Leonard Lipshitz: Artin Approximation, recursion relations, decidability and definability in local rings and fields.

Approximation theorems for henselian local rings often yield algorithms for deciding whether a system of equations has a solution. It is natural to ask when there is an algorithm for deciding whether a system of equations and inequalities has a solution. More generally one can ask whether, at least in special cases (for example in one dimension, or even for $\mathbb{Q}_p$ or $\mathbb{C}_p$), there a simple description of all the first order definable sets (in the algebraic case the semi-algebraic sets, and in the analytic case the subanalytic sets). I will give a survey of results on these and related topics.