Three-dimensional vortex structures in a rotating dipolar Bose-Einstein condensate

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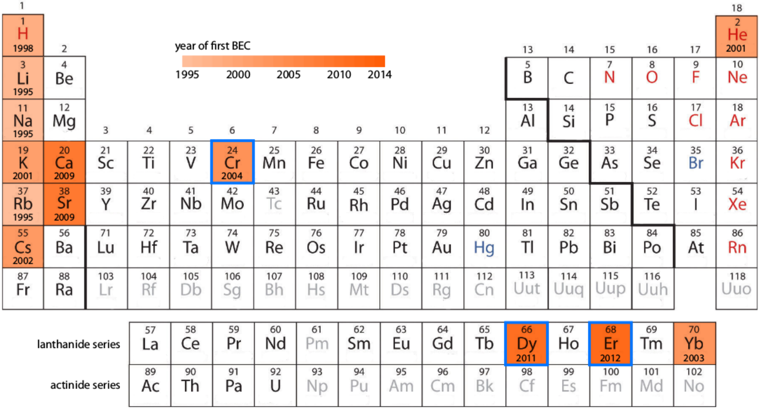
- Introduction
- Dipolar Bose-Einstein Condensate
- Vortex structures in Dipolar BECs
- Summary and conclusion

Bose condensed elements in the periodic table

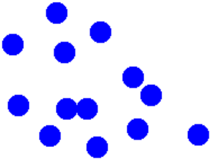


Interactions in BEC

Contact interaction

- Short range
- Isotropic

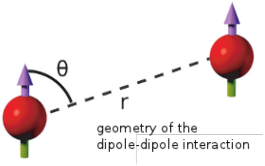
$$V_{\mathrm{vdW}}(\mathbf{r}) = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r})$$



Dipole-Dipole interaction (DDI)

- Long range
- Anisotropic

$$V_{\mathrm{dd}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\mu^2\frac{1-3\cos^2\theta}{\mathbf{r}^3}$$



Magnetic dipolar systems

- Interaction strength : $C_{\rm dd}^{\mathcal{B}} = \mu_0\mu^2$, $\mu \approx 1$ to $10\mu_B$ (μ_B is the Bohr magneton)
- Bose-Einstein condensates realized in ^{52}Cr, ^{164}Dy, and ^{168}Er

Griesmaier et al., *Phys. Rev. Lett.* **94**, 160401 (2005)

Lu et al., *Phys. Rev. Lett.* **107**, 190401 (2011)

Aikawa et al., *Phys. Rev. Lett.* **108**, 210401 (2012)

Electric dipolar systems

- Interaction strength : $C_{\rm dd}^{\varepsilon} = 4\pi d^2$, $d \approx 1$ Debye
- Bosonic polar molecule realized in, ^{41}K^{87}Rb

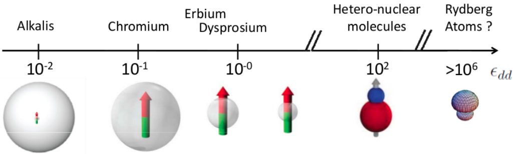
Aikawa et al., *New J. Phys.* **11**, 055035 (2009)

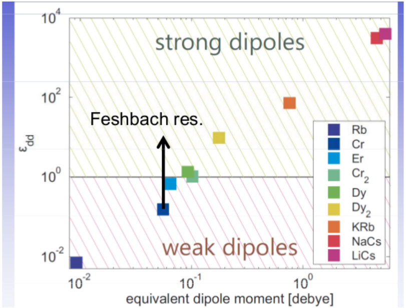
K. K. Ni et al., *Nature* **464**, 1324-1328 (2010)

Ratio between magnetic and electric systems, $C_{\rm dd}^{\mathcal{B}}/C_{\rm dd}^{\varepsilon} \approx 10^{-4}$

Characteristic change in dipolar strength

dipolar interaction $\varepsilon_{dd} = \dfrac{\mu_0 \mu^2 m}{12\pi\hbar^2 a_{bg}}$

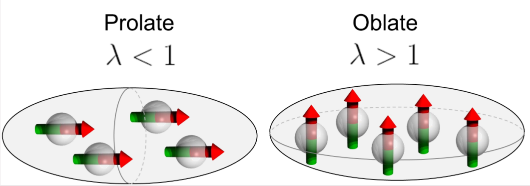




Trapping and Interaction potentials

Harmonic trapping potential

$$V_{trap}(\mathbf{r}) = \frac{\mathbf{M}}{\mathbf{2}}\left(\omega_\rho^{\mathbf{2}}(\mathbf{x^2}+\mathbf{y^2})+\omega_\mathbf{z}^{\mathbf{2}}\mathbf{z^2}\right);\ \lambda = \frac{\omega_\mathbf{z}}{\omega_\rho}$$



Attraction Repulsion

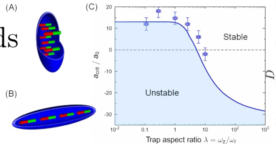
Interaction potential

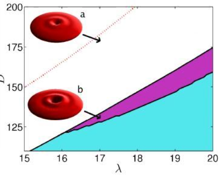
$$V_{int}(\mathbf{r}) = \frac{\mathbf{4}\pi\hbar^{\mathbf{2}}\mathbf{a}}{\mathbf{m}}\delta(\mathbf{r}) + \frac{\mu_\mathbf{0}}{\mathbf{4}\pi}\mu^{\mathbf{2}}\frac{\mathbf{1}-\mathbf{3}\cos^{\mathbf{2}}\theta}{\mathbf{r^3}}$$

New physical effects

Anisotropy

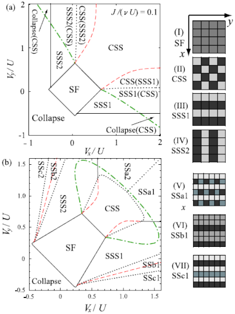
- condensate stability depends on the trap geometry
- new dispersion relations of elementary excitations
- new equilibrium shapes





Long range: New quantum phases

- supersolid
  (superfluid phase with a modulation)
- checkerboard
  (isolating phase, one atom every second site)



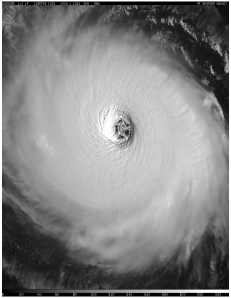
Koch et al., *Nature Phys.* **4**, 218 (2008)

Wilson et al., *Phys. Rev. Lett.* **104**, 094501 (2010)

Danshita et al., *Phys. Rev. Lett.* **103**, 225301 (2009)

Vortex in classical fluids

Smoke Ring



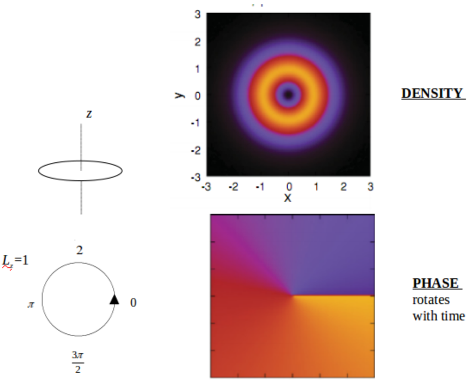
Huricane

Tornado

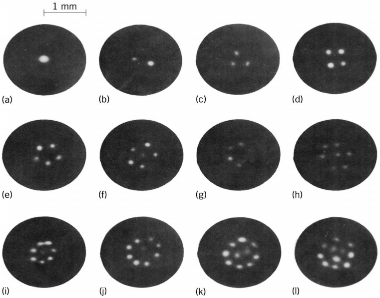
Whirlpool

Vortex in Quantum Systems

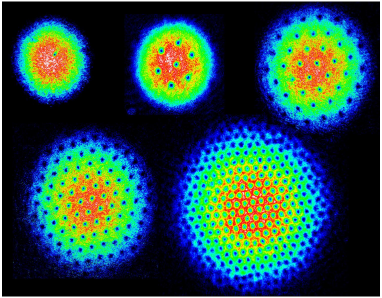
- Excited state of superfluid would represent the creation of vortices
- Quantized vortices are observed in superconductors, liquid helium, and BEC



Vortex in Quantum Systems



Superfluid ^{4}He



^{87}Rb BEC

E. J. Yarmchuk et al., *Phys. Rev. Lett.*, **43**, 214-217 (1979)

M.R. Matthews, et al., *Phys. Rev. Lett.* 83, 2498 (1999)

I. Coddington et al., *Phys. Rev. A* **70**, 063607 (2004)

Techniques used to observe vortices in BEC

- Rotating traps or rotating thermal cloud
- Stirring a laser through condensate
- Formation of vortices from dynamical instabilities
- Imprinting vortices using topological phases
- Superimposing an oscillating excitation to the trapping potential
- Applying artificial magnetic fields

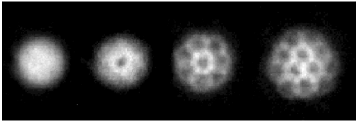
K. W. Madison et al., *Phys. Rev. Lett.* **84**, 806 (2000)

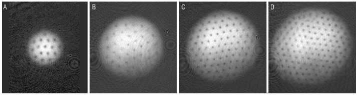
A. L. Fetter, *Rev. Mod. Phys.* **81** 647 (2009)

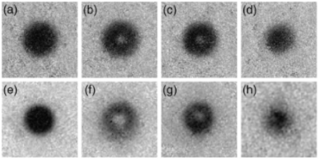
H. Saarikoski et al., *Rev. Mod. Phys.* **82** 2785 (2010)

J. Dalibard et al., *Rev. Mod. Phys.* **83** 1523 (2011)

Methods of vortex nucleation in BECs







Laser stirring

K. W. Madison et al., *Phys. Rev. Lett.* **84**, 806 (2000)

Rotating the magnetic trap

J. R. Abo-Shaeer et al., *Science* **292**, 476-479 (2001)

Imprinting vortices using topological Phases

A. E. Leanhardt et al., *Phys. Rev. Lett.* **89**, 190403 (2002)

$$\mathrm{i}\frac{\partial \phi(\mathbf{r},t)}{\partial t} = \left[-\frac{1}{2}\nabla^2 + V_{\mathrm{trap}} - \Omega L_z + g|\phi(\mathbf{r},t)|^2 + F\right]\phi(\mathbf{r},t)$$

- $\phi(\mathbf{r},t)$: condensate wave function
- $g = 4\pi a N/l$; $(l = \sqrt{\hbar/m\omega})$
- a : atomic scattering length
- N : number of atoms in the condensate
- θ : angle between $\mathbf{R}$ and the polarization direction $\hat{z}$
- $\bar{\mu}$: dipole moment of an atom.
- $\alpha = (3\cos^2\varphi - 1)/2 \implies -0.5 \leq \alpha \leq 1$

$$V_{\mathrm{trap}} = \frac{1}{2}\left(\gamma^2 x^2 + \nu^2 y^2 + \lambda^2 z^2\right)$$

$$\gamma = \omega_x/\omega;\ \nu = \omega_y/\omega;\ \lambda = \omega_z/\omega$$

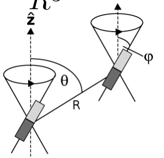
$$L_z = -\mathrm{i}\hbar(x\partial_y - y\partial_x)$$

$$F = \int U_{dd}(\mathbf{r}-\mathbf{r}')|\phi(\mathbf{r}',t)|^2 d\mathbf{r}'$$

$$U_{dd}(\mathbf{R}) = g_{dd}\frac{(1-3\cos^2\theta)}{R^3}$$

$$g_{dd} = 3a_{dd}N\alpha$$

$$a_{dd} = \frac{\mu_0\bar{\mu}^2 m}{12\pi\hbar^2}$$



Vortices in Dipolar BECs : Previous observation

- Craterlike structure while dipoles polarized along the rotation axis
- Strength of dipolar interaction affects critical rotation frequency (Ω_c) for the formation of vortices
- Ω_c for vortex nucleation increases with dipolar interaction strength
- Vortex states due to the presence of the dipolar interaction

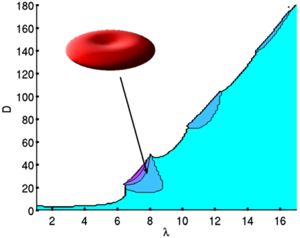
S. Yi and H. Pu *Phys. Rev. A* **73**, 061602R (2006)

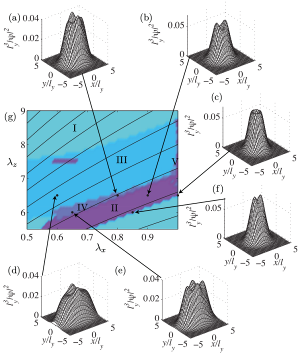
F. Malet et al., *Phys. Rev. A* **83** 033628 (2011)

What is new?

- Previous studies on vortices in dipolar BECs are based on 2D models
- More realistic to investigate the stationary vortex structures in full 3D
- Vortices in pure dipolar BECs are not discussed

Structured states in dipolar BEC

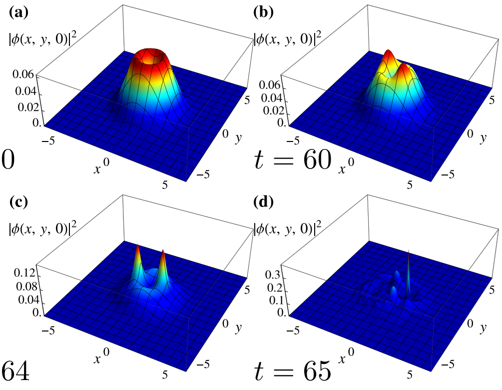




S. Ronen et al., *Phys. Rev. Lett.* 98, 030406 (2007)

A. D. Martin and P. B. Blakie *Phys. Rev. A* 86, 053623 (2012)

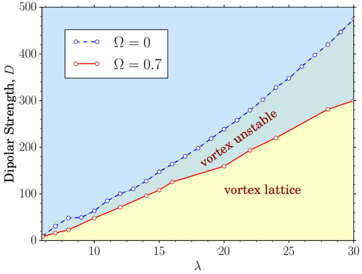
Collapse of rotating biconcave shaped dipolar BEC



Condensate density $|\phi(x, y, 0)|^2$ of the ^{52}Cr for $\lambda = 7$ and $D = 3Na_{\mathrm{dd}} = 30.4$ in xy plane showing the collapse dynamics of rotating biconcave shaped condensate with $\Omega = 0.5$ in real-time.

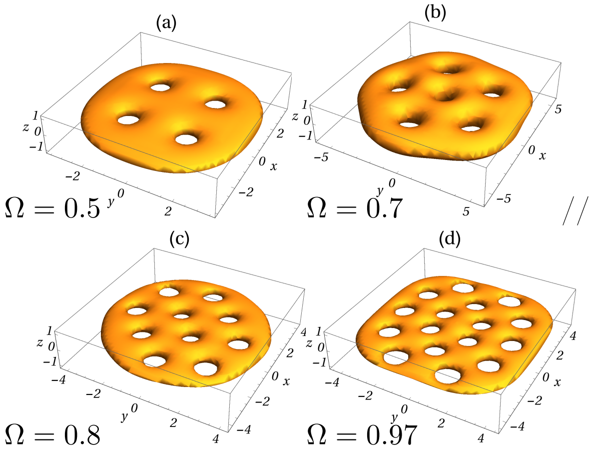
R. K. Kumar, T. Sriraman, H. Fabrelli, P. Muruganandam and **AG**, arXiv:1506.08184, accepted JPB

Stability phase plot for the vortex states

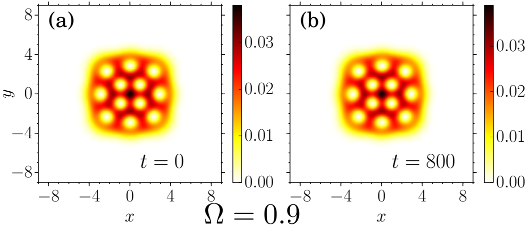


Stability diagram of a purely dipolar BEC as a function of λ and dipolar interaction parameter (D). Below the dotted blue line non-vortex state is stable while below the solid red line is the region of stable vortex lattice.

R. K. Kumar, T. Sriraman, H. Fabrelli, P. Muruganandam and **AG**, arXiv:1506.08184



3D view of condensate density with vortex lattice of purely dipolar BECs with $a_{dd} = 16\,a_0$, $a = 0$, $D = 38$, $\lambda = 10$.

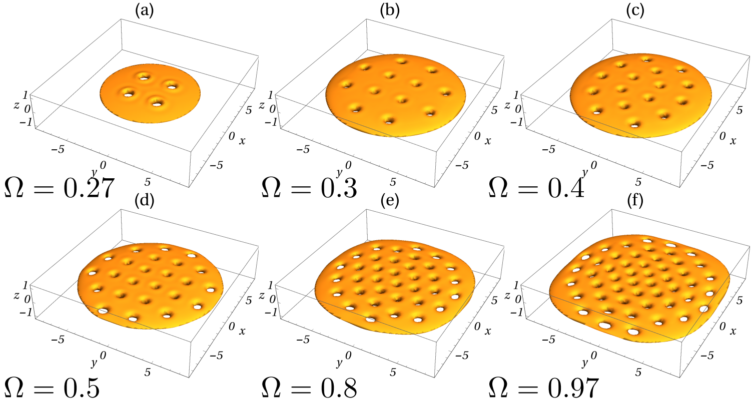


$\Omega = 0.9$

Two-dimensional view of the stable evolution of the condensate density $|\phi(x, y, 0)|^2$ with vortices arranged in a square lattice with $a_{dd} = 16\, a_0$, $a = 0$, $\lambda = 10$, $D = 38$.

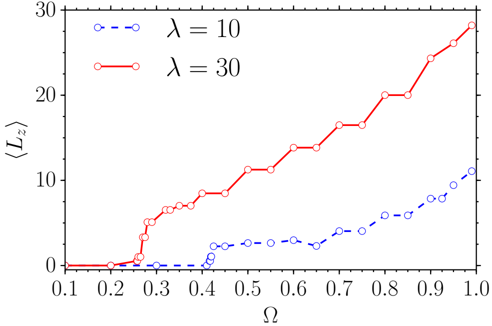
R. K. Kumar, T. Sriraman, H. Fabrelli, P. Muruganandam and **AG**, arXiv:1506.08184

Vortex lattice structures in Pancake trap $\lambda = 30$



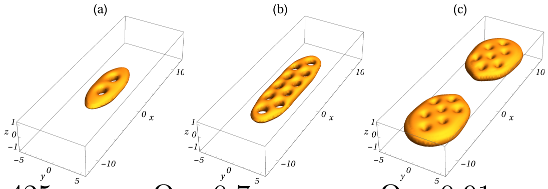
3D view of condensate density with vortex lattice of purely dipolar BECs with $a_{dd} = 16\, a_0$, $a = 0$, $D = 300$, $\lambda = 30$ at different rotation frequencies.

Angular momentum and Number of vortices



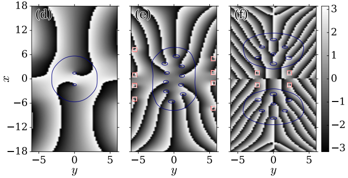
Expectation value of angular momentum $\langle L_z \rangle$ as a function of Ω.

R. K. Kumar, T. Sriraman, H. Fabrelli, P. Muruganandam and **AG**, arXiv:1506.08184



$\Omega = 0.425$ $\Omega = 0.7$ $\Omega = 0.91$

phase(z=0)

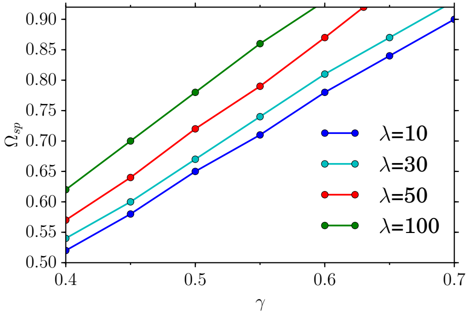


— density contour lines
○ position of vortices
□ position of vortices

Vortex lattices in asymmetric trap with $\gamma = 0.7$, $\lambda = 10$ and $D = 38$.

R. K. Kumar, T. Sriraman, H. Fabrelli, P. Muruganandam and **AG**, arXiv:1506.08184

Critical rotation frequency for the splitting due to rotation



Critical rotation frequency (Ω_{sp}) for the splitting of the condensate with γ of the trap along x-direction for the parameters $\nu = 1$, $D = 38$ and for different trap aspect ratios $\lambda = 10, 30, 50$ and 100.

R. K. Kumar, T. Sriraman, H. Fabrelli, P. Muruganandam and **AG**, arXiv:1506.08184

Numerical scheme: Split-step Crank-Nicolson method

$$\mathrm{i}\frac{\partial}{\partial t}\varphi(\mathbf{r};t) = H_i\varphi(\mathbf{r};t), \quad i = 1,2,3,4 \tag{1}$$

where $\mathbf{r} \equiv (x, y, z)$

$$H_1 = V_{trap} + 4\pi a N\,|\varphi(\mathbf{r};t)|^2 + 3Na_{\mathrm{dd}}\int V_{\mathrm{dd}}^{3D}(\mathbf{R})|\phi(\mathbf{r}',t)|^2 d\mathbf{r}'$$

$$\begin{aligned} H_2 &= -\frac{1}{2}\frac{\partial^2}{\partial x^2} - \mathrm{i}\Omega y\frac{\partial}{\partial x} \\ H_3 &= -\frac{1}{2}\frac{\partial^2}{\partial y^2} + \mathrm{i}\Omega x\frac{\partial}{\partial y} \\ H_4 &= -\frac{1}{2}\frac{\partial^2}{\partial z^2} \end{aligned} \tag{2}$$

Split-step Crank-Nicolson method

$$\varphi^{n+\frac{1}{3}} = \mathcal{O}_{\mathrm{nd}}(H_1)\varphi^n \equiv \mathrm{e}^{-\mathrm{i}\Delta H_1}\varphi^n,$$

$\mathcal{O}_{\mathrm{nd}}(H_1)$ – time-evolution operation with H_1

- φ^n is the discretized wave function at time t_n
- φ^n is advanced over the time step Δ at time t_n by solving (1) with H_1
- an intermediate solution $\varphi^{n+\frac{1}{3}}$
- time propagation with H_2, H_3 & H_4 – semi-implicit Crank-Nicolson scheme

Split-step Crank-Nicolson method

Discretization along x direction

$$\frac{\mathrm{i}(\varphi_{i,j,k}^{n+\frac{2}{3}} - \varphi_{i,j,k}^{n+\frac{1}{3}})}{\Delta} = -\frac{1}{2h_x^2}\Bigg[\left(\varphi_{i+1,j,k}^{n+\frac{2}{3}} - 2\varphi_{i,j,k}^{n+\frac{2}{3}} + \varphi_{i-1,j,k}^{n+\frac{2}{3}}\right) + \left(\varphi_{i+1,j,k}^{n+\frac{1}{3}} - 2\varphi_{i,j,k}^{n+\frac{1}{3}} + \varphi_{i-1,j,k}^{n+\frac{1}{3}}\right)\Bigg] - \frac{\mathrm{i}\Omega y_j}{4h_x}\Big[(\varphi_{i+1,j,k}^{n+\frac{2}{3}} - \varphi_{i-1,j,k}^{n+\frac{2}{3}}) + (\varphi_{i+1,j,k}^{n+\frac{1}{3}} - \varphi_{i-1,j,k}^{n+\frac{1}{3}})\Big], \tag{3}$$

- $\varphi_{i,j,k}^{n} = \varphi(x_i, y_j, z_k; t_n)$
- $x \equiv x_i = ih$, $i = 0, 1, 2, ..., N_x$ and h_x – space step (x)
- tridiagonal sets of eqs (3) in $\varphi_{i+1,j,k}^{n+\frac{2}{3}}$, $\varphi_{i,j,k}^{n+\frac{2}{3}}$, and $\varphi_{i-1,j,k}^{n+\frac{2}{3}}$
- solved using proper boundary conditions

Split-step Crank-Nicolson method

- error $\sim \Delta^2$
- stable
- preserve normalization
- major part of the Hamiltonian treated fairly accurately
- can deal with large nonlinearity

Split-step Crank-Nicolson method

$$A_i^- \varphi_{i-1,j,k}^{n+\frac{2}{3}} + A_i^0 \varphi_{i,j,k}^{n+\frac{2}{3}} + A_i^+ \varphi_{i+1,j,k}^{n+\frac{2}{3}} = b_i \quad (4)$$

$$b_i = \frac{i\Delta}{2h_x^2}\left(\varphi_{i+1,j,k}^{n+\frac{1}{3}} - 2\varphi_{i,j,k}^{n+\frac{1}{3}} + \varphi_{i-1,j,k}^{n+\frac{1}{3}}\right) \quad (5)$$
$$-\frac{\Delta\Omega y_i}{4h_x}\left(\varphi_{i+1,j,k}^{n+\frac{1}{3}} - \varphi_{i-1,j,k}^{n+\frac{1}{3}}\right) + \varphi_{i,j,k}^{n+\frac{1}{3}}$$

$$A_i^0 = 1 + \frac{\mathrm{i}\Delta}{h_x^2}$$
$$A_i^- = -\frac{\Delta}{2h_x}\left(\frac{\mathrm{i}}{h_x} + \frac{\Omega y_i}{2}\right)$$
$$A_i^+ = -\frac{\Delta}{2h_x}\left(\frac{\mathrm{i}}{h_x} - \frac{\Omega y_i}{2}\right)$$

Split-step Crank-Nicolson method

one-term forward recursion relation

$$\varphi_{i+1,j,k}^{n+\frac{2}{3}} = \alpha_i \varphi_{i,j,k}^{n+\frac{2}{3}} + \beta_i, \tag{6}$$

α_i and β_i – coefficients to be determined.
Substituting (6) in (4)

$$A_i^- \varphi_{i-1,j,k}^{n+\frac{2}{3}} + A_i^0 \varphi_{i,j,k}^{n+\frac{2}{3}} + A_i^+ (\alpha_i \varphi_{i,j,k}^{n+\frac{2}{3}} + \beta_i) = b_i,$$

Kumar et al., *Fortran and C programs for the time-dependent dipolar Gross-Pitaevskii equation in an anisotropic trap*, preprint (2015)

Split-step Crank-Nicolson method

leads to the solution

$$\varphi_{i,j,k}^{n+\frac{2}{3}} = \gamma_i(A_i^- \varphi_{i-1,j,k}^{n+\frac{2}{3}} + A_i^+ \beta_i - b_i), \qquad (7)$$

with

$$\gamma_i = -1/(A_i^0 + A_i^+ \alpha_i), \qquad (8)$$

the backward recursion relations for the coefficients α_i and β_i

$$\alpha_{i-1} = \gamma_i A_i^-, \quad \beta_{i-1} = \gamma_i(A_i^+ \beta_i - b_i). \qquad (9)$$

$\alpha_{N_x-1} = 0, \beta_{N_x-1} = \varphi_{N_x}^{n+\frac{2}{3}}$ and $\varphi_0^{n+\frac{2}{3}} \sim 0$

Kumar et al., Comp. Phys. Commun. (2015) submitted.

Koonin and Meredith, *Computational Physics: Fortran version*, Addison-Wesley, Reading, 1990.

Split-step Crank-Nicolson method

Discretization along y direction

$$\frac{\mathrm{i}(\varphi_{i,j,k}^{n+1} - \varphi_{i,j,k}^{n+\frac{2}{3}})}{\Delta} = -\frac{1}{2h_y^2}\Bigg[\Big(\varphi_{i,j+1}^{n+1} - 2\varphi_{i,j,k}^{n+1} + \varphi_{i,j-1,k}^{n+1}\Big) + \Big(\varphi_{i,j+1,k}^{n+\frac{2}{3}} - 2\varphi_{i,j,k}^{n+\frac{2}{3}} + \varphi_{i,j-1,k}^{n+\frac{2}{3}}\Big)\Bigg] + \frac{\mathrm{i}\Omega x_i}{4h_y}\Big[(\varphi_{i,j+1,k}^{n+1} - \varphi_{i,j-1,k}^{n+1}) + (\varphi_{i,j+1,k}^{n+\frac{2}{3}} - \varphi_{i,j-1,k}^{n+\frac{2}{3}})\Big] \quad (10)$$

- $y_j = jh_y$, $j = 0, 1, 2, ..., N_y$, h_y – space step along y.

Split-step Crank-Nicolson method

$$C_i^- \varphi_{i,j-1,k}^{n+1} + C_i^0 \varphi_{i,j,k}^{n+1} + C_i^+ \varphi_{i,j+1,k}^{n+1} = d_i, \tag{11}$$

$$\begin{aligned} d_i = \frac{i\Delta}{2h_x^2}\left(\varphi_{i,j+1,k}^{n+\frac{2}{3}} - 2\varphi_{i,j,k}^{n+\frac{2}{3}} + \varphi_{i,j-1,k}^{n+\frac{2}{3}}\right) \\ -\frac{\Delta\Omega y_i}{4h_x}\left(\varphi_{i,j+1,k}^{n+\frac{2}{3}} - \varphi_{i,j-1,k}^{n+\frac{2}{3}}\right) + \varphi_{i,j,k}^{n+\frac{2}{3}} \end{aligned} \tag{12}$$

Solving dipolar integral using FFT

The integral can be simplified in Fourier space by means of convolution as

$$\int d\mathbf{r}' V_{\rm dd}^{3D}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{V}_{\rm dd}^{3D}(\mathbf{k})\widetilde{n}(\mathbf{k},t), \quad (13)$$

where $n(\mathbf{r},t) = |\phi(\mathbf{r},t)|^2$. The Fourier transformation (FT) and inverse FT, respectively, are defined by

$$\widetilde{A}(\mathbf{k}) = \int d\mathbf{r} A(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}, \quad A(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}\widetilde{A}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}. \quad (14)$$

The FT of the dipole potential can be obtained analytically

$$\widetilde{V}_{\rm dd}^{3D}(\mathbf{k}) \equiv \frac{4\pi}{3} h_{3D}(\mathbf{k}) = \frac{4\pi}{3}\left(\frac{3k_z^2}{\mathbf{k}^2} - 1\right), \quad (15)$$

so that

$$\int d\mathbf{r}' V_{\rm dd}^{3D}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t) = \frac{4\pi}{3}\int \frac{d\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{r}} h_{3D}(\mathbf{k})\widetilde{n}(\mathbf{k},t). \quad (16)$$

The FT of density $n(\mathbf{r})$ is evaluated numerically by means of a

Summary and Conclusion

- Vortex lattice structures in purely dipolar BEC of ^{52}Cr atoms are studied by considering the full 3D GP equation
- Stability regimes observed for vortex states in purely dipolar BECs with respect to λ and dipolar interaction.
- Square lattice is observed
- In fully anisotropic trap breaking up of the condensates into two parts with the equal number of vortices is observed due to strong dipolar strength and high rotation frequency.

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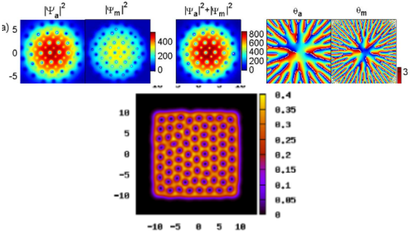
Thank You!

Square Lattice

Dipolar BECs under rotation can produce square vortex lattices. This was found with two-component BECs and recently in dipolar Fermi gas.



Square lattice in 2C BEC and dipolar Fermi system

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