Counterflowing superfluids

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Theory of solutions of a superfluid Fermi liquid in a superfluid Bose liquid

G. E. Volovik, V. P. Mineev, and I. M. Khalatnikov

L. D. Landau Theoretical Physics Institute, USSR Academy of Sciences
(Submitted March 7, 1975)
Zh. Eksp. Teor. Fiz. 69, 675–687 (August 1975)

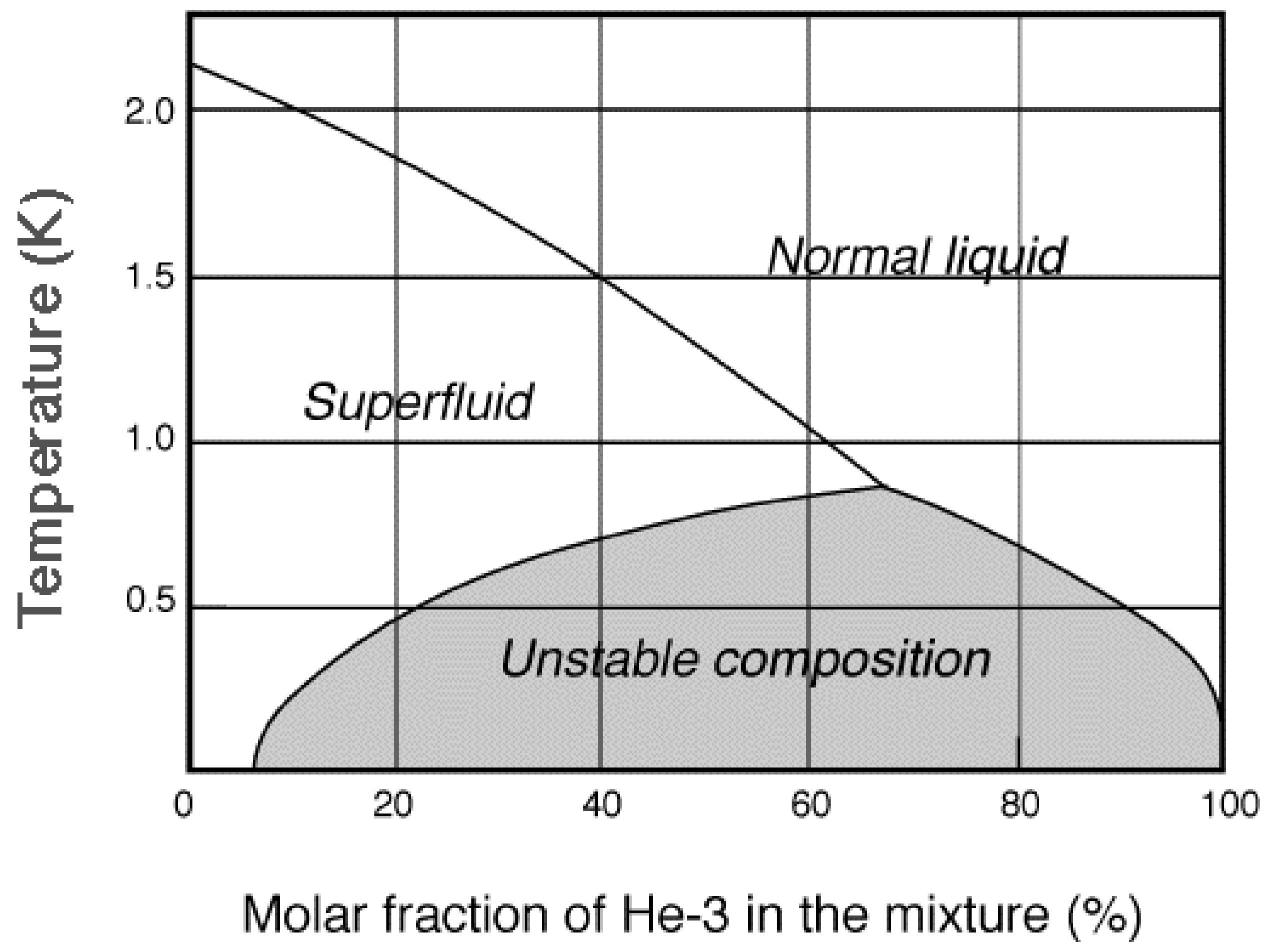
There also exists the possibility of an analogous phase transition in liquid ^{3}He dissolved in superfluid ^{4}He. The point is that a liquid solution of ^{3}He in ^{4}He does not separate into pure components, even at absolute zero temperature and normal pressure, for ^{3}He concentrations up to 6% (when the pressure is raised, the concentration can be increased to 10%). In order that ^{3}He dissolved in ^{4}He become superfluid, Cooper pairing of ^{3}He atoms is necessary. The possibility of pairing depends on the sign of λ—the scattering amplitude for mutual scattering of ^{3}He atoms dissolved in superfluid ^{4}He at small momentum transfers. If $\lambda < 0$, the ^{3}He atoms attract each other and a phase transition of ^{3}He to a superfluid state occurs at a certain temperature T_c. The calculations of λ and T_c are complicated problems. The estimates existing at present mainly lead to an anomalously small value of T_c (cf.[2] and the references therein). We shall not be interested in this question here. In the hope that a solution of a superfluid Fermi liquid in a superfluid Bose liquid will sometime become available to the experimentalists, we shall construct a semi-microscopic theory of such a solution, in the spirit of the Landau Fermi-liquid theory.

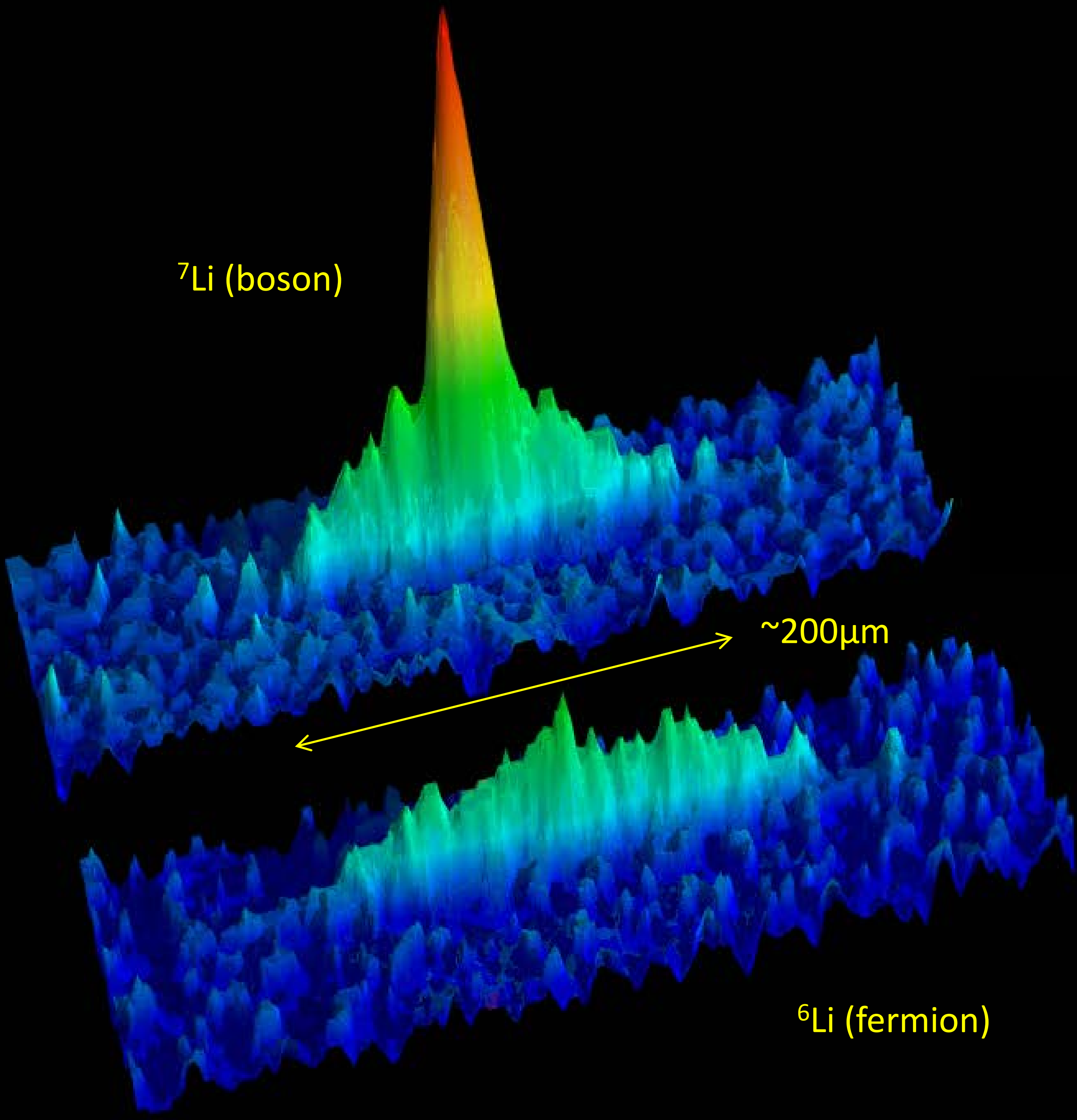
quasi-particle distribution function. A superfluid Fermi liquid can be described in the framework of a two-fluid model, all the thermodynamic quantities of which are expressed in terms of the Fermi-liquid constants of the Landau theory. The system of equations obtained in[11] makes it possible to calculate the first-, second- and fourth-sound velocities in the limit $\omega\tau \ll 1$ and the zero-sound velocity in the limit $\omega\tau \gg 1$ in a superfluid Fermi liquid (cf.[12]).

The purpose of the present article is to construct an analogous theory for a solution of a superfluid Fermi liquid in a superfluid Bose liquid. By means of the kinetic equation and the expression for the energy of the Fermi quasi-particles, the three-fluid hydrodynamics is derived in the hydrodynamic regime $\omega\tau \ll 1$ and the velocities of the first, second, third and two fourth sounds are calculated. In the collisionless regime expressions are obtained for the velocities of the two high-frequency sounds, one of which goes over into the ordinary zero sound in a normal Fermi liquid when $T > T_c$. All physical quantities are expressed in terms of the Fermi-liquid constants.

The following assumptions have been used in the paper.

$^3He/^4He$ phase diagram





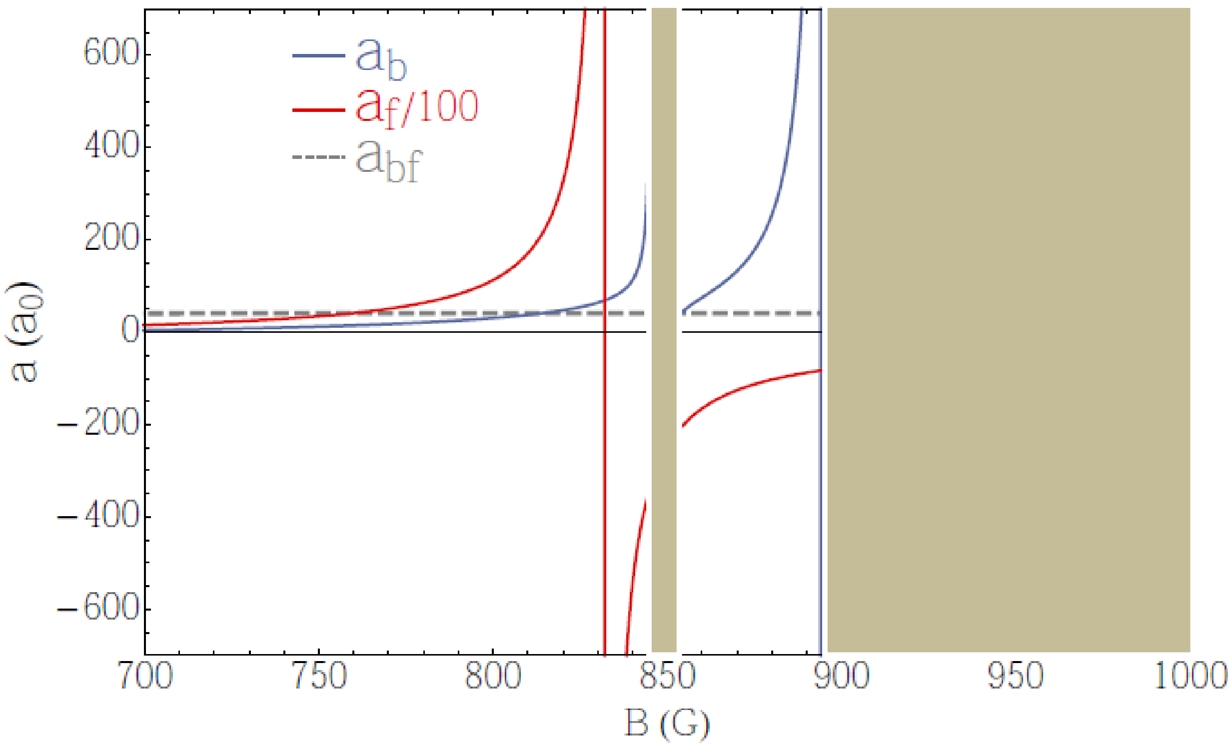
^{7}Li (boson)
~200μm
^{6}Li (fermion)

Achieving double superfluidity with cold atoms

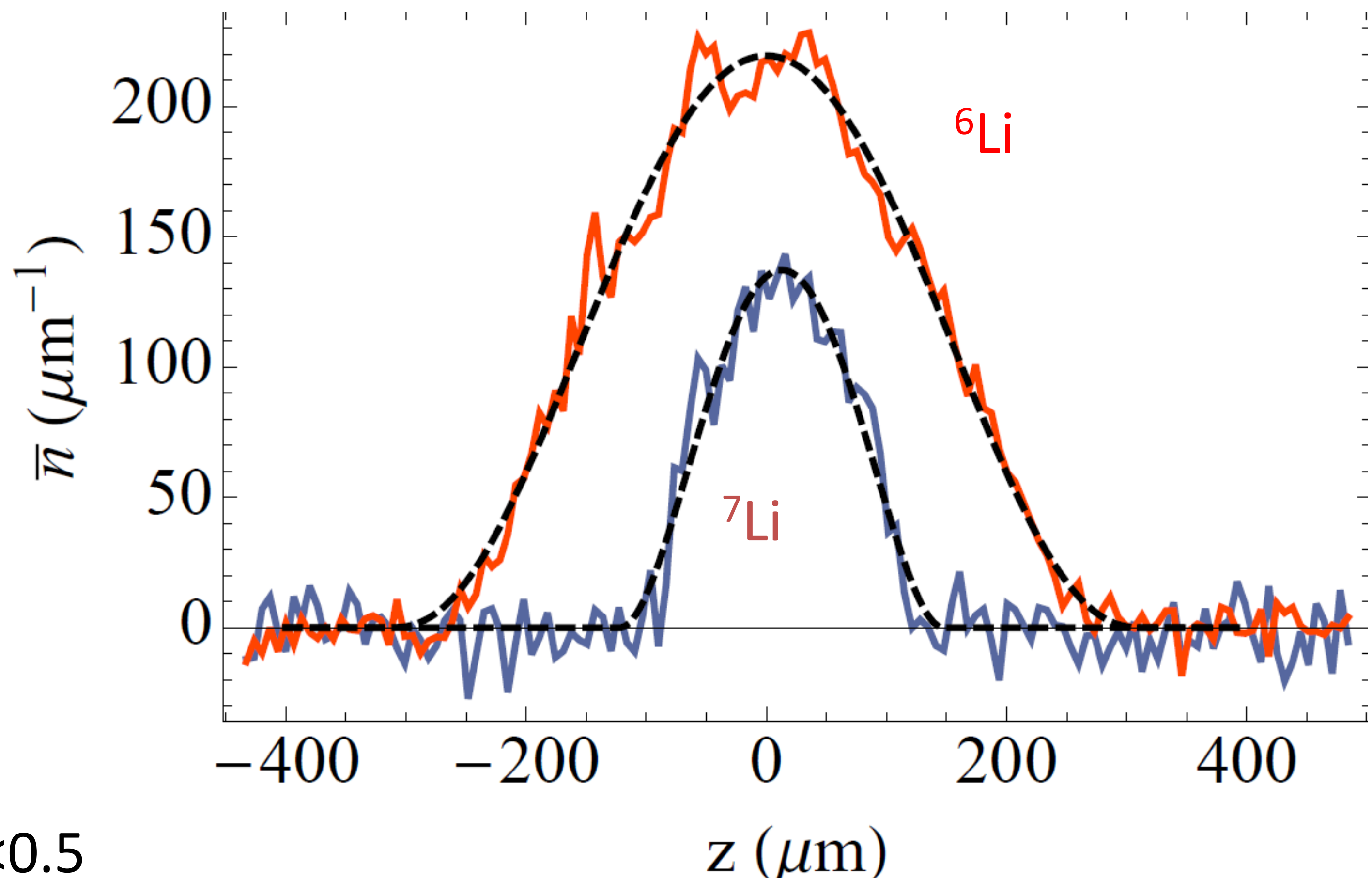
Requirements:

- Low a_{bf} (no interspecies demixing)
- High $|a_{ff}|$ (high fermionic Tc)
- Positive a_{bb} (stable BEC)

^{6}Li – ^{7}Li mixture in the $|1\rangle_f$, $|2\rangle_f$ and $|2\rangle_b$



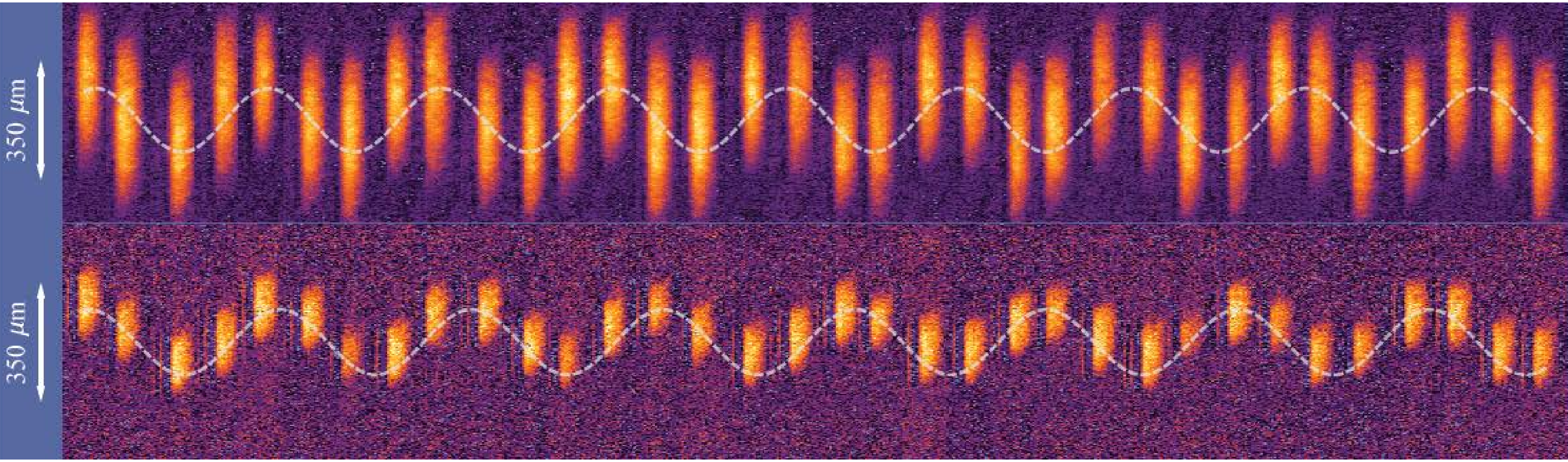
Superfluid mixture @ B=832G



$T/T_{cb,f}<0.5$

*I. Ferrier-Barbut, et al., Science **345**, 1035 (2014)*

Dynamics of the Mixtures



$$\omega_6 = 2\pi \times 16.80(2) Hz \qquad \tilde{\omega}_6 = 2\pi \times 17.06(1) Hz$$

$$\omega_7 = 2\pi \times 15.00(2) Hz \qquad \tilde{\omega}_7 = 2\pi \times 15.40(1) Hz$$

Single Superfluid

Ratio = $(7/6)^{1/2} = (m_7/m_6)^{1/2}$

Coupled Superfluids

See also C. Hammer *et al* Phys. Rev. Lett. **106**, 065302 (2011) for boson-boson superfluid counterflow

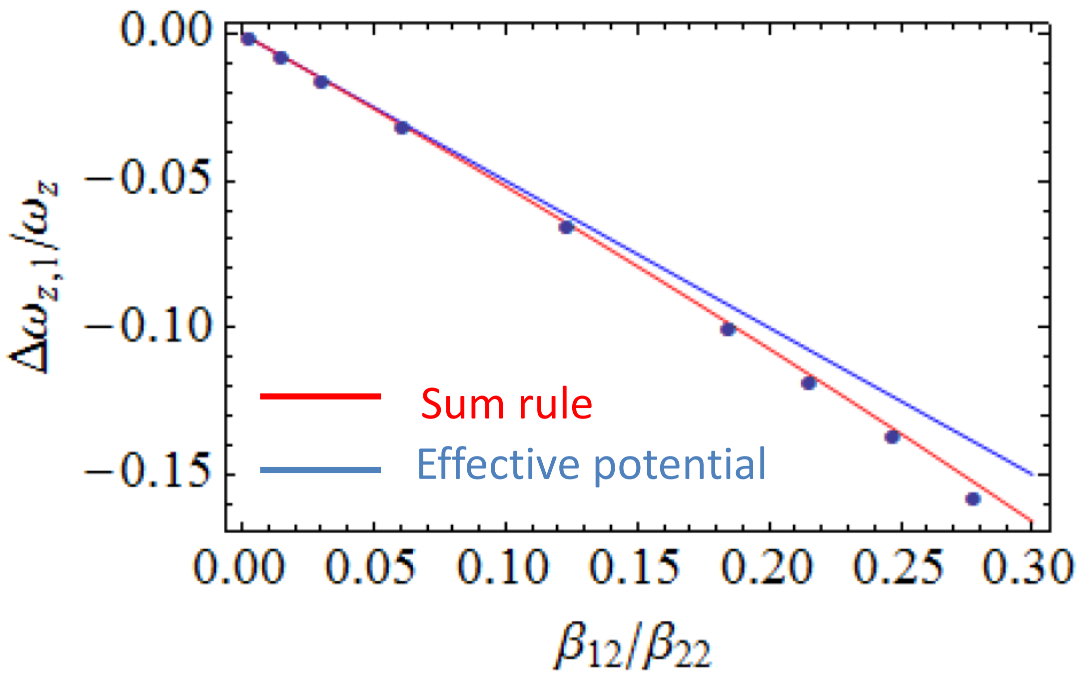
Frequency shift

$$V_{\mathrm{eff},7} = V(\mathbf{r}) + g_{67} n_6(\mu_6(\mathbf{r})) \qquad \mu_6(n_6(\mathbf{r})) = \mu_6^0 - V(\mathbf{r})$$ (Local Density Approximation)

$$\approx g_{67} n_6(\mu_6^0) + V(\mathbf{r})\left(1 - g_{67}\frac{\partial n_6}{\partial \mu_6}\right)$$

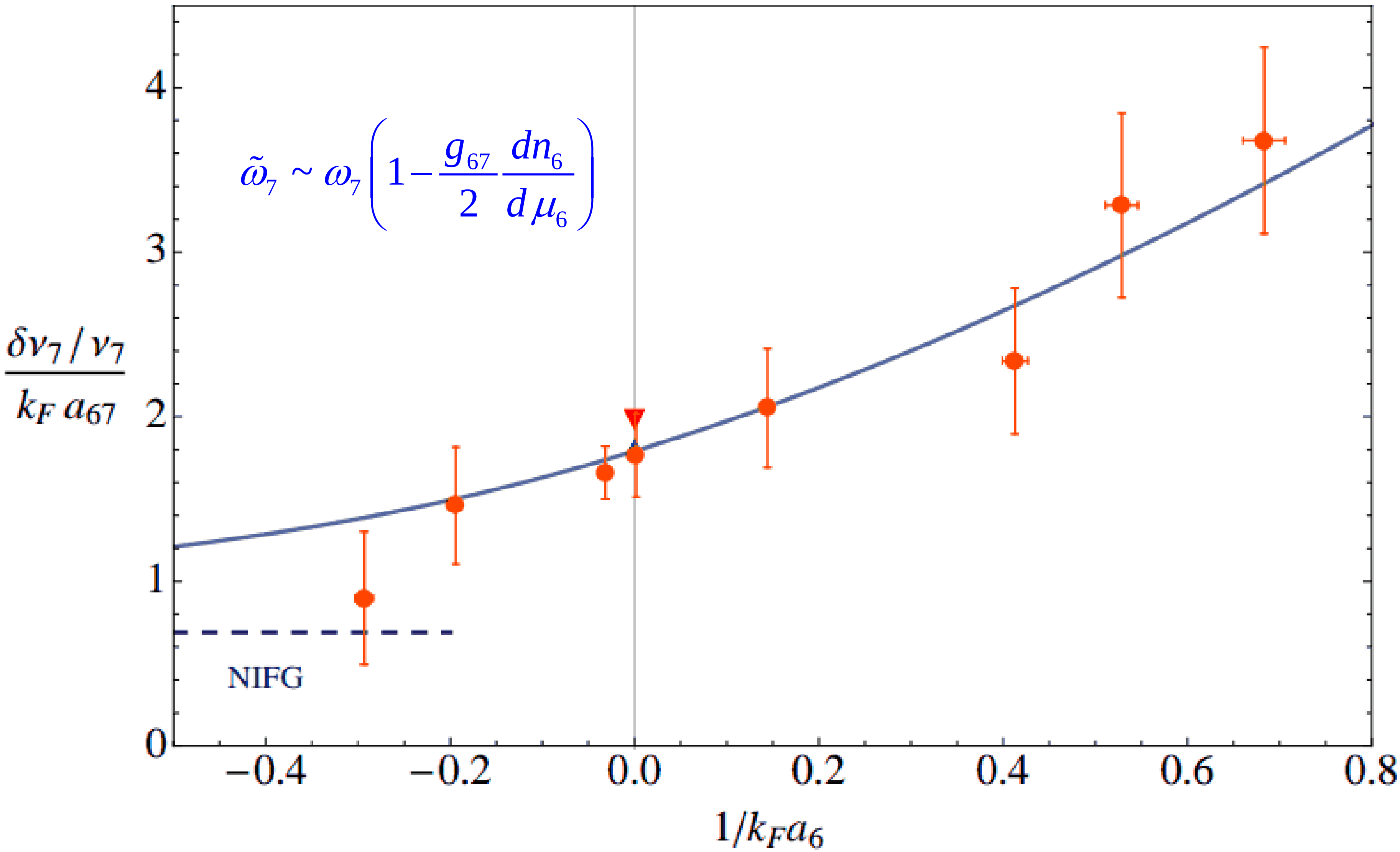
Harmonic trap: $$\frac{\Delta\omega_7}{\omega_7} \approx -\frac{g_{67}}{2}\frac{\partial n_6}{\partial \mu_6}$$

Benchmark: Numerical solution of GPE (P. Parnaudeau/I. Danaila/A. Suzuki)

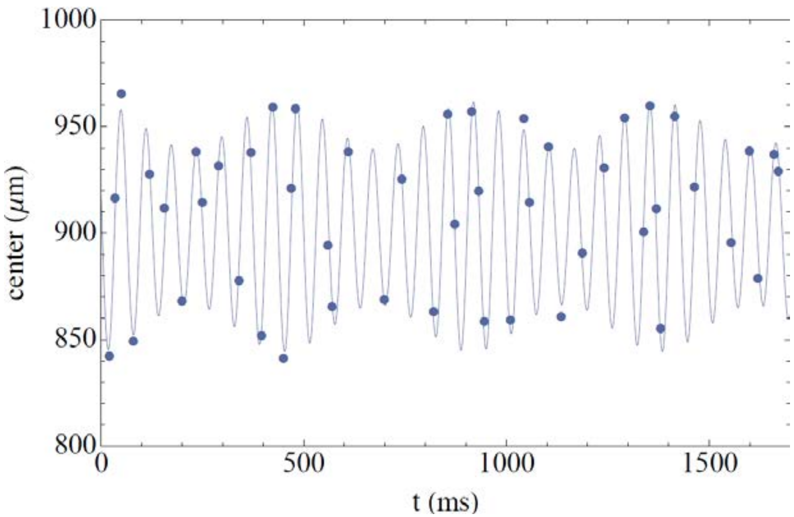


Oscillation Frequency of the BEC

Weak frequency shift (few percents) of the bosons due to the fermions

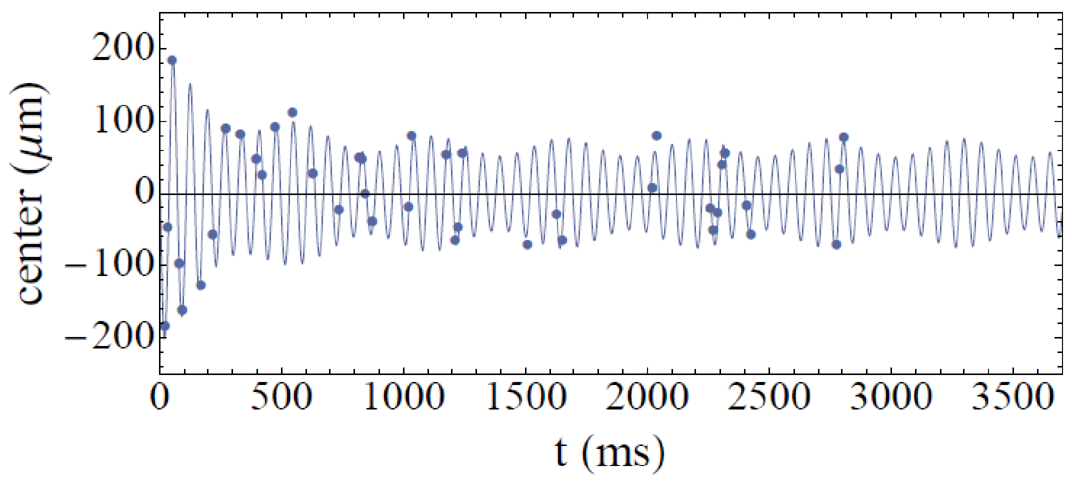


Oscillations of ^{7}Li

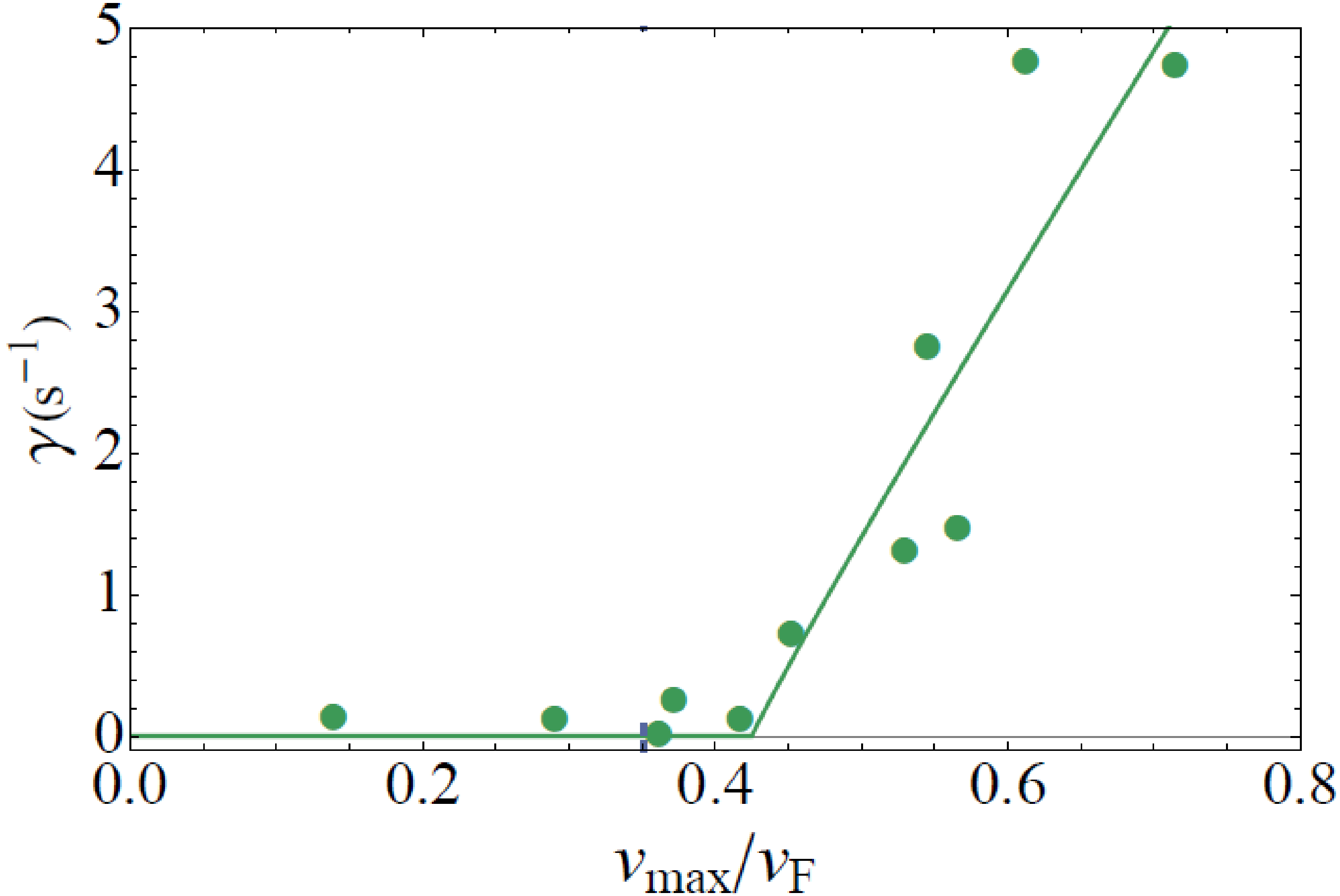


Small initial displacement:
•Almost no damping (decay time>4s)
•Beatnote (coherent coupling between the two oscillators)

Large initial displacement:
Damped oscillations



Critical velocity



Landau's Criterion

Momentum Conservation : $M\mathbf{V} = M\mathbf{V}' + \hbar\mathbf{k}$

Energy Conservation : $MV^2/2 = MV'^2/2 + \varepsilon_\mathbf{k}$

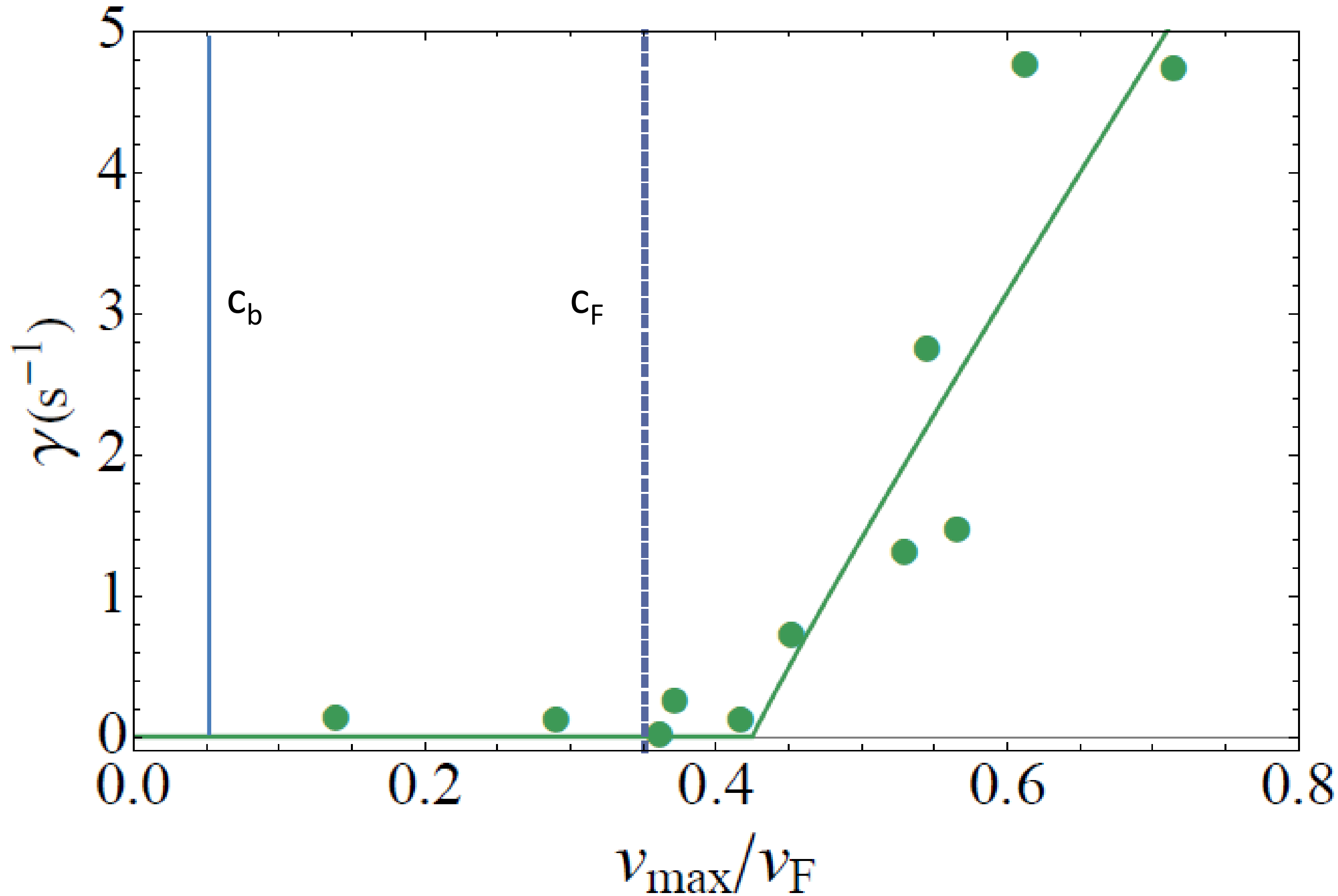
$$\hbar k V \geq \hbar\mathbf{k}.\mathbf{V} = \varepsilon_k + \hbar^2 k^2/2m \geq \varepsilon_k$$

The motion of the impurity is damped by the creation of elementary excitations if

$$V \geq V_c = \min_k \left(\frac{\varepsilon_k}{\hbar k} \right)$$

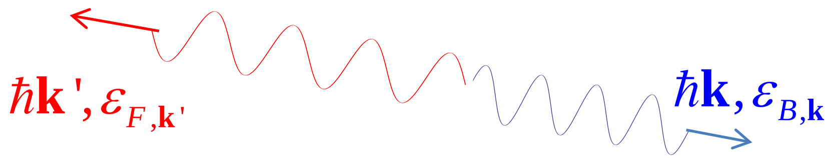
= sound velocity for a linear excitation spectrum $\varepsilon = \hbar k c$

Critical velocity



Landau criterion for a superfluid Mixture

(Castin *et al.* Comptes Rendus Physique **16,** 241 (2015) **arXiv:1408.1326**)



1 Excitation in the bosonic superfluid $$E_{\mathrm{B},\mathbf{k}} = \varepsilon_{\mathrm{B},\mathbf{k}} + \hbar \mathbf{k} \cdot \mathbf{V}_{\mathrm{B}}$$

1 Excitation in the fermionic superfluid $$E_{\mathrm{F},\mathbf{k}} = \varepsilon_{\mathrm{F},\mathbf{k}'} + \hbar \mathbf{k}' \cdot \mathbf{V}_{\mathrm{F}}$$

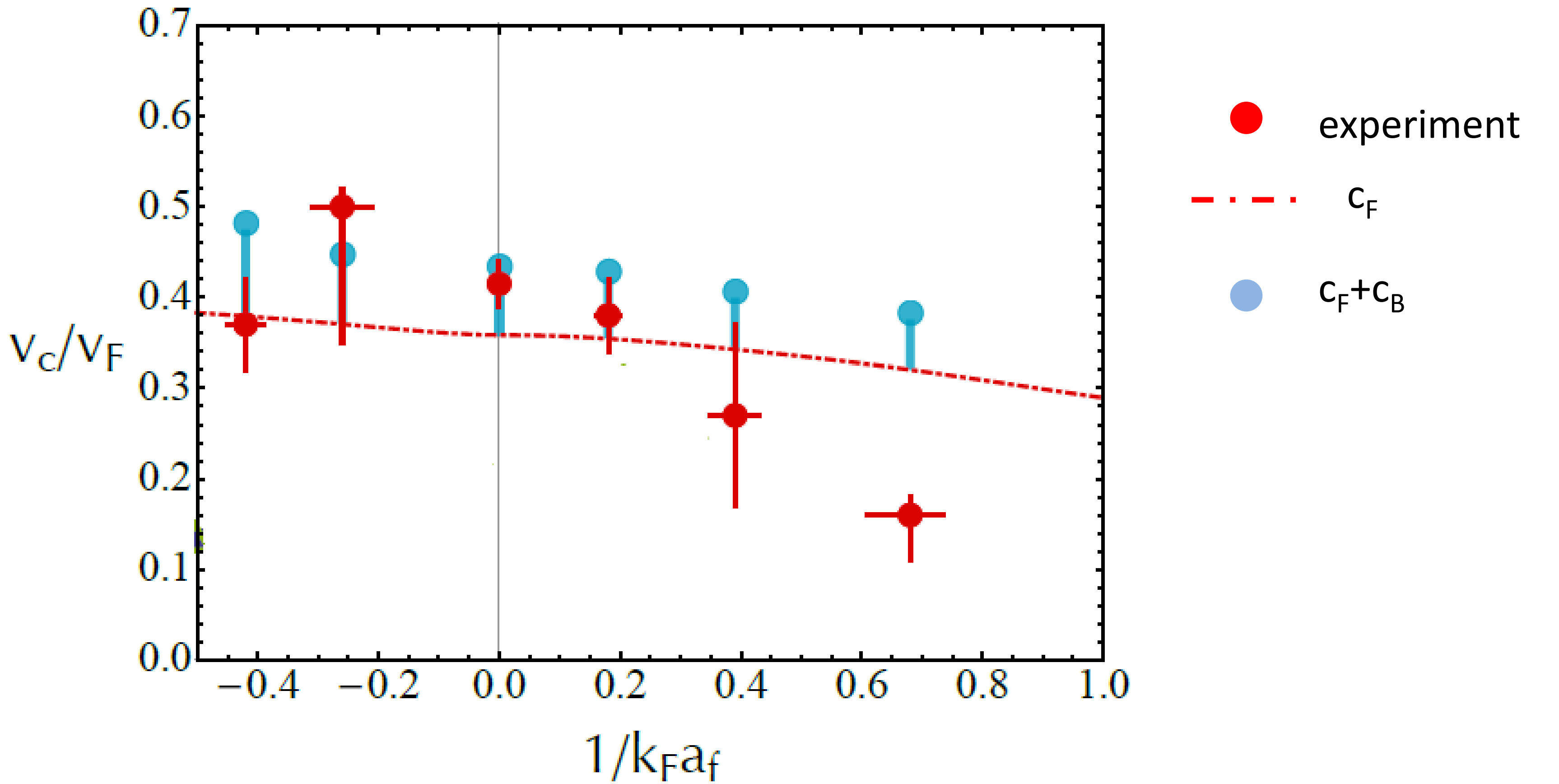
Energy-momentum conservation: $$E_{\mathrm{B},\mathbf{k}} + E_{\mathrm{F},\mathbf{k}'} = 0 \qquad \mathbf{k} + \mathbf{k}' = 0$$

$$|\mathbf{V}_B - \mathbf{V}_F| \geq \min_k \left(\frac{\varepsilon_{B,k} + \varepsilon_{F,-k}}{\hbar k} \right)$$

Acoustic Modes: $V_c = \cancel{c_B} + c_F$

See also Abbad et al. EPJD 69, 126 (2015), F. Chevy PRA **91**, 063606 (2015), W. Zheng et H. Zhai, Phys. Rev. Lett. 113, 265304 (2014)

Critical Velocity



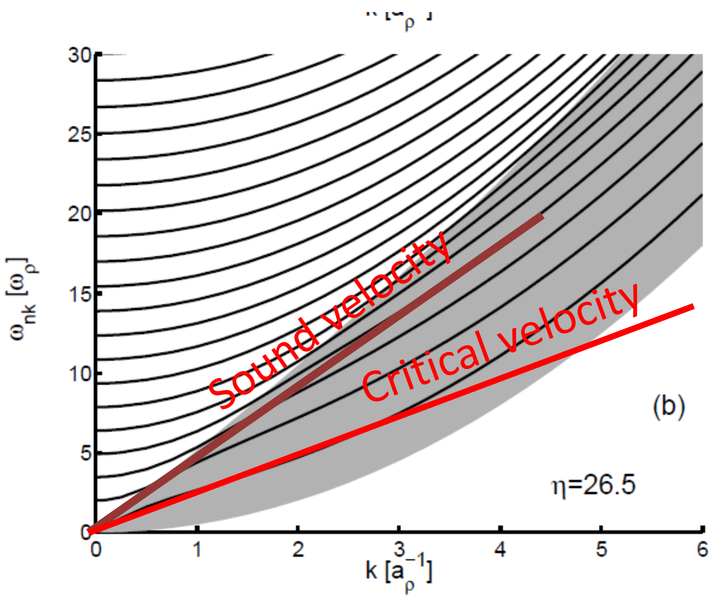
Similar reduction of v_c at MIT and Hamburg for fermions.
Possible explanations: finite temperature, vortex nucleation…

Validity of Landau's argument?

(see also V.P. Singh *et al.* arXiv:1509.02168)

- Argument valid for a **constant** velocity in an **homogeneous** medium.
- But:

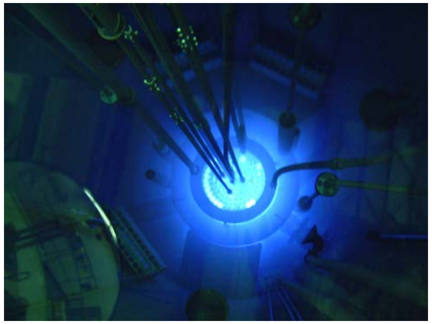
Trapping potential



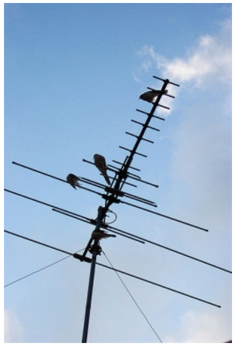
Cozzo & Dalfovo, NJP (2003)

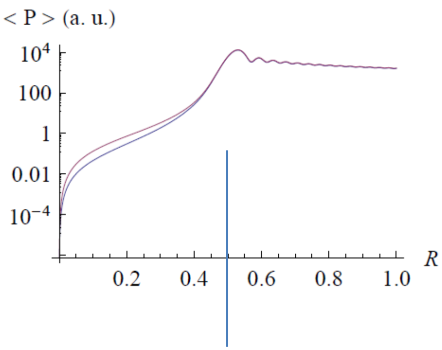
Crépin, Leyronas & FC under submission

Oscillatory motion



vs





No critical velocity!

THANKS FOR YOUR ATTENTION!