Modeling and transmission dynamics of mosquito-borne diseases

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HALE Conference: Dynamics of Evolution Equations
March 21-25, 2016 CIRM, Luminy,France

Plan of the talk

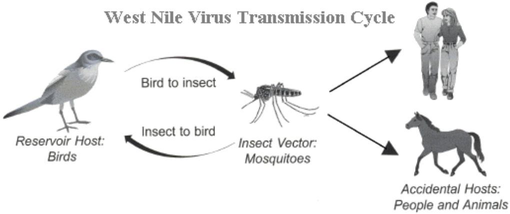
1. Introduction of mosquito-borne diseases (MBDs)
2. Dynamical modeling and Triggering mechanisms of an outbreak
3. Temperature and transmission dynamics, recurrence
4. Fast-slow dynamics of mosquito-borne diseases

1. Mosquitoes and mosquito-borne diseases

1.1 Malaria, dengue fever, West Nile virus (WNV), Chikungunya, ..., Zika virus

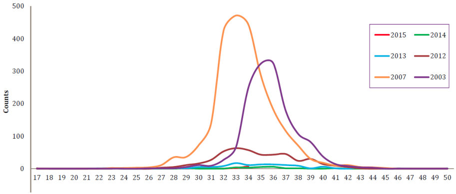
- Mosquito-borne pathogens
- Vector population: species of mosquitoes
- Host population: human, birds, animals, ...
- Environmental factors: weather and landscape

1.2 West Nile virus in USA/Canada

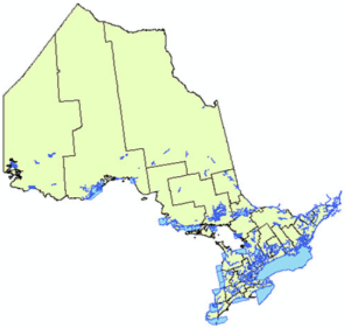


- Originated in Uganda in 1937
- Emerged in New York in 1999
- Arrived in Ontario/Canada in 2000
- Keep spreading ....USA/Canada

West Nile virus in Canada



1.3. Mosquito surveillance program and data in Ontario



- Peel region
- GTA (5 regions)
- Ontario (36 Health units)

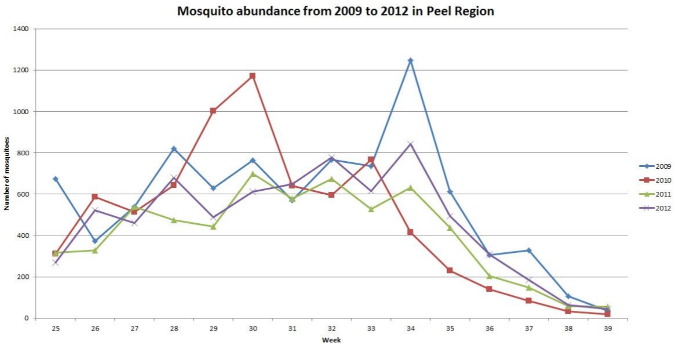
Trapping and data

Carbon dioxide-baited light traps to attract/collect adult mosquitoes; Identified and WNV tested.

Mosquito surveillance data

1	Health Unit	Week Code	Date Collected	Site Code	Latitude	Longitude	Cx. pipiens/restuans	Cx. salinarius	Cx. tarsalis	Ae. vexans vexans	Ae. vexans/cantator	Ae. Albopictus	Cq. perturbans
2	PEE	25	2008-06-17	Y08P2	43.735	-79.7817	2	0	0	0	0	0	50
3	PEE	25	2008-06-17	Y08Q1	43.675	-79.6915	1	0	0	0	0	0	0
4	PEE	25	2008-06-17	Y08O1	43.659	-79.743	1	0	0	2	0	0	0
5	PEE	25	2008-06-17	Y08P1	43.642	-79.7958	3	0	0	1	0	0	0
6	PEE	25	2008-06-17	Y08H1	43.552	-79.6721	4	0	0	2	0	0	7
7	PEE	25	2008-06-17	Y08Z1	43.61	-79.7197	0	0	0	0	0	0	0
8	PEE	25	2008-06-17	Y08D1	43.603	-79.6376	0	0	0	1	0	0	0
9	PEE	25	2008-06-17	Y08G1	43.553	-79.6423	8	0	0	0	0	0	0
10	PEE	25	2008-06-17	Y08H2	43.537	-79.7127	0	0	0	0	0	0	0
11	PEE	25	2008-06-17	Y08J2	43.598	-79.7625	1	0	0	0	0	0	0
12	PEE	25	2008-06-17	Y08A2	43.58	-79.5776	1	0	0	3	0	0	0

- 42 identified mosquito species in Peel Region Culex pipiens/restuans adult mosquito, primary WNV vector in Canada
- Surveillance period: middle of June to the end of September (Week 25-39)



Mosquito abundance from 2009 to 2012 in Peel Region
Number of mosquitoes
1400
1200
1000
800
600
400
200
0
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
Week
2009
2010
2011
2012

1.4 Prevention and Control:

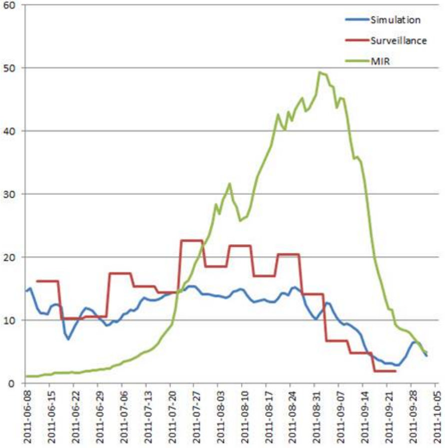
Public Health Agency of Canada / Agence de santé publique du Canada

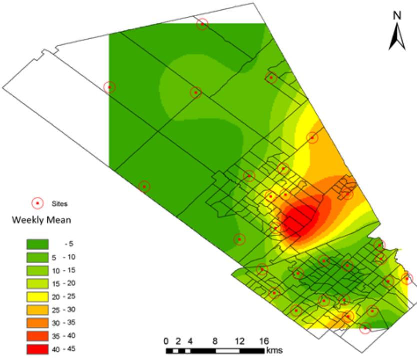
Forecasting and Decision making:

- Mosquito abundance and distribution
- West Nile virus risk, Minimum Infection Rate (MIR)
- Hot spot detection
- Short and long term

CIHR IRSC
Canadian Institutes of Health Research / Instituts de recherche en santé du Canada
Public Health Agency of Canada / Agence de la santé publique du Canada
Ontario
NSERC CRSNG
INNOVATION.CA
CANADA FOUNDATION FOR INNOVATION | FONDATION CANADIENNE POUR L'INNOVATION

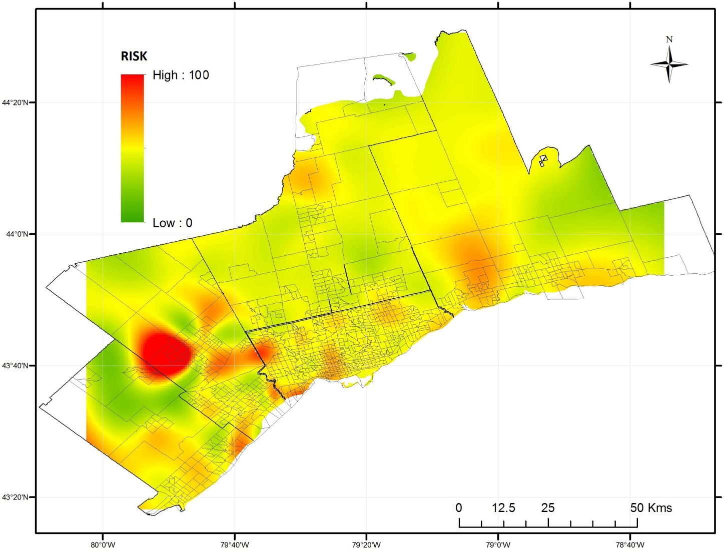
1.5 Weekly forecasting of mosquito-abundance in GTA since 2011





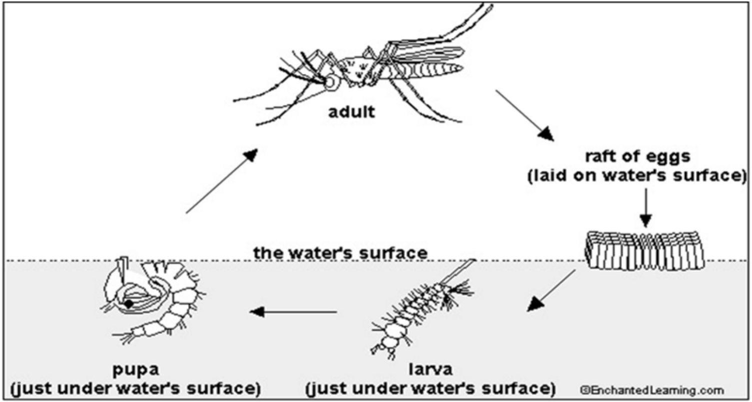
http://www.lamps.yorku.ca/

1.6 Weekly forecasting of WNV risk in GTA



2. Dynamical modeling and triggering mechanisms of an outbreak

2.1 Life cycle of culex mosquitoes



Epidemic models for vector-borne diseases

- Vector population (mosquitoes):
  susceptible S_v, exposed E_v and infected I_v,
- Host population (birds or humans):
  susceptible S_h, exposed E_h infected I_h and recovered R_h

State variables	Vector	Host
Susceptible	S_v	S_h
Exposed	E_v	E_h
Infectious	I_v	I_h
Recovered		R_h
Total	N_v	N_h

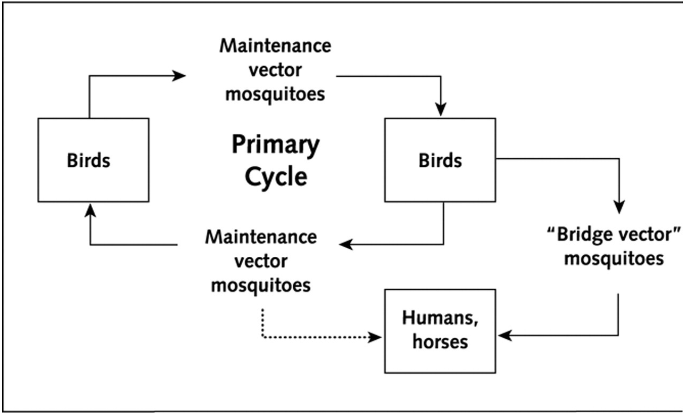
Table : State variables modeling the transmission dynamics of VBDs

Modeling studies of the transmission dynamics of WNv

- Wonham, de Camino-Beck Lewis (2004). An epidemiological model for WNV: Invasion analysis and control applications.
- Bowman, Gumel, van den Driessche, Wu and Zhu (2005). Model for assessing control strategies for WNV. vir
- Cruz-Pacheco, Esteva, Montao-Hirose, Vargas et al. (2005) Modelling the dynamics of WNV.
- Lewis, Renclawowicz, van den Driessche (2006). Traveling waves and spread rates for WNV
- Liu, Shuai, Wu and Zhu (2006). Modeling spatial spread of WNV: directional dispersal of birds.
- Fan, van den Driessche, Wu and Zhu (2010). Maturation delay of mosquitoes and transmission of WNV
- Abdelrazec, Lenhart and Zhu (2013). Transmission of WNV with Corvids and Non-Corvids.
- Abdelrazec, Lenhart and Zhu (2015): Transmission of WNV and seasonality.
- Abdelrazec, Belair, Shan and Zhu (2015): Control of dengue considering impact of public health resource.
- Fan and Zhu (2016). Temperature and transmission dynamics for WNV
- Lin and Zhu (2016). Spatial spreading model and dynamics of WNV with free boundary.

2.2 Compartmental models for transmission of WNV

Abdelrazec, Suzanne and Zhu (2014). Transmission dynamics of West Nile virus in mosquitoes and corvids and non-corvids. Journal of Mathematical Biology.



Modeling of mosquito population

Consider two stages of mosquito development model:

- Aquatic stage (larval L)
- Adult mosquitoes (M)

Then

$$\begin{cases} \dfrac{dL}{dt} = \gamma_m(M) - (m_L + d_L)L, \\ \dfrac{dM}{dt} = m_L L - d_m M. \end{cases} \tag{1}$$

Egg laying function $\gamma_m(M)$:

- recruitment rate $\gamma_m(M) = constant$
- linear reproduction $\gamma_m(M) = r_m M$.
  Assume that all parameters satisfy $r_m m_L = d_m(m_L + d_L)$.

$$
\left\{
\begin{aligned}
\frac{dM_s}{dt} &= (r_m M_s + (1-q) r_m M_i)\left(1 - \frac{N_m}{K_m}\right) - d_m M_s - \beta_m b_m \frac{B_{1i}+B_{2i}}{N_b+A} M_s, \\
\frac{dM_i}{dt} &= q r_m M_i \left(1 - \frac{N_m}{K_m}\right) - d_m M_i + \beta_m b_m \frac{B_{1i}+B_{2i}}{N_b+A} M_s, \\
\frac{dB_{1s}}{dt} &= \gamma_{b1} - d_b B_{1s} - \beta_b b_m \frac{B_{1s}}{N_b+A} M_i, \\
\frac{dB_{1i}}{dt} &= -(d_b + \mu_1 + \nu_1) B_{1i} + \beta_b b_m \frac{B_{1s}}{N_b+A} M_i, \\
\frac{dB_{1r}}{dt} &= -d_b B_{1r} + \nu_1 B_{1i}, \\
\frac{dB_{2s}}{dt} &= \gamma_{b2} - d_b B_{2s} - \beta_b b_m \frac{B_{2s}}{N_b+A} M_i, \\
\frac{dB_{2i}}{dt} &= -(d_b + \mu_2 + \nu_2) B_{2i} + \beta_b b_m \frac{B_{2s}}{N_b+A} M_i, \\
\frac{dB_{2r}}{dt} &= -d_b B_{2r} + \nu_2 B_{2i},
\end{aligned}
\right. \tag{2}
$$

Total number of mosquitoes N_m satisfies

$$\frac{dN_m}{dt} = r_m N_m \left(1 - \frac{N_m}{K_m}\right) - d_m N_m. \tag{3}$$

For any given positive initial condition, the total number of mosquitoes approaches a steady equilibrium $\tilde{M} = (1 - \frac{d_m}{r_m})K_m$ if $d_m < r_m$.

For the two species of birds, their totals satisfy

$$\frac{dN_{bj}}{dt} = \gamma_{bj} - d_b N_{b_j} - \mu_i B_{ji}, \quad j = 1, 2, \tag{4}$$

respectively. If there is no virus involved ($B_{ji} = 0$), the total populations of corvids and non-corvids will approach a constant respectively.

2.3 Dynamics of the model

Equilibria and Reproduction Number

The full model has two disease free equilibrium (DFE) points: $E_0 = (0, 0, \tilde{B}_1, 0, 0, \tilde{B}_2, 0, 0)$ and $E_1 = (\tilde{M}, 0, \tilde{B}_1, 0, 0, \tilde{B}_2, 0, 0)$.

DFE E_0 is a hyperbolic saddle.

The local stability of E_1 is governed by the basic reproduction number R_0.

Next generation matrix method leads to

$$R_0 = \frac{q}{2} + \frac{1}{2}\sqrt{q^2 + 4\Re^2}. \tag{5}$$

where

$$\Re = \sqrt{\beta_m \beta_b b_m^2 \frac{\tilde{M}}{d_m \tilde{B}^2} \left(\frac{\tilde{B}_1}{\delta_1} + \frac{\tilde{B}_2}{\delta_2} \right)}. \tag{6}$$

Backward bifurcation and subthreshold condition for R_0:

$$R_0^1 = \frac{q + \sqrt{q^2 + \frac{((1-q)d_m+k)^2}{kd_m}\left(\frac{4k(1-q)d_m}{((1-q)d_m+k)^2} - \left(1 - \frac{(1-q)d_m-k}{(1-q)d_m+k}\frac{\beta_b b_m}{d_b}\frac{\tilde{M}}{\tilde{B}}\right)^2\right)}}{2}. \quad (7)$$

Thus, the backward bifurcation scenario involves the existence of a subcritical transcritical bifurcation at $R_0 = 1$ and of a saddle-node bifurcation at $R_0 = R_0^1 < 1$.

Backward bifurcation and subthreshold condition for R_0:

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Thus, the backward bifurcation scenario involves the existence of a subcritical transcritical bifurcation at $R_0 = 1$ and of a saddle-node bifurcation at $R_0 = R_0^1 < 1$.

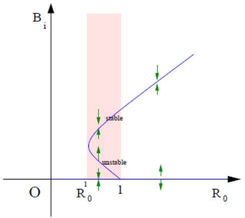
Theorem

For the full model with positive parameters. If

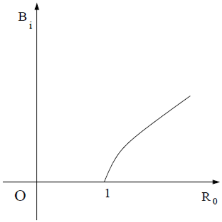
$$A < \left(\mu_1 - (\nu_1 + d_b(1 + \frac{\beta_m b_m}{(1-q)d_m}))\right)\frac{\tilde{B_1}}{\delta_1} + \left(\mu_2 - (\nu_2 + d_b(1 + \frac{\beta_m b_m}{(1-q)d_m}))\right)\frac{\tilde{B_2}}{\delta_2}, \tag{9}$$

then the model undergoes a backward bifurcation when $R_0 = 1$.

The bifurcation diagrams at $R_0 = 1$.



(a) Backward bifurcation.



(b) Forward bifurcation.

2.4 Conclusion remarks

- In order to induce an outbreak,
  - $R_0 > 1$: enough number of mosquitoes over the amplification host (birds);
  - When $R_0 < 1$: Backward bifurcation: enough vector-mosquitoes, enough number of host birds with higher mortality rate (American crows: corvids).
- Ross 1911 (A simple model): To control malaria, control and reduce the vector mosquitoes.

Risk assessment of public health units: MIR

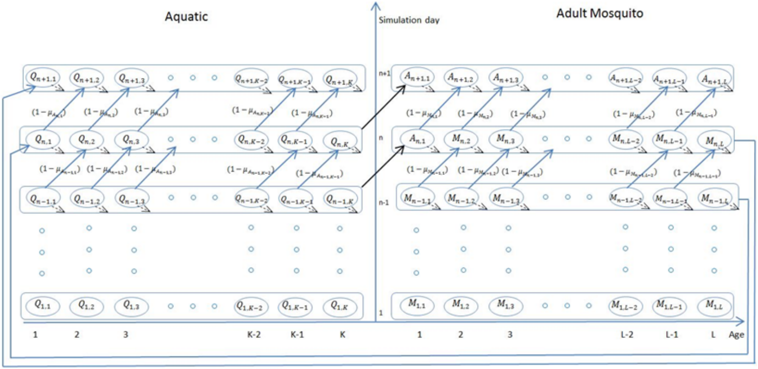
Minimum Infection Rate:

$$MIR = \frac{Number\ \ of\ \ positive\ \ pools}{number\ \ of\ \ tested\ \ mosquitoes} \times 1000.$$

2.5 A simulation model considering stage and age with daily temperature and precipitation

Flow diagram with age structure

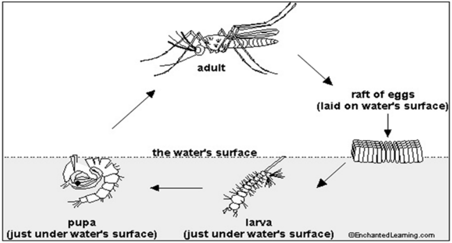
of mosquito eggs and adult mosquitoes



Vertical: time, day; Horizontal: age

3. Modeling the impact of temperature on the transmission of WNV

Life cycle of mosquito species (culex mosquitoes)



Assuming that the daily temperature of the season remains a constant, then the maturation (development) delay of the mosquitoes:

$$\tau = \frac{Total\ accummulative\ degree\ day\ temperature\ required}{daily\ average\ temperature}$$

3.1 Model for WNV with delay for mosquitoes

Fan, van den Driessche, Wu and Zhu (2010) The impact of maturation delay of mosquitoes on the transmission of West Nile virus.

Fan, Shan and Zhu (2015): Oscillation and driving mechanism in a model of West Nile virus with time delay.

$$
\begin{cases}
\dfrac{dM_s(t)}{dt} &= r_m M_s(t-\tau)e^{-d_j\tau}e^{-\alpha N_M(t-\tau)} - \beta_m\kappa\dfrac{M_s(t)B_i(t)}{N_b(t)} \\
& \quad +(1-q)r_m M_i(t-\tau)e^{-d_j\tau}e^{-\alpha N_M(t-\tau)} - d_m M_s(t), \\
\dfrac{dM_i(t)}{dt} &= q r_m M_i(t-\tau)e^{-d_j\tau}e^{-\alpha N_m(t-\tau)} - d_m M_i(t) \\
& \quad +\beta_m\dfrac{M_s(t)\kappa B_i(t)}{N_b(t)}, \\
\dfrac{\mathrm{d}B_s(t)}{\mathrm{d}t} &= r_b - \frac{\kappa\beta_b M_i(t)B_s(t)}{N_b(t)} - d_b B_s(t), \\
\dfrac{\mathrm{d}B_i(t)}{\mathrm{d}t} &= \frac{\kappa\beta_b M_i(t)B_s(t)}{N_b(t)} - (\mu+\nu+d_b)B_i(t), \\
\dfrac{\mathrm{d}B_r(t)}{\mathrm{d}t} &= (\mu+\nu)B_i(t) - d_b B_r(t),
\end{cases}
$$

Total mosquitoes $N_m = M_s + M_i$

$$\frac{dN_m(t)}{dt} = r_m N_m(t-\tau)e^{-d_j\tau}e^{-\alpha N_m(t-\tau)} - d_m N_m(t).$$

Then the full model becomes

$$\begin{cases} \dfrac{dN_m(t)}{dt} = r_m N_m(t-\tau)e^{-d_j\tau}e^{-\alpha N_m(t-\tau)} - d_m N_m(t), \\ \dfrac{dM_i(t)}{dt} = q r_m M_i(t-\tau)e^{-d_j\tau}e^{-\alpha N_m(t-\tau)} - d_m M_i(t) \\ \qquad + \beta_m \frac{(N_m - M_i(t))\kappa B_i(t)}{N_b(t)}, \\ \dfrac{\mathrm{d}B_s(t)}{\mathrm{d}t} = r_b - \frac{\kappa\beta_b M_i(t) B_s(t)}{N_b(t)} - d_b B_s(t), \\ \dfrac{\mathrm{d}B_i(t)}{\mathrm{d}t} = \frac{\kappa\beta_b M_i(t) B_s(t)}{N_b(t)} - (\mu + \nu + d_b)B_i(t), \\ \dfrac{\mathrm{d}N_b(t)}{\mathrm{d}t} = r_b - \mu B_i(t) - d_b N_b(t). \end{cases} \tag{10}$$

Consider the equation for total number of vector mosquitoes

$$\frac{dN_m(t)}{dt} = r_m N_m(t-\tau) e^{-d_j \tau} e^{-\alpha N_m(t-\tau)} - d_m N_m(t). \qquad (11)$$

- Without survival term $e^{-d_j \tau}$:
  (Smith 1995, Hu and Yuan 2003, and Chen 2003, Wei and Li 2005)
- With survival term, Hopf bifurcation occurs (Cooke et al. 1999)

If define

$$\tau_1 = \big(\ln(r_m/d_m) - 2\big)/d_j.$$

Unique positive equilibrium N_m^* which is locally stable for $\tau < \tau_1$. The characteristic equation at N_m^*

$$\mathcal{F}_1(\lambda) = -d_m - \lambda + d_m(1 - \alpha N_m^*)e^{-(\lambda + d_j)\tau}$$

Define

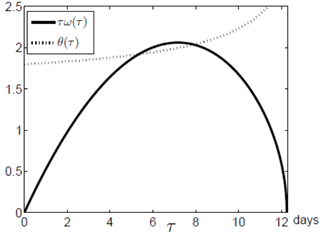
$$\omega(\tau) = d_m \sqrt{\big(\ln(r_m/d_m) - d_j\tau\big)\big(\ln(r_m/d_m) - d_j\tau - 2\big)},$$
$$\theta(\tau) = \arccos\left((1 + d_j\tau - \ln(r_m/d_m))^{-1}\right).$$

Lemma

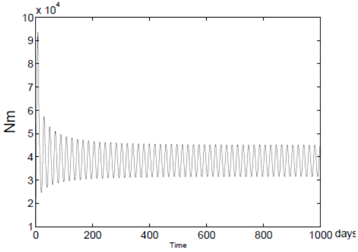
Consider equation (11). Assume $r_m > e^2 d_m$ *so that* $\tau_1 > 0$*. If function* $\theta(\tau)$ *and* $\tau\omega(\tau)$ *has an intersection at* $\tau^* \in (0, \tau_1]$*, and* $(\tau\omega(\tau^*))' \neq \theta'(\tau^*)$*, equation* $\mathcal{F}_1(\lambda) = 0$ *has a pair of simple pure imaginary roots* $\lambda = \pm i\omega(\tau^*)$ *crossing the imaginary axis. Equation (11) has a Hopf bifurcation at* $\tau = \tau^*$ *and a small amplitude periodic solution bifurcated from equilibrium* N_m^**.*

If $F_1(\lambda) = 0$ has a pair of pure imaginary roots crossing, the system (10) can have a Hopf bifurcation.

Intersections of $\theta(\tau)$ and $\tau\omega(\tau)$ indicate where Hopf bifurcations occur. A stable periodic solution for the mosquito population.



(a) Plot of $\theta(\tau)$ and $\tau\omega(\tau)$.



(b) A periodic solution for N_m.

3.2 Oscillation of the full model due to delay

For the positive equilibrium, the characteristic equation,

$$\mathcal{F}_1(\lambda)\mathcal{F}_2(\lambda) = 0 \tag{12}$$

where

$$\begin{aligned} \mathcal{F}_2(\lambda) = \quad & (d_b + \lambda)\big[\lambda^3 + (A + D)\lambda^2 + (F + AD - B)\lambda \\ & + AF - BC - qd_m e^{-\lambda\tau}(\lambda^2 + D\lambda + F)\big] \end{aligned} \tag{13}$$

and

$$\begin{aligned} A &= d_m + \frac{\beta_m \kappa B_{i2}}{N_{b2}}, \\ B &= \frac{\kappa^2 \beta_m \beta_b B_{s2}(N_m^* - M_{i2})}{N_{b2}^2}, \\ C &= r_b / N_{b2}, \\ D &= \mu + \nu + d_b + r_b / B_{s2}, \\ F &= \frac{\mu + \nu + d_b}{B_{s2} N_{b2} d_b}(r_b^2 - \mu(\mu + \nu + d_b)B_{i2}^2), \end{aligned} \tag{14}$$

Lemma

Assume that $q = 0$. All roots of $\mathcal{F}_2(\lambda) = 0$ have negative real parts.

Theorem

Assume that $0 < q << 1$, equation $\mathcal{F}_2(\lambda) = 0$ admits no pure imaginary root.

3.3 Simulations

Parameters

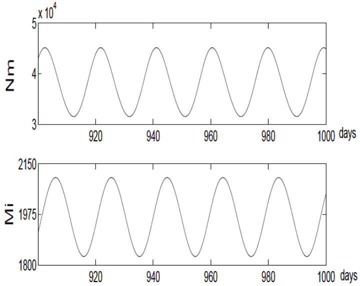
For mosquitoes: $r_m = 30$, $d_m = 0.13$, $d_j = 0.281$, and $\alpha = 0.0001$.

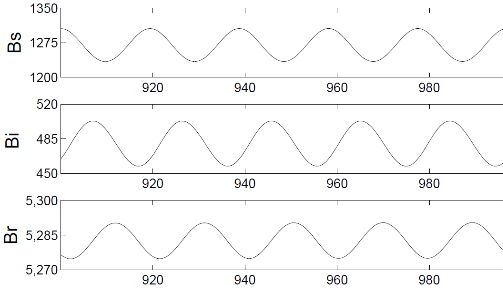
For such choice of parameters, we plot $\theta(\tau)$ and $\tau\omega(\tau)$ as delay varies.

The figure shows that there exist two intersections at $\tau \approx 5.32$ and $\tau \approx 7.89$ days. By Theorem 2, we obtain that the mosquito population undergoes a Hopf bifurcation at $\tau \approx 5.32$ and $\tau \approx 7.89$ days. For any τ in between, numerical simulations show that there exists a stable periodic solution (see Fig. for $\tau = 6$ days) with an approximate period 20 days

The full model with periodic solutions forced by mosquito population.

Take $\mu = 0.27,\ \nu = 0.11,\ r_b = 200,\ d_b = 0.01$ for birds and $\beta_m = 0.16,\ \beta_b = 0.88,\ \kappa = 0.6,\ q = 0.007,\ \tau = 6\ days.$ But $\mathcal{F}_2(\lambda) = 0$ has no pure imaginary roots.





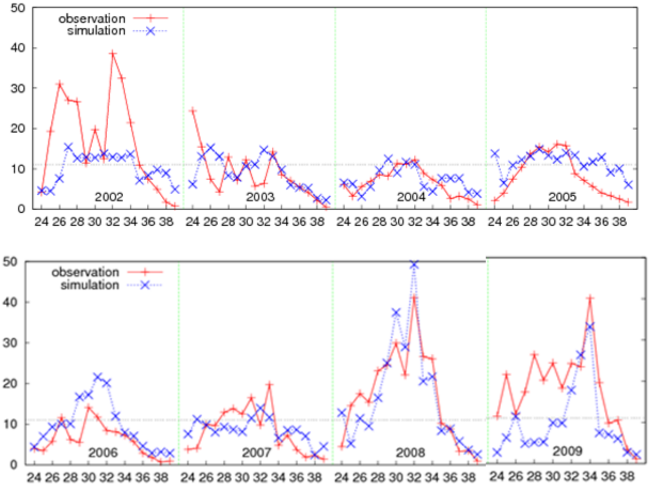
Conclusion: *The incidence interaction between vector-hosts does not generate oscillations for the endemic, BUT the oscillation is driven by the mosquitoes (environment).*
Abdelrazec, Lenhart and Zhu (2015): Dynamics and Optimal Control of a West Nile Virus Model with Seasonality.

3.4 Driven mechanisms for the recurrence in MBDs/VBDs

How about we change the demographics for mosquitoes? [Jiang, Li, Zhu 2015]

$$
\begin{cases}
\dfrac{dM_s}{dt} &= r_m\Big(M_s(t-\tau)+(1-q)M_i(t-\tau)\Big)\left(1-\dfrac{N_m}{K_m}\right) \\
&\quad -b_1\beta_m\dfrac{M_sB_i}{N_b}-d_mM_s, \\
\dfrac{dM_i}{dt} &= qr_mM_i(t-\tau)\left(1-\dfrac{N_m}{K_m}\right)+b_1\beta_m\dfrac{M_sB_i}{N_b}-d_mM_i, \\
\dfrac{dB_s}{dt} &= r_b-b_1\beta_b\dfrac{M_iB_s}{N_b}-d_bB_s, \\
\dfrac{dB_i}{dt} &= b_1\beta_b\dfrac{M_iB_s}{N_b}-(d_b+d_i+\nu)B_i, \\
\dfrac{dB_r}{dt} &= \nu B_i-d_bB_r.
\end{cases}
\tag{15}
$$

3.5 A simulation model considering stage and age with daily temperature and precipitation Wang, Ogden and Zhu (2011). The impact of weather conditions on Culex pipiens and Culex restuans (Diptera: Culicidae) abundance: A case study in Peel region. Journal of Medical Entomology.



Wang, J., N.H. Ogden, and H. Zhu, The Impact of Weather Conditions on the abundance of culex mosquitoes. *J. of Medical Entomology* 2010

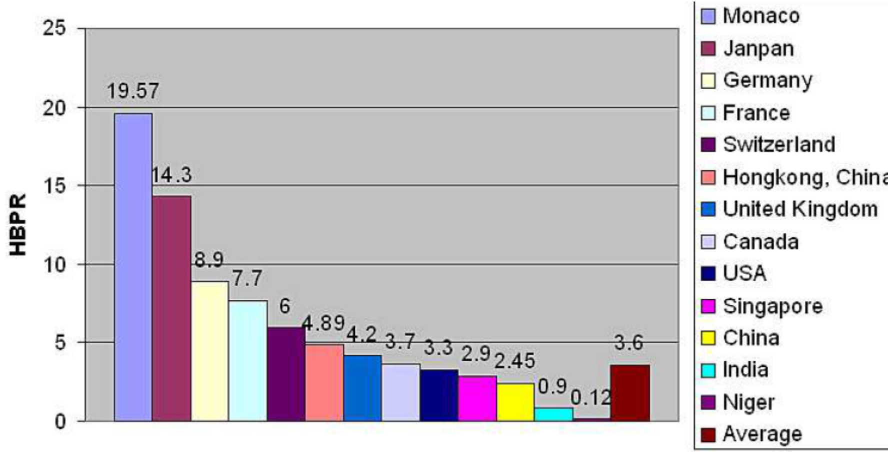
4. Fast-slow dynamics of mosquito-borne diseases

4.1 Model for Dengue with limited health resources The driven mechanisms for the oscillation in mosquito-bonne diseases (WNV, dengue, malaria): always due to vector-mosquitoes?

Abdelrazec, Belair, Shan, Zhu (2015): Modeling the Spread and Control of Dengue with Limited Public Health Resources.

Hospital bed-population ratio (HBPR)

Number of in-patient beds available per 10,000 people in the population served by the hospital.

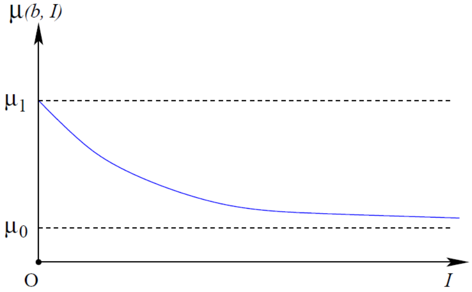

25
20
15
10
5
0
HBPR
19.57
14.3
8.9
7.7
6
4.89
4.2
3.7
3.3
2.9
2.45
0.9
0.12
3.6
Monaco
Janpan
Germany
France
Switzerland
Hongkong, China
United Kingdom
Canada
USA
Singapore
China
India
Niger
Average

For the recover rate μ, consider

- $\mu(b, I) > 0$ and $\mu(b, 0) = \mu_1 > 0$. and μ_1 is the maximum recovered rate.
- $\frac{\partial \mu(b,I)}{\partial I} < 0$, $\lim_{I \to \infty} \mu(b, I) = \mu_0$ and $\lim_{I \to 0} \mu(b, I) = \mu_1$. Here μ_0 is the minimum recovered rate due to the limited clinical resources.
- $\frac{\partial \mu(b,I)}{\partial b} > 0$, $\lim_{b \to \infty} \mu(b, I) = \mu_1$ and $\lim_{b \to 0} \mu(b, I) = \mu_0$.

We will take

$$\mu = \mu(b, I) = \mu_0 + (\mu_1 - \mu_0)\frac{b}{I + b},$$



DF model with nonlinear recovery rate

$$
\begin{cases}
\dfrac{dS}{dt} & = & d_h N - d_h S - \beta_h \dfrac{SM_i}{N}, \\
\dfrac{dI}{dt} & = & \beta_h \dfrac{SM_i}{N} - d_h I - \mu(b, I)I, \\
\dfrac{dR}{dt} & = & \mu(b, I)I - d_h R, \\
\dfrac{dL_s}{dt} & = & r(M_s + (1-q)M_i) - (m_l + d_l)L_s, \\
\dfrac{dL_i}{dt} & = & rqM_i - (m_l + d_l)L_i, \\
\dfrac{dM_s}{dt} & = & m_l L_s - d_m M_s - \beta_m \dfrac{IM_s}{N}, \\
\dfrac{dM_i}{dt} & = & m_l L_i + \beta_m \dfrac{IM_s}{N} - d_m M_i,
\end{cases}
$$

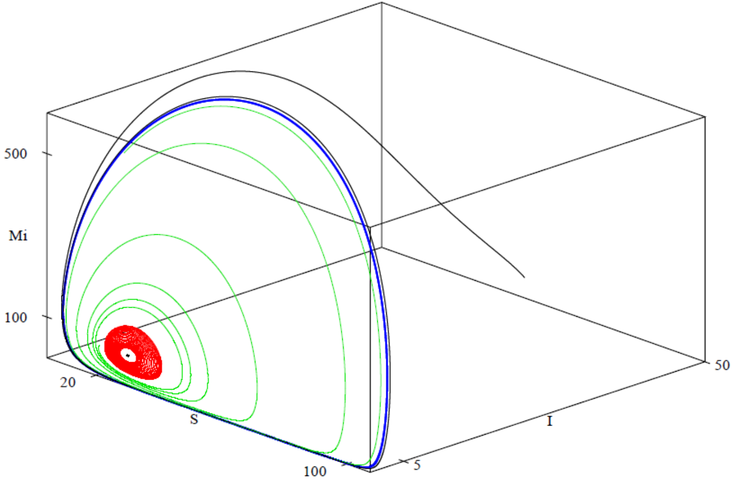
A simplified model for DF

Assuming the vector-mosquito population remains a constant:

$$\begin{cases} \dfrac{dS}{dt} & = & d_h N - d_h S - \beta_h \dfrac{SM_i}{N}, \\ \dfrac{dI}{dt} & = & \beta_h \dfrac{SM_i}{N} - d_h I - \mu(b, I)I, \\ \dfrac{dL_i}{dt} & = & rqM_i - (m_l + d_l)L_i, \\ \dfrac{dM_i}{dt} & = & m_l L_i + \beta_m \dfrac{I(M - M_i)}{N} - d_m M_i, \end{cases}$$

where N, L, M are constants and $R = N - S - I$.

4.2 Some interesting observation



Huzak and Zhu: Fast-slow dynamics and bifurcation in mosquito-borne diseases.

Thank you

for your attention!