Controllability of Partial Differential Equations and Applications
9 - 13 November, 2015

Fabio Ancona: On the optimization of traffic flow at a junction.

We consider a traffic flow model at a junction where the dy- namics follows the classical conservation law formulation introduced by Lighthill, Whitham and Richards. The usual approach to achieve unique- ness of solutions is based on a pointwise maximization criteria involving the solutions of boundary Riemann problems. We shall address a general optimization problem where the boundary data and the distributional pa- rameters at the junction are regarded as controls. The goal is to select, on a given time interval [0, T], (possibly non unique) solutions which maxi- mize a suitable functional of the flux traces of the incoming edges (as maps over the whole interval [0, T]), among all entropy admissible solutions that preserve the conservation of cars through the junction and satisfy some distributional rules.
(Collaboration avec Annalisa Cesaroni, Giuseppe M. Coclite et Mauro Garavello)

Farid Ammar-Khodja: Exact boundary controllability of a system of mixed order with essential spectrum in 2-D

We address in this work the null boundary controllability of a linear hyperbolic system of the form $u_{tt} + Au = 0$ with $u = (u_1; u_2)$ posed in $(0;T) \times (0;1)^2$. A denotes a self-adjoint operator of mixed order, that usually appears in the modelization of linear elastic membrane shell. The operator A possesses an essential spectrum which prevents the null-controllability to hold uniformly with respect to the initial data $(u^0; u^1)$. We show in this work that the null controllability holds by a one Dirichlet control acting on the first variable u_1 for any initial data $(u^0; u^1)$ generated by the eigenfunctions corresponding to the discrete part of the spectrum $\sigma(A)$. The proof relies on a suitable observability inequality obtained by the way of a full spectral analysis and the adaptation of an Ingham type inequality for the Laplacian in two space dimension. This work provides a non trivial example of controlled system by a number of controls strictly lower than the number of components.
(joint work with K. Mauffrey and A. Münch).

Boris Andreianov: Two results of controllability of hyperbolic conservation laws.

I will present results about attainability for scalar conservation laws with and without source followed by two applications. The first application is based on a general result of controllability to trajectories of mild solutions of abstract evolution equations governed by m-accretive operators: the control by distributed source can be realized thanks to a variant of the nudging strategy. The second application concerns special "triangular" hyperbolic systems combining linear continuity equations with velocity field determined by the solution of one one nonlinear scalar conservation law with strictly convex flux. E.g., the Keyfitz-Kranzer system and the system of multicomponent chromatography fit this framework. Numerical aspects of both constructions will also be presented. The talk is based on joint works with Adimurthi, C. Donadello, S.S. Ghoshal and U. Razafison.

Claude Bardos: Observation of solutions of the Neutron Transport Equation in the diffusion limit.

This talk is a report on an ongoing work with Kim Dang Phung. It concerns the observation of the solution of the neutron transport equation.
As such this project has two goals.
First establish error estimates which may have further application. And this is done through the diffusion approximation and the classical observability results for the diffusion equation.
The second goal is through the analysis of these error estimates trying to contribute to the understanding of the role of Carleman estimates or other standard tools for the diffusion approximation.

Karine Beauchard: Control of degenerate parabolic equations of hypoelliptic type.

For evolution equations associated with hypoelliptic operators, analysis and control properties are less understood than for uniformly parabolic equations. Recent studies proved that a few results from the uniformly parabolic case still hold in hypoelliptic setting, but new behaviours also appear: a positive minimal time and/or a geometric control condition can be required for the null controllability. This talk will present the state of the art on this topic, focussing on Grushin type operators, for which a rather complete analysis is available, and Heisenberg and Kolmogorov operators, for which investigation is still at an earlier stage.

Sébastien Benzekry: An optimal control problem for the treatment of metastatic cancers.

The post-surgical development of metastases (secondary tumors spread from a primary one) represent the major cause of death from a cancer disease. In order to try to control the spread of the malignancy, patients often receive systemic chemical therapy that targets the population of (possibly occult) metastatic tumors. While tools from optimal control theory have been classically applied to several problems of primary tumor control, no study has yet investigated the control of a population of (several) tumors, as well the dissemination of the disease. In this talk, we will introduce a modeling framework for description of metastatic dissemination and colonization as well as the effect of several anti-cancerous therapies (cytotoxic chemotherapy and anti-angiogenic therapy). This modeling approach is based on structured partial differential equations. Then, we will formulate an optimal control problem and illustrate by simulations in a simplified case how the scheduling of the therapy can differently impact growth of a (single) primary tumor and development of an organ or organism- scale population of metastases.

Giuseppe Buttazzo: Dirichlet-Neumann shape optimization problems.

We consider spectral optimization problems of the form

$$\min\Big\{\lambda_1(\Omega;D):\ \Omega\subset D,\ |\Omega|=1\Big\},$$

where D is a given subset of the Euclidean space $\mathbf{R}^d$. Here $\lambda_1(\Omega;D)$ is the first eigenvalue of the Laplace operator $-\Delta$ with Dirichlet conditions on $\partial\Omega\cap D$ and Neumann or Robin conditions on

$\partial\Omega \cap \partial D$. The equivalent variational formulation

$$\lambda_1(\Omega; D) = \min\Big\{ \int_\Omega |\nabla u|^2\,dx + k\int_{\partial D} u^2\,d\mathcal{H}^{d-1} \ : \ u \in H^1(D),\ u = 0 \text{ on } \partial\Omega \cap D,\ \|u\|_{L^2(\Omega)} = 1\Big\}$$

reminds the classical drop problems, where the first eigenvalue replaces the total variation functional. We prove an existence result for general shape cost functionals and we show some qualitative properties of the optimal domains. The case of Dirichlet condition on a *fixed* part and of Neumann condition on the *free* part of the boundary is also considered

Piermarco Cannarsa: Control and stabilization of degenerate evolution equations in one space dimension.

This talk aims to discuss analogies and differences in the study of controllability problems for evolution equations in one space dimension which exhibit boundary degeneracy. Given a function $a \in \mathcal{C}\big([0,1]\big) \cap \mathcal{C}^1\big(]0,1]\big)$ such that $a(x) > 0$ for all $x \in]0,1]$ and $a(0) = 0$, we are interested in the controllability properties of the parabolic operator

$$Pu(t,x) = u_t(t,x) - \big(a(x)u_x(t,x)\big)_x \qquad (t,x) \in]0,T[\times]0,1[$$

as well as the hyperbolic one

$$Hu(t,x) = u_{tt}(t,x) - \big(a(x)u_x(t,x)\big)_x \qquad (t,x) \in]0,T[\times]0,1[\,,$$

with suitable boundary conditions. The parabolic case will be considered first with a boundary control located at $x = 0$, that is, the degenerate part of the boundary. We shall present the main results obtained for this problem in collaboration with P. Martinez and J. Vancostenoble, which concern the sharp evaluation of the cost of control with respect to degeneracy parameters. Then, the controllability problem will be approached for operator H with a boundary control located at $x = 1$, as was done in a recent joint work with F. Alabau-Boussouira and G. Leugering. In this case, boundary stabilization results will also be derived.

Jean-Michel Coron: Linear transformations for the stabilization of nonlinear partial differential equations.

We start by presenting on hyperbolic systems some strange phenomena which appear for the stability of nonlinear partial equations. We then present methods to transform a given linear control system into new ones for which the stabilization is easy to get. We apply these methods to the rapid stabilization of nonlinear control systems modeled by partial differential equations as Korteweg de Vries, Kuramoto-Sivashinsky equations, Schrödinger equations and $1-D$-hyperbolic systems.

Sylvain Ervedoza: On the stabilization of incompressible Navier-Stokes equations in a 2d channel with a normal control.

In this talk, I will present some recent work on the stabilization of the incompressible Navier-Stokes equations in a 2d channel with periodic boundary conditions in the longitudinal variable when the control acts on the upper horizontal boundary only on the normal component of the velocity. In that case, the divergence free condition imposes that the control necessarily is of 0 mean value, so that the 0-mode of the linearized equations (corresponding to an infinite dimensional space) is independent of the control function. Therefore, following an idea previously used by E. Cerpa, J.-M. Coron, and E. Crépeau for the control of the Korteweg-de-Vries equation, we expand the solution as $u = \varepsilon u_1 + \varepsilon^2 u_2$ for $\varepsilon > 0$ small and construct an exponentially stable controlled solution for which the advection term $(u \cdot \nabla)u$ controls the most unstable eigenspace among the 0-mode. *Joint work with Shirshendu Chowdhury and Jean-Pierre Raymond.*

Olivier Glass: Control of the motion of a set of particles.

We consider the problem of lagrangian controllability for two models of fluids. The lagrangian controllability consists in the possibility of prescribing the motion of a set of particle from one place to another in a given time. The two models under view are the Euler equation for incompressible inviscid fluids, and the quasistatic Stokes equation for incompressible viscous fluids. These results were obtained in collaboration with Thierry Horsin (Conservatoire National des Arts et Métiers, Paris)

Manuel González-Burgos: Controllability of linear parabolic systems: New phenomena.

In this talk we will exhibit some new phenomena that arise in the framework of the controllability of coupled parabolic systems when the number of (distributed or boundary) controls exerted on the system is less than the number of equations. To this end, we will study the controllability properties of some simple examples of parabolic systems. As a consequence, we will see that, in general,

(1) The distributed and boundary null controllability are not equivalent.
(2) The null controllability is not equivalent to the approximated controllability.
(3) The null controllability only holds if the final time T is large enough (minimal time of controllability).
(4) The distributed null controllability depends on the position of the control open set.

Florence Hubert: Mathematical modeling of the microtubule dynamic instabilities.

The aim of our group is to design some pertinent mathematical /computational models of the pharmacological effects of microtubule-targeted drugs, which are powerful anti-mitotic drugs used in human cancers. Those drugs induce important perturbations on microtubule dynamic instabilities. As these instabilities play a key role in cancer progression: i.e cell proliferation/division and cell migration, any contribution on the comprehension of their effects could be helpfull. I will review in this talk differents issues for mathematicians in the modeling such dynamics.

Hiroshi Isozaki: Asymptotic properties of solutions to the elastic equation in a half space.

In a perturbed half space in $\mathbf{R}^3$, with crack or local perturbation of the surface, we study the asymptotic expansion at infinity for solutions to the stationary elastic equation $(P - \lambda)u = 0$

using the framework of the Besov type space B^*. Main purpose is the asymptotic behavior of the Rayleigh wave near the surface as well as those of the P-wave and S- wave near the critical direction. This is a joint work with M. Kadowaki and M. Watanabe.

Gilles Lebeau, Chaves-Silva Felipe: Controllability for the Stokes system.

Let Ω be a regular bounded domain and ω an open non void subset of Ω. From parabolic Carleman estimates, it is known that the controllability problem for the Stokes system

$$y_t - \triangle y + \nabla p = 1_\omega f, \quad div(y) = 0, \quad y|_{\partial\Omega} = 0$$

$$y|_{t=0} = y_0 \in H, \quad y|_{t=T} = 0$$

admits for any $T > 0$ a solution f such that $\|f 1_\omega\|_{L^2} \leq C_T \|y_0\|_H$. Our main result is a proof of the following small time estimate:

$$C_T \leq A e^{B/T}.$$

We will describe in the talk the proof of the spectral inequality for the Stokes system which leads to the above estimate.

Arnaud Münch: Inverse problems for linear PDEs using mixed formulations.

We explore a direct method allowing to solve numerically inverse type problems for hyperbolic type equations. We first consider the reconstruction of the full solution of the equation posed in $\Omega \times (0, T)$ - Ω a bounded subset of $\mathbb{R}^N$ - from a partial distributed observation. We employ a least-squares technic and minimize the L^2-norm of the distance from the observation to any solution. Taking the hyperbolic equation as the main constraint of the problem, the optimality conditions are reduced to a mixed formulation involving both the state to reconstruct and a Lagrange multiplier. Under usual geometric optic conditions, we show the well-posedness of this mixed formulation (in particular the inf-sup condition) and then introduce a numerical approximation based on space-time finite elements discretization. We show the strong convergence of the approximation and then discussed several examples for $N = 1$ and $N = 2$. The reconstruction of both the state and the source term is also discussed, as well as the boundary case. The parabolic case - more delicate as it requires the use of appropriate weights - will be also addressed. Joint works with Nicolae Cîndea and Diego Araujo de Souza. Details can be found in [**?**, **?**, **?**] using arguments developed in [**?**, **?**].

Maria Grazia Naso: Some results on Timoshenko systems.

The last decades have witnessed a rapid development in high technologies using beams. This has prompted great interest and several results have been published. In the wide literature on this field, most of papers deal with Euler-Bernoulli models and only few of them are devoted to Timoshenko ones, though, for instance, it was recently shown that the beam (plate) model of Timoshenko type has a wider range of applicability than Euler-Bernoulli model. In this talk we will present some recent results concerning boundary stabilization, contact problems and bridge systems applied to a Timoshenko beam.

Cristina Pignotti: Stabilization of wave equations with switching time delay.

Time-delay effects are frequently present in applications and physical problems. On the other hand, it is well-known that they may induce some instability. We will discussexistence and stability properties for wave equations intermittently damped/delayed. Under suitable conditions asymptotic/exponential stability results are proved.

Luc Robbiano: A spectral inequality for the bi-Laplace operator.

In this talk we present a inequality obtained with Jérôme Le Rousseau, for sum of eigenfunctions for bi-Laplace operator with clamped boundary condition. These boundary conditions do not allow to reduce the problem for a Laplacian with adapted boundary condition. The proof follow the strategy used for Laplacian, namely we consider a problem with an extra variable and we prove Carleman estimates for this new problem. The main difficulty is to obtain a Carleman estimate up to the boundary.

Julien Salomon: Some numerical methods for optimal control problems in quantum chemistry.

During the last 20 years, Quantum chemistry has made significant progresses and gives now rise to practical applications. From the mathematical point of view, the issues are often formalized as bilinear control problems. Because of the complex structure of these problems, specific algorithms have been developed. In this talk, we discuss the state-of-the-art in this field. Among others, we present a class monotonic schemes, and ways to extend and parallelize them.

Luz de Teresa: On hierarchic control for coupled heat equations.

A hierarchic control problem is a control problem where multi-objectives are present. There is a leader control and one or more followers. Each of them determine a strategy. The first results on hierarchic control for pde were obtained mainly by J.L. Lions in the context of approximate controllability and some of the recent results for some equations continue to be in this context. Recently Araruna, Fernández-Cara and Santos presented a hierarchic control result for one scalar heat equation where one of the objectives is an exact control to trajectories. In this talk we will present an (exact) hierarchic control for two coupled heat equations. In our result we need one of the coupling parameters to be small. The strategy is to write controls and solutions as a series of powers of this parameter an then to control the system corresponding to each power. This method was introduced by Castro and de Teresa for a system of thermoelasticity. In the present situation the key point will be controlling a system to zero with an exponential decay. To this end we will combine different techniques, including decoupling, Carleman estimates, and solving a fourth order system.
This work has been done in collaboration with Víctor Hernández Santamaría.

Djamel Eddine Teniou: Null controllability for the Stokes system interacting with the heat equation.

We consider the Stokes system with dynamic boundary conditions of type heat equation. We show that the problem is well posed ant that the solution is "smooth". Then we establish a Carleman estimate for this system. Finally, we write inequality observability that provides null controllability.

Ouahiba Zair: Null controllability of heat equation in the presence of singularities.

In this work we are interested in the study of controllability for the solutions of the heat equation in the presence of singularities. The singularity theory of elliptic problems knows a long history, both from the point of view of the engineer that of the mathematician. In the first part of this work, we are interested in the behavior of the solution in the neighborhood of singularities which is due either to the geometry of the domain (corners, edges, vertex, ...etc) or to a sudden change in the boundary conditions. The aim is to explicitly give singular functions that allow to decompose the solution into a singular and a regular part. We will also give a maximal regularity result for the Laplace equation in fractional spaces and a weak resolution for mixed problems of the heat equation. As reference, we have chosen the Grisvard approach. In the second part of this work, we will give null controllability results, for the heat equation in the presence of these singularities. The method is based on the use of particular Carleman estimates. These results will be given in the following cases:

(1) Case of Dirichlet boundary conditions in a polygonal domain.
(2) Case of a cracked domain.
(3) Case of mixed boundary conditions in a regular domain.