Classical and non-classical flows of superfluids

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Motivation

- Formation of coherent BEC from a thermal Bose gas
- Experiments on thermally quenched fluids, Kibble-Zurek mechanism, topological defects (solitons, quantum vortices) (Bäuerle & al 1996, Weiler & al 2008, Chomaz & al 2015, Lamporesi & al 2014)
- Paradigm: evolution of homogeneous weakly interacting Bose gas from non-equilibrium initial condition to a state of

  quantum turbulence

  to a final vortex-free state (Berloff & Svistunov 2002)
- **Question:** is the **quantum turbulence** which follows a thermal quench similar to **ordinary turbulence** ?

  (a natural question, given the observed similarities between quantum turbulence in superfluid helium and classical turbulence in ordinary fluids, see CFB, Skrbek & Sreenivasan PNAS 2014)

Gross-Pitaevskii equation

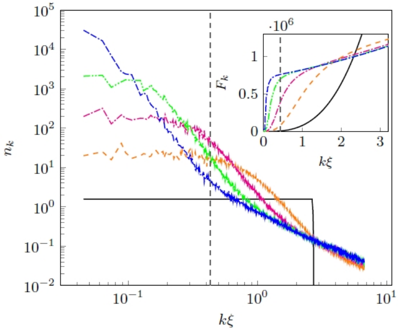
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi$$

$$H = \int \left(\frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{g}{2}|\psi|^4\right) dV$$

- GPE usually models a $T = 0$ condensate, but also models a finite-temperature gas, provided modes are highly occupied
- Highly nonequilibrium initial condition $\psi(\mathbf{r}, 0) = \sum_{\mathbf{k}} a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$ (uniform $n_{\mathbf{k}} = |a_{\mathbf{k}}|^2$, random phases)
- Number density N/D^3, energy density $\langle H\rangle/D^3$
- Condensate fraction ρ_0/ρ (where ρ_0 = quasi-condensate density)

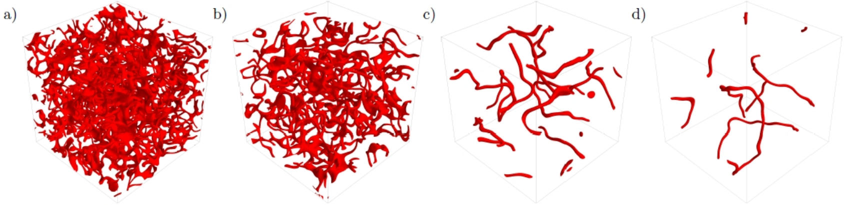
The quasi-condensate

Evolution of occupation numbers $n_{\mathbf{k}}$ and of integral distributions $F_k = \sum_{k'<k} n_{\mathbf{k}'}$



- Growth of occupation of low-k modes associated with condensate. High-k modes associated with thermal excitations.
- Quasi-condensate ψ' (containing the long-wavelength part of the classical field) defined by cutoff $k_c = 10(2\pi/D)$ and $a'_{\mathbf{k}} = a_{\mathbf{k}}\ max\{1 - k^2/k_c^2, 0\}$.

Evolution of vortex tangle (here for $\rho_0/\rho = 0.22$)



Question:

How does this turbulence compare with ordinary turbulence ?

The Kolmogorov spectrum in classical turbulence

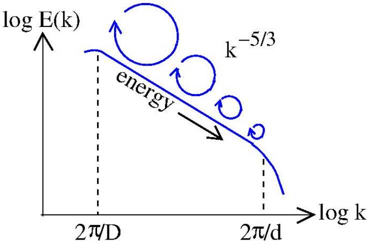
Energy spectrum $E(k)$ describes the distribution of kinetic energy over the length scales $2\pi/k$ (where $k=$ wavenumber)

$$E = \frac{1}{V}\int_V \frac{\mathbf{v}^2}{2} dV = \int_0^\infty E(k) dk$$

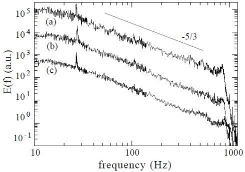
Forward **energy cascade**: large eddies break up into smaller eddies

$E(k) \sim k^{-5/3}$
Kolmogorov's 5/3 law

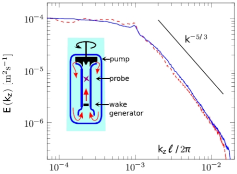
$D =$ system size
$d =$ dissipation length scale



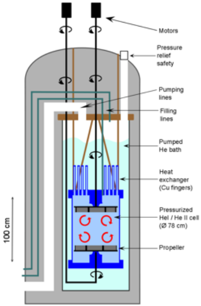
The Kolmogorov spectrum in turbulent superfluid helium



Maurer & Tabeling 1998



Salort &al 2012

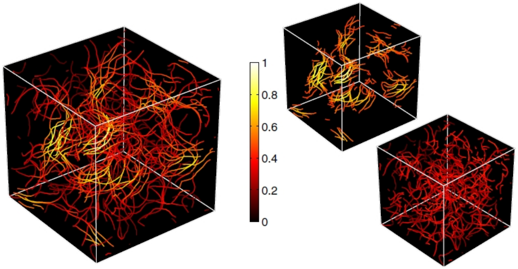


SHREK, Grenoble

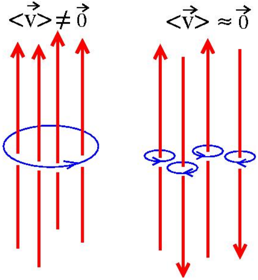
The Kolmogorov spectrum in turbulent superfluid helium

The Kolmogorov spectrum is associated with (metastable) coherent bundles of vortex lines.

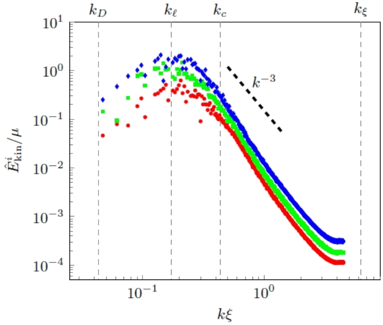
Decompose the vortex lines (according to locally coarse-grained vorticity) into **polarized lines** (yellow) containing most of energy and causing the $k^{-5/3}$ law, and **unpolarized lines** (red).



(Baggaley, Laurie & CFB 2012)

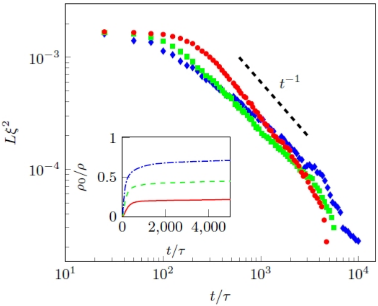


Energy spectrum following thermal quench



- No pile-up of energy at largest length scales ($k \approx 1/D$), no Kolmogorov $k^{-5/3}$ law
- Expected k^{-3} behavior at $k \approx 1/\xi$, hint of (expected) k^{-1} behaviour at $k \approx 1/\ell$

Decay of turbulence following thermal quench



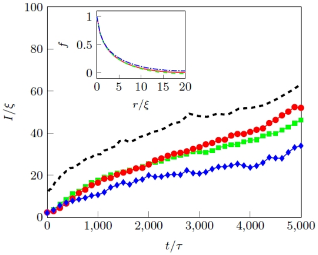
- Vortex line density decays as $L \sim t^{-1}$
- (Kolmogorov turbulence would decay as $L \sim t^{-3/2}$ as observed in superfluid helium by Walmsley & Golov 2008)

Correlation function following thermal quench

Velocity correlation function $f(r,t)$ and integral length scale $I(t)$:

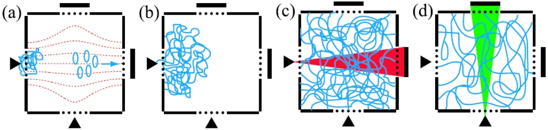
$$f(r,t) = \frac{v_x(\mathbf{r},t)v_x(\mathbf{r}+r\widehat{\mathbf{e}}_x,t)}{\langle v_x(\mathbf{r},t)^2\rangle}, \qquad I(t) = \int_0^\infty f(r)dr$$

No correlation at distances larger than ℓ,
and $I(t) < \ell/2$ at all t



Quasi-classical vs ultra-quantum

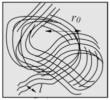
Turbulence generated in ^{4}He by injecting vortex rings
(Walmsley & Golov 2008)



Depending on the injection rate, they found two turbulence states:

(1) quasi-classical (Kolmogorov) turbulence: decays as $L \sim t^{-3/2}$

(2) ultra-quantum (Vinen) turbulence: decays as $L \sim t^{-1}$

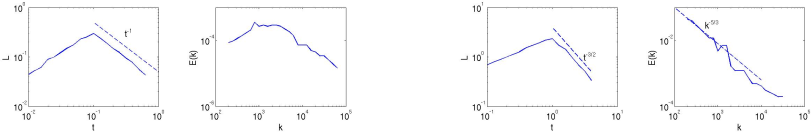


quasi-classical
(Kolmogorov)

ultra-quantum
(Vinen)

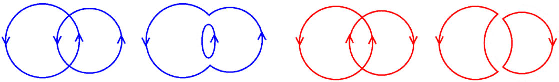
(Volovik 2003)

Walmsley & Golov's experiment is reproduced by numerical simulations (Baggaley, Sergeev & CFB 2012)



ultra-quantum (Vinen) quasi-classical (Kolmogorov)

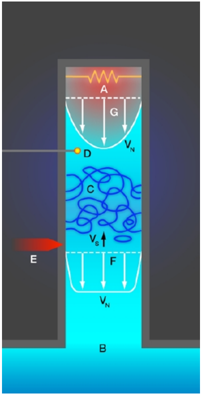
The inverse 3D energy cascade which creates ultra-quantum turbulence depends crucially on vortex reconnections "from behind" (left) rather than "head on" (right).

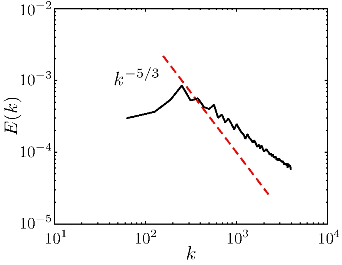


Another ultra-quantum regime: thermal counterflow

Heat transfer turbulence in ^{4}He (thermal counterflow), first studied by Vinen:

Sherwin, Baggaley, Sergeev, CFB 2012: As the ultra-quantum regime, counterlow turbulence lacks pile-up of energy at the smallest k and the $k^{-5/3}$ law





Conclusions

Quantum turbulence induced by thermal quench = **prototype ultra-quantum (Vinen) turbulence**:

- Energy concentrated at intermediate scales, not at the largest length scales
- No $k^{-5/3}$ energy spectrum, no energy cascade
- Decay as $L \sim t^{-1}$, not as $L \sim t^{-3/2}$
- Spatially random and homogeneous, without coherent structures.