ABSTRACTS Conference on Noncommutative Geometry CIRM, November 2 - 6, 2015

CLAIRE ANANTHARAMAN

TITLE: From groups to semigroups and groupoids.

ABSTRACT: In this overview, limited to discrete semigroups and étale groupoids for simplicity, we shall begin by giving examples of the many situations where these mathematical objects take part and prove to be very useful. Although often considered as mere (albeit technical) generalizations of discrete groups, we shall discuss questions where they do not behave in the same way as discrete groups.

Saad Baaj

TITRE : Quelques applications de l'équivalence monoidale des groupes quantiques localement compacts.

RÉSUMÉ: Généralisant le cas des groupes quantiques compacts monoidalement équivalents, nous établissons une équivalence fonctorielle entre les actions continues de deux groupes quantiques localement compacts G_1 et G_2 , réguliers et monoidalement équivalents au sens de de Commer. Nous déduisons de ce résultat, une équivalenve canonique des catégories KK^{G_1} et KK^{G_2} de Kasparov.

(Travail en collaboration avec J.Crespo).

TEODOR BANICA

TITLE: Liberation questions in noncommutative geometry.

ABSTRACT: Given a real algebraic manifold X, one interesting question is that of replacing the commutation relations ab=ba between the standard coordinates by some weaker relations (e.g. abc=cba, half-liberation) or by nothing at all (liberation). Thanks to about 20 years of efforts, this is now relatively well understood in the compact Lie group case. I will discuss what happens for more general manifolds (NC spheres, homogeneous spaces) where some preliminary explorations are now available. Of course the interesting case would be the one when X has some supplementary, more specialized structure (e.g. Riemannian), and I will discuss some basic open questions here.

Alain Connes

TITRE : Unification et modèle spectral.

RÉSUMÉ: J'exposerai les résultats très récents obtenus en collaboration avec Chamseddine et Suijlekom sur l'unification des constantes de couplage dans l'approche de la physique par la géométrie noncommutative .

JOACHIM CUNTZ

TITLE: C^* -algebras associated with algebraic actions.

ABSTRACT: We give a survey of work in which the author was involved in recent years on C*-algebras constructed from representations of one or several algebraic endomorphisms of a compact abelian group - or, dually, of a discrete abelian group. In our survey we do not try to describe the entire scope of the methods and results but we concentrate on the important thread coming from the action of the multiplicative semigroup of a Dedekind ring on its additive group. Representations of such actions give rise to particularly intriguing problems and has motivated many of the new methods and general results obtained in this area.

YVES DE CORNULIER

TITLE: Central extensions of current Lie algebras.

ABSTRACT: Let \mathfrak{g} be a Lie algebra over a field K of characteristic zero. Given a commutative K-algebra A, the so-called current Lie algebra is the Lie algebra $A \otimes_K \mathfrak{g}$, viewed as a Lie algebra over K. I will survey results about the description of the second homology space $H_2(A \otimes \mathfrak{g})$ in terms of invariants of \mathfrak{g} and A. One important ingredient is the so-called Koszul homomorphism, which maps the space of invariant bilinear forms on \mathfrak{g} to the third cohomology space $H^3(\mathfrak{g})$.

Mikael de la Salle

TITLE: Group actions on Banach spaces.

ABSTRACT: Kazhdan's property (T) has long been a fundamental concept in various parts of mathematics, but it is only quite recently that variants for group actions on Banach spaces have been studied. Such a study has been simultaneously initiated by Lafforgue and Bader–Furman–Gelander–Monod, and it turns out that there are several ways of defining property a Banach space version of property (T). I will explain the connections between these properties, and then focus on the case of higher rank groups and their lattices. The last part of the talk will be partly based on joint works with Tim de Laat and Masato Mimura.

DAMIEN GABORIAU

TITLE: Geometric dimension and approximations in orbit equivalence.

ABSTRACT: My talk will consist in two parts.

I will present a joint work in progress with Clinton Conley, Andrew Marks and Robin Tucker-Drob about the geometric dimension of probability measure preserving (pmp) group actions. Treeability coincides with geometric dimension = 1 and it consists in equipping, in a measurable way, every orbit with the structure of a tree. We show for instance that:

- Every (!) free pmp action of the fundamental group of a compact surface is treeable.

- Every free pmp action of the fundamental group of an aspherical 3-dimensional closed orientable manifold either is amenable or has geometric dimension = 2.

In a joint work with Robin Tucker-Drob we initiate a general study of the notion of increasing approximation for pmp equivalence relations, just like hyperfiniteness (which plays a central role in orbit equivalence), consists in approximation by finite subrelations. We obtain for instance a non-approximability result for non-amenable groups generated by two commuting finitely generated infinite subgroups, one of them action in a strongly ergodic manner. This result has the consequence in Bernoulli percolation on Cayley graphs of such groups, that the uniqueness threshold p_u doesn't itself belong to the uniqueness phase.

Emmanuel Germain

TITLE: KK-theory for reduced free product C^* -algebras.

ABSTRACT: In a joint work with Pierre Fima, we show how the point of view brought by the fundamental C^* -algebra of a graph of C^* -algebras allows to solve old conjectures about reduced free product C^* -algebras.

NIGEL HIGSON

TITLE: The geometry and noncommutative geometry of parabolic induction.

ABSTRACT: The functor of parabolic induction is a central operation in the Harish-Chandra-style representation theory of a real reductive group. It may be represented by a bimodule and a tensor product operation, much as Marc Rieffel represented ordinary induction. The C*-algebra version of this bimodule is a Hilbert module, and has been studied by Pierre Clare, but this talk will be about a Fréchet space version that is related to Clare's bimodule in the same way that Harish-Chandra's Schwartz space is related to the reduced group C*-algebra. Rather unexpectedly, in this context the functor parabolic induction has a right adjoint; one might call it parabolic restriction. This is a new construction in representation theory, discovered using noncommutative geometry. I'll discuss some of the features and potential uses of parabolic restriction.

Cyril Houdayer

TITLE: Asymptotic structure and rigidity of free product von Neumann algebras.

ABSTRACT: I will give an overview of recent results obtained in joint work with Yoshimichi Ueda on the asymptotic structure and the rigidity of arbitrary free product von Neumann algebras. First, I will explain that in any free product von Neumann algebra, any amenable von Neumann subalgebra that has a diffuse intersection with one of the free components is necessarily contained in this free component. This result completely settles the problem of maximal amenability inside free product von Neumann algebras. Then I will present new Kurosh-type rigidity results for free product von Neumann algebras. Namely, I will explain that for any family of nonamenable factors belonging to a large class of (possibly type III) factors including nonprime factors, nonfull factors and factors with a Cartan subalgebra, the corresponding free product von Neumann algebra with respect to arbitrary states retains the cardinality of the family as well as each factor up to stable inner conjugacy, after permutation of the indices.

Gennadi Kasparov

TITLE: On the L^p Baum-Connes conjecture.

ABSTRACT: The right side of the Baum-Connes conjecture is the K-theory of the reduced C^* -algebra $C^*_{red}(G)$ of the group G. This algebra is the completion of the algebra $L^1(G)$ in the norm of the algebra of operators acting on $L^2(G)$. If we complete the algebra $L^1(G)$ in the norm of the algebra of operators acting on $L^p(G)$ we will get the Banach algebra $C^{*,p}_{red}(G)$. The K-theory of this algebra serves as the right side of the L^p -version of the Baum-Connes conjecture. The construction of the left and the assembly map in this case requires a little bit of techniques of asymptotic morphisms for Banach algebras. A useful category of Banach algebras for this purpose includes all algebras of operators acting on L^p -spaces (which may be called L^p -algebras).

The current joint work in progress with Guoliang Yu aims at proving the following result:

The L^p -version of the Baum-Connes conjecture with coefficients in any L^p -algebra is true for any discrete group G which admits an affine-isometric, metrically proper action on the space $X = l^p(Z)$, where Z is a countable discrete set, so that the linear part of this action is induced by a measure-preserving action of G on Z.

I will discuss the techniques involved in this work.

VINCENT LAFFORGUE

TITRE : Quelques aperçus du programme de Langlands.

RÉSUMÉ. Après une introduction générale, j'énoncerai des conjectures du programme de Langlands, notamment celles qui sont les plus proches des algèbres d'opérateurs. Puis j'expliquerai la preuve de la paramétrisation de Langlands dans le cas des corps de fonctions.

VICTOR NISTOR

TITLE: Representations of groupoid C^* -algebras and invertibility.

ABSTRACT: A little over one hundred years ago, Fredholm, Hilbert, and Schmidt each published a paper that can be regarded among the precursors of Operator Algebras, because they underscored the importance of compact and Fredholm operators. To this day, Operator Algebras continue to play an important role in certain analysis questions. In my talk, I will present some results that use properties of groupoids and of their representations to decide whether certain differential operators are invertible. Several results of Skandalis are used in a crucial way in the proofs. These are joint works with Georgescu and Prudhon.

PAOLO PIAZZA

TITLE: Indomitable rho-invariants.

ABSTRACT: The classic Atiyah-Patodi-Singer index theorem was published precisely 40 years ago. One fundamental contribution presented in the three seminal papers published by these authors, was the definition of the rho-invariant, a secondary invariant

particularly suited to geometric applications to riemannian geometry and differential topology. Since then, many other contributions have been given to the definition of invariants of rho-type and to their applications to geometry. The goal of this talk is to present this exciting story, with emphasis on recent results and future directions.

Sorin Popa

TITLE: Approximating freeness under constraints, with applications.

ABSTRACT: I will discuss a method for constructing a Haar unitary u in a subalgebra B of a II₁ factor M that's "as independent as possible" (approximately) with respect to a given finite set of elements in M. The technique consists of "patching up infinitesimal pieces" of u. This method had some striking applications over the years:

- 1. vanishing of the 1-cohomology for M with values into the compact operators (1985);
- 2. reconstruction of subfactors through amalgamated free products and axiomatisation of standard invariants (1990-1994).
- 3. first positive results on Kadison-Singer type paving (2013);
- 4. vanishing of the continuous version of Connes-Shlyakhtenko 1-cohomology (with Vaes in Jan. 2014) and of smooth 1-cohomology (with Galatan in June 2014).

JEAN RENAULT

TITRE : Semigroupes, P-graphes et groupoïdes.

RÉSUMÉ : Les groupoïdes permettent d'unifier un certain nombre de constructions qui associent une C*-algèbre à des objets de nature algébrique comme les semigroupes, combinatoire comme les graphes de rang supérieur, ou encore dynamique comme les actions de semigroupes. Ces constructions sont souvent limitées à des classes de semigroupes, par exemple les semigroupes de Ore ou les semigroupes de Clifford. Je montrerai pourquoi elles sont valables en général. Je présenterai quelques exemples.

Mikael Rørdam

TITLE: Supramenable groups and their actions on locally compact spaces and C^* -algebras.

ABSTRACT: Supramenable groups, introduced by Rosenblatt in 1974, is the class of groups that have no paradoxical subsets. Non-supramenable groups are thus groups that carry some paradoxicality, and they are in this way a natural generalisation of non-amenable groups. Rosenblatt proved that solvable groups are supramenable if and only if they have polynomial growth. Non-supramenable groups have exponential growth, however, it is not known if the class of supramenable groups coincides with the class of groups with sub-exponential growth.

In this talk we discuss geometric properties of supramenable groups, and how these groups can be characterized by their actions and partial actions on locally compact Hausdorff spaces and on C^* -algebras.

This is joint work with Kellerhals and Monod, and with Matui. I will also report on more recent work by my PhD student Eduardo Scarparo.

THOMAS SCHICK

TITLE: Homotopy invariants of closed manifolds through higher signatures.

ABSTRACT: In the classification of closed manifolds, the surgery exact sequence provides a powerful tool to understand the collection of all manifolds homotopy equivalent to a given one (together with fine ways to distingish manifolds which are not homotopy equivalent). To efficiently extract this information, the signature operator and its index invariants (taking values in the K-theory of suitably associated C*-algebras) are particularly useful.

We present joint work with Paolo Piazza how to use these methods to systematically map the surgery exact sequence to a corresponding sequence in the K-theory of operator algebras (due to Higson-Roe). Then we describe new developments how to get extract the relevant information from this K-theory groups via new pairings with cyclic cocycles (joint with Higson and Piazza). If time permits, we will also present a somewhat mysterious addition (likely) homotopy invariant based on the signature operator of codimension 2 hypersurfaces.

Dimitri Shlyakhtenko

TITLE: Free probability of type B and asymptotics of finite-rank perturbations of random matrices.

ABSTRACT: We show that finite rank perturbations of certain random matrices fit in the framework of infinitesimal (type B) asymptotic freeness. This can be used to explain the appearance of free harmonic analysis (such as subordination functions appearing in additive free convolution) in computations of outlier eigenvalues in spectra of such matrices.

STEFAAN VAES

TITLE: Cohomology and L^2 -Betti numbers for subfactors and quasi-regular inclusions.

ABSTRACT: I present a joint work with S. Popa and D. Shlyakhtenko introducing a cohomology theory for quasi-regular inclusions of von Neumann algebras. In particular, we define L²-cohomology and L²-Betti numbers for such inclusions. Applying this to the symmetric enveloping inclusion of a finite index subfactor, we get a cohomology theory and a definition of L²-Betti numbers for finite index subfactors, as well as for arbitrary rigid C^{*}-tensor categories. For the inclusion of a Cartan subalgebra in a II₁ factor, we recover Gaboriau's L²-Betti numbers for equivalence relations.

Alain Valette

TITLE: Expanders and box spaces.

ABSTRACT: Expanders, especially those coming from box spaces of residually finite groups, have been used to test various forms of the coarse Baum-Connes conjecture. The first construction of a pair of expanders, one not coarsely embedding in the other, was provided by Mendel and Naor in 2012. This was extended by Hume in 2014 who constructed a continuum of expanders with unbounded girth, pairwise not coarsely equivalent. In joint work with A. Khukhro, we construct a continuum of expanders with geometric property (T) of Willett-Yu, as box spaces of $SL(3,\mathbb{Z})$. We also show that, for $m > n \ge 2$, box spaces of $SL(m,\mathbb{Z})$ are not coarsely equivalent to box spaces of $SL(n,\mathbb{Z})$.

DAN-VIRGIL VOICULESCU

TITLE: The bi-free extension of free probability.

ABSTRACT: I will discuss the recent extension of free probability to systems with left and right variables. This includes the R-, S- and T-transforms for the computation of the three bi-free convolutions in the plane and extreme values.

GUOLIANG YU

TITLE: Dimension and K-theory.

ABSTRACT: I will introduce a notion of dynamics asymptotic dimension and discuss its application to K-theory in both the analytic and algebraic settings. This is joint work with Erik Guentner, Rufus Willett, and Zhizhang Xie. If time permits, I will also discuss my joint work with Herve Oyono-Oyono on a concept of asymptotic dimension for C*-algebras and its application to the Kunneth formula.