Université Lille 1 Sciences et Technologies, Lille, France

Laboratoire de Physique des Lasers, Atomes et Molécules

Équipe Chaos Quantique

17 years of experiments on the atomic kicked rotor!

Symmetries and dynamics in a quantum-chaotic system

Engineering Hamiltonians and symmetries

Jean-Claude Garreau

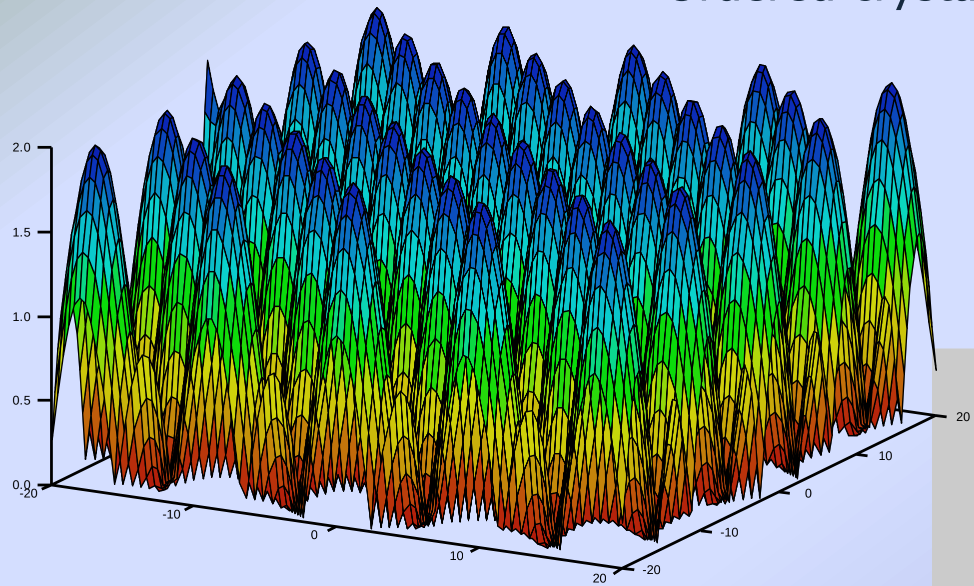
Nouveaux défis dans la modélisation mathématique et la simulation numérique de systèmes superfluides

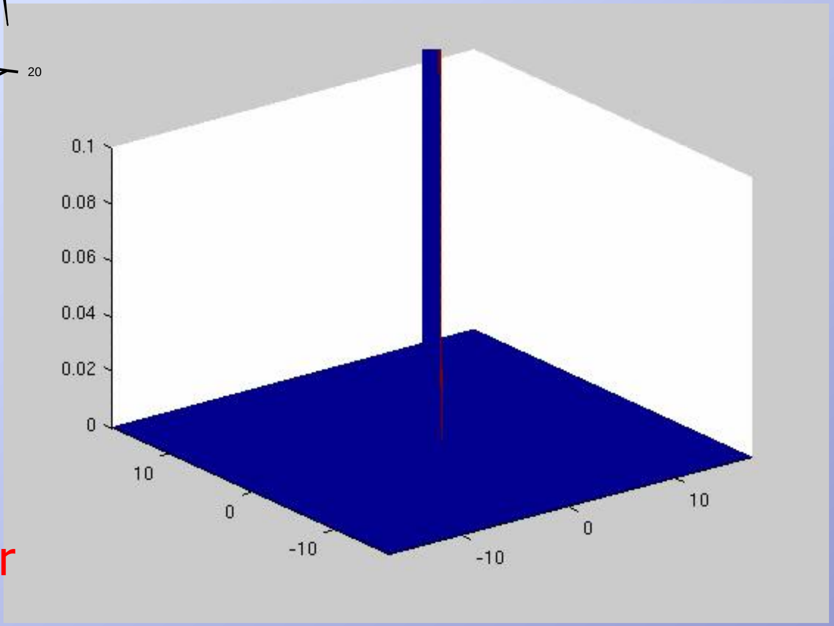
CIRM – Luminy – 30 June 2016

Quantum disorder

Multiple scattering and interference

Ordered crystal



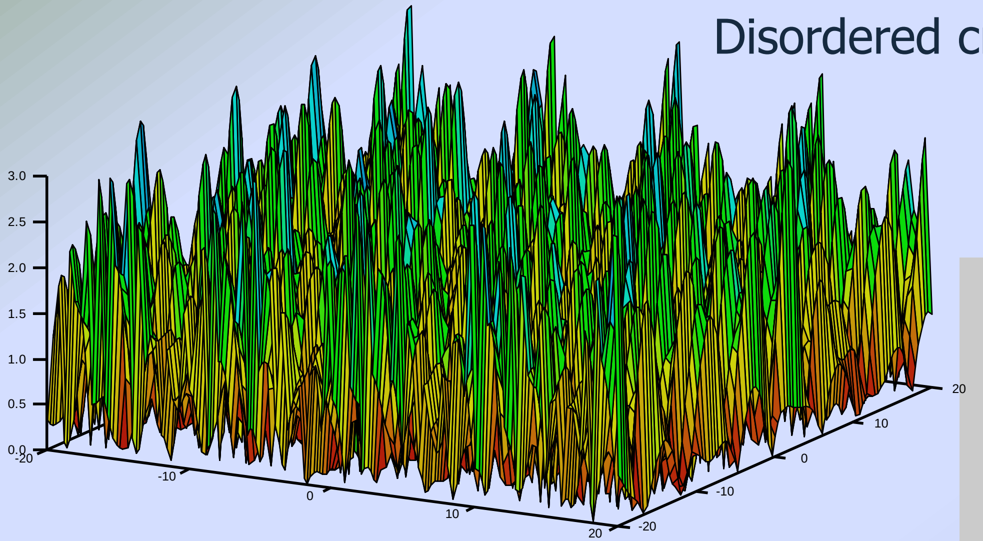


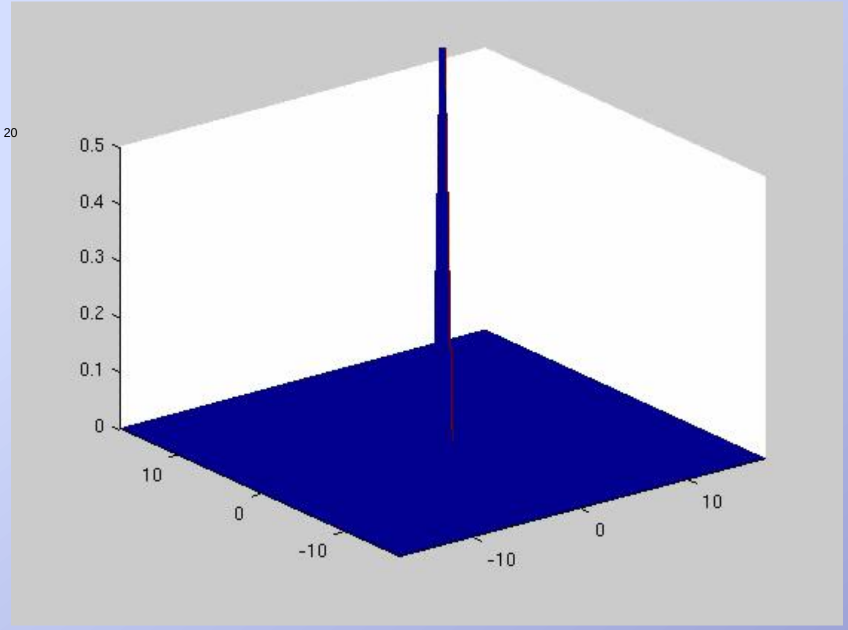
Conductor

Perfect crystal: Delocalized Bloch waves → diffusive dynamics

Strong disorder, strong localization – Anderson localization

Disordered crystal





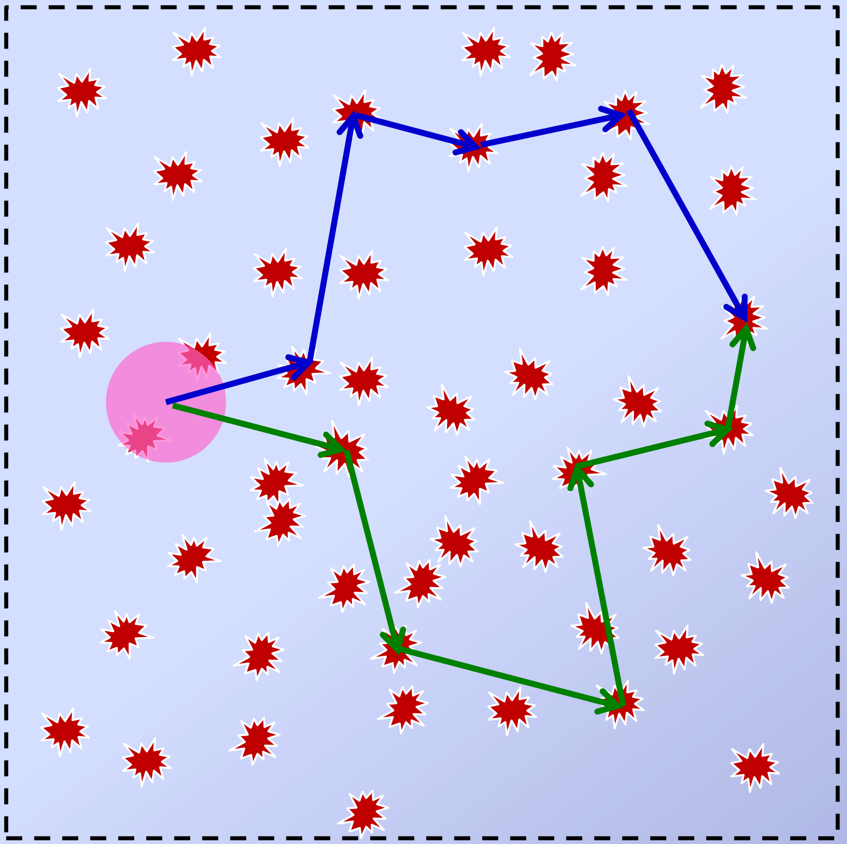
Insulator

Localized states: Anderson (or strong) localization

Strong localization: Complete destructive interference

(Too) many (all) interference paths contribute

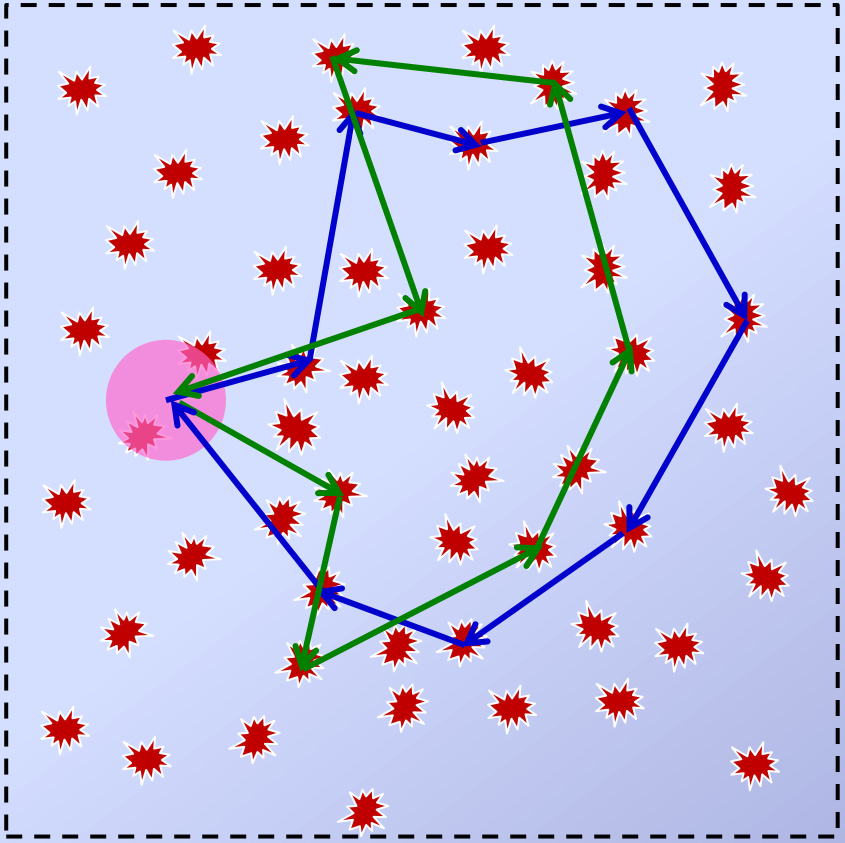
P. W. Anderson, *Absence of Diffusion in Certain Random Lattices*, Phys. Rev. **109**, 1492--1505 (1958)



$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\varphi_1 - \varphi_2) \approx |A_1|^2 + |A_2|^2$$

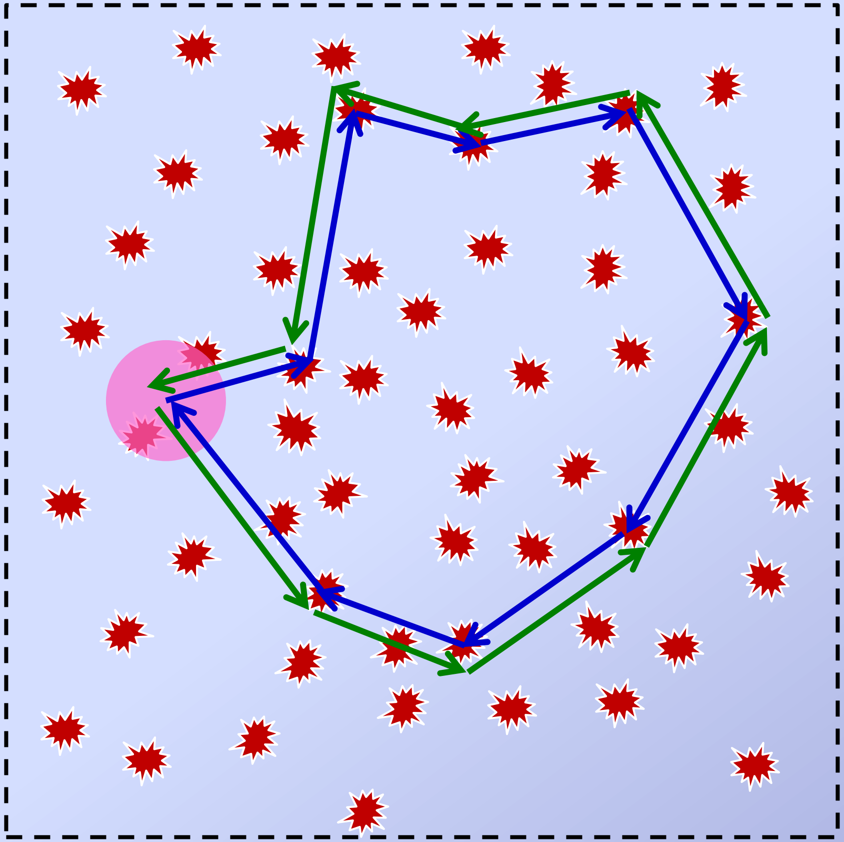
General rule is phase averaging among different paths (no interference)

Disordered media: Return to the origin



General rule: Still no interference

Disordered media: systems invariant under time-reversal



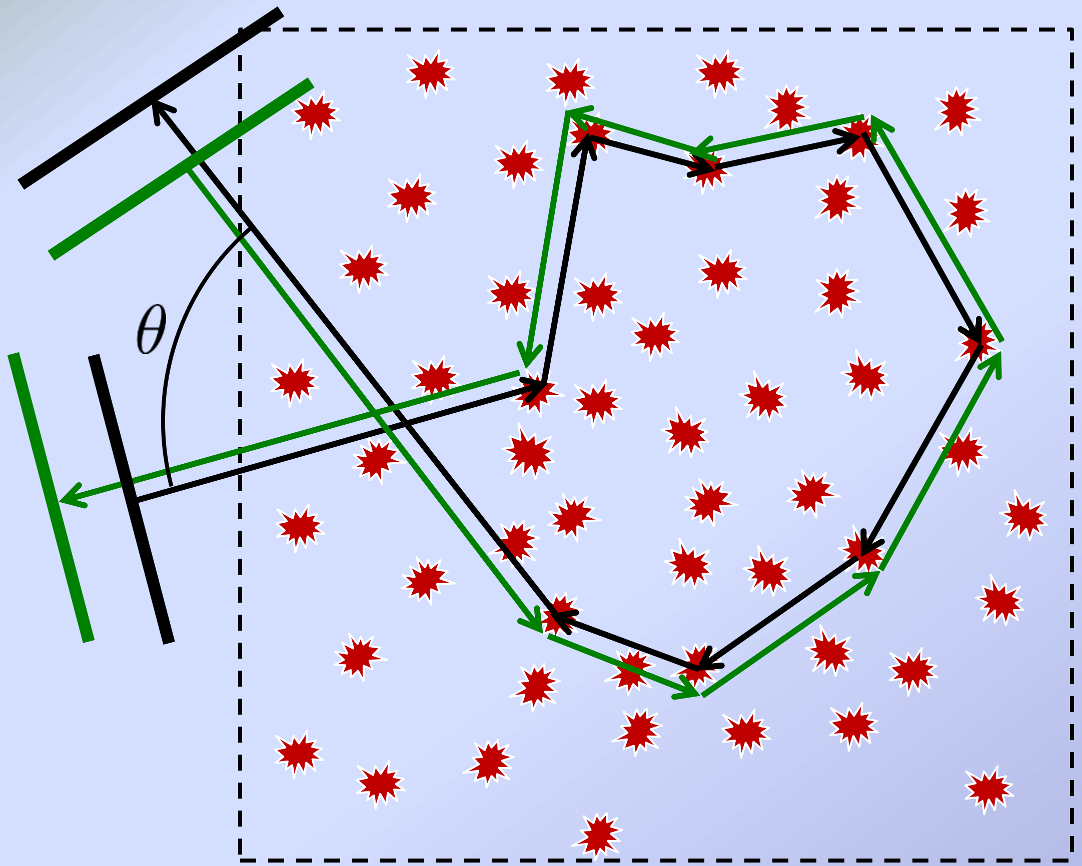
Identical paths, inverted sense

$$|A_1 + A_2|^2 = 2|A_1|^2 + 2|A_1|^2 \cos(\varphi_1 - \varphi_2)$$

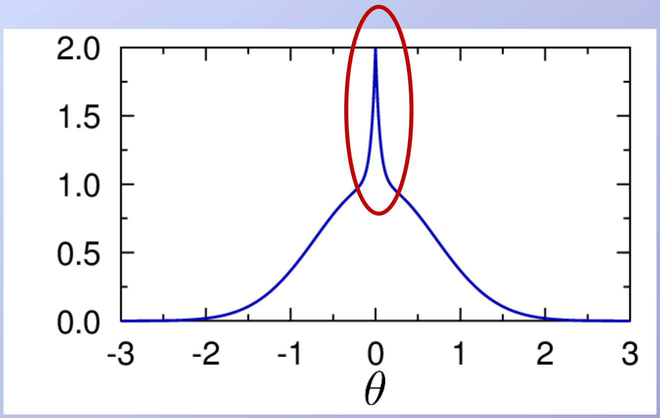
Time-reversal invariant system

$$|A_1 + A_2|^2 = 4|A_1|^2 = 2 \times \text{"incoherent contribution"}$$

"Enhanced return to the origin"



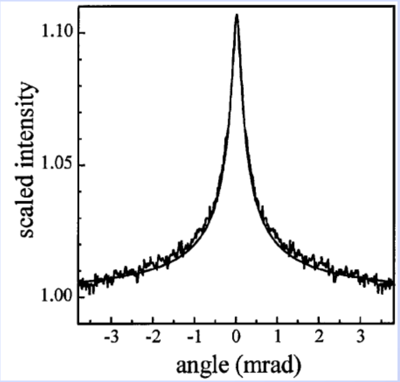
Scattering back in the initial direction is 2 x more intense!



"Coherent backscattering"

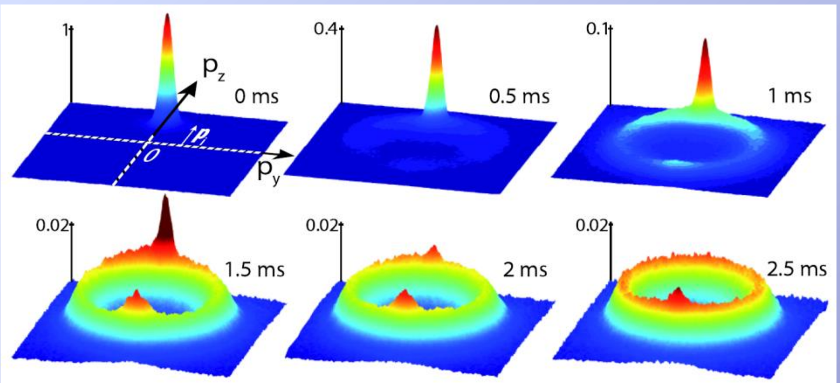
- CBS and enhanced return to origin are manifestations of "weak localization": A disordered system presents a diffusion coefficient reduced by interference effects.
- CBS was observed with kinds of wave: Light (laser light diffused by milk!), microwaves, acoustic waves, seismic waves and matter waves

CBS of light by cold atoms!



G. Labeyrie *et. al.*, *Coherent backscattering of light by cold atoms*, Phys. Rev. Lett. **83**, 5266--5269 (1999)

CBS of (ultra)cold atoms by light!



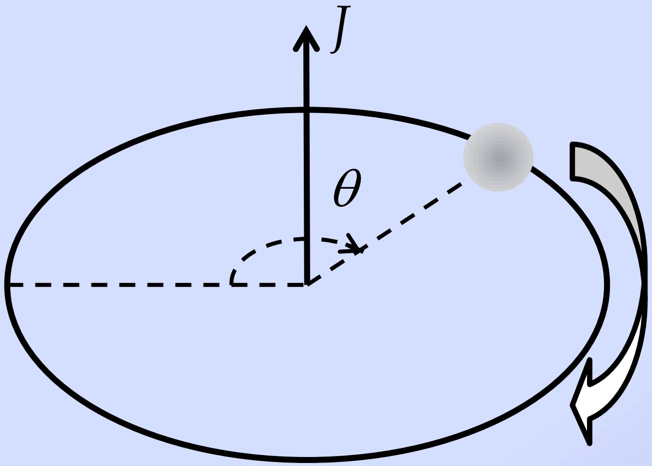
F. Jendrzejewski *et al.*, *Coherent Backscattering of Ultracold Atoms*, Phys. Rev. Lett. **109**, 195302 (2012)

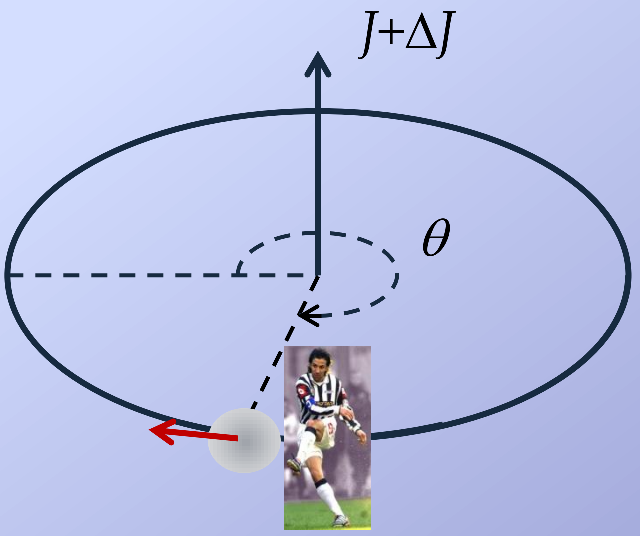
Enhanced return to the origin was never directly observed with matter waves

The kicked rotor:
A paradigm of classical and quantum chaos

Free motion

Kick

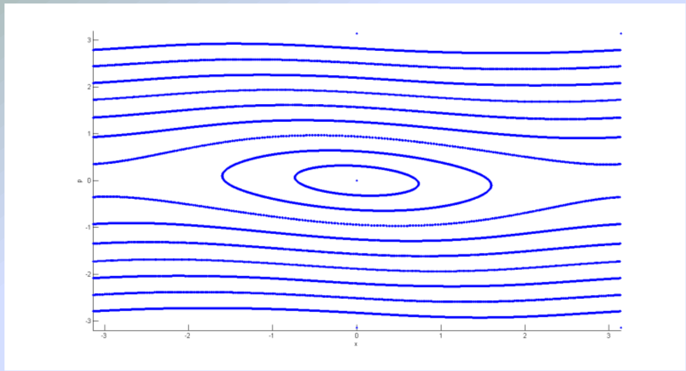




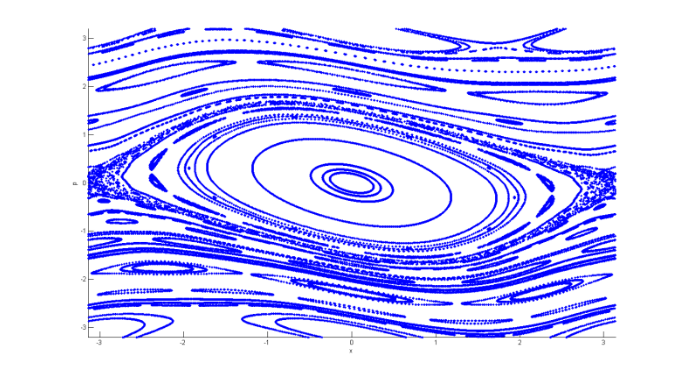
$$H = \frac{J^2}{2I} + K \cos\theta \sum_n \delta(t - nT)$$

Classical chaos: phase portraits for the classical KR

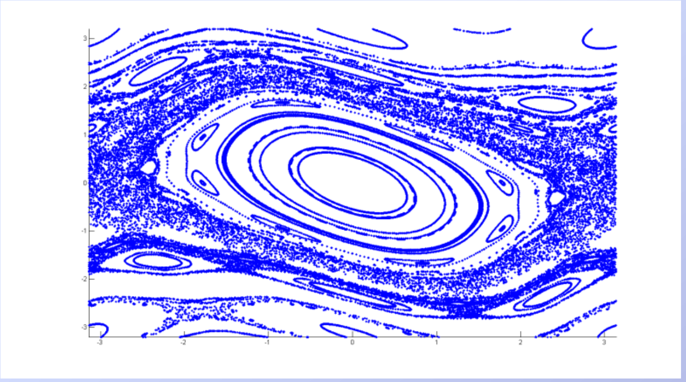
$K = 0.2$



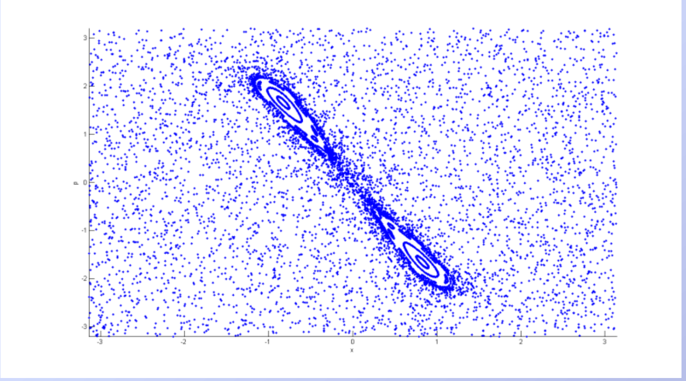
$K = 0.6$



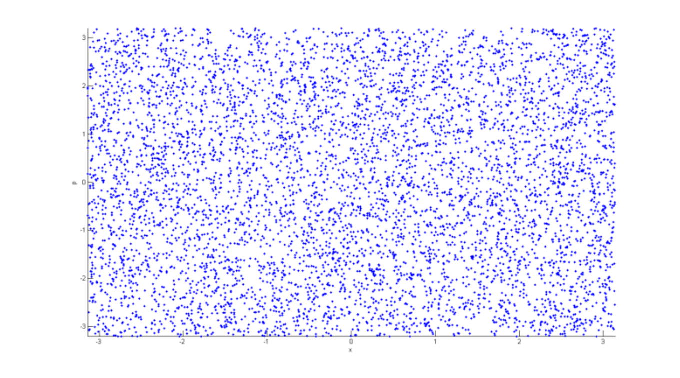
$K = 1 > 0.9716...$



$K = 4.5$



$K = 6$

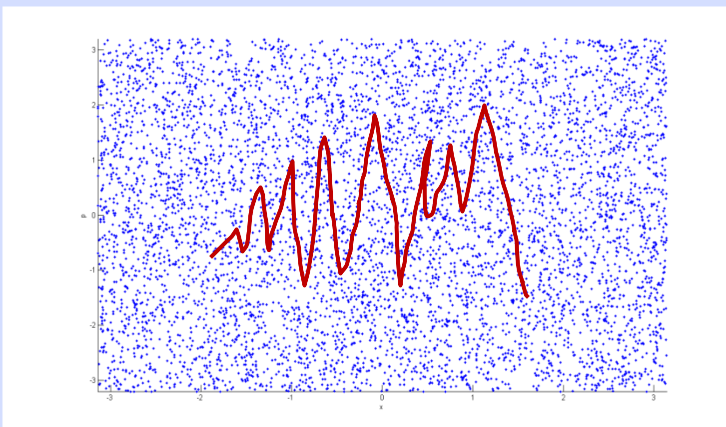


B. V. Chirikov, *A universal instability of many-dimensional oscillator systems*, Phys. Rep. **52**, 263--379 (1979)

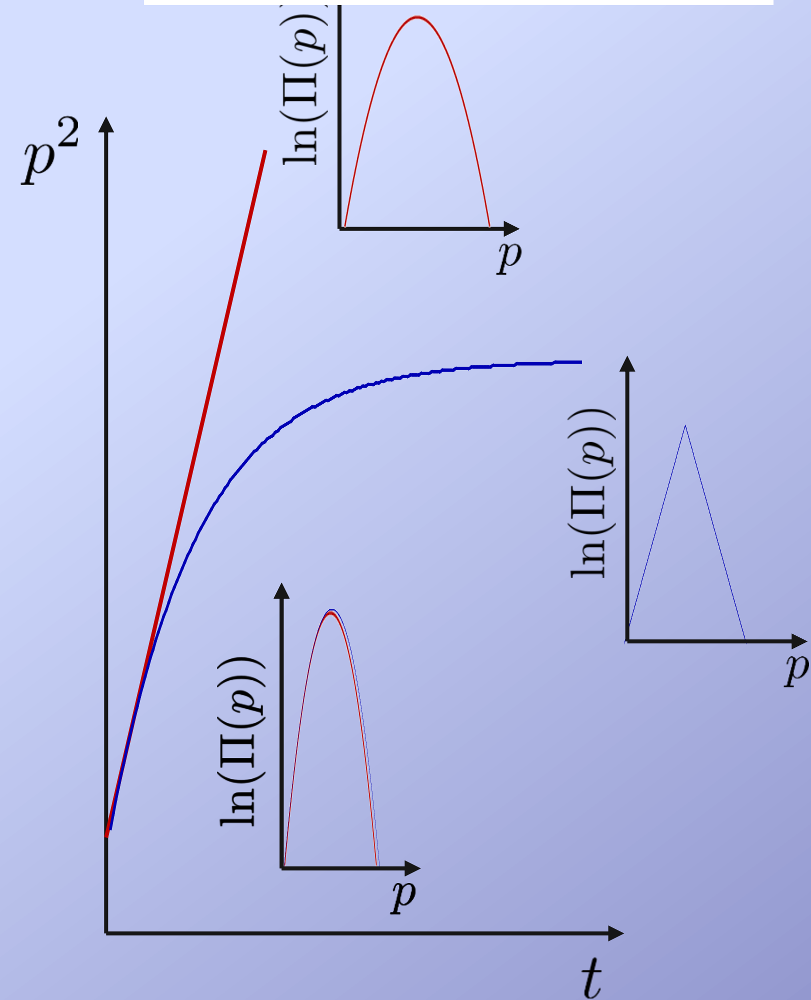
iKcked rotor: Classical and quantum dynamics

$$H = \frac{J^2}{2I} + K\cos\theta \sum_n \delta(t - nT)$$

$$K \geq 5$$



"Dynamical" localization



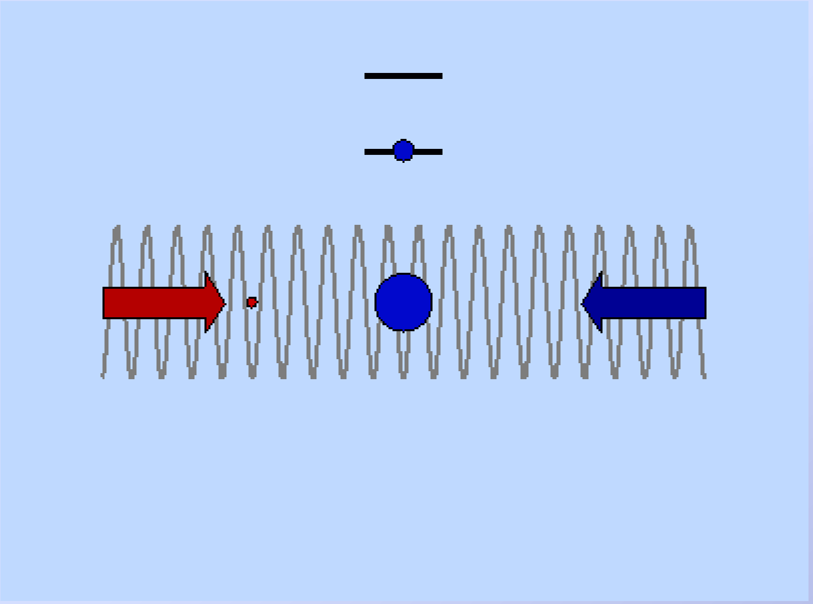
Quantum behavior can be mapped to an Anderson pseudo-random model

S. Fishman, D. R. Grempel and R. E. Prange, *Chaos, Quantum Recurrences, and Anderson Localization*, Phys. Rev. Lett. **49**, 509—512 (1982)

G. Casati *et al.*, *Stochastic behavior of a quantum pendulum under periodic perturbation* in Stochastic Behavior in Classical and Quantum Systems, Lect. Notes Phys. **93**, 334 (1979)

The atomic kicked rotor: An almost ideal "quantum simulator"

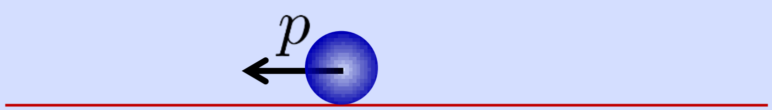
Standing wave far from resonance (no spontaneous emission)



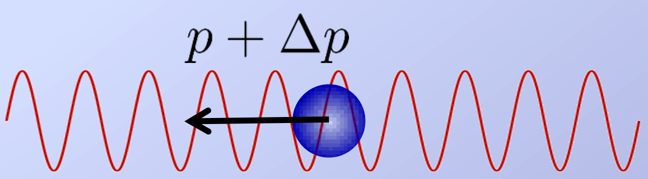
$$p_{\rm after} = p_{\rm before} + 2\hbar k$$

$$V(x) \propto \frac{I}{\Delta} \propto 1 + \cos(2kx)$$

Free motion



Kick



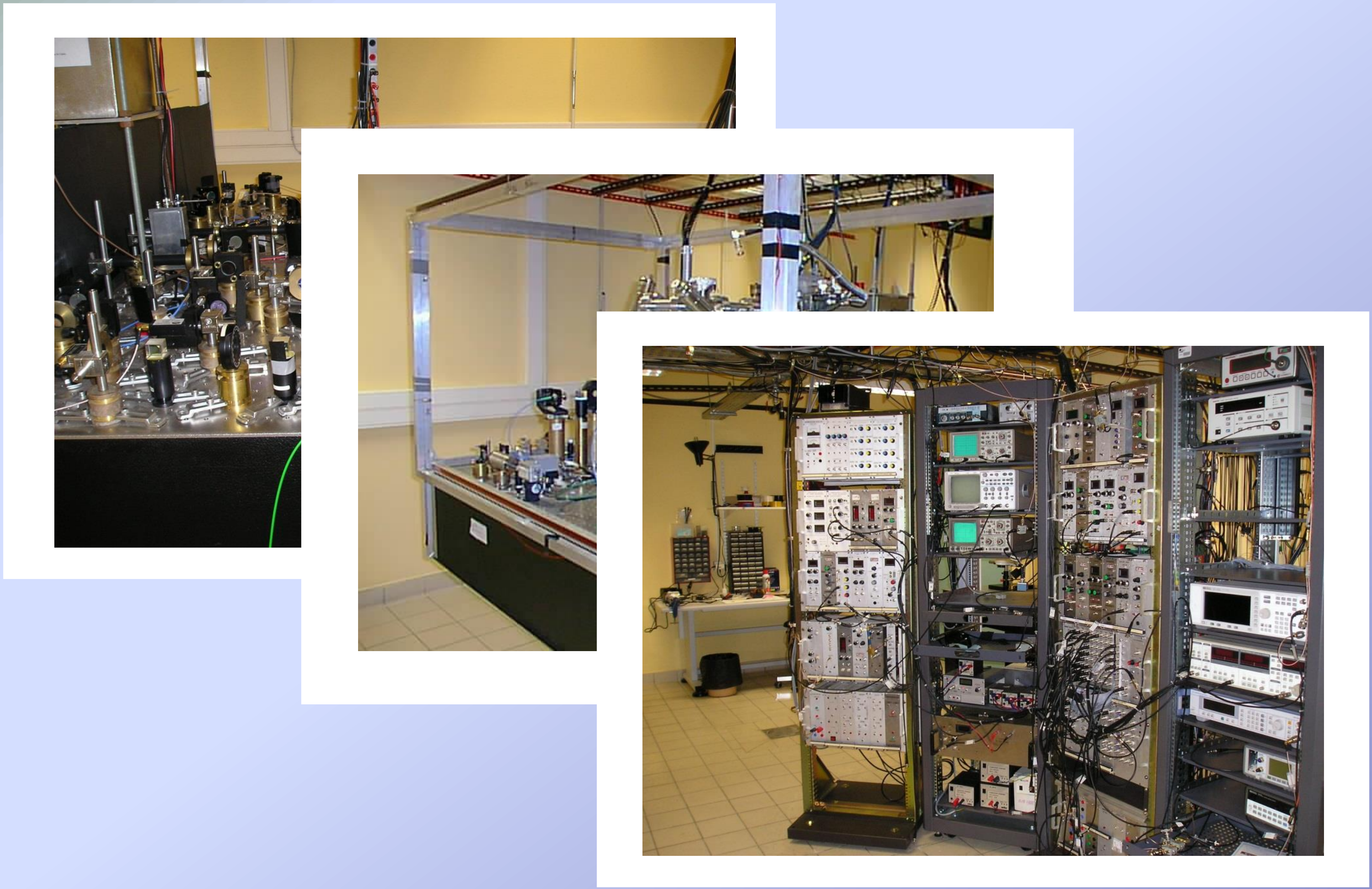
$$H = \frac{p^2}{2} + K \cos x \sum_n \delta(t-n)$$

$$[x, p] = i\hbar\!\!\!/$$

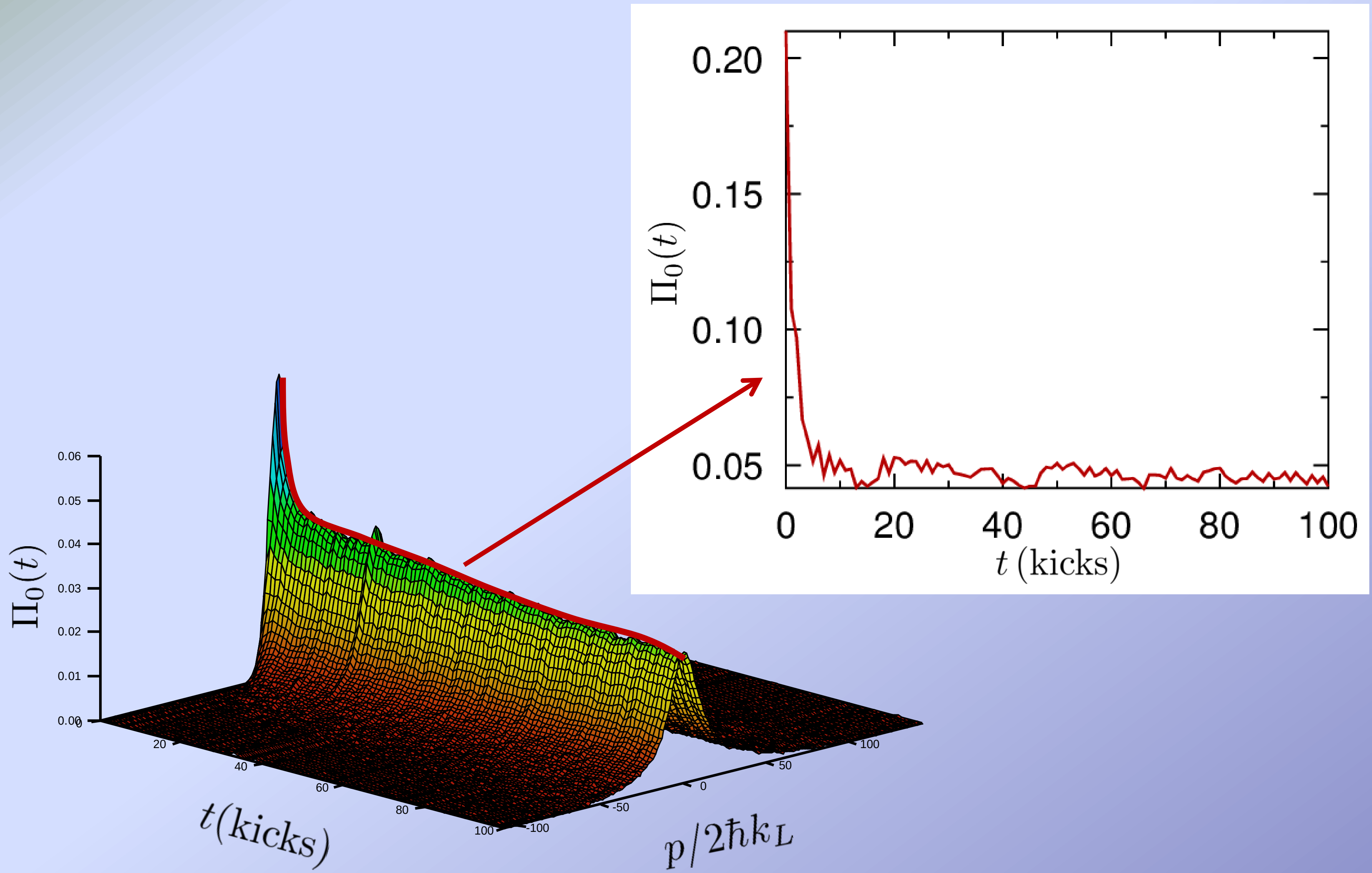
$$\hbar\!\!\!/ = 4\hbar k_L^2 T/M$$

F. L. Moore *et al.*, *Atom optics realization of the quantum δ-kicked rotator*, Phys. Rev. Lett. **75**, 4598 (1995)

Doing it with cold atoms



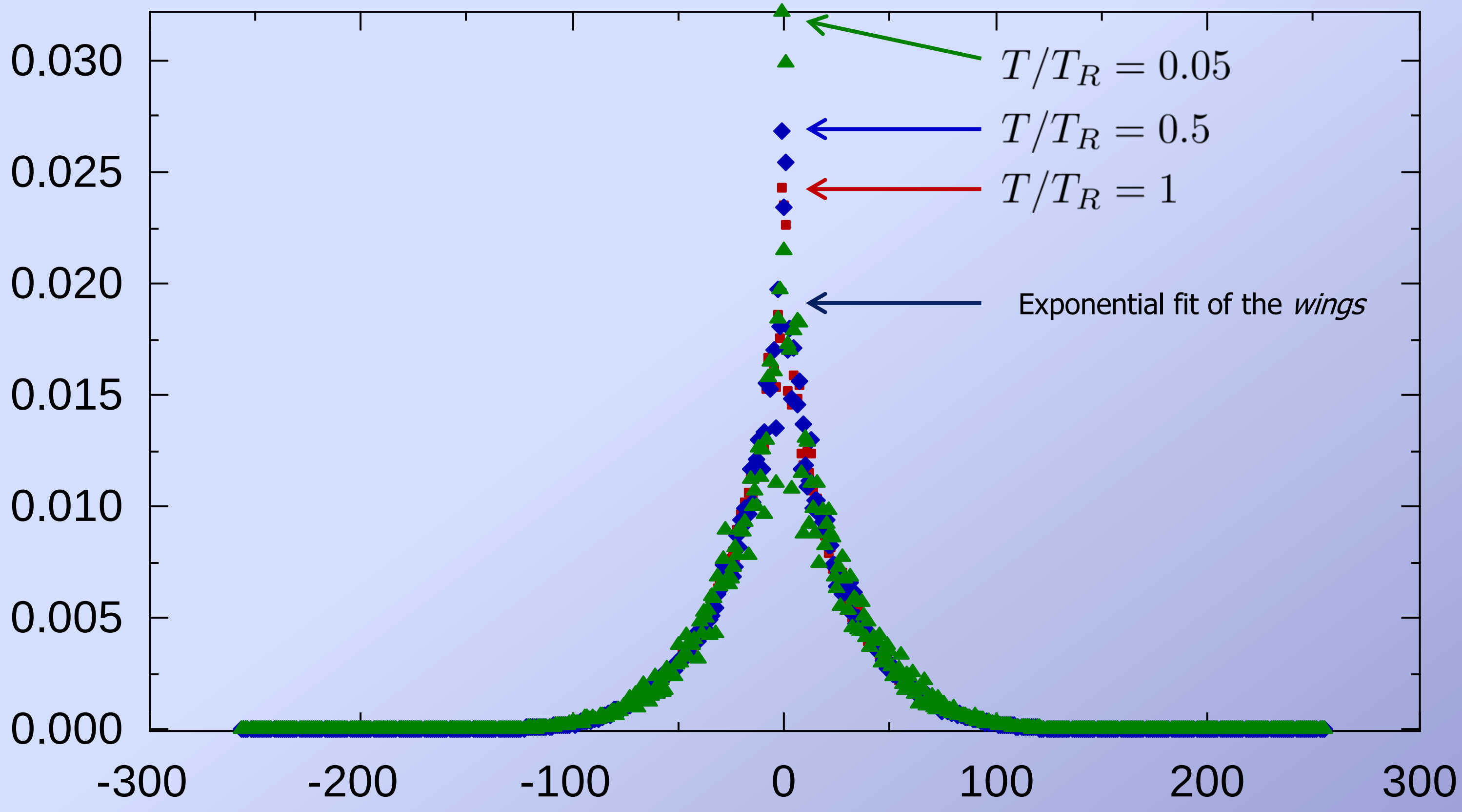
Dynamical localization, experiment with the atomic kicked rotor



Weak localization effects in the atomic kicked rotor

Enhanced return to the origin

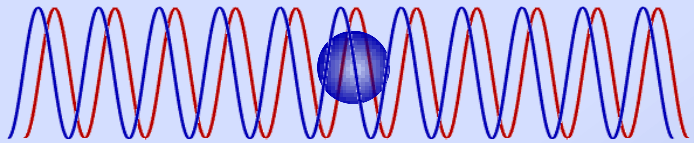
$$\frac{1}{2}k_B T_R = \frac{\hbar^2 k_L^2}{2M}$$



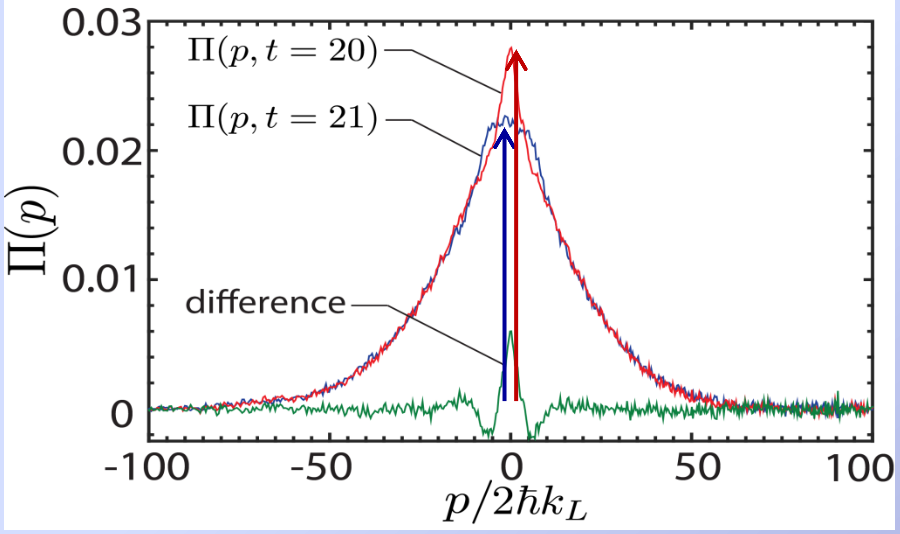
C. Hainaut *et al.*, submitted to PRL

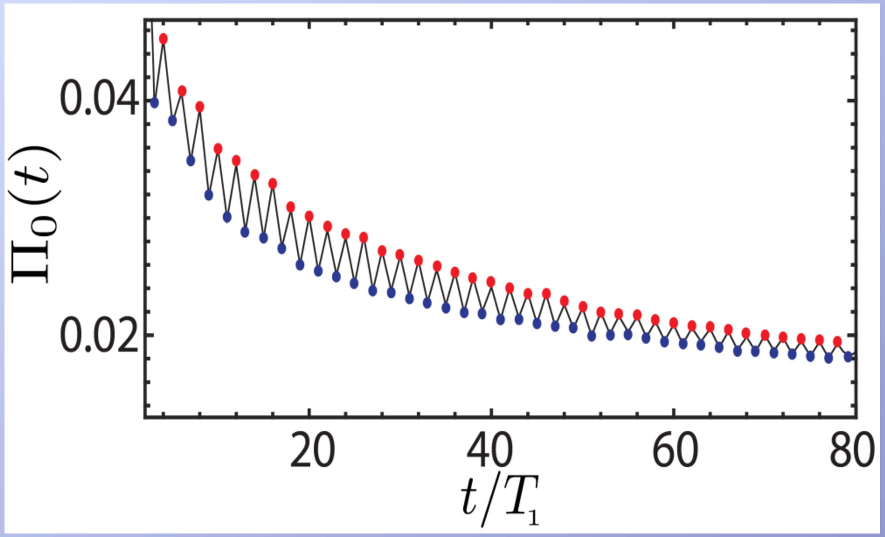
Hamiltonian engineering: The periodically-shifted kicked rotor

$$H = \frac{p^2}{2} + K\left(\cos(x - a/2)\sum_n \delta(t - 2n) + \cos(x + a/2)\sum_n \delta(t - 2n + 1)\right)$$



Period-2 system: Enhanced return to the origin after two *kicks*





C. Hainaut *et al.*, *Return to the Origin as a Probe of Atomic Phase Coherence*, arXiv:1606.07237 (2016)

Kicked rotor dynamics and symmetries

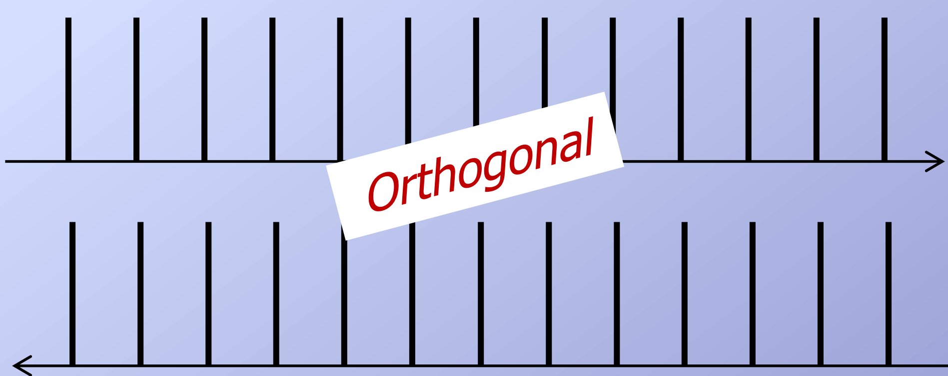
“Universality” classes

- *Orthogonal:* *Spinless systems invariant under time reversal*
- *Unitary:* *Spinless systems not invariant under time reversal*
- *Symplectic:* *Spin systems invariant under time reversal, and under spin rotation*

$$H = \frac{p^2}{2} + K\cos x \sum_n \delta(t-n)$$

$$\tilde{H} = \frac{(-p)^2}{2} + K\cos x \sum_n \delta(-t-n) = \frac{p^2}{2} + K\cos x \sum_n \delta(t-n) \; = H$$

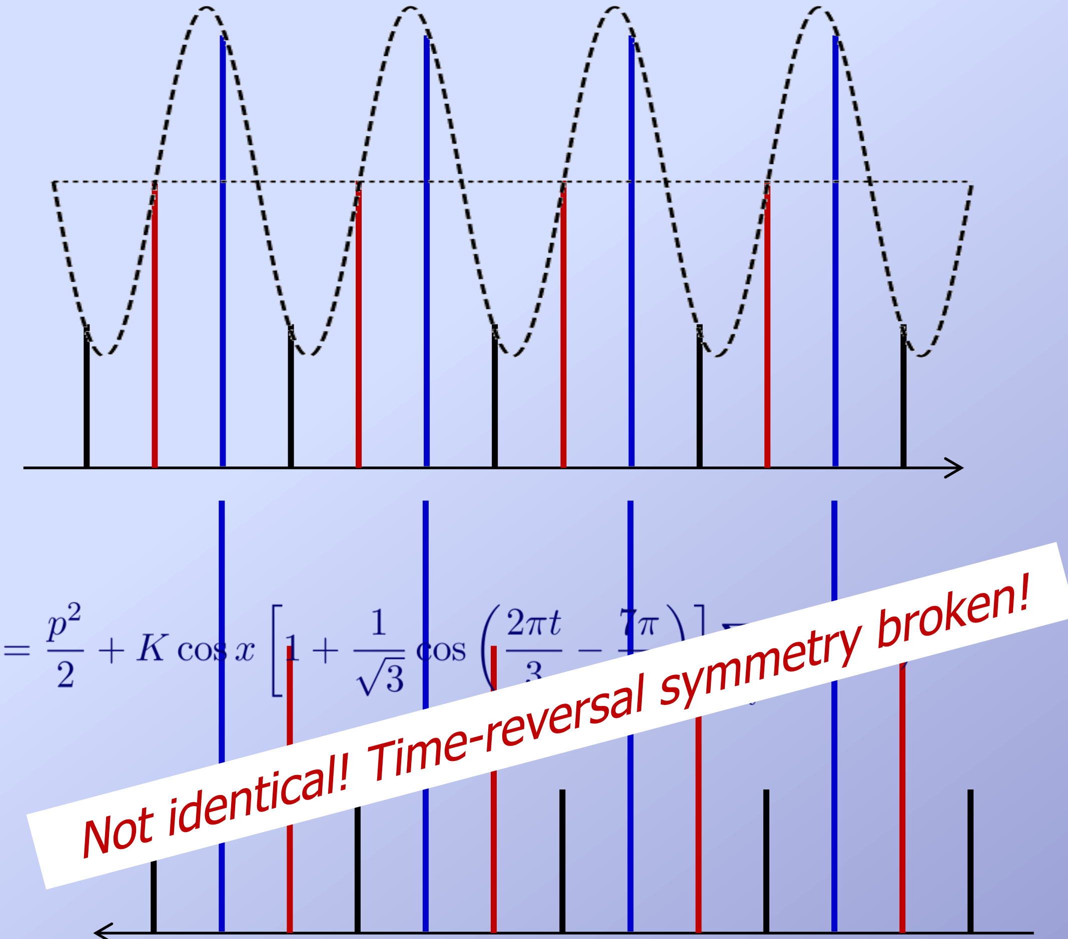
The (standard) kicked rotor belongs to the orthogonal class: The kick sequence is invariant under time reversal



How to obtain a unitary kicked rotor?

Break time-reversal symmetry

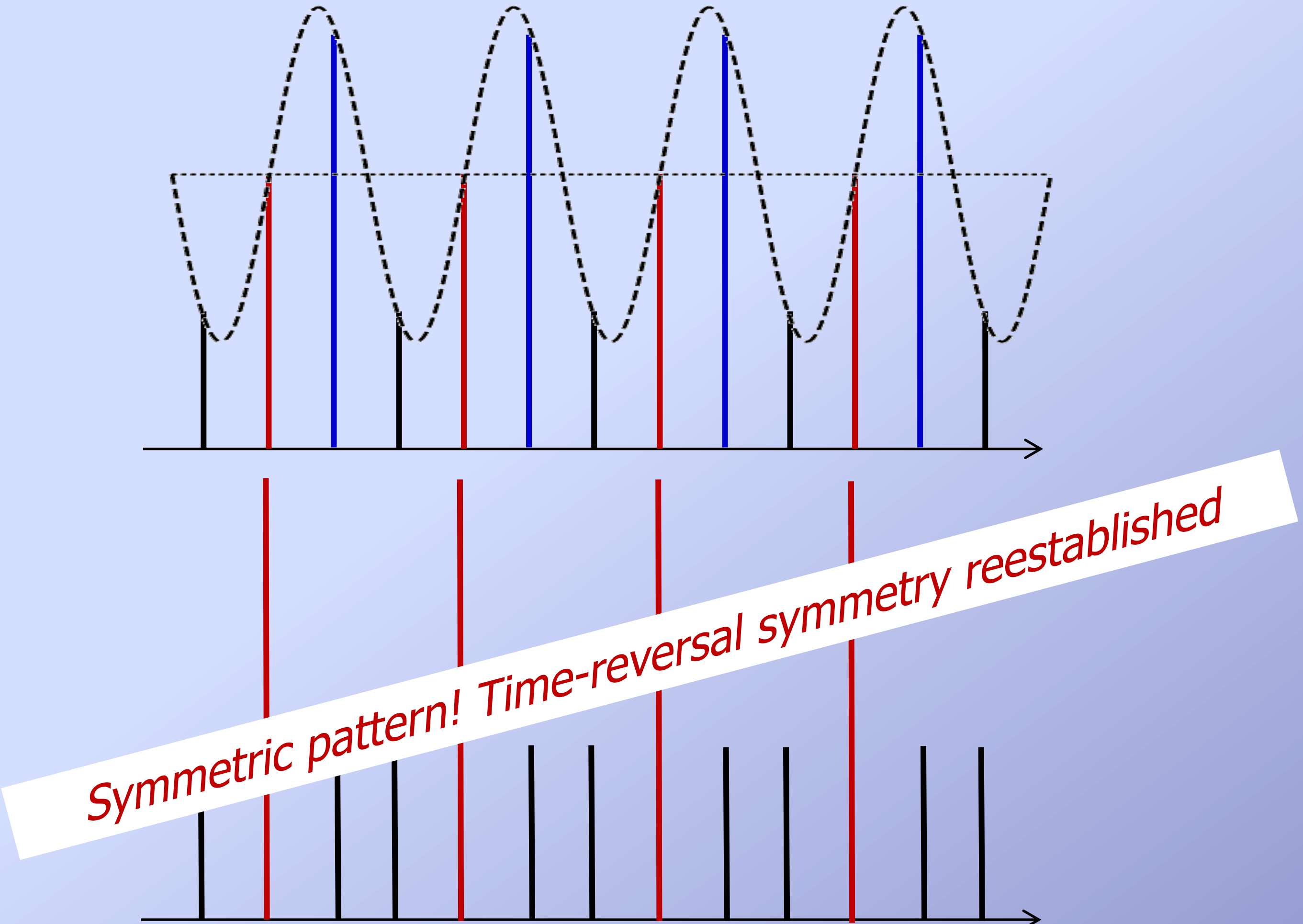
With period-3 sequences!



$$H = \frac{p^2}{2} + K\cos x\left[1+\frac{1}{\sqrt{3}}\cos\left(\frac{2\pi t}{3}-\frac{7\pi}{\ldots}\right)\right]\ldots$$

Not identical! Time-reversal symmetry broken!

Breaking symmetry in a controlled way



One can construct symmetry-breaking Hamiltonians

- Sequences must be periodic (condition for dynamical localization)
- But need not to be time-symmetric

$$H = \frac{p^2}{2} + K\cos x \left[1 + \varepsilon \cos\left(\frac{2\pi t}{T_2} + \varphi\right)\right] \sum_n \delta(t-n)$$

We can "engineer" more complicate ones

e.g. combine with periodically-shifted sequences

$$H = \frac{p^2}{2} + K \left[1 + \varepsilon \cos\left(\frac{2\pi t}{5} + \varphi\right)\right] \sum_n \left[\cos(x - a/2)\,\delta(t-2n) + \cos(x+a/2)\,\delta(t-2n+1)\right]$$

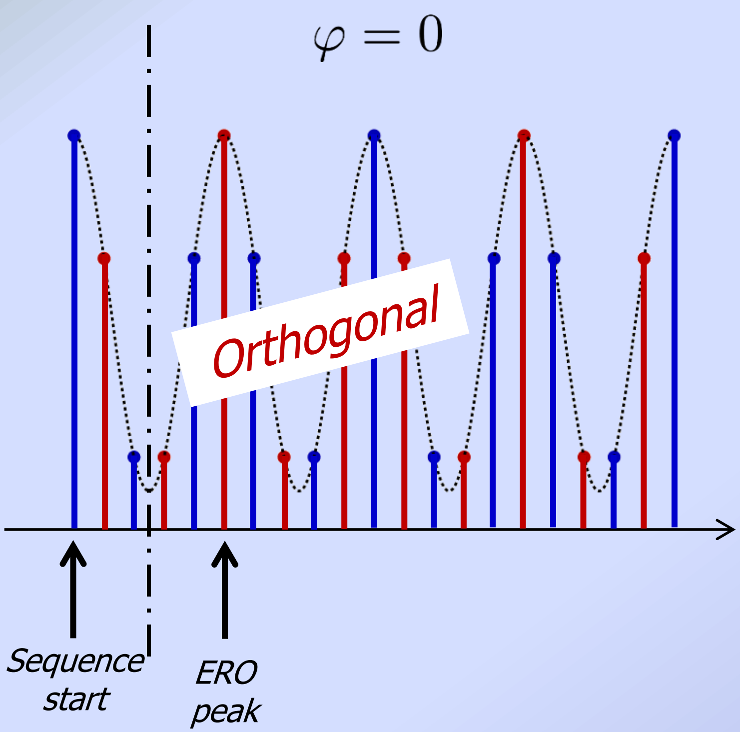
Period-10 Hamiltonian!

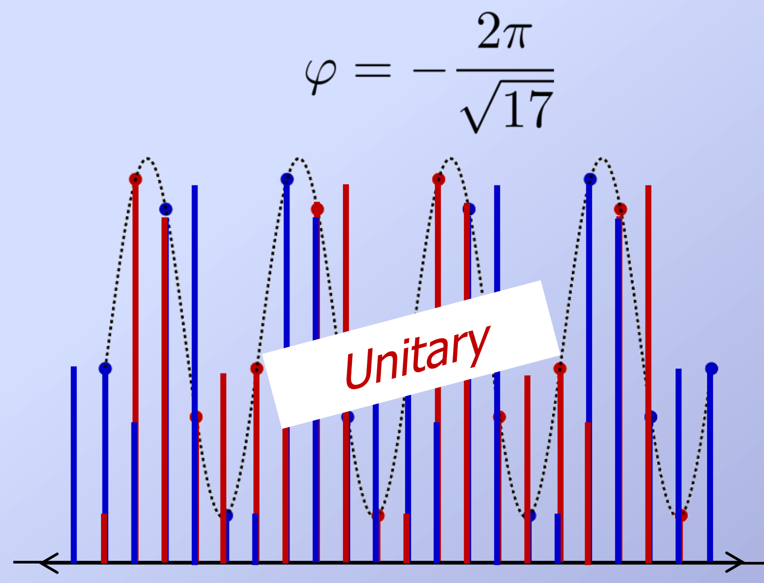
The pertinent symmetry is PT

C. Hainaut *et al.*, to be published

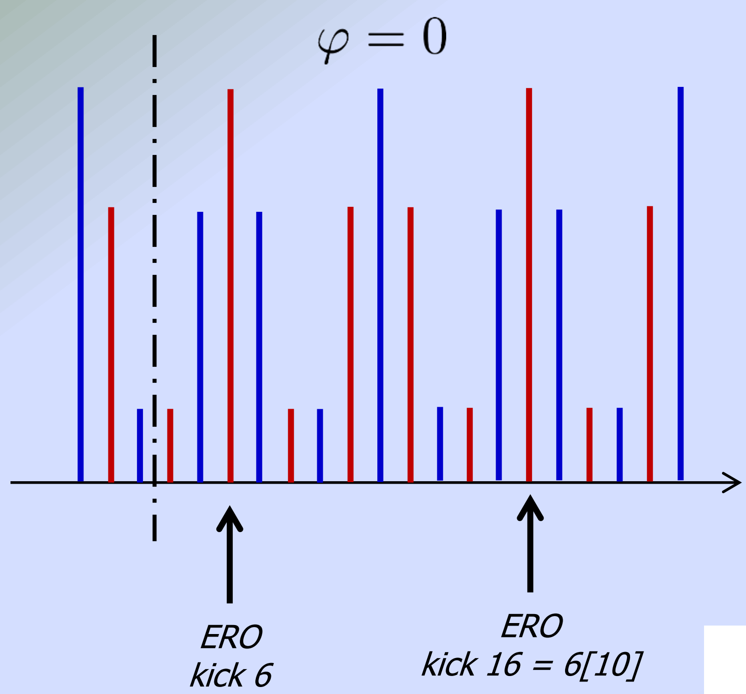
Symmetry engineering

$$H = \frac{p^2}{2} + K\left[1 + \varepsilon\cos\left(\frac{2\pi t}{5} + \varphi\right)\right]\sum_n \left(\cos(x - a/2)\,\delta(t - 2n) + \cos(x + a/2)\,\delta(t - 2n + 1)\right)$$

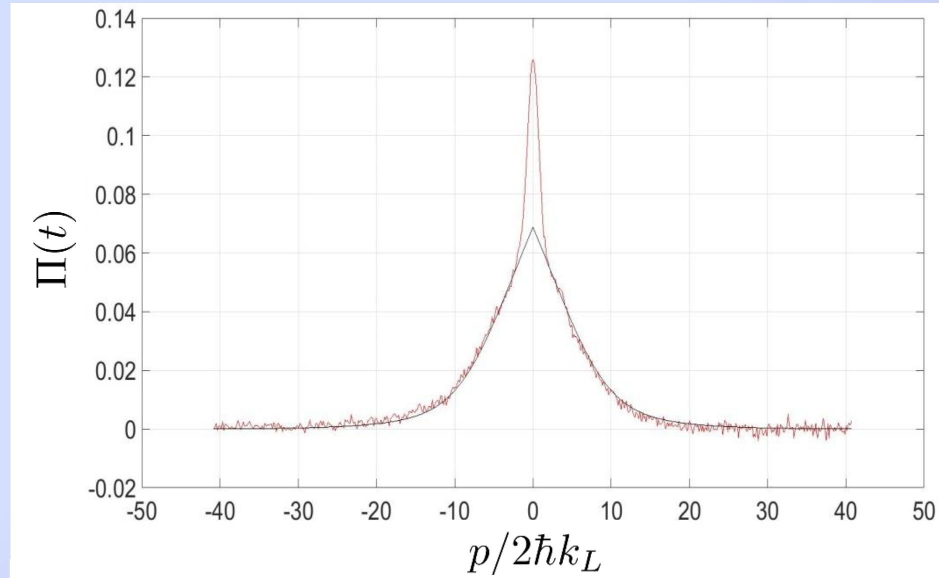


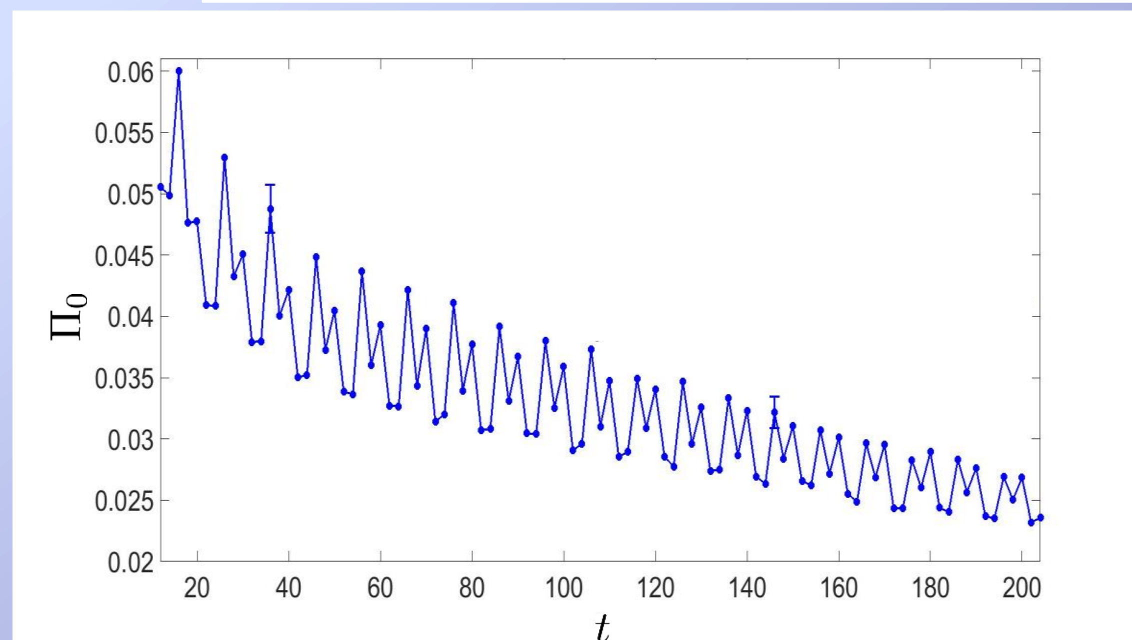


PT-invariant case



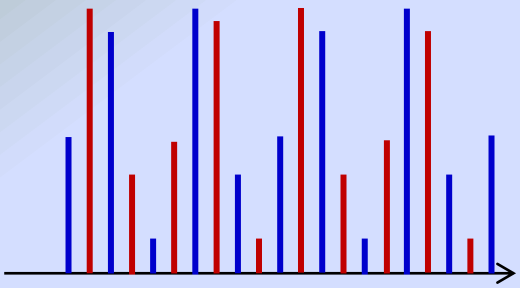
Peaks at kicks $t = 6[10]$

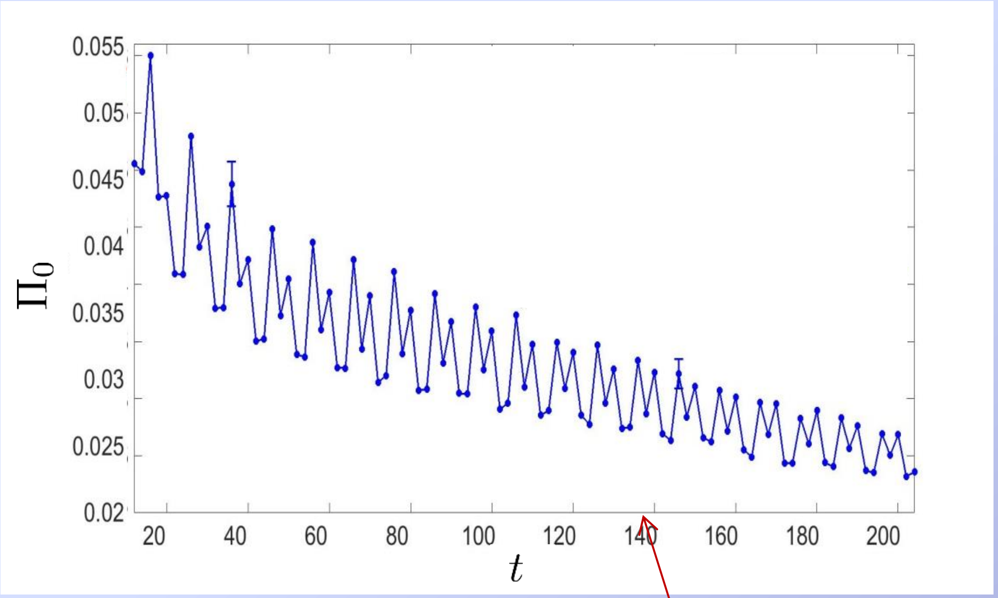




PT symmetry breaking

$$\varphi = -\frac{2\pi}{\sqrt{17}}$$

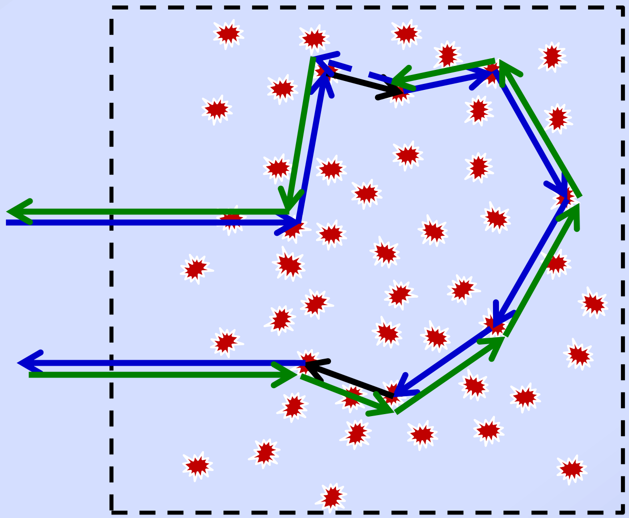


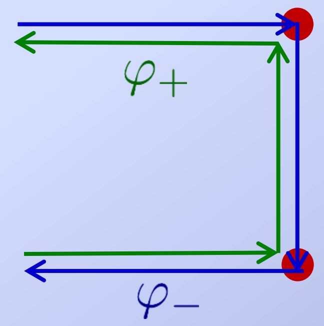


- No peaks at kicks $t[10] \neq 0$: symmetry broken, unitary class
- "Mysterious" peaks appear at the period of the system
- "Mysterious" peaks increase with time for $t \lesssim t_{\text{loc}}$

C. Hainaut *et al.*, to be published

Coherent backscattering loops





φ_- φ_+

Time-reversal:

$-\varphi_-$ $-\varphi_+$

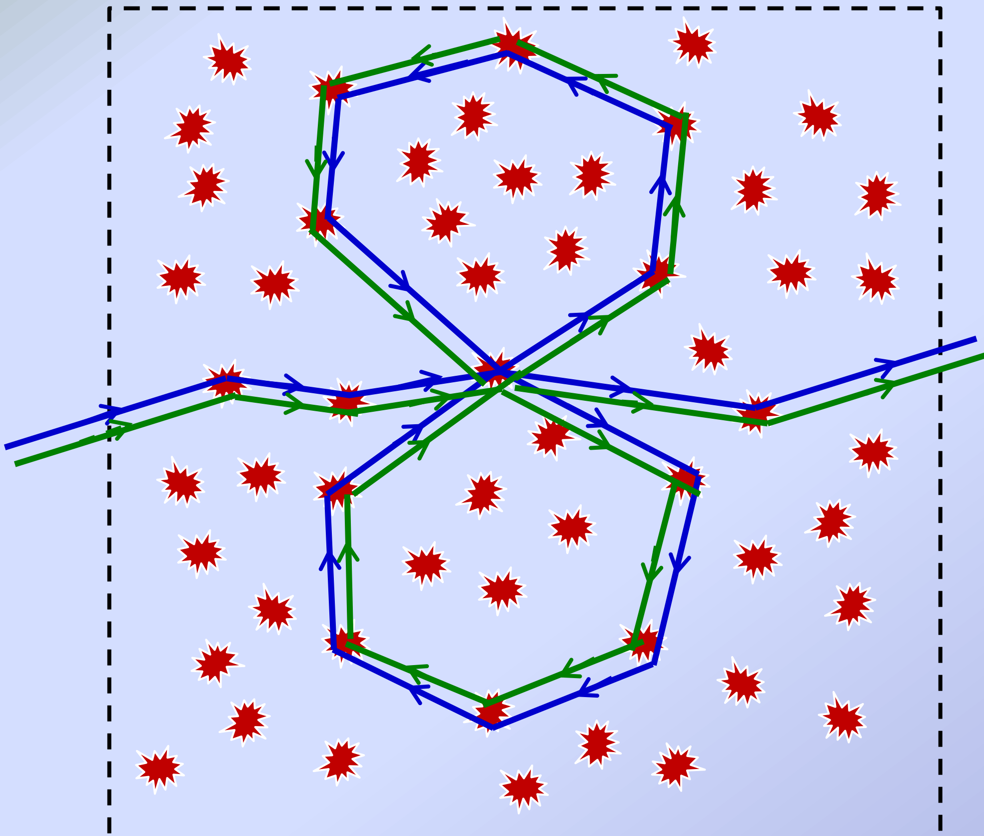
Time-reversal symmetry:

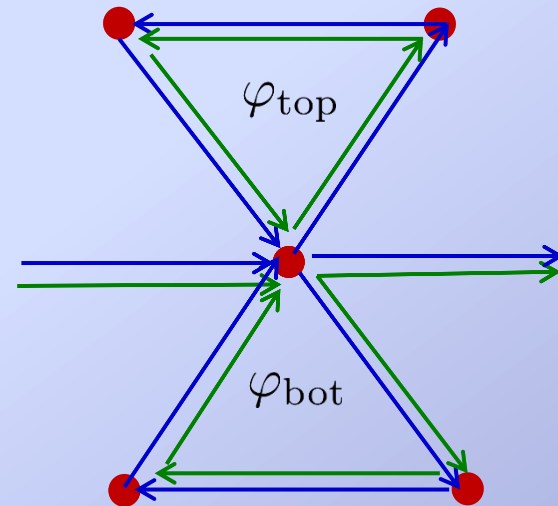
$$\varphi_- = \varphi_+ \; [2\pi]$$

$\Rightarrow$The scattering matrix is *real symmetric*

These loops cannot contribute in the *unitary* case

More complex loops





$\varphi_{top} + \varphi_{bot}$ (blue)

$\varphi_{bot} + \varphi_{top}$ (green)

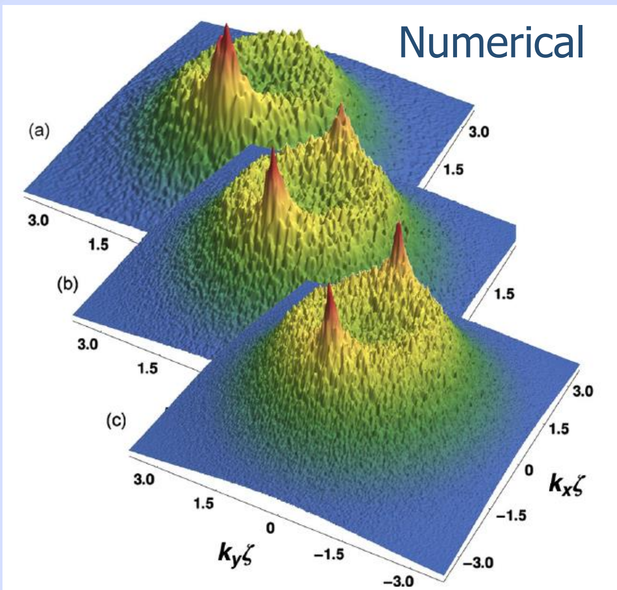
Time-reversal:

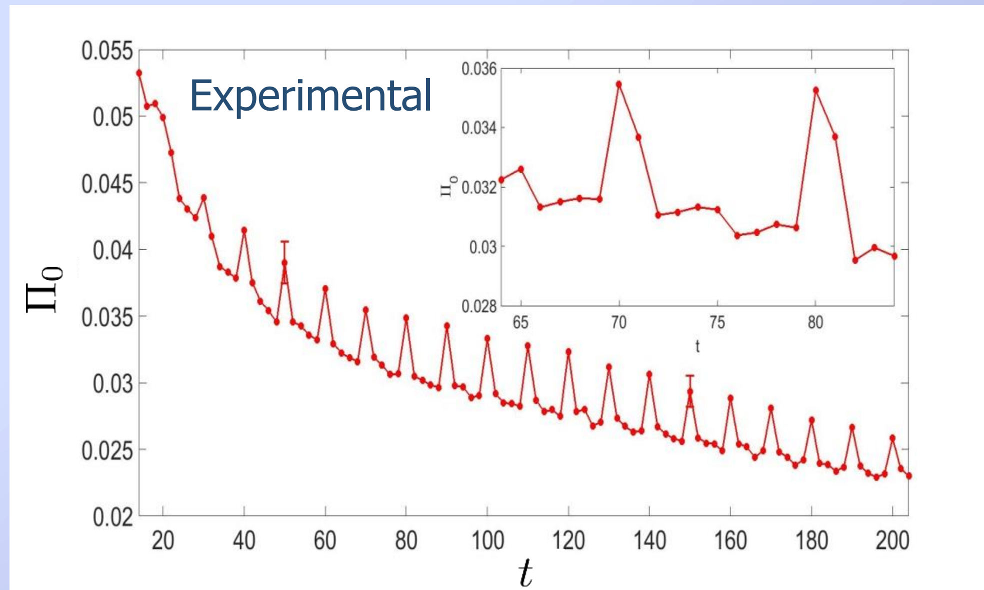
$-\varphi_{bot} - \varphi_{top}$ (blue)

$-\varphi_{top} - \varphi_{bot}$ (green)

Coherent forward *scattering*

T. Karpiuk, N. Cherroret, K. L. Lee, B. Grémaud, C. A. Müller and C. Miniatura,
Coherent Forward Scattering Peak Induced by Anderson Localization
Phys. Rev. Lett. **109**, 190601 (2012)





Time-reversal symmetry breaking in closed systems: Magnetic fields

Interpretation in terms of *artificial* gauge fields?

See J. Dalibard “Magnétisme artificiel pour les gaz d’atomes froids”, lectures at Collège de France (2014)

Most probably yes!

Map QKR to a 2D tight-binding (Anderson) model

Look for an effective Aharonov-Bohm flux on plaquettes

Conclusion

- Ultracold atom physics is very powerful tool for the study of fundamental properties of quantum systems
- The kicked rotor is an excellent system to study interference, decoherence, symmetries…
    - *Symplectic universality class*

      R. Scharf, *Kicked rotator for a spin-1/2 particle*, J. Phys. A: Math. Theor. **22**, 4223–4242 (1989)
    - *Other symmetry-breaking strategies*

      R. Blümel and U. Smilansky, *Symmetry breaking and localization in quantum chaotic systems*, Phys. Rev. Lett. **69**, 217–220 (1992)
    - *Measurement of the* $\beta(g)$ *function*
    - *Higher-dimension Anderson model: Measurement of critical exponents*

      $\nu_{\mathrm{orht}} \approx 1.57$ $\nu_{\mathrm{exp}} = 1.63 \pm 0.05$

      J. Chabé *et al.*, Phys. Rev. Lett. **101**, 255702 (2008)
      M. Lopez *et al.* New J. Phys **15**, 065013 (2013)

      $\nu_{\mathrm{unit}} \approx 1.43$

      $\nu_{\mathrm{sym}} \approx 1.375$

Thank you very much!