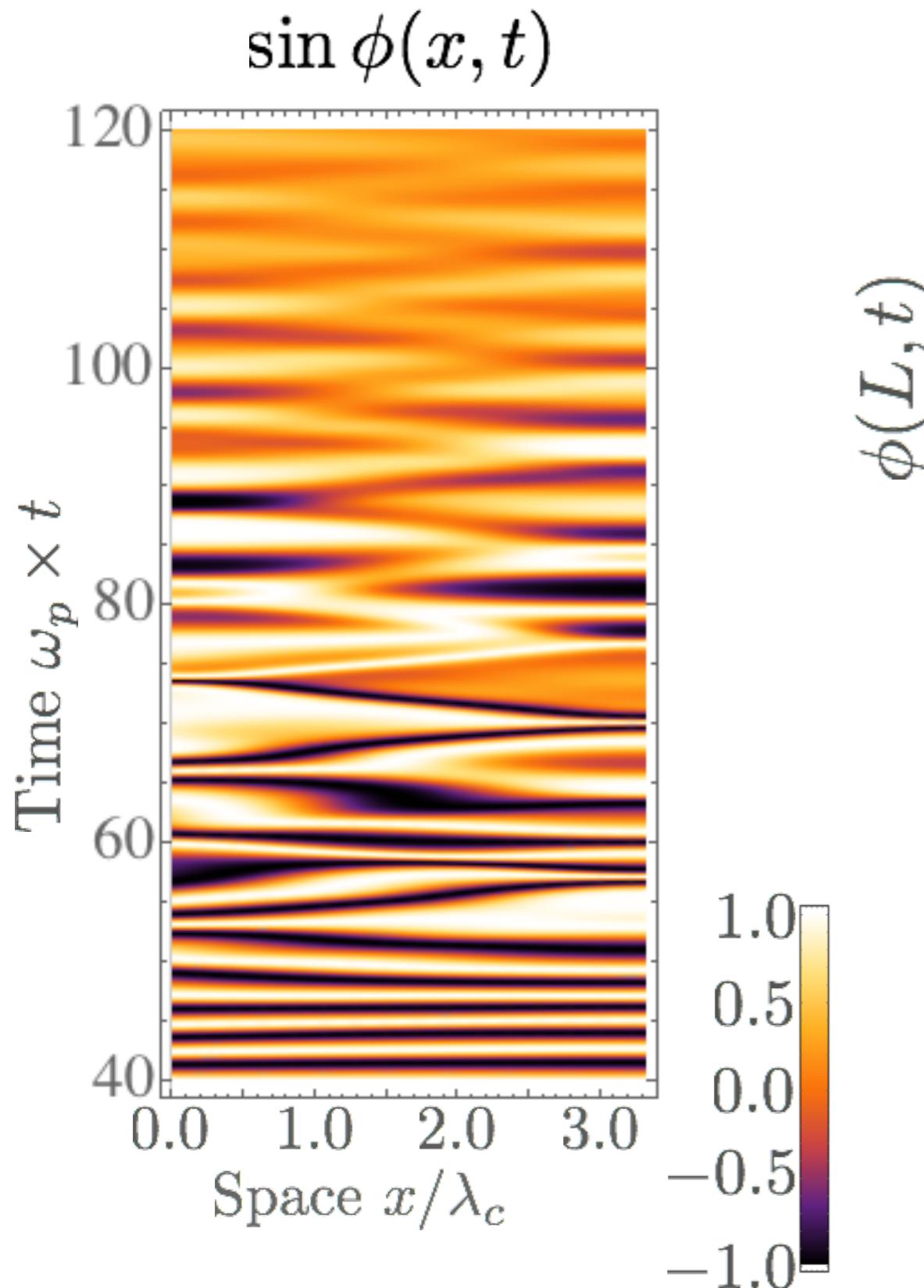
Electric field $\propto \phi'(t)$

resistive state

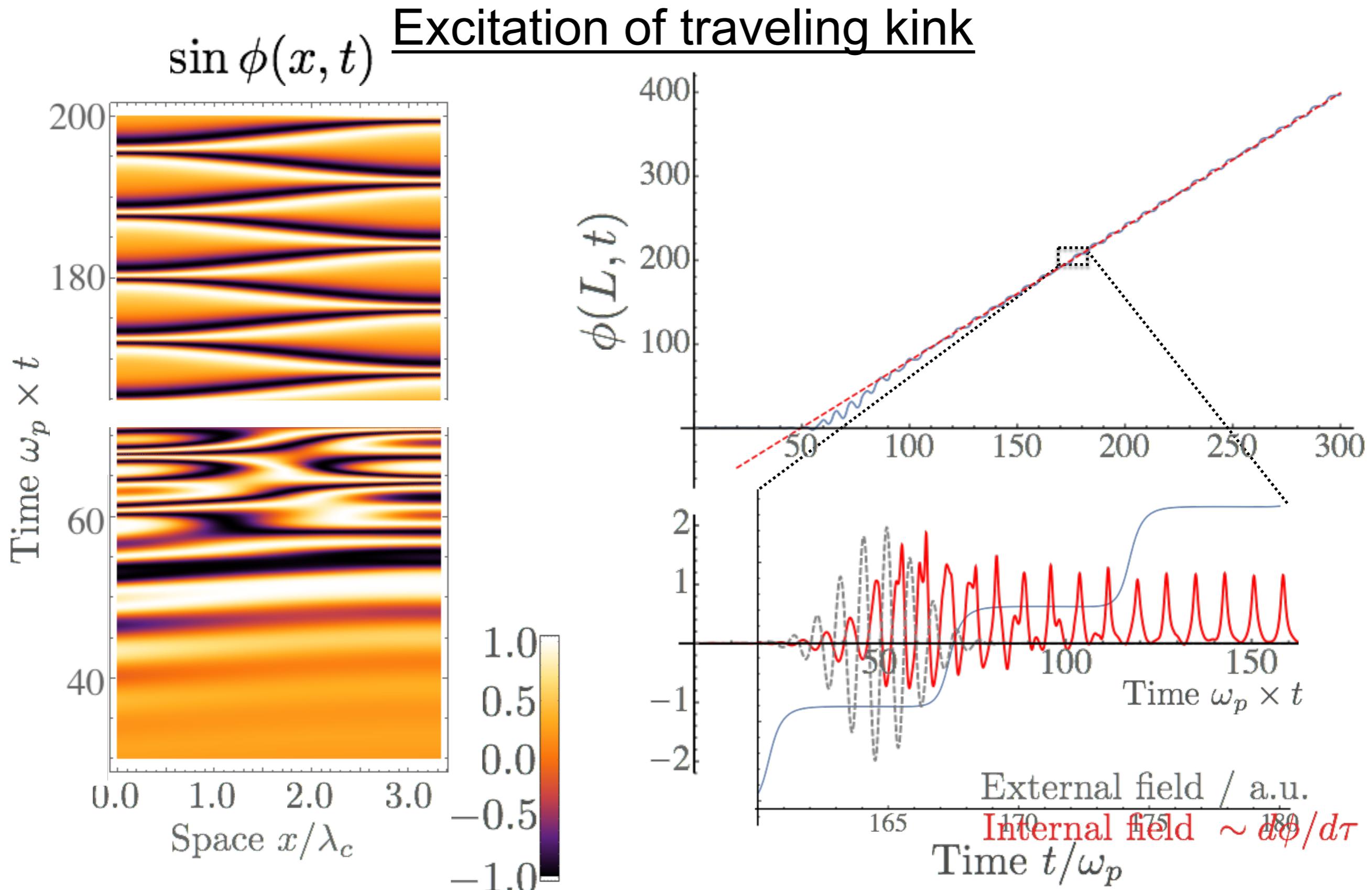
Switching I: SC \rightarrow resistive state



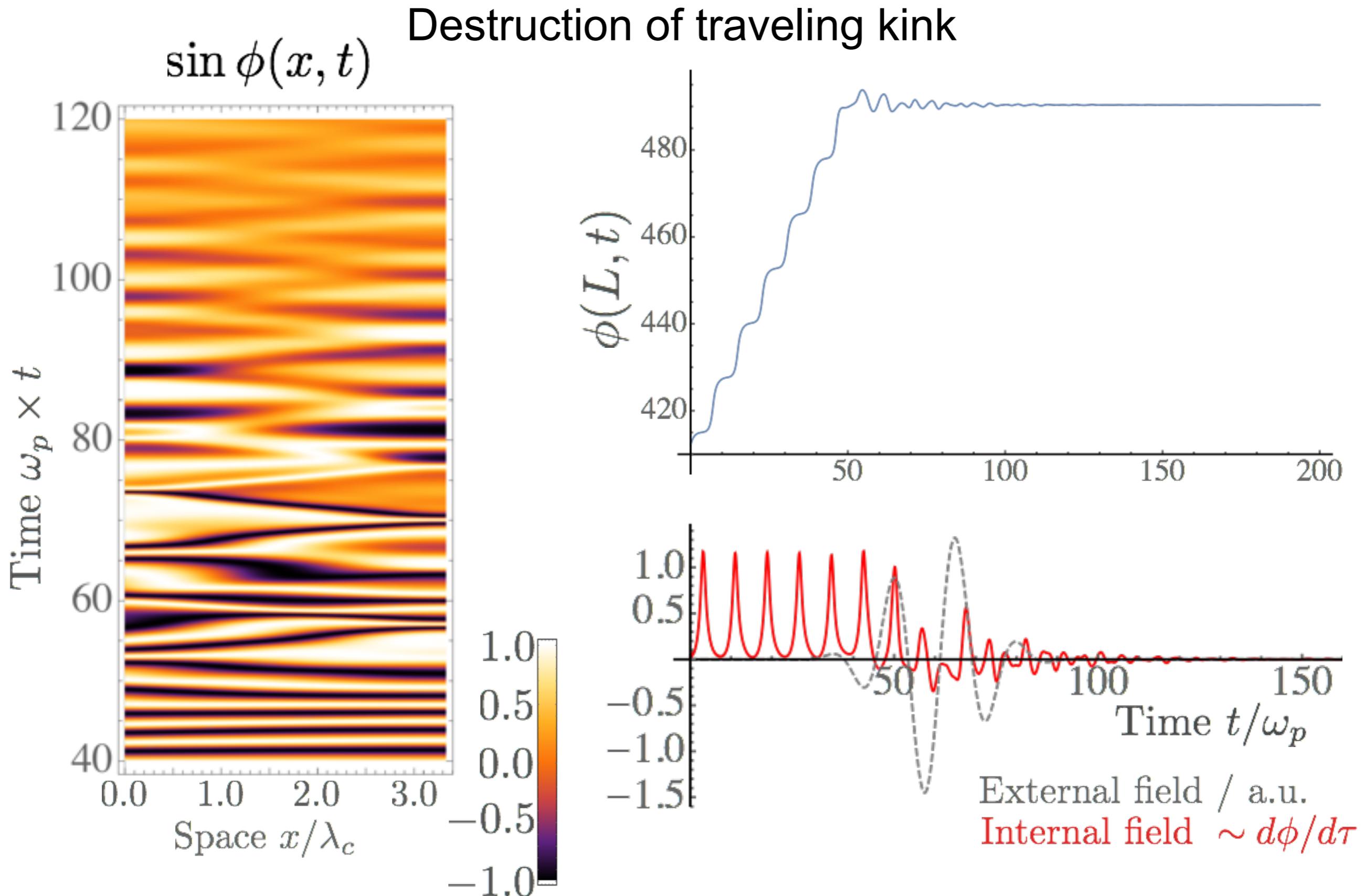
Switching I: resistive state \rightarrow SC



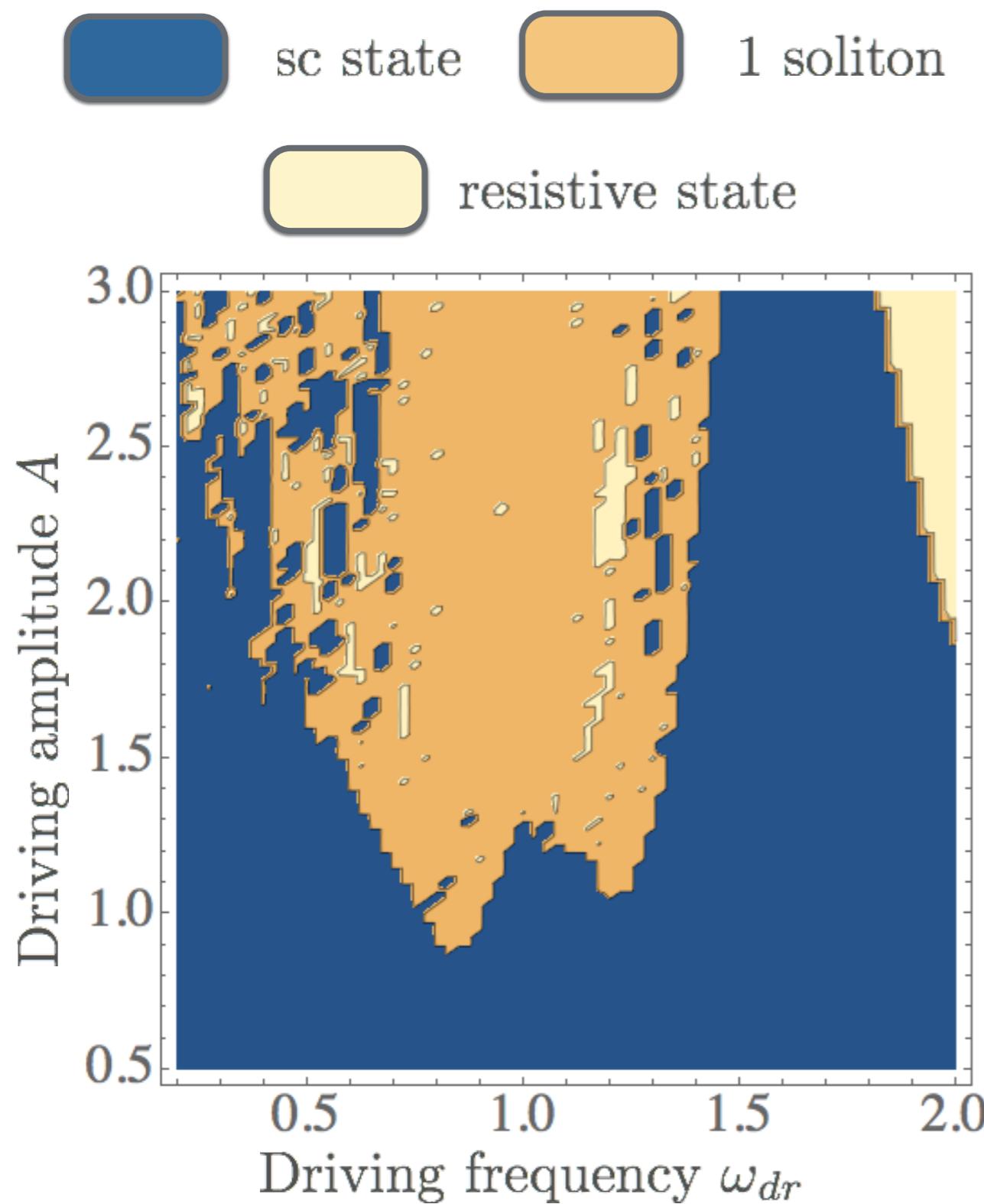
Switching II: SC \rightarrow kink



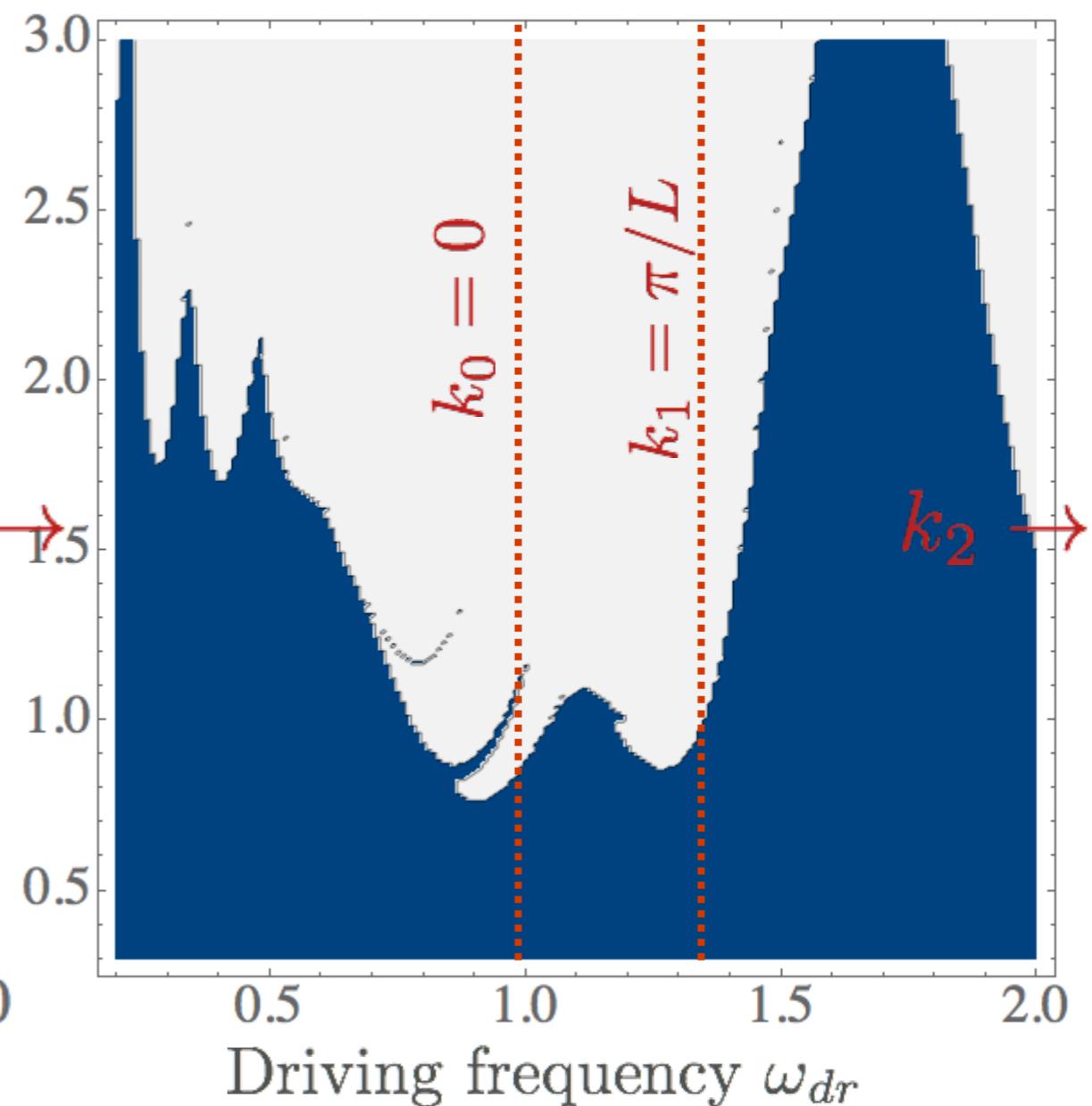
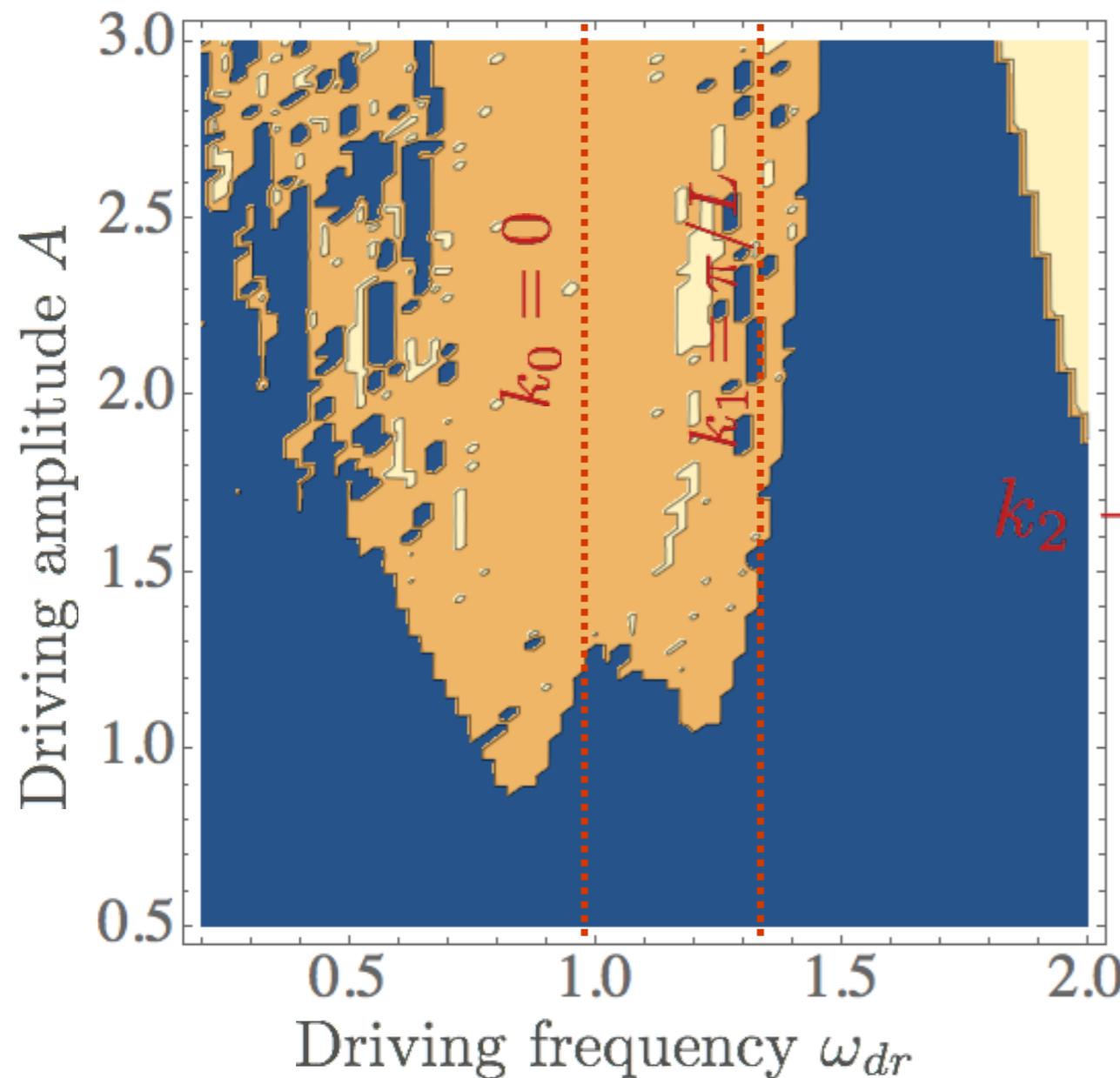
Switching II: kink \rightarrow SC



Parameter space I

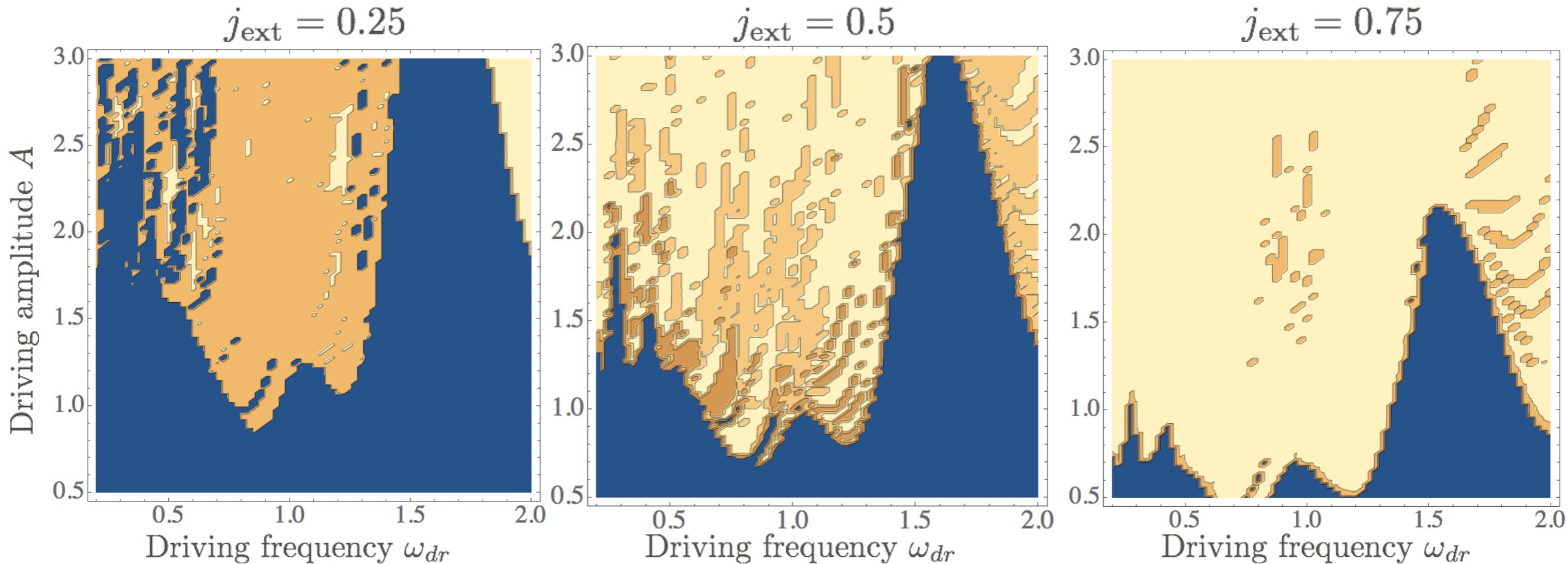


Parameter space I: SC \rightarrow ?



Parameter space II

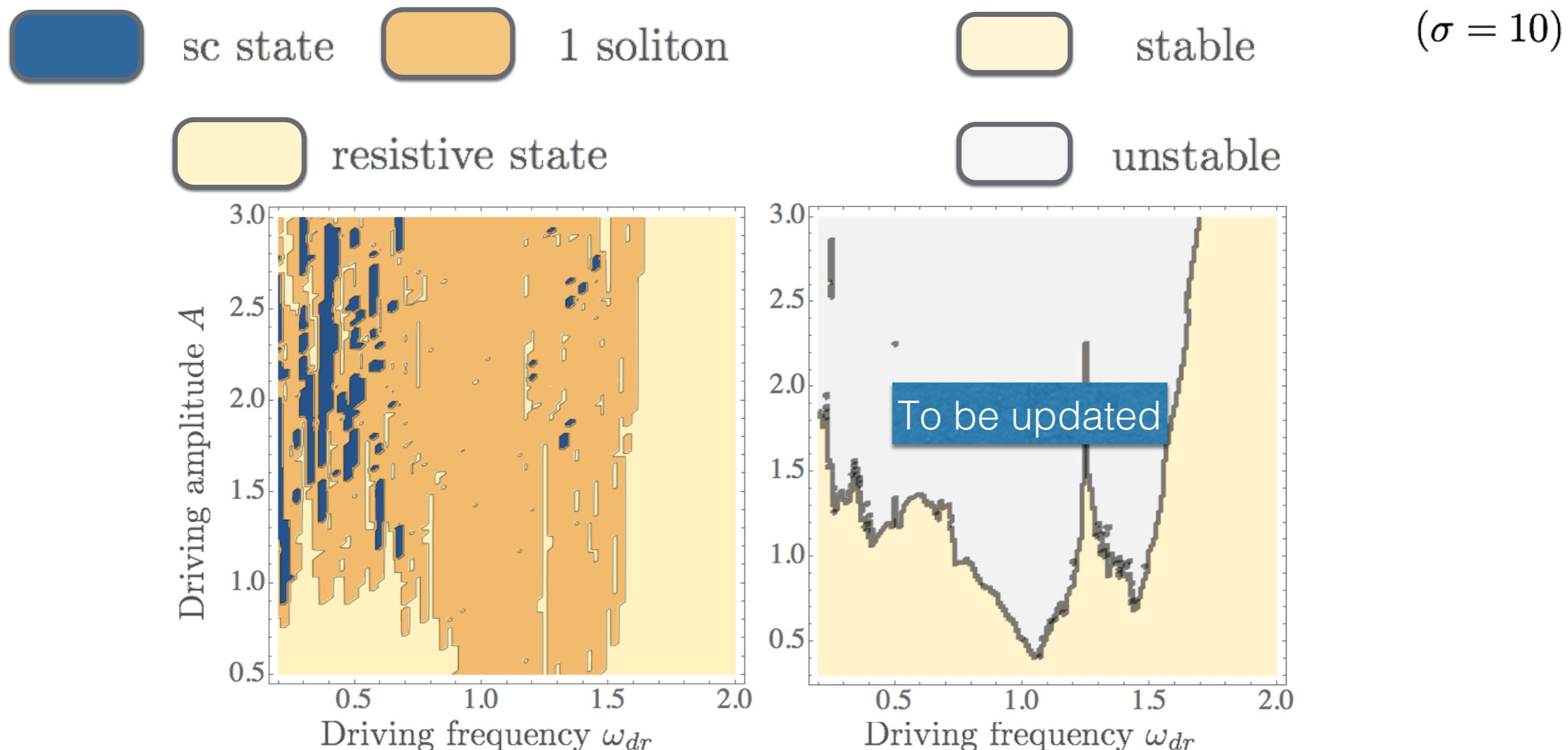
sc state 1 soliton resistive state ($\sigma = 10$)



Initial SC state: $\phi_0 = \arcsin j_{\text{ext}}$

External driving: $h_{\text{ext}}(t) = A \sin(\omega_{dr} t + \gamma) e^{-(t-t_0)^2/(2\sigma^2)}$

Parameter space: resistive → ?



Initial resistive state: $\phi_{res}(t) = \omega_0 t + \Im\{e^{i\omega_0 t}/(\omega^2 - i\nu_c \omega)\}$

External driving: $h_{ext}(t) = A \sin(\omega_{dr} t + \gamma) e^{-(t-t_0)^2/(2\sigma^2)}$

Equations of motion

$$\frac{\partial^2 \phi_n}{\partial \tau^2} + \nu_c \frac{\partial \phi_n}{\partial \tau} + \sin(\phi_n) - \frac{\partial h_n}{\partial x} = j_{\text{ext}} + \eta(x, \tau)$$

$$(\nabla_n^2 - \frac{1}{\ell^2})h_n + \frac{\partial \phi_n}{\partial x} + \nu_{ab} \frac{\partial}{\partial \tau} \left(\frac{\partial \phi_n}{\partial x} - \frac{h_n}{\ell^2} \right) = 0$$

Open Boundaries: $\frac{\ell}{\sqrt{\epsilon_c}} \frac{\partial \phi_n}{\partial \tau} = h_n - 2H_{\text{ext}}(\tau)$

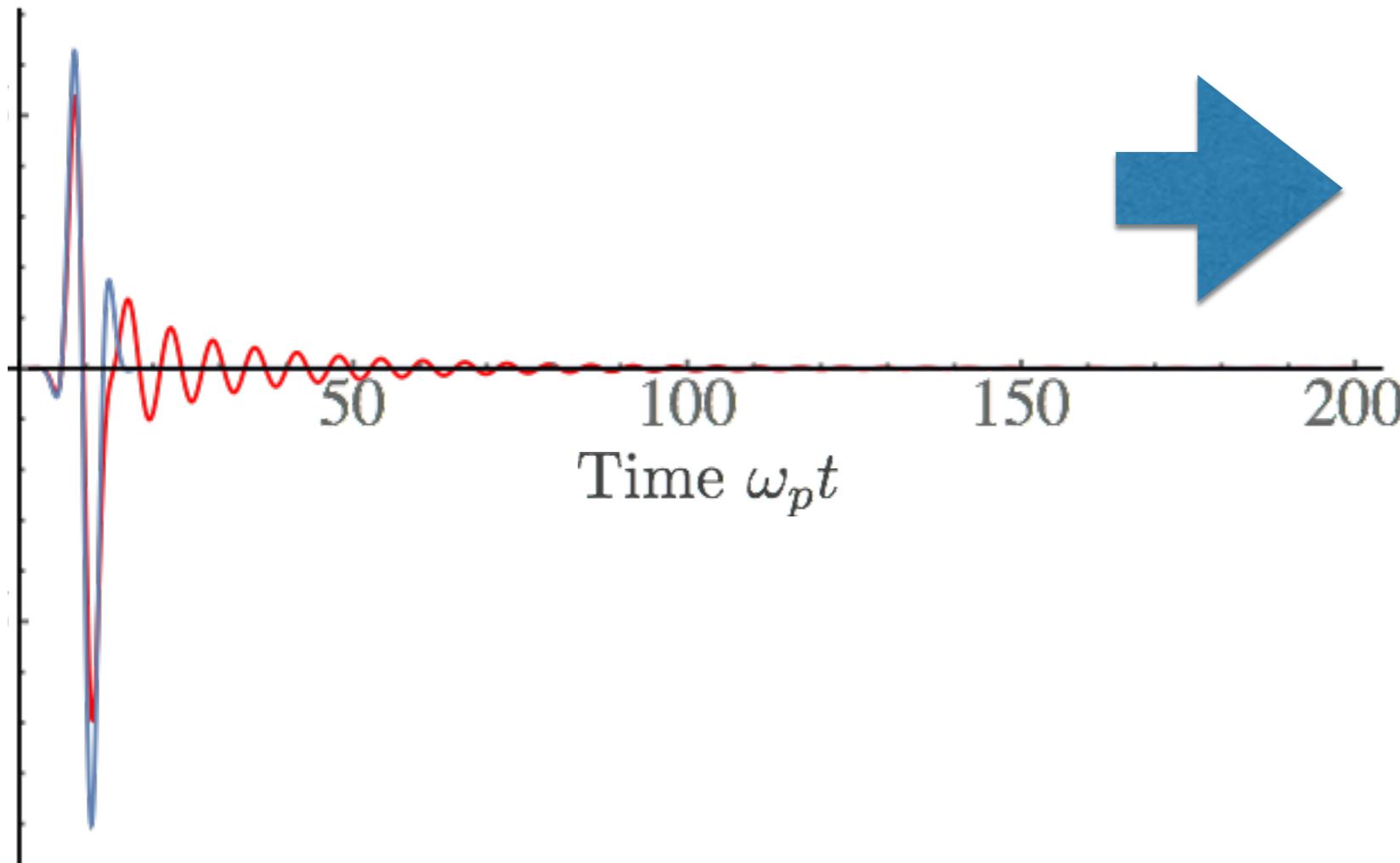
Closed Boundaries: $h_n = 2H_{\text{ext}}(\tau)$

$h \downarrow n$ magnetic field in y-direction in units of critical field

The system is solved by discretising the spatial dimension, and the use of the IDA (Implicit Differential-Algebraic solver) package, developed at the Center for Applied Scientific Computing of Lawrence Livermore National Laboratory.

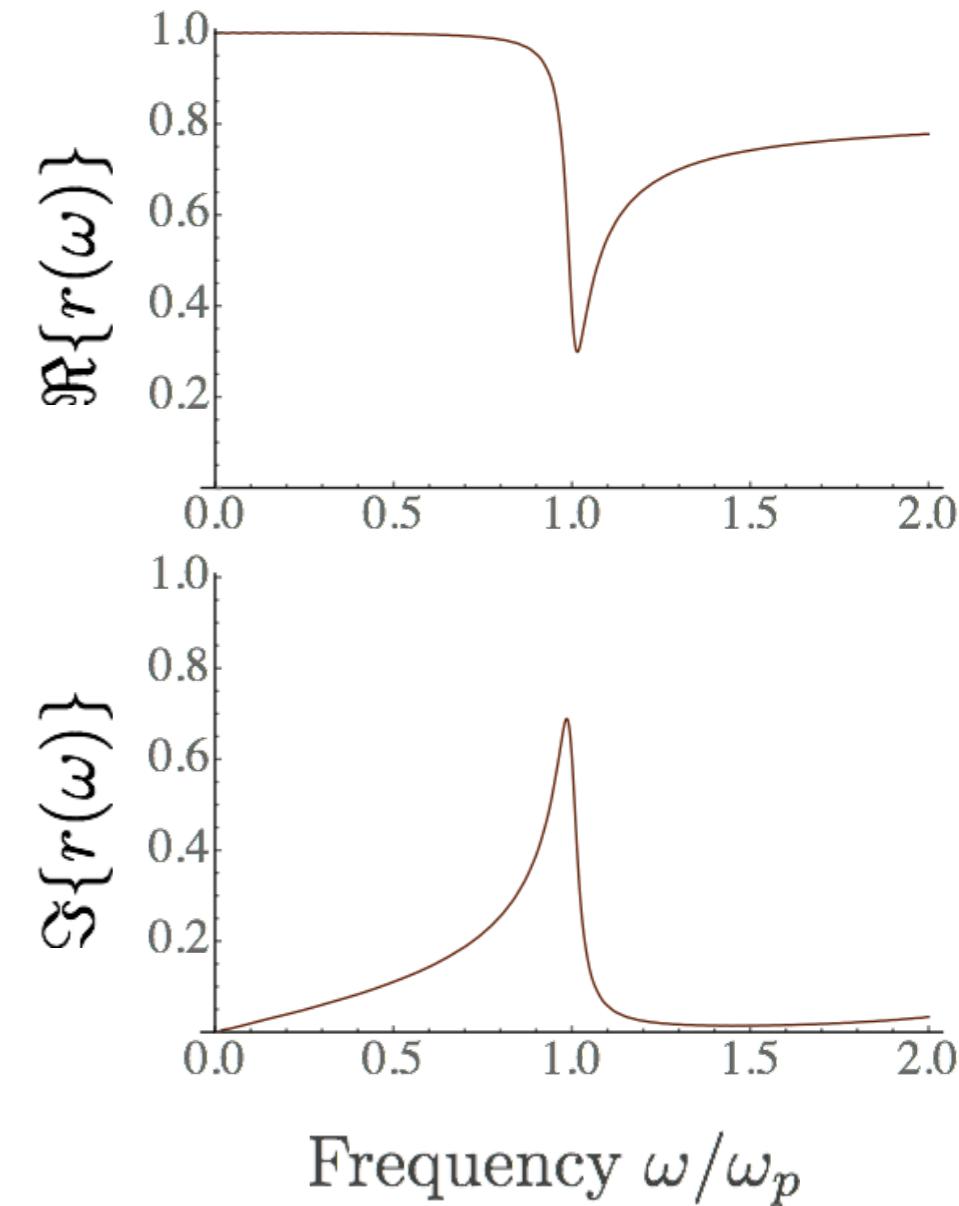
Problems: Resolution

Calculating optical signals - Reflectivity



Incident electric field

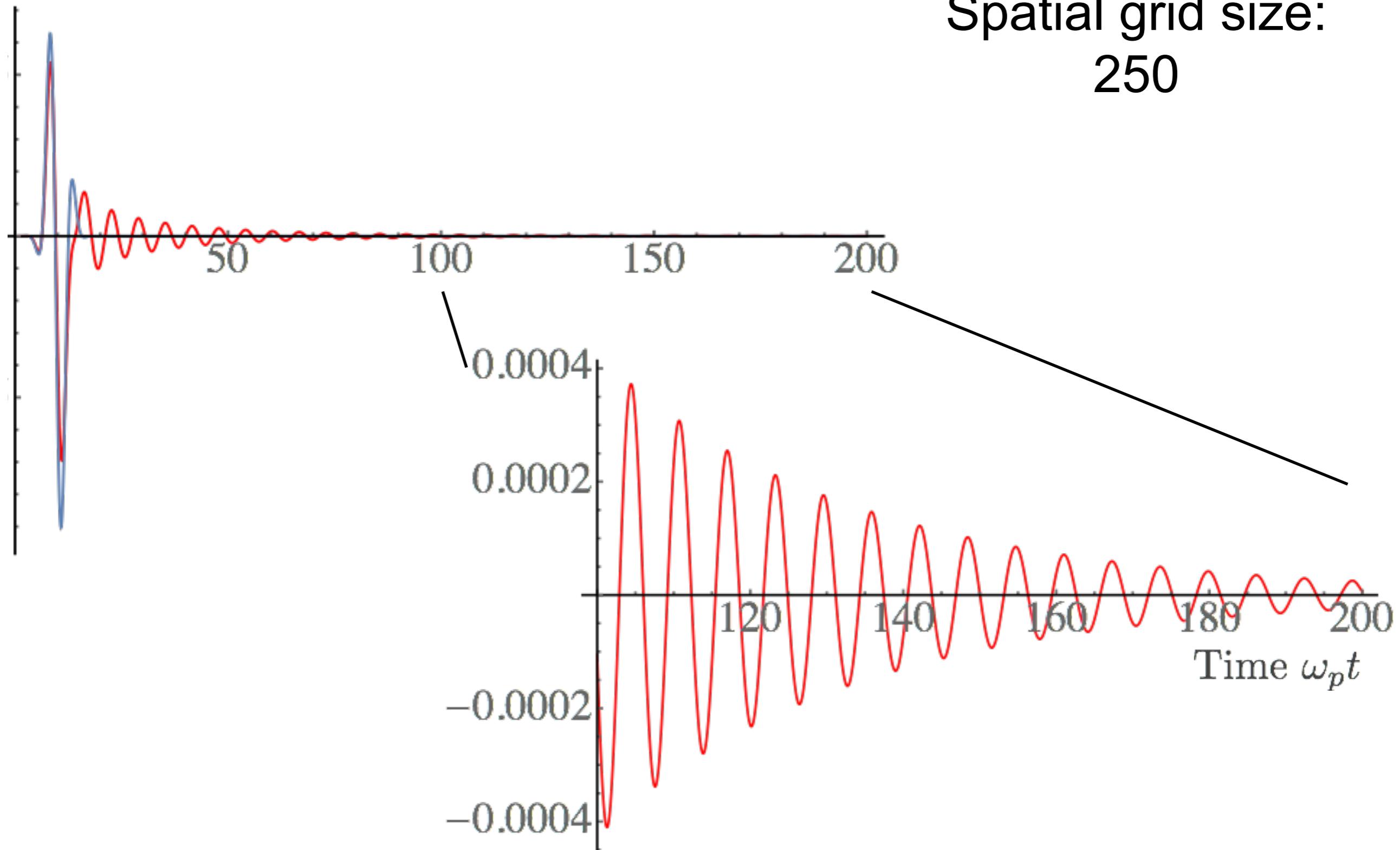
Reflected electric field $\propto \frac{\partial \phi_n}{\partial \tau}$



Problems: Resolution

Close-up to small values

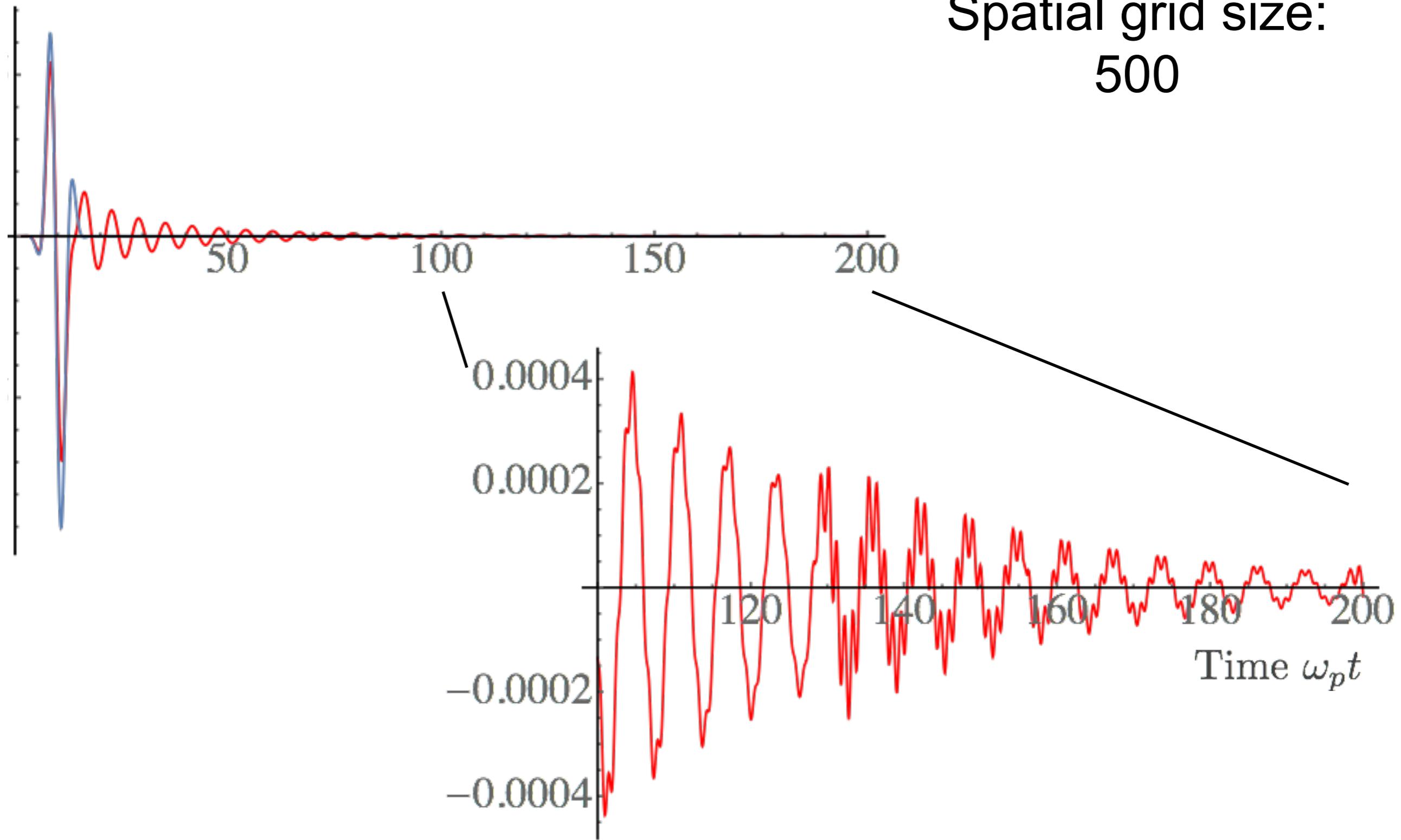
Spatial grid size:
250



Problems: Resolution

Close-up to small values

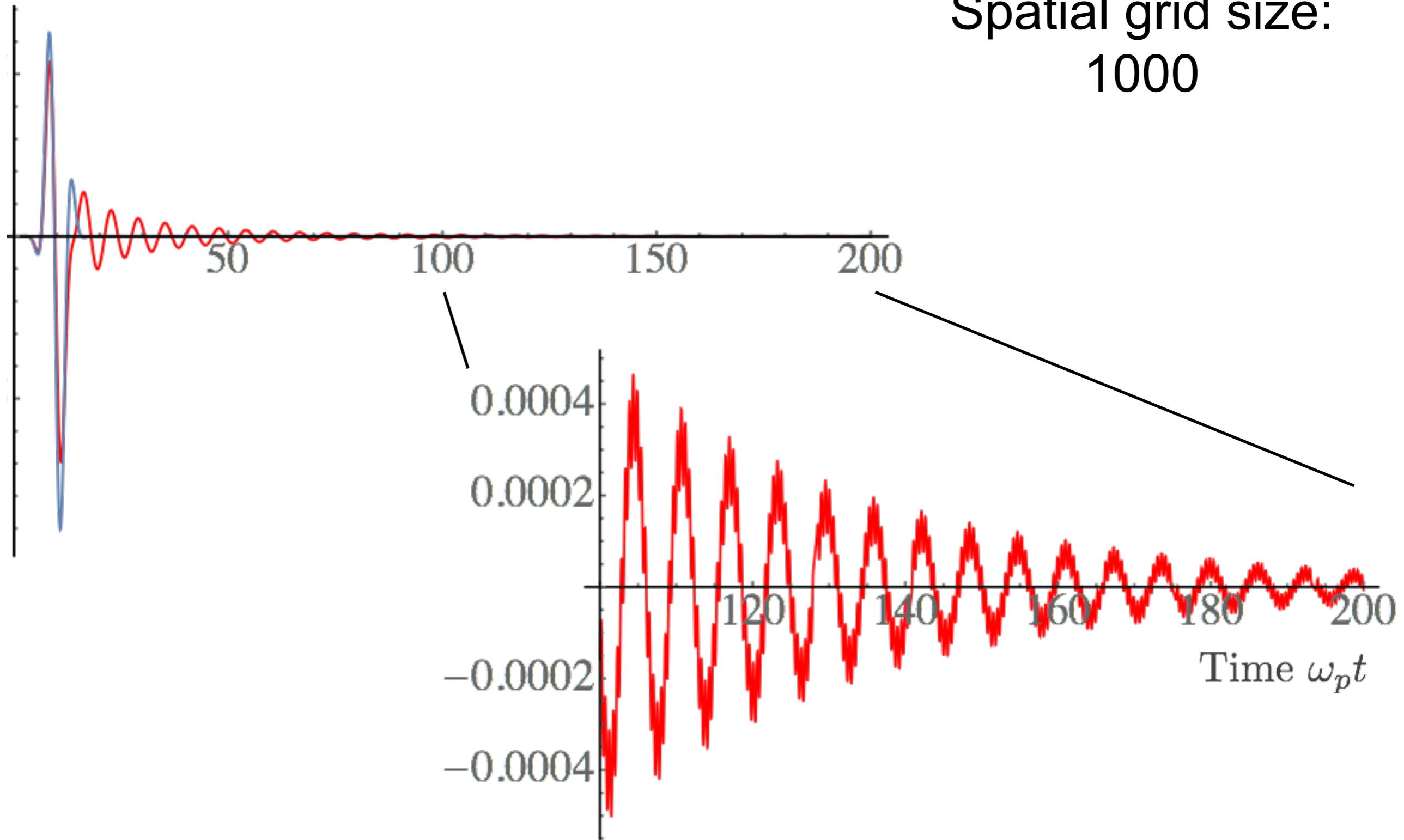
Spatial grid size:
500



Problems: Resolution

Close-up to small values

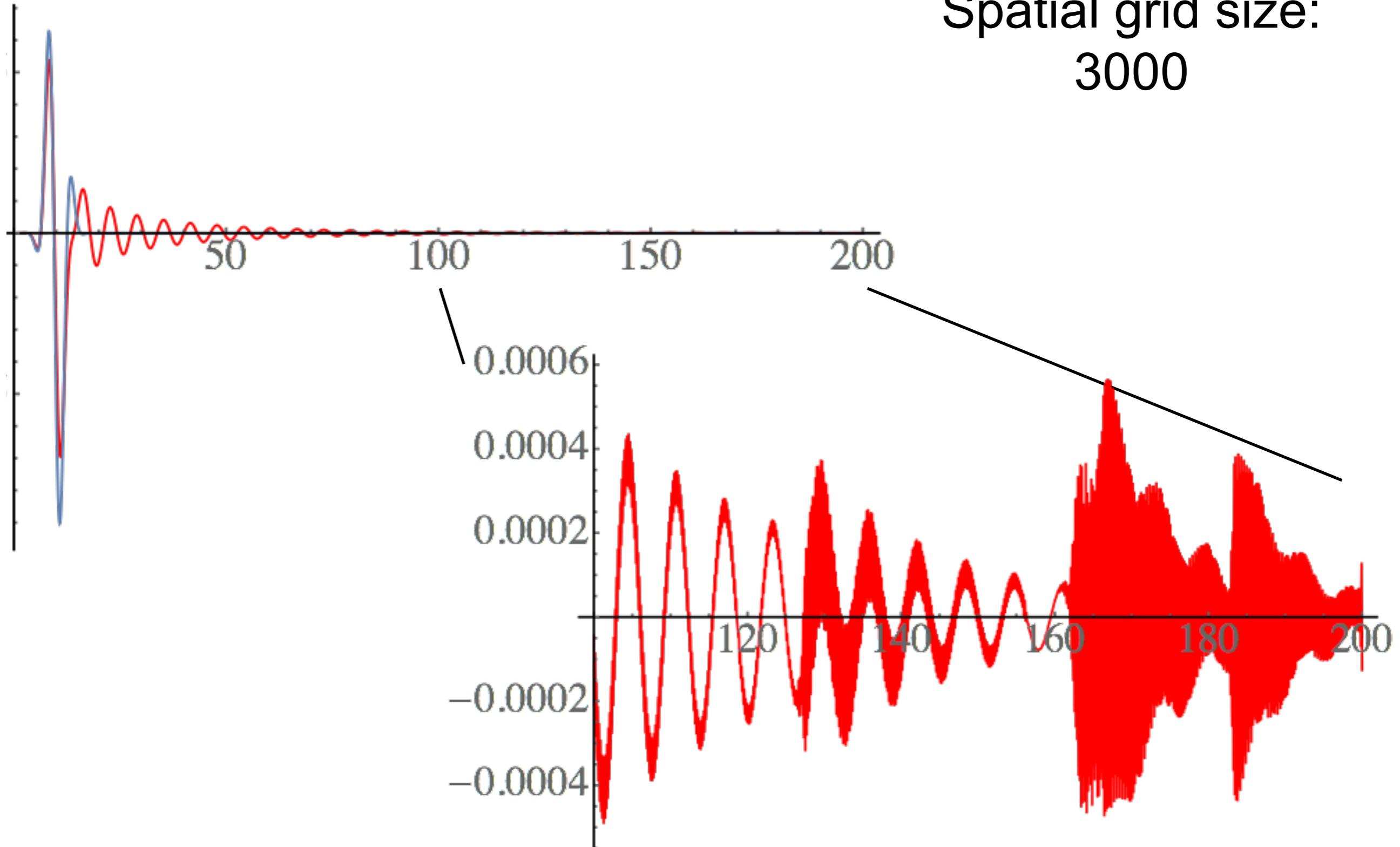
Spatial grid size:
1000



Problems: Resolution

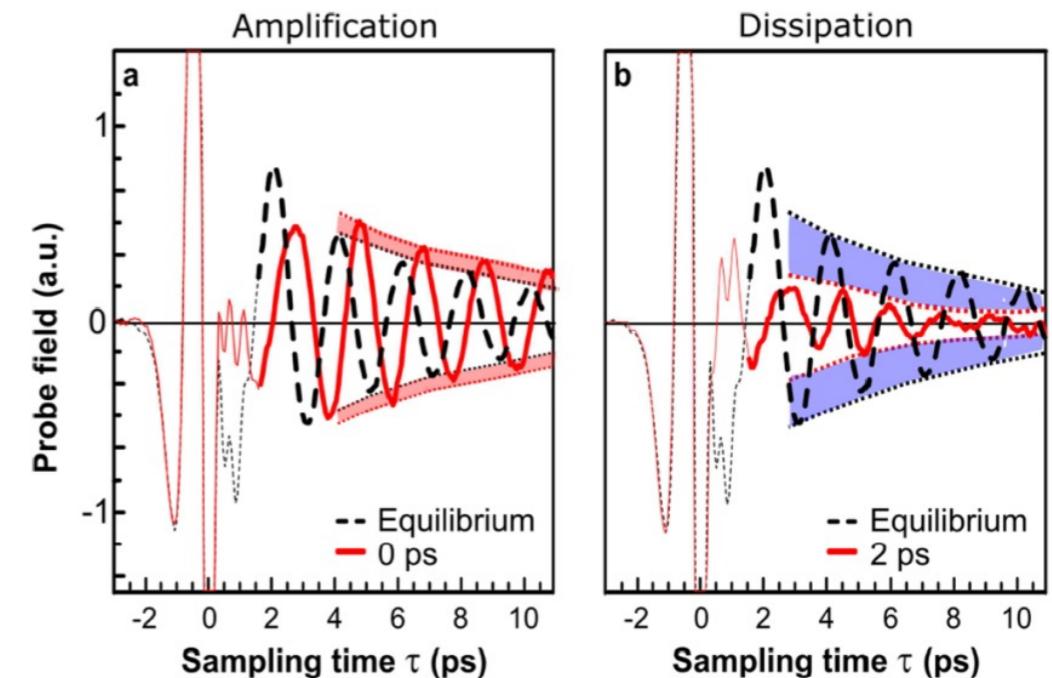
Close-up to small values

Spatial grid size:
3000



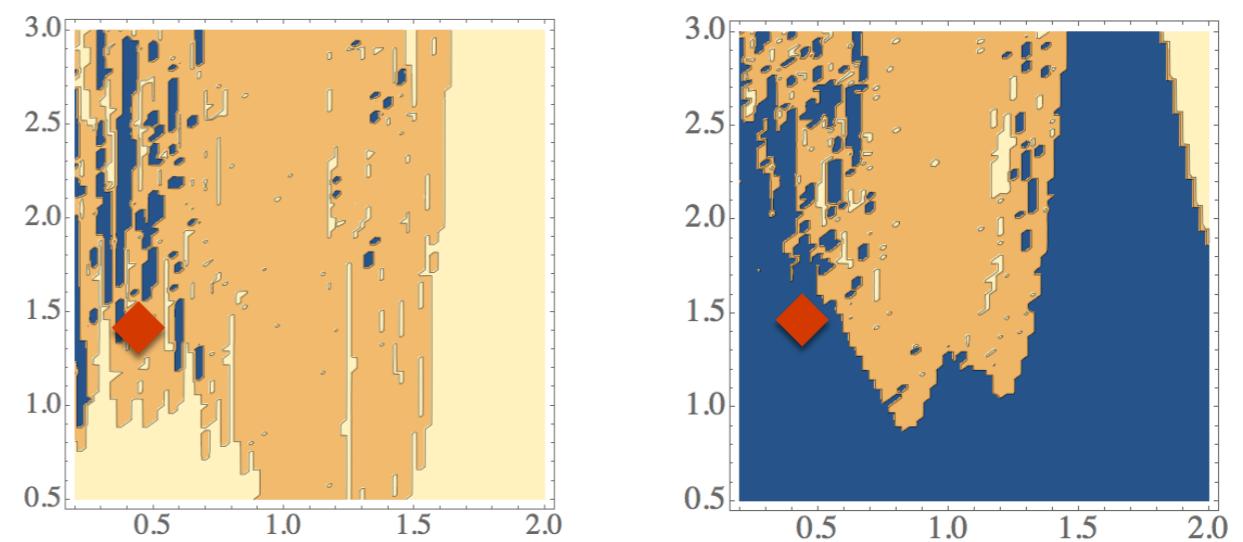
Summary

- ▶ Amplification of plasma waves
- ▶ Manipulation of currents with THz pulses
- ▶ Control of the system's phases



Challenge:

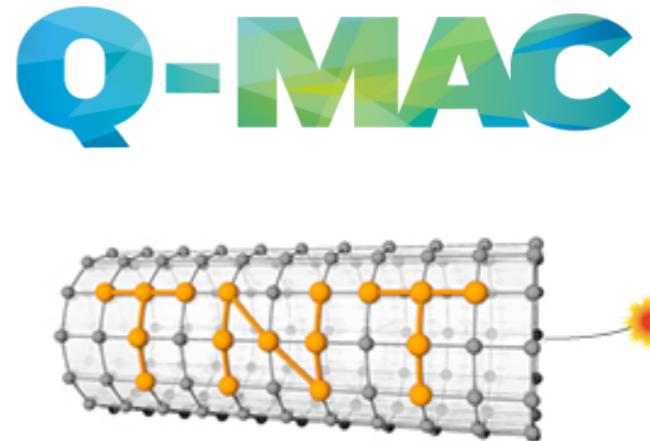
- ▶ Get accurate numerical results in the highly non-linear regime



Thanks to the team and collaborators



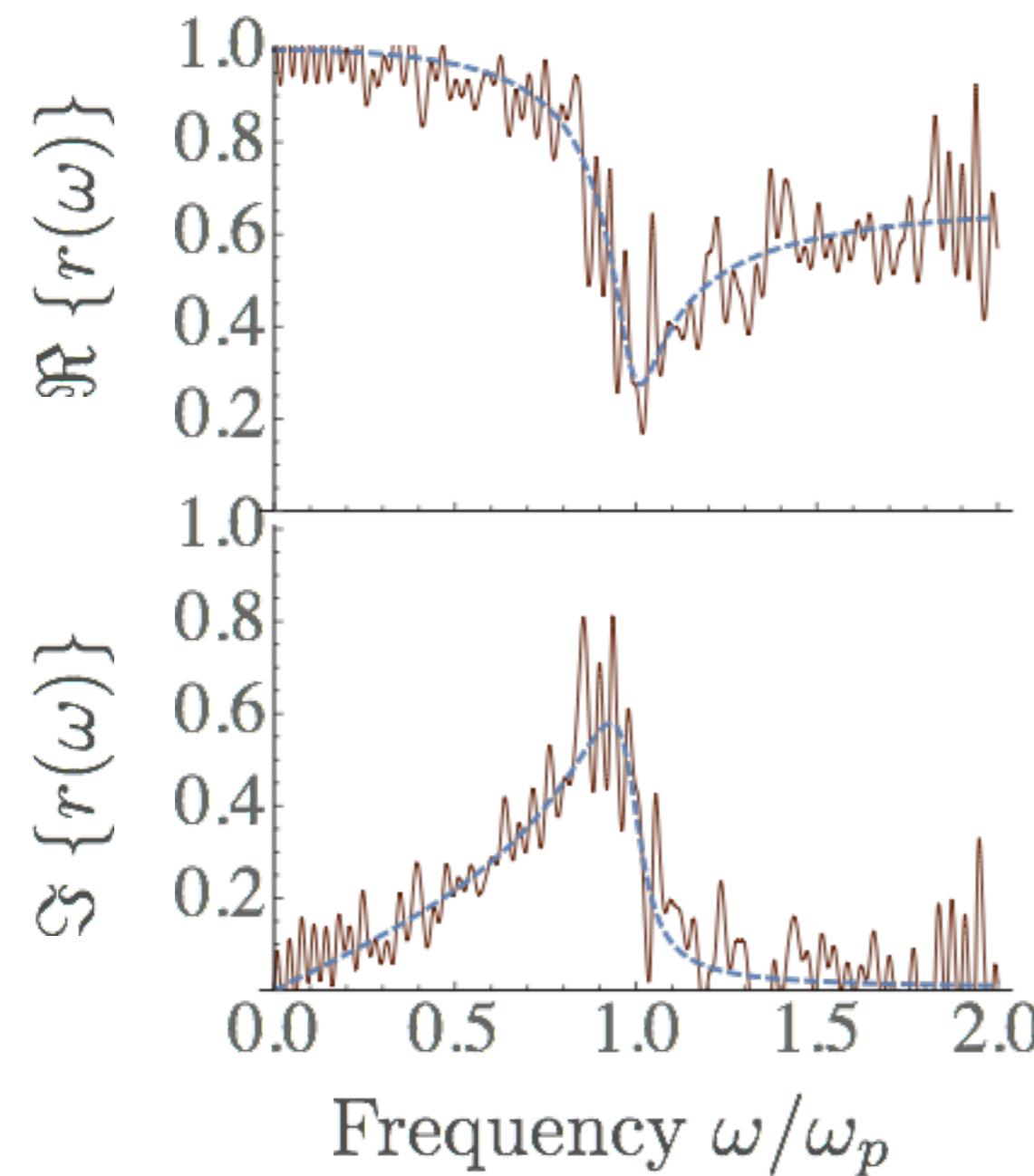
- Post-Docs
 - Dr Stephen Clark → Bath
 - Dr Sarah Al-Assam
 - Dr Martin Kiffner
 - Dr Tomi Johnson → London
 - **Dr Frank Schlawin**
 - Dr Juan Jose Mendoza
 - Dr Anna-Maija Uimonen
 - Dr Jordi Mur Petit
 - Dr Florian Pinsker
- DPhil students
 - Jonathan Coulthard
 - Paolo Rosson
 - Anastasia Dietrich
 - Samuel Denny
 - Juha Kreula
- Collaborators
 - **Andrea Cavalleri**
 - **Jean-Marc Triscone**
 - **Antoine Georges**
 - Wenhui Li
 - Dimitris Angelakis
 - Weizhu Bao
 - Ian Walmsley
 - Axel Kuhn
 - Igor Mekhov
 - Chris Foot
 - Sabrina Maniscalco
 - Andreas Buchleitner
 - Stefan Kuhr
 - Martin Plenio
 - Javier Prior
 - Numerical Algorithms Group



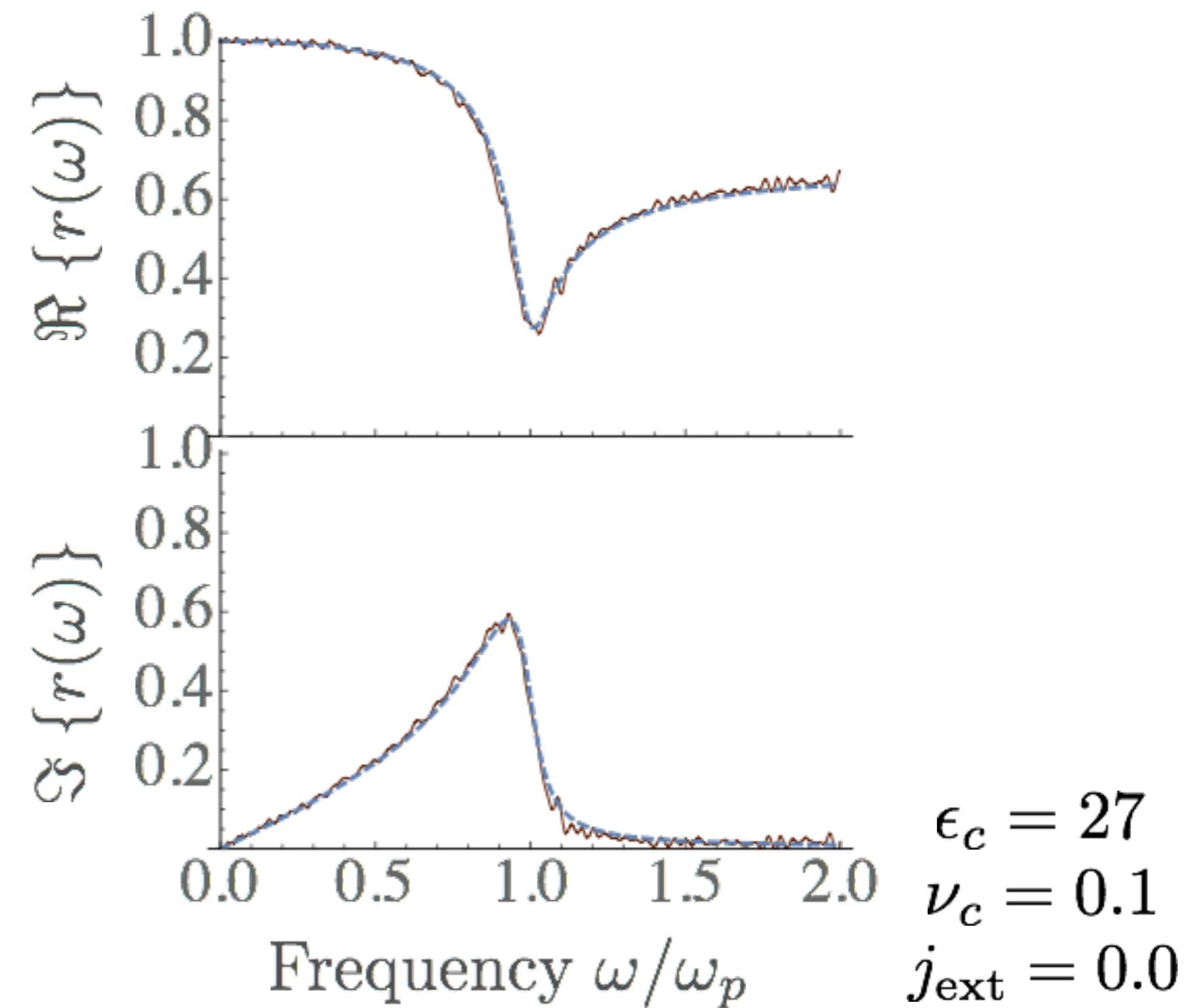
Noise

Dashed line: $\epsilon_0(\omega) = \epsilon_c \left(\sqrt{1 - j_{\text{ext}}^2} - \frac{1}{\omega^2} + i \frac{\nu_c}{\omega} \right)$ ($A = 0.5$)

Single realisation:

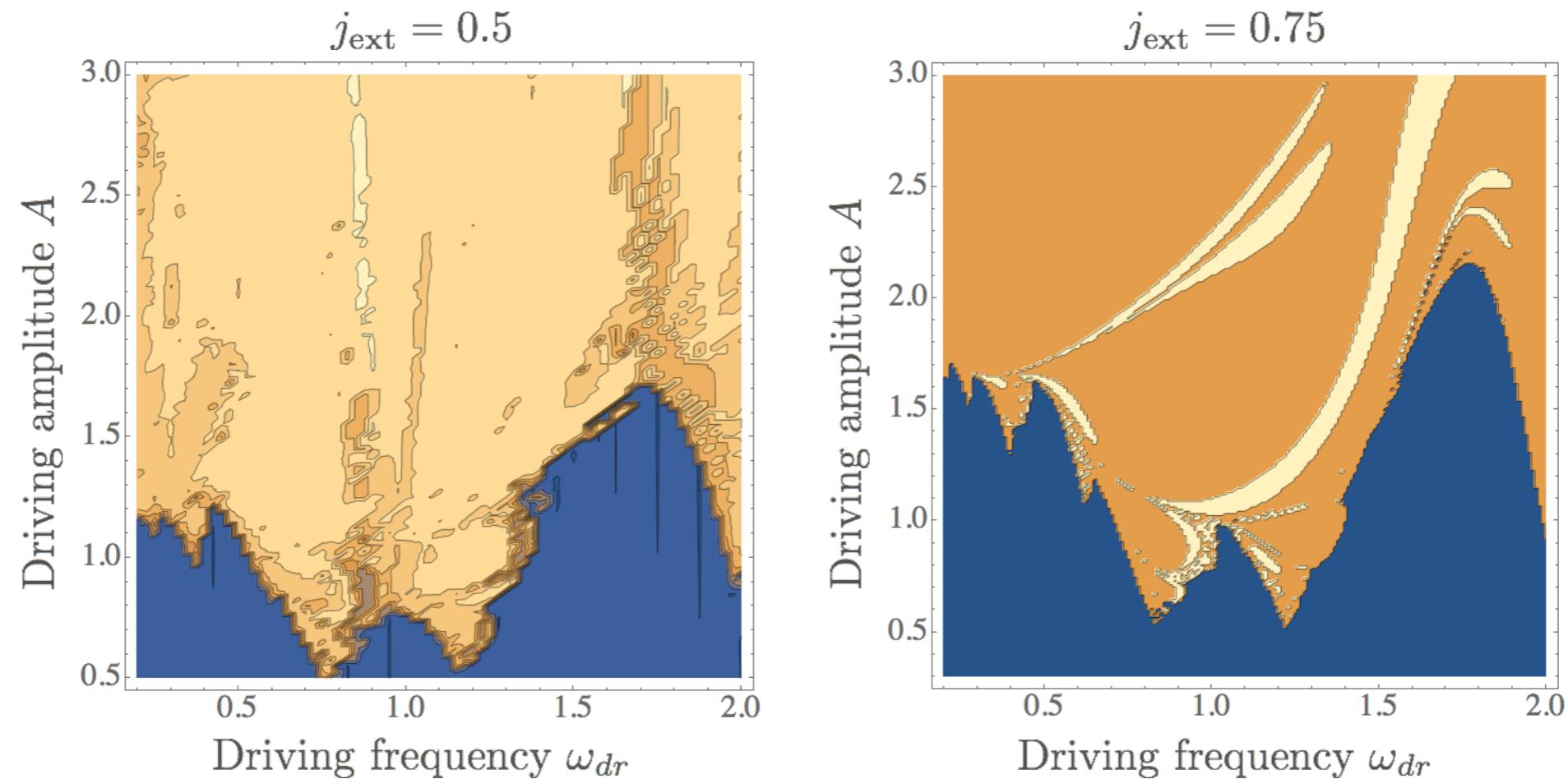


Average:



Parameter space II

($\sigma = \infty$)

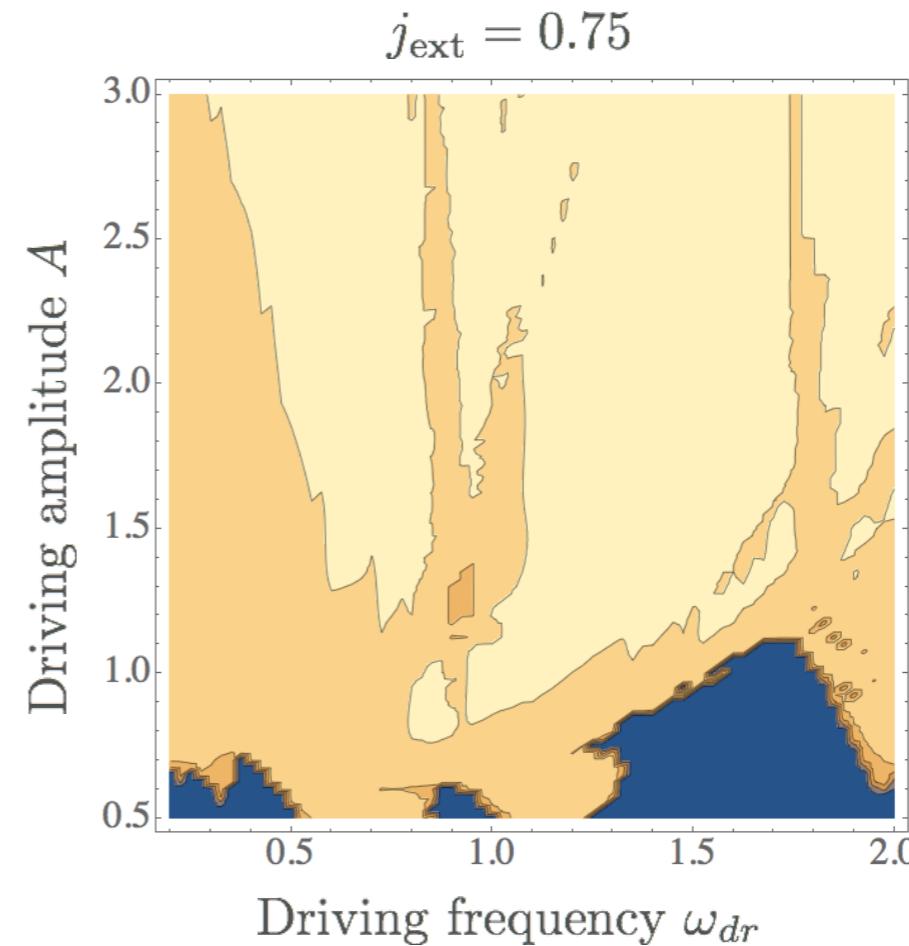
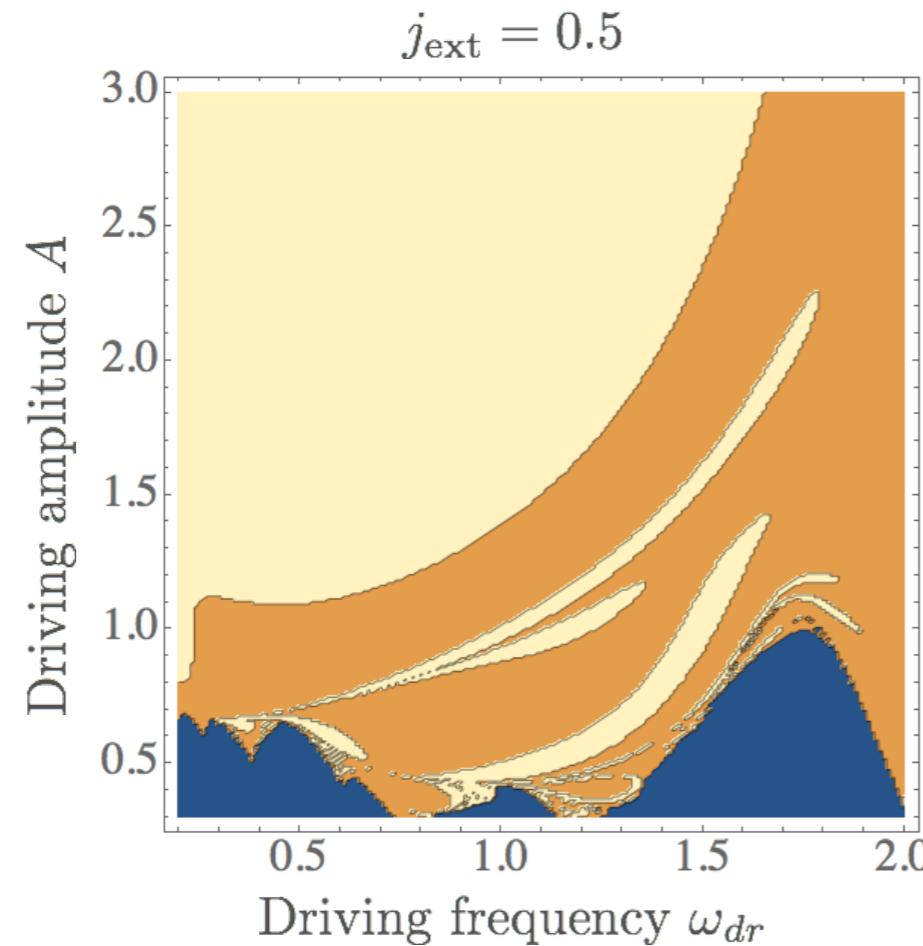


Initial sc state: $\phi_0 = \arcsin j_{\text{ext}}$

External driving: $h_{\text{ext}}(t) = A \sin(\omega_{dr} t)$

Parameter space II

($\sigma = \infty$)



Initial sc state: $\phi_0 = \arcsin j_{\text{ext}}$

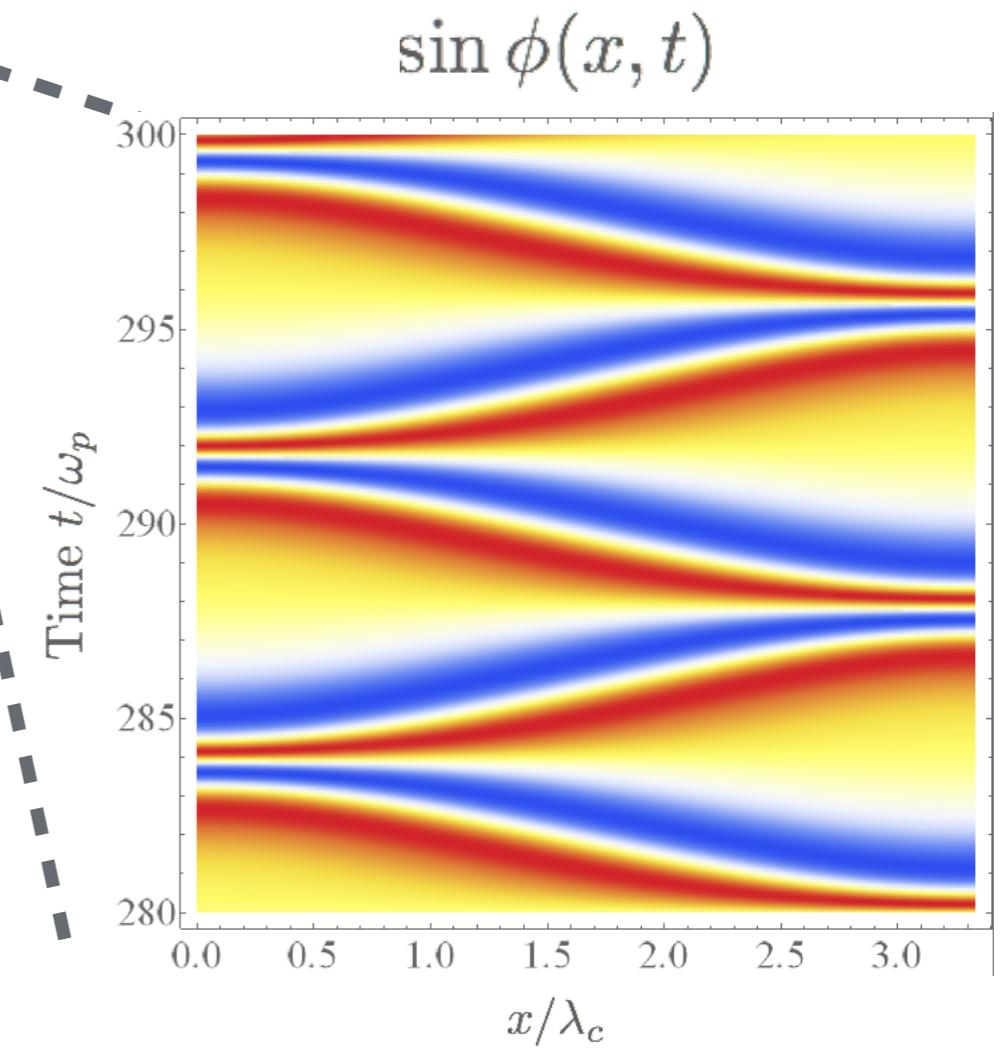
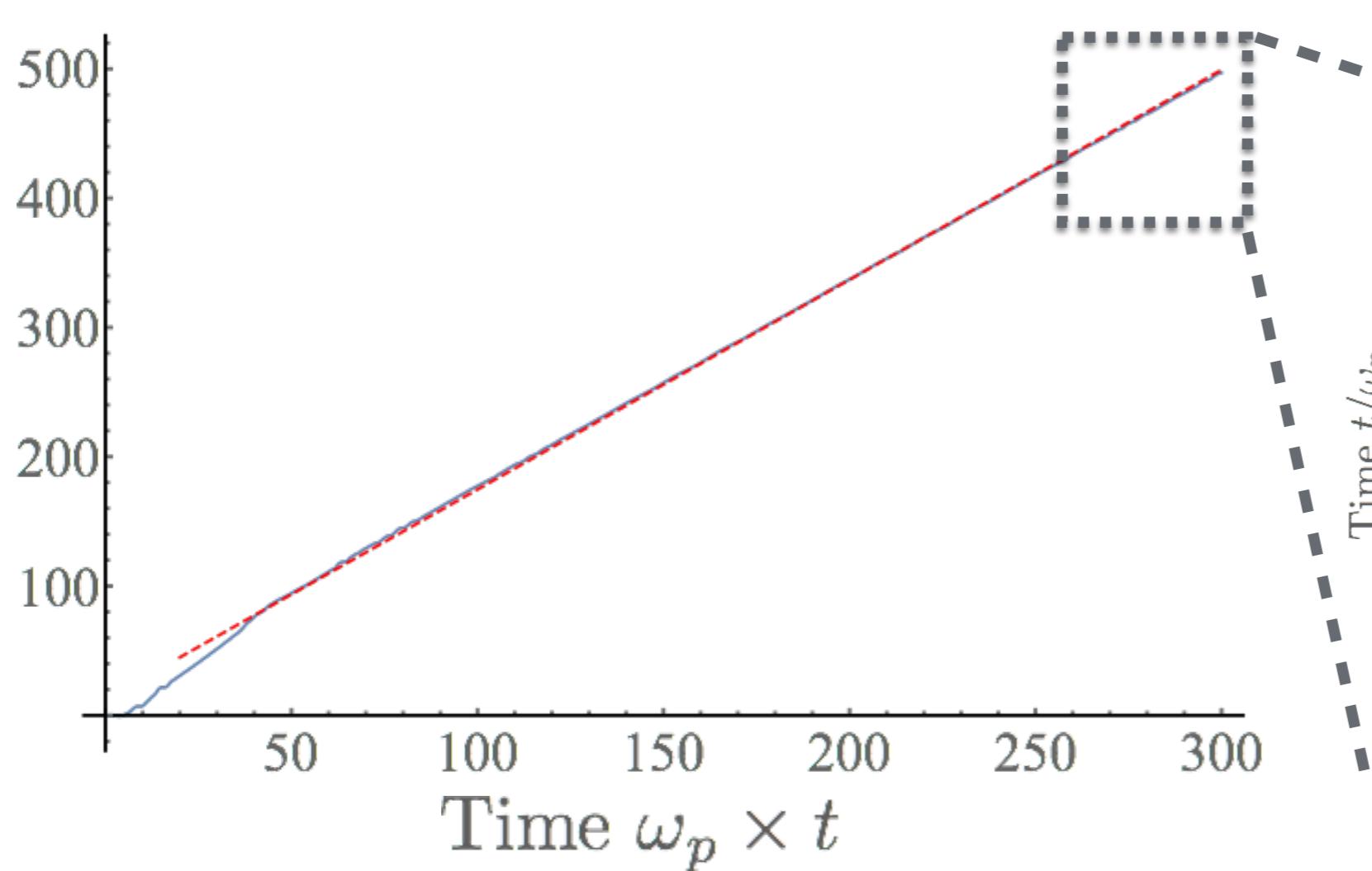
External driving: $h_{\text{ext}}(t) = A \sin(\omega_{dr} t)$

Simplified model
overestimates
excitation amplitude

Long junctions: traveling kinks

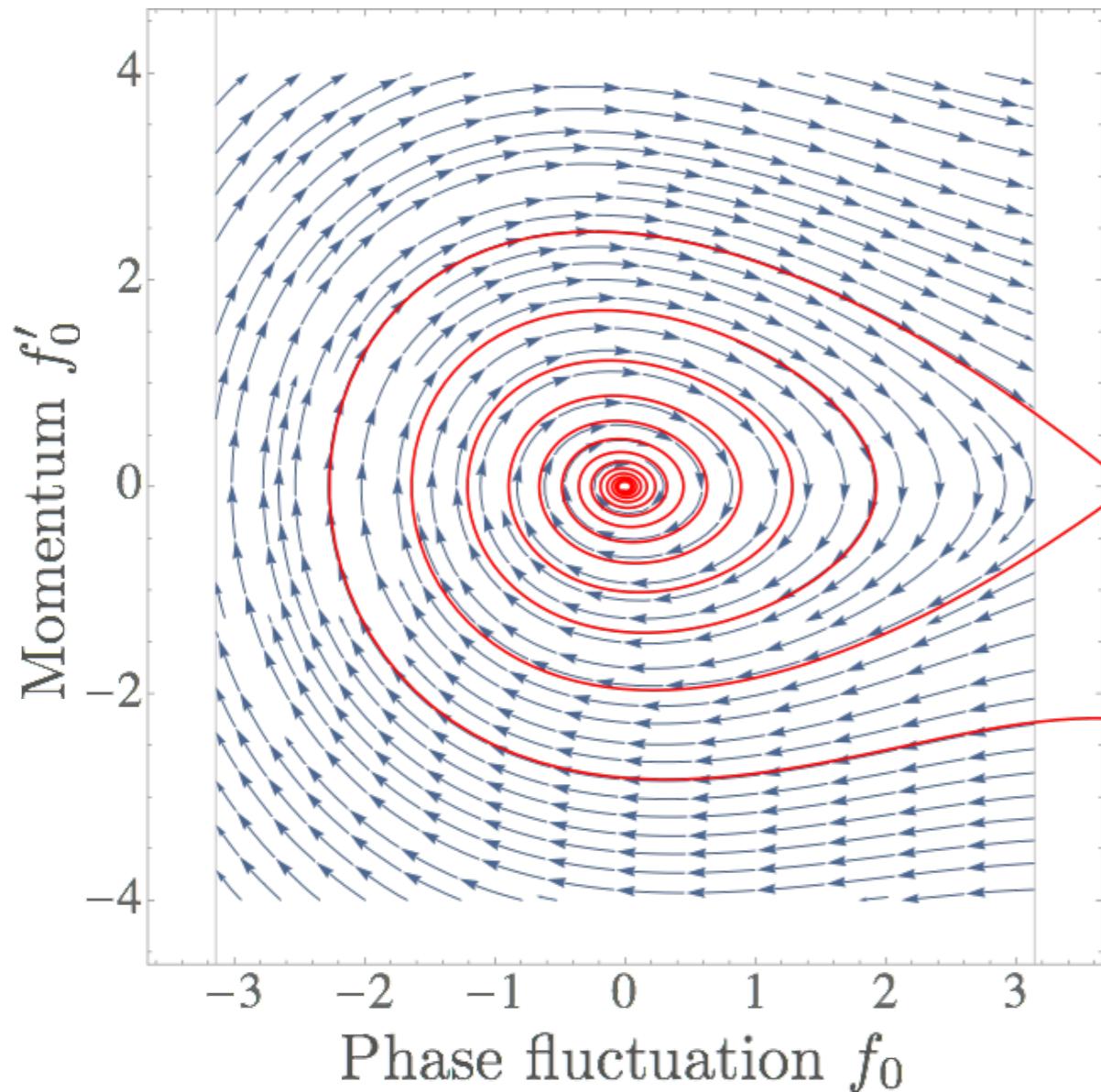
Traveling kink:

$$\phi(x, t) = 4 \tan^{-1} \left[\exp \left(\pm \frac{x - ut}{\sqrt{1 - u^2}} \right) \right] \quad 0 \leq |u| \leq 1$$



$(\nu_{ab} \approx 0)$

Parameter space I



Evolution of uniform oscillations:

$$f_0''(t) + \nu_c f_0'(t) + \sqrt{1 - j_{\text{ext}}^2} f_0 - \frac{j_{\text{ext}}}{2} f_0^2 = f_{dr}(t)$$

