

# ANALYSIS AND GEOMETRY OF RESONANCES / ANALYSE ET GÉOMÉTRIE DES RÉSONANCES

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## ABSTRACTS

- **Viviane Baladi.** *The spectrum of Sinai billiard flows*  
(Joint work with M. Demers and C. Liverani)

Sinai billiard maps in dimension two have been proved to be exponentially mixing by L.-S. Young twenty years ago. Recent work of Demers and Zhang gives a spectral gap for their natural transfer operators on a suitable anisotropic Banach space. Understanding Sinai billiard flows is more difficult. I will present recent results on the spectrum of the generator of the semi-group of their transfer operators on an anisotropic space.

- **Yannick Bonthonneau.** *Resonances-free regions for cusp manifolds*

I will explain how one can construct a parametrix for the scattering determinant at high frequencies, for cusp manifolds — finite volume, hyperbolic ends, negative curvature. The main consequence is that the resonances are either in a vertical strip near the axis  $\operatorname{Re} s = 1/2$ , or outside of some log region. When the curvature is constant, it has been known since Selberg that all the resonances really are in a vertical strip. However, in variable curvature, a variety of behaviour is possible for the set of resonances that lie outside of the strip, as I propose to show.

- **David Borthwick.** *Symmetry factorization of Selberg zeta functions and distributions of resonances* (Joint work with Tobias Weich)

We discuss a factorization of the Selberg Zeta function that applies to hyperbolic manifolds generated by Schottky groups and possessing discrete symmetries. The factorization results in dramatically improved convergence rates for calculations of resonances. We apply these methods to shed new light on the phenomenon of resonance chains in hyperbolic surfaces, and on the recent essential spectral gap conjecture of Jakobson and Naud.

- **Nicolas Burq.** *Euclidean scattering, resolvent estimates and evolution PDE's.*
- **Gilles Carron.** *Riesz transform on manifolds with quadratic curvature decay.*

We will describe some  $L^p$  soundness of the Riesz transform  $d\Delta^{-1/2}$  on manifolds with quadratic curvature decay. This class of manifolds includes manifolds with conics ends (also called scattering metrics) but there are manifolds with infinite topological type that satisfies this properties.

- **Kiril Datchev.** *Semiclassical resolvent bounds in general trapping situations*

We give a new, simplified proof of Burq's and Cardoso and Vodev's resolvent estimates for scattering on manifolds with boundary. The proof also allows lower regularity on the coefficients of the scattering problem. Globally, the resolvent norm grows exponentially in the inverse semiclassical parameter, and near infinity it grows linearly.

- **Semyon Dyatlov.** *A microlocal toolbox for hyperbolic dynamics.*

I will discuss recent applications of microlocal analysis to the study of hyperbolic flows, including geodesic flows on negatively curved manifolds. The key idea is to view the equation  $(X + \lambda)u = f$ , where  $X$  is the generator of the flow, as a scattering problem. The role of spatial infinity is taken by the infinity in the frequency space. We will concentrate on the case of noncompact manifolds, featuring a delicate interplay between shift to higher frequencies and escaping in the physical space. I will show meromorphic continuation of the resolvent of  $X$ ; the poles, known as Pollicott-Ruelle resonances, describe exponential decay of correlations. As an application, I will prove that the Ruelle zeta function continues meromorphically for flows on non-compact manifolds (the compact case, known as Smale's conjecture, was recently settled by Giulietti-Liverani-Pollicott and a simple microlocal proof was given by Zworski and the speaker). Joint work with Colin Guillarmou.

- **Frédéric Faure** *The asymptotic spectral gap of hyperbolic dynamics. Tentative to improve the estimates.*

Hyperbolic (Anosov or Axiom A) flows have discrete Ruelle spectrum. For contact Anosov flows, e.g. geodesic flows, where a smooth contact one form is preserved, the trapped set is a smooth symplectic manifold, normally hyperbolic, and M. Tsujii, S. Nonnenmacher and M. Zworski, have given an estimate

for the asymptotic spectral gap, i.e. that appears in the limit of high frequencies in the flow direction. We will propose a different approach that may improve this estimate. This will be presented on a simple toy model, partially expanding maps. Work with Tobias Weich.

- **Peter Hintz.** *Asymptotics for the wave equation on differential forms on Kerr-de Sitter space.*

As the simplest model for non-scalar waves on a geometric class of spacetimes including Schwarzschild-de Sitter and Kerr-de Sitter black holes, we study the wave equation on differential forms of any degree, which as a very special case includes Maxwell's equations. We prove that waves decay exponentially in time to stationary, 'resonant' states, and identify the space of resonant states in a canonical way with certain cohomology groups of the underlying spacetime. Combined with a framework developed in a recent paper, this immediately implies the global solvability of suitable quasilinear wave equations on differential forms, and is strongly tied to the black hole stability problem. Joint work with Andras Vasy.

- **Lizhen Ji.** *Geometry and analysis of locally symmetric spaces of infinite volume.*

For any symmetric space  $X$  of noncompact type, its quotients by torsion-free discrete isometry groups  $\Gamma$  are locally symmetric spaces. One problem is to understand the geometry and analysis, especially the spectral theory, and interaction between them of such spaces.

Two classes of infinite groups  $\Gamma$  have been extensively studied:

- (1)  $\Gamma$  is a lattice, and hence  $\Gamma \backslash X$  has finite volume.
- (2)  $X$  is of rank 1, for example, when  $X$  is the real hyperbolic space,  $\Gamma$  is geometrically finite and  $\Gamma \backslash X$  has infinite volume.

When  $\Gamma$  is a nonuniform lattice in case (1) or any group in case (2), compactification of  $\Gamma \backslash X$  and its boundary play an important role in the geometric scattering theory of  $\Gamma \backslash X$ .

When  $X$  is of rank at least 2, quotients of  $X$  of finite volume have also been extensively studied. There has been a lot of recent interest and work to understand quotients  $\Gamma \backslash X$  of infinite volume. For example, there are some generalizations of convex cocompact groups, but no generalizations yet of geometrically finite groups. They are related to the notion of thin groups.

One naturally expects that these locally symmetric spaces should have real analytic compactifications with corners (with codimension equal to the rank),

and their boundary should also be used to parametrize the continuous spectrum and to understand the geometrically scattering theory. These compactifications also provide a natural class of manifolds with corners. In this talk, I will describe some questions, open problems and results.

- **Roberto Miatello.** *Resolvent and lattice points on symmetric spaces of negative curvature.*

We will describe the meromorphic continuation of the resolvent on symmetric spaces of negative curvature, giving applications to the asymptotics of the counting function of lattice points in such spaces.

- **Werner Müller.** *Resonances for the Laplacian on locally symmetric spaces of finite volume.*

For locally symmetric spaces of finite volume the theory of Eisenstein series provides an explicit description of the spectral resolution of the absolute continuous part of the Laplacian. In the case of arithmetic quotients, the scattering matrices can be expressed in terms special Dirichlet series which generalize Dirichlet L-series. The scattering resonances correspond to zeros of these Dirichlet series. Methods of analytic number theory can be used to study their location and distribution. I will give an overview of results and discuss some problems.

- **Frédéric Naud.** *Nodal lines and domains for Eisenstein series on surfaces.*

Eisenstein series are the natural analog of "plane waves" for hyperbolic manifolds of infinite volume. These non- $L^2$  eigenfunctions of the Laplacian parametrize the continuous spectrum. In this talk we will discuss the structure of nodal sets and domains for surfaces. Upper and lower bounds on the number of intersections of nodal lines with "generic" real analytic curves will be given, together with similar bounds on the number of nodal domains inside the convex core. The results are based on equidistribution theorems for restriction of Eisenstein series to curves that bear some similarity with the so-called "QER" results for compact manifolds.

- **Martin Olbrich.** *Transfer operators, resonances, and group cohomology for hyperbolic manifolds of infinite volume.*

- **Aprameyan Parthasarathy.** *Scattering poles for rank one symmetric spaces.*

Taking a harmonic analytic viewpoint to scattering theory on symmetric spaces in the spirit of Semenov Tian-Shansky, we will discuss the correspondence between scattering poles and the poles of the resolvent of the Laplacian for rank one symmetric spaces. This is joint work with Sanke Hansen and Joachim Hilgert.

- **Peter Perry.** *Scattering Resonances on hyperbolic manifolds as a model of chaotic scattering.*

In this survey talk we will review the basics of scattering theory on geometrically finite, real hyperbolic manifolds. In particular we will discuss the connection of scattering resonances with Selberg's zeta function and known results relating the distribution of resonances to properties of the classical geodesic flow. We will briefly discuss problems and results for the class of asymptotically hyperbolic manifolds introduced by Mazzeo and Melrose.

- **Anke Pohl.** *Transfer operators for Riemann surfaces of finite and infinite area with cuspidal ends.*

Hecke triangle groups form a one-parameter family of Fuchsian groups. This family starts with the modular group  $\mathrm{PSL}(2, \mathbb{Z})$  which has one cusp and two elliptic points, one of which is of order 2, the other one is of order 3. By iteratively increasing the order of the second elliptic point, this point is slowly pulled to infinity until it turns into a second cusp. Then this cusp is opened to form a funnel of increasing width. We will use this family to discuss parallel "slow" and "fast" discretizations for the geodesic flows on the Hecke triangle surfaces as well as the billiard flows on the underlying triangle surfaces. We will see that the fast discretizations serve for thermodynamic formalism approaches to dynamical zeta functions. The transfer operators arising from the slow discretizations however allow (classical dynamical) characterizations of Laplace eigenfunctions in the finite area case. Moreover, these results lead to natural conjectures on resonances and vector-valued automorphic forms.

- **Mark Pollicott.** *Zeta functions, decay of correlations and resonances.*

I will give an exposition of some basic results on the zeros of dynamically defined zeta functions, the Fourier transform of the correlation function for mixing, and associated resonances. I will also try to relate this to some more recent developments.

- **Pablo Ramacher.** *Quantum ergodicity and symmetry reduction.*

We study the ergodic properties of eigenfunctions of Schrödinger operators on a closed connected Riemannian manifold in case that the underlying Hamiltonian system possesses certain symmetries, relying on recent results on singular equivariant asymptotics. More precisely, we prove an equivariant quantum ergodicity theorem assuming that the symmetry-reduced Hamiltonian flow on the principal stratum of the singular symplectic reduction is ergodic by deducing an equivariant version of the semiclassical Weyl law. The theorem then implies an equivariant version of the Shnirelman-Zelditch-Colin-de-Verdiere theorem. The guiding idea is that symmetries imply the existence of conserved quantities and partial integrability of the Hamiltonian flow, forcing the system to behave less chaotically. By dividing out the symmetries, one is able to study the symmetry-reduced ergodic properties of the corresponding quantum system.

- **Alexander Strohmaier.** *Resonances and symmetric spaces.*
- **Duc Viet Vu.** *Asymptotic number of scattering resonances for generic Schrödinger operators*

Let  $-\Delta + V$  be the Schrodinger operator acting on  $L^2(\mathbb{R}^d; \mathbb{C})$  with  $d \geq 3$  odd. Here  $V$  is a bounded real or complex function vanishing outside the closed ball of center 0 and of radius  $a$ . Let  $n_V(r)$  denote the number of resonances of  $-\Delta + V$  with modulus less than  $r$ . We show that if the potential  $V$  is generic in a sense of pluripotential theory, then

$$n_V(r) = c_d a^d r^d + O(r^{d-3/16+\epsilon})$$

as  $r \rightarrow \infty$  for any  $\epsilon > 0$  where  $c_d$  is a dimensional constant.

- **Tobias Weich.** *Resonance chains on Schottky surfaces.*

Recently David Borthwick discovered through numerical calculations surprising chain structures in the resonance spectrum of certain Schottky surfaces. In this talk we will see that these resonance chains have the same origin as the resonance chains in the classical and quantum mechanical spectrum of the three disk system and we will see that they are related to a clustering in the length spectrum. Finally the existence of these chains will be proven for three funneled Schottky surfaces in a certain geometrical limit in the Teichmüller space. Joint work with S. Barkhofen and F. Faure.

- **Polina Vytnova.** *Estimating fast dynamo growth rate using Zeta functions*

Kinematic fast dynamo theory is an area of magnetohydrodynamics which subject is the origin and maintenance of magnetic field of large astrophysical objects. I will explain how one can estimate the growth rate of magnetic field in several simple scenarios, using traditional approach of Grothendieck and Ruelle to calculating the largest eigenvalue of the Dynamo operator.

- **Chengbo Zhu.** *Conservation relations for local theta correspondence.*

The theory of theta correspondence, initiated by R. Howe, gives one of the few general methods of constructing automorphic forms of groups over number fields. The talk is about the local theory, called the local theta correspondence, which is a correspondence between admissible representations of two groups of a dual pair. A key question is the non-vanishing and the fundamental principle governing non-vanishing is a conjecture of Kudla and Rallis from the mid 90's, known as the conservation relations. I will discuss my recent work (joint with B. Sun) which establishes this conjecture in full generality.

- **Maciej Zworski.** *Resonances as viscosity limits.*

In practically all situations resonances can be defined as limits of L2 eigenvalues of operators which regularize the Hamiltonian at infinity. For instance, Pollicott–Ruelle resonances in the theory of dynamical systems are given by viscosity limits: adding a Laplacian to the generator of an Anosov flow gives an operator with a discrete spectrum; letting the coupling constant go to zero turns eigenvalues into the resonances (joint work with S Dyatlov). This principle seems to apply in all other settings where resonances can be defined and I will explain it in the case of black box Euclidean scattering (after reviewing that general set-up). The method has also been numerically investigated in the chemistry literature as an alternative to complex scaling.