

Mark Spivakovsky: Popescu's theorem and (nested) Artin approximation.

Popescu's theorem can be viewed as an analogue for rings of Lazard's theorem for modules: any flat module over a commutative ring A is a filtered direct limit of free finitely generated A -modules. Flat modules are acyclic objects for Tor , they are characterized by the vanishing of the higher Tor functors. The analogue of Tor functors for algebras are the André homology modules, whose vanishing characterizes regular ring homomorphisms. Popescu's theorem says that given a regular ring homomorphism $A \rightarrow B$, the A -algebra B is a filtered direct limit of smooth A -algebras of finite type.

Popescu's theorem easily implies the Artin approximation theorem in the strongest possible form as well as nested Artin approximation. It has other applications such as the Bass-Quillen conjecture under some restrictions.

This talk is meant for a fairly broad audience, so an effort will be made to make it as self-contained as possible. All the definitions will be given from scratch.