Higher Dimensional Birational Geometry and Characteristic $p > 0$

CIRM Luminy, September 12–16, 2016
Manuel Blickle, Karl Schwede, Chenyang Xu

All talks are sheduled to last 50 minutes (+discussion). There are coffee brakes at 10:00 in the morning and 15:00 in the afternoon. Lunch will be served at 12:30, dinner at 19:30.

Monday

 9:00 **Christian Liedtke** Crystalline cohomology, period maps, and applications to K3 surfaces

10:30 **Tommaso de Fernex** Towards a link theoretic characterization of smoothness

11:30 **Remy van Dobben de Bruyn** The Hodge ring of varieties in positive characteristic

12:30 Lunch

16:00 **Yoshinori Gongyo** Rational points on log Fano 3-fold over a finite field

17:30 **Adrian Langer** Endomorphisms of Drinfeld half-spaces over a finite field

Tuesday

 9:00 **Christian Liedtke** Crystalline cohomology, period maps, and applications to K3 surfaces

10:30 **Johannes Nicaise** The non-archimedean SYZ fibration and Igusa zeta functions

11:30 **Paolo Cascini** Singularities of log pairs in characteristic two

12:30 Lunch

16:00 **Sofia Tirabassi** Fourier-Mukai partners of canonical covers in positive characteristic

17:30 **Stefan Kebekus** Higgs sheaves on singular spaces and the Miyaoka-Yau Inequality for minimal varieties of general type

Wednesday

 9:00 **Christian Liedtke** Crystalline cohomology, period maps, and applications to K3 surfaces

10:30 **Johannes Nicaise** The non-archimedean SYZ fibration and Igusa zeta functions

11:30 **Linquan Ma** Deformation of F-injectivity

12:30 Lunch

free afternoon

Thursday

9:00   **Mircea Mustata** Hodge ideals

10:30   **Johannes Nicaise** The non-archimedean SYZ fibration and Igusa zeta functions

11:30   **Shunsuke Takagi** General hyperplane sections of canonical 3-folds in positive characteristic

12:30   Lunch

16:00   **Yuan Wang** On the characterization of abelian varieties for log pairs in zero and positive characteristic.

17:10   **Sergey Rybakov** On zeta functions of cubic surfaces over finite fields

18:20   **Roberto Svaldi** Log birational boundedness of Calabi-Yau pairs

Friday

9:00   **Vasudevan Srinivas** Stratified vector bundles on simply connected varieties in positive characteristic

10:30   **Zsolt Patakfalvi** On projectivity of the moduli space of stable surfaces in characteristic $p > 5$

11:30   **Kevin Tucker** Fundamental Groups of F-regular Singularities via F-Signature

12:30   Lunch

13:30   **Bhargav Bhatt** The direct summand conjecture and its derived variant

Abstracts

Cristian Liedkte: Crystalline cohomology, period maps, and applications to K3 surfaces

I will first introduce K3 surfaces and determine their algebraic deRham cohomology. Next, we will see that crystalline cohomology (no prior knowledge assumed) is the "right" replacement for singular cohomology in positive characteristic. Then, we will look at one particular class of K3 surfaces more closely, namely, supersingular K3 surfaces. These have Picard rank 22 (note: in characteristic zero, at most rank 20 is possible) and form 9-dimensional moduli spaces. For supersingular K3 surfaces, we will see that there exists a period map and a Torelli theorem in terms of crystalline cohomology. As an application of the crystalline Torelli theorem, we will show that a K3 surface is supersingular if and only if it is unirational.

Tommaso de Fernex: Towards a link theoretic characterization of smoothness

A theorem of Mumford states that, on complex surfaces, any normal isolated singularity whose link is diffeomorphic to a sphere is actually a smooth point. While this property fails in higher dimensions, McLean asks whether the contact structure that the link inherits from its embedding in the variety may suffice to characterize smooth points among normal isolated singularities. He proves that this is the case in dimension 3. In joint work with Yu-Chao Tu, we use techniques from birational geometry to extend McLean's result to a large class of higher dimensional singularities. We also introduce a more refined invariant of the link using CR geometry, and conjecture that this invariant is strong enough to characterize smoothness in full generality.

Remy van Dobben de Bruyn: The Hodge ring of varieties in positive characteristic

Which Hodge diamonds can occur as a sum of Hodge diamonds of varieties? Are there any linear relations between the Hodge numbers of a variety in characteristic p, besides the ones coming from Serre duality? Which linear combinations of Hodge numbers are birational invariants? These questions were answered over the complex numbers by D. Kotschick and S. Schreieder. In this talk, we will discuss the version in characteristic p, where the answers are different due to the failure of Hodge symmetry.

Yoshinori Gongyo: Rational points on log Fano 3-fold over a finite field

I talk about the $W\mathcal{O}$-rationality of klt threefolds and the rational chain connectedness of klt Fano threefolds over a perfect field of characteristic $p > 5$. As a consequence, we show that any klt Fano threefold over a finite field of characteristic $p > 5$ has a rational point.

Adrian Langer: Endomorphisms of Drinfeld half-spaces over a finite field

Rémy, Thuillier and Werner proved that every automorphism of Drinfeld's half space over a finite field extends to an automorphism of the projective space. I will show a construction of a purely inseparable endomorphism of Drinfeld's half-space that does not extend to an endomorphism of the projective space. The construction uses an inseparable analogue of the Cremona transformation and it gives some higher height foliations on the projective space.I will use this to illustrate various examples of interesting varieties in positive characteristic.

Johannes Nicaise: The non-archimedean SYZ fibration and Igusa zeta functions

The SYZ fibration is a conjectural geometric explanation for the phenomenon of mirror symmetry for maximal degenerations of complex Calabi-Yau varieties. I will explain Kontsevich and Soibelman's construction of the SYZ fibration in the world of non-archimedean geometry, and its relations with the Minimal Model Program and Igusa's p-adic zeta functions. No prior knowledge of non-archimedean geometry is assumed. These lectures are based on joint work with Mircea Mustata and Chenyang Xu.

Paolo Cascini: Singularities of log pairs in characteristic two

We show that many results regarding singularities of log pairs which appear naturally in the minimal model programme, do not extend to varieties defined over an algebraically closed field of characteristic two. Joint work with H.Tanaka.

Sofia Tirabassi: Fourier-Mukai partners of canonical covers in positive characteristic

We show that surfaces arising as canonical covers of Enriques and bielliptic surfaces do not have any non-trivial Fourier–Mukai partner, extending result of Sosna for complex surfaces. This is a joint work with K. Honigs and L. Lombardi

Stefan Kebekus: Higgs sheaves on singular spaces and the Miyaoka-Yau Inequality for minimal varieties of general type

We establish the Miyaoka-Yau inequality for the tangent sheaf of any minimal, complex, projective variety X of general type, with only klt singularities. In the case of equality, we prove that the canonical model of X has only quotient singularities and is uniformized by the unit ball. Joint with Greb, Peternell, Taji.

Linquan Ma: Deformation of F-injectivity

A local singularity is called F-injective if the Frobenius acts injectively on the local cohomology supported at the point. This is the (conjectured) characterisitic $p > 0$ analog of DB singularities. A natural and interesting question asks whether F-injective singularities deform, and this is supported by the fact that DB singularities deform. In this talk we introduce and study a surjectivity condition on local cohomology, which is motivated by DB singularities, and we use it to show that F-injecitivity deforms in large characteristics. We also provide more evidence on the general case. The talk is based on joint work with Karl Schwede and Kazuma Shimomoto, and with Pham Hung Quy.

Mircea Mustata: Hodge ideals

Given a reduced effective divisor D on a smooth variety, the localization along this divisor is endowed with a Hodge filtration via Saito's theory of mixed Hodge modules. The information encoded by this filtration translates in a sequence of ideals, the first of which is the multiplier ideal of (a small perturbation of) D. I will discuss some general properties of these invariants. This is based on joint work with Mihnea Popa.

Shunsuke Takagi: General hyperplane sections of canonical 3-folds in positive characteristic

Since the Bertini theorem fails in positive characteristic, it is not clear whether a general hyperplane section of a canonical 3-fold in positive characteristic has only canonical singularities or not. I will report some recent progress on this issue. This talk is based on joint work with Kenta Sato.

Yuan Wang: On the characterization of abelian Varieties for log pairs in characteristic zero and $p > 0$.

Let X be a projective variety and D an effective $\mathbb{Q}$-divisor on X. A celebrated theorem of Kawamata says that if X is smooth and $\kappa(X) = 0$ then the Albanese morphism of X is an algebraic fiber space. Later it was shown by Zhang that if (X, D) is a log canonical pair and $-(K_X + D)$ is nef then the Albanese morphism of X is a fiberspace map. In this talk I will further discuss the relationship between $\kappa(K_X + D) = 0$, positivity of $-(K_X + D)$ and the Albanese map of X in both characteristic 0 and characteristic $p > 0$, and present some related results. In particular, I will present a result in characteristic $p > 0$ and dimension 3 that is a positive characteristic analog of Zhang's result, and another result in characteristic 0 that generalizes Kawamata's result to klt pairs.

Sergey Rybakov: On zeta functions of cubic surfaces over finite fields

A list of possible zeta functions of cubic surfaces over finite fields was obtained by Manin and Swinnerton-Dyer 50 years ago. To obtain a full classification of zeta functions one has to find a cubic surface with a given zeta function from the list. If the cubic surface is not minimal, then it is a blow up of a del Pezzo surface of higher degree, and in this case the construction of a surface was known. We prove that over many finite fields there exist minimal cubic surfaces with desired zeta functions. This is a joint project with Andrey Trepalin.

Roberto Svaldi: Log birational boundedness of Calabi-Yau pairs

I will discuss joint work with Gabriele Di Cerbo on boundedness of Calabi-Yau pairs. Recent works in the minimal model program suggest that pairs with trivial log canonical class should satisfy some boundedness properties. I will show that Calabi-Yau pairs which are not birational to a product are indeed log birationally bounded, if the dimension is less than 4. In higher dimensions, the same statement can be deduced assuming the BAB conjecture. If time permits, I will discuss applications of this result to elliptically fibered Calabi-Yau manifolds.

Vasudevan Srinivas: Stratified vector bundles on simply connected varieties in positive characteristic

This talk is a report on some joint work with Helene Esnault. We showed that on a smooth quasi-projective variety over the algebraic closure of a finite field, with trivial etale fundamental group, all stratfifed vector bundles are trivial, provided the variety admits a normal compactification with codimension 2 boundary. I will give some background, and discuss the steps going into the proof, one of which is a variant of the Grothendieck LEFF condition due to J.-B. Bost.

Zsolt Patakfalvi: On projectivity of the moduli space of stable surfaces in characteristic $p > 5$

Stable varieties are higher dimensional generalizations of stable curves. Their moduli space contains an open locus parameterizing varieties of general type up to birational equivalence, just as the space of stable curves contains the space of smooth curves in dimension one. Furthermore, also similarly to the one dimensional picture, it provides a compactification of the above locus, the construction of which is known in characteristic zero but it is only conjectural in positive characteristic in dimension at least two. I will present a work in progress aiming to prove the projectivity of every proper subspace of the moduli space of stable surfaces in characteristic greater than 5.

Kevin Tucker: Fundamental Groups of F-regular Singularities via F-Signature

The F-signature is a numerical invariant of singularities which measures the asymptotic number of splittings of iterates of Frobenius. The positivity of the F-signature characterizes F-regular singularities, which are closely related to KLT singularities in characteristic zero. After giving an overview, I will discuss new transformation rules for F-signature under finite maps. These transformation rules allow us to show finiteness of the etale local fundamental group for F-regular singularities, analogous to results of Xu and Kreb-Kebekus-Peternell for KLT singularities in characteristic zero.

Bhargav Bhatt: The direct summand conjecture and its derived variant

In the late 60s, Hochster conjectured that every regular ring is a direct summand, as a module, of any finite extension. Soon thereafter, Hochster himself proved his conjecture in equicharacteristic. In the mixed characteristic setting, Hochster's conjecture was settled very recently by Yves Andre using perfectoid geometry. I'll discuss a second proof (which is related to, but simplifies, Andre's proof) of Hochster's conjecture, and explain why the ideas going into the new proof also help establish a derived variant of the conjecture put forth by de Jong.