Group Representations in Dynamical Systems and Geometry
29 June - 3 July, 2015

SHORT COURSES

Amos Nevo: Representation theory, effective ergodic theorems, and applications

Our first purpose is to show how aspects of the representation theory of (non-amenable) algebraic groups can be utilized to derive effective ergodic theorems for their actions. Our second purpose is to demonstrate some the many interesting applications that ergodic theorems with a rate of convergence have in a variety of problems. We will start by a discussion of property T and show how to extend the spectral estimates it provides considerably beyond their usual formulations. We will also show how to derive best possible spectral estimates via representation theory in some cases. In turn, such spectral estimates will be used to derive effective ergodic theorems. Finally we will show how the rate of convergence in the ergodic theorem implies effective solutions in a host of natural problems, including the non-Euclidean lattice point counting problem, fast equidistribution of lattice orbits on homogenous spaces, and best possible exponents of Diophantine approximation on homogeneous algebraic varieties.

Alain Valette: Groups with the Haagerup property (4 lectures).

A locally compact group has the Haagerup property (or: is a-(T)-menable) if it admits a proper affine isometric action on a Hilbert space. The Haagerup property is a weak form of amenability: the class of Haagerup groups contains amenable groups, but also free groups, Coxeter groups, closed subgroups of SO(n,1) and SU(n,1), etc... Quoting from the Wikipedia page: "The Haagerup property is interesting to many fields of mathematics, including harmonic analysis, representation theory, operator K-theory, and geometric group theory."

Here is a rough outline of the four lectures:

1) Unitary representations vs affine isometric actions; definitions and first examples.
2) Geometric characterizations; more examples.
3) Why do we care? A glimpse into the Baum-Connes conjecture.
4) Permanence properties of the class of Haagerup groups.

SPEAKERS

Javier Aramayona: The abelianization of automorphism groups of right-angled Artin groups.

Automorphism groups of right-angled Artin groups form an interesting class of groups, as they “interpolate" between the two extremal cases of (F_n) and $GL(n, \mathbb{Z})$. In this talk we will discuss some conditions on a simplicial graph G which imply that the automorphism group of the associated right-angled Artin group has (in)finite abelianization. As a direct consequence, we obtain families of such automorphism groups that do not have Kazhdan’s property (T). This is joint work with Conchita Martinez-Perez.

Philippe Biane : Representations of symmetric groups and free cumulants.

I will give an overview of the work on asymptotics of characters of symmetric groups related to free probability theory and leading to new exact formulas for characters.

Paul Bourgade: Local quantum unique ergodicity for random matrices

For generalized Wigner matrices, I will explain a probabilistic version of quantum unique ergodicity at any scale, and gaussianity of the eigenvectors entries. The proof relies on analyzing the effect of the Dyson Brownian motion on eigenstates. Relaxation to equilibrium of the eigenvectors is related to a new multi-particle random walk in a random environment, the eigenvector moment flow. This is joint work with H.-T. Yau.

Reda Chhaibi: A probabilistic approach to Whittaker functions, in both Archimedean and non-Archimedean cases

Following Hervé Jacquet, Whittaker functions are matrix coefficients in the principal series representations. By relating them to certain random walks on the group, we provide Poisson kernel formulas which can be explicitly computed. This probabilistic approach allows a unified point of view, whether the group is taken with Archimedean or non-Archimedean coefficients

Michael Cowling: On matrix coefficients of unitary representations of semisimple Lie groups

An action of a semisimple Lie group G on a measure space X gives rise to a unitary representation π of G on $L^2(X)$. It is of interest to know which irreducible unitary representations of G appear in the decompostion of π. It is sometimes relatively easy to find some information about the behaviour of the matrix coefficents $\langle \pi(\cdot)\xi, \eta\rangle$, where ξ and η lie in a dense subset of $L^2(X)$. The problem is then to translate this into information about the decomposition of π. This talk describes what I know about this problem.

Rostislav Grigorchuk: Random subgroups, totally non free actions and factor representations

I will present results of three studies, performed in collaboration with M.Benli, L.Bowen, A.Dudko, R.Kravchenko and T.Nagnibeda, concerning the invariant and characteristic random subgroups in some groups of geometric origin, including hyperbolic groups, mapping class groups, groups of intermediate growth and branch groups. The role of totally non free actions will be emphasized. This will be used to explain why branch groups have infinitely many factor representations of type II_1.

Vadim Kaimanovich : Stationarity and stationarizing.

In the absence of an invariant probability measure a natural replacement is a measure invariant with respect to a single operator related to the structure of the state space. Such constructions are, for instance, known for group actions (Furstenberg's μ-stationary measures) and for foliations (Garnett's harmonic measures). Quasi-regular representations with respect to stationary measures play an important role in the representation theory. The opposite problem is to describe all probability measures μ on the group which make a given measure on the action space μ-stationary (or, at least, to find some of them). We shall describe a procedure which allows one to obtain a lot of measures which have the same Poisson boundary (and in particular, stationarize the same harmonic measure on the Poisson boundary), and show that in certain situations which procedure is exhaustive.

Igor Krasovsky: Asymptotic behaviour of a sine-kernel determinant which appears in the theory of random matrices and log-gases.

We consider the Fredholm determinant $\det(I - aK(x,y))$ on an interval of length 's', where $K(x,y) = (Sin(x-y))/(Pi(x-y))and0 < a < 1$. This determinant arises in the theory of log-gases and in random matrix theory. Its asymptotics for large 's' and fixed 'a' have been known previously. There is a great difference in the asymptotic behaviour for a<1 and a=1. The aim of the talk will be to present recent joint results of the speaker with T. Bothner, P. Deift, and A.Its on the description of the transition between a<1 and a=1. The transition, first considered by Dyson, involves oscillatory behaviour via theta-functions.

Julien Marché: Dynamics of the mapping class group in character varieties of genus 2 surfaces.

The following question was popularized by Goldman: given a genus g surface S and a group G, the mapping class group Mod(S) acts on the character variety $X(S,G)$ of conjugacy classes of representations of the fundamental group of S into G. When G is compact, Goldman and Xia-Pickrell showed that the action is ergodic whereas when $G = PSL_2(R)$, some component of $X(S,G)$ is the Teichmuller space, and the action is proper and discontinuous. In a work with M. Wolff we prove that the action on the remaining components is ergodic. We will prove one case with techniques mixing hyperbolic and symplectic geometry.

Sevak Mkrtchyan: The entropy of Schur-Weyl measures

Relative dimensions of the isotypic components of the N-th order tensor representations of the symmetric group on n letters give rise to a Plancherel-type measure, called the Schur- Weyl measure, on the space of Young diagrams with n cells and at most N rows. We obtain logarithmic, order-sharp bounds for the maximal dimensions of the isotypic components of the tensor representations, and prove that the typical dimensions, after appropriate normalization, converge to a constant with respect to the Schur-Weyl measures.

Tomoyuki Shirai : Persistent homology and minimum spanning acycle for random simplicial complexes.

Persistent homology appeared around 2000 as an algebraic method which measures topological features of objects or point cloud data. Recently, much attention has been paid to it in the context of Topological Data Analysis. Persistent homology theory is, roughly speaking, a time-dependent version of homology theory or homology theory for time-dependent objects. The 0th homology describes the situation of connected components of objects. It has been studied for random objects in detail in probability theory, especially, percolation theory, random geometric graph theory and study of the Erdös–Rényi random graph. So we can expect that analysis of higher-dimensional homology and also persistent homology for random objects might give us hints of a natural generalization of such theories mentioned above. In this talk, we would like to discuss the relationship between persistent homology and minimum spanning acycle in simplicial complexes, and also apply it to study certain random simplicial complex processes, especially the Linial-Meshulam simplicial complex process, which is one of the natural higher dimensional generalizations of the Erdös–Rényi random graph process. This talk is based on a joint work with Yasuaki Hiraoka (AIMR, Tohoku Univ.)

Tim Steger: On Certain Tempered Unitary Representations of Hyperbolic Groups
TBA

Benjamin Weiss: On weak mixing properties for non-singular actions

For a probability measure preserving action of a locally compact group G, there are various characterizations of weak mixing. Analogous definitions can be given when the action merely preserves the measure class and I will discuss some implications that hold between these properties (based on joint work with Eli Glasner).

Yasushi Yamashita: The geometric and dynamical decompositions of the character veriety of the free group of rank two

Let X be the $SL(2, C)$ character variety of the free group F_2 of rank two. We can decompose X "geometrically" into two parts by asking whether the corresponding representation is discrete or not. On the other hand, Minsky proposed a "dynamical" decomposition of X using the OUT(F_2) action on X. In this talk, we present computer algorithms to study these decompositions and compare them computationally.