Deciding the Bell number for hereditary graph properties

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Abstract: A graph property is a set of graphs closed under isomorphism. A property is hereditary if it is closed under taking induced subgraphs. Given a graph property $\mathcal{X}$, we write $\mathcal{X}_n$ for the number of graphs in $\mathcal{X}$ with vertex set $\{1, 2, \ldots, n\}$ and following [1] we call $\mathcal{X}_n$ the speed of the property $\mathcal{X}$.

The paper [2] identifies a jump in the speed of hereditary graph properties to the Bell number B_n and provides a partial characterization of the family of minimal classes whose speed is at least B_n. In the present work we give a complete characterization of this family. Since this family is infinite, the decidability of the problem of determining if the speed of a hereditary class is above or below the Bell number is questionable. We answer this question positively by showing that there exists an algorithm which, given a finite set F of graphs, decides whether the speed of the class of graphs containing no induced subgraphs from the set F is above or below the Bell number. For properties defined by infinitely many minimal forbidden subgraphs, the speed is known to be above the Bell number.

By the structural results obtained, it turns out that the boundary of the Bell number is a partial boundary for well-quasi-ordering by the induced subgraph relation. We show that all the classes below the Bell number are defined by finitely many minimal forbidden induced subgraphs and are all well-quasi-ordered by the induced subgraph relation, while the finitely defined classes which are above the Bell number and have finite distinguishing number are not well-quasi-ordered by the induced subgraph relation. This result gives us some insight how one can approach the question of deciding well-quasi-ordering by the induced subgraph relation in its full generality.

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Recoloring bounded treewidth graphs

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Abstract: Let k be an integer. Two vertex k-colorings of a graph are *adjacent* if they differ on exactly one vertex. A graph is *k-mixing* if any proper k-coloring can be transformed into any other through a sequence of adjacent proper k-colorings. Any graph is $(tw+2)$-mixing, where tw is the treewidth of the graph (Cereceda 2006). We prove that the shortest sequence between any two $(tw+2)$-colorings is at most quadratic, a problem left open in Bonamy et al. (2012).

Jerrum proved that any graph is k-mixing if k is at least the maximum degree plus two. We improve Jerrum's bound using the grundy number, which is the worst number of colors in a greedy coloring.

Induced cycles and coloring

• M. Chudnovsky – Princeton University
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Abstract: A *hole* in a graph is an induced cycle of length at least four, and an *odd hole* is a hole of odd length. A famous conjecture of A. Gyárfás [1] from 1985 asserts:

Conjecture 1: For all integers k, l there exists $n(k, l)$ such that every graph G with no clique of carnality more than k and no odd hole of length more than l has chromatic number at most $n(k, l)$.

In other words, the conjecture states that the family of graphs with no long odd holes is χ-bounded. Little progress was made on this problem until recently Scott and Seymour proved that Conjecture 1 is true for all pairs (k, l) when $l = 3$ (thus excluding *all* odd holes guarantees χ-boundedness) [3].

No other cases have been settled, and here we focus on the case $k = 2$. We resolve the first open case, when $k = 2$ and $l = 5$, proving that

Theorem 1. *Every graph with no triangle and no odd hole of length* > 5 *is* 82200*-colorable.*

Conjecture 1 has a number of other interesting special cases that still remain open; for instance

- **Conjecture:** For all l every triangle-free graph G with sufficiently large chromatic number has an odd hole of length more than l;
- **Conjecture:** For all k, l every graph with no clique of size more than k and sufficiently large chromatic number has a hole of length more than l.

We prove both these statements with the additional assumption that G contains no 5-hole. (The latter one was proved, but not published, by Scott earlier, improving on [2]).

All the proofs follows a similar outline. We start with a *leveling* of a graph with high chromatic number, that is a classification of the vertices by their distance from a fixed root. Then the graph undergoes several rounds of "trimming" that allows us to focus on a subgraph M with high chromatic number that is, in some sense, minimal. We also ensure that certain pairs of vertices with a neighbor in M can be joined by a path whose interior is anticomplete to M. It is now enough to find two long paths between some such pair of vertices, both with interior in M and of lengths of different parity, to obtain a long odd hole.

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Comparing tree-width and clique-width for degree-constrained graphs

• Bruno Courcelle – Bordeaux

Abstract: We will review some results that bound clique-width in terms of tree-width and vice-versa, for graphs of bounded degree and for incidence graphs. For an example, if a graph has tree-width k and maximal degree d, then its clique-width is at most $20.d.(k+1)+2$ (which is better than the general exponential bound).

We also examine how the trees underlying the graph decompositions are transformed in the corresponding proofs. This aspect is important for the construction (or the comparison) of FPT graph algorithms using tree-width or clique-width as parameters because, in most cases, input graphs must be given to the algorithms by their decompositions or by algebraic terms representing them.

The clique-width of the *incidence graph* of a graph of tree-width k is at most $k+3$ (T. Bouvier, 2014). This result makes possible to check *monadic second-order* (MSO) properties expressed with *edge quantifications* for graphs of bounded tree-width with the existing tools (finite automata that compute their transitions, called *fly-automata*) developped for checking MSO properties expressed *without* edge quantifications for graphs of bounded clique-width.

Computational Complexity of Threshold Editing

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Abstract: We show that the problem of *editing to a threshold graph*, i.e., adding and deleting as few edges as possible to obtain a threshold graph is NP-complete, thereby solving a long-standing open problem in the field of graph modification problems. This problem has been repeatedly stated as open [2, 5, 6, 8], and renewed interest appeared very recently in the field of social network theory [1], where it has been suggested as a good basis for an axiomatic centrality measure. Coincidentally, the related problem Trivially Perfect Editing, which was recently shown NP-hard, and to admit a polynomial kernel [3], has recently been suggested as a good measure for hierarchyness of social networks [7]. Both these classes are chordal cographs, and the main technique applied for obtaining polynomial kernels is that of a *vertex modulator* which allows for extracting structure.

Theorem 1. Threshold Editing *is* NP-*complete.*

More interestingly, on the positive side we show that the problems Threshold Editing, Completion, and Deletion all admit polynomial kernels with $O(k^2)$ vertices. This answers a recent question by Liu, Wang and Guo [4], who asked whether the previously known kernel for Threshold Completion could be improved from $O(k^3)$ to $O(k^2)$.

Theorem 2. Threshold Editing *admits a quadratic kernel.*

Finally, we show that we can solve Threshold Editing in parameterized subexponential time $2^{O(\sqrt{k}\log k)} \cdot n^{O(1)}$. The subexponential time algorithm uses a decomposition of almost-threshold graphs; We are able to decompose any yes instance into subexponentially many "unbreakable" segments, each of which we are able to solve in subexponential time. Applying dynamic programming, we manage to glue a select few such unbreakable segments back together to obtain our target graph.

Theorem 3. Threshold Editing *is solvable in* $2^{O(\sqrt{k}\log k)} \cdot \mathrm{poly}(n)$ *time.*

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Strongly sublinear separators and polynomial expansion

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Abstract: A *k-minor of a graph* G is any graph obtained from G by contracting pairwise vertex-disjoint subgraphs of radius at most k and removing vertices and edges. A graph G has *expansion bounded by function* $f : \mathbf{N} \to \mathbf{R}$ if for every $k \geq 0$, every k-minor of G has average degree at most $f(k)$.

There is a natural connection between bounded expansion and small separators. Building upon a result of Plotkin et al., Nešetřil and Ossona de Mendez proved that for every subexponential function $f : \mathbf{N} \to \mathbf{R}$, there exists a sublinear function $s : \mathbf{N} \to \mathbf{N}$ such that every graph G with expansion bounded by f has a balanced separator of order at most $s(|V(G)|)$. We prove an approximate converse to this claim: For every $\varepsilon > 0$ and a function $s(n) = O(n^{1-\varepsilon})$, there exists a polynomial f such that if every subgraph $H \subseteq G$ has a balanced separator of order at most $s(|V(H)|)$, then G has expansion bounded by f.

Uniform Kernelization Complexity of Hitting Forbidden Minors

• Archontia Giannopoulou – Durham University

Abstract: The F-Minor-Free Deletion problem asks, for a fixed set F and an input consisting of a graph G and integer k, whether k vertices can be removed from G such that the resulting graph does not contain any member of F as a minor. It generalizes classic graph problems such as Vertex Cover and Feedback Vertex Set. Fomin et al. (FOCS 2012) showed that the special case Planar-F-Minor-Free Deletion (when F contains at least one planar graph) has a kernel of polynomial size: instances (G, k) can efficiently be reduced to equivalent instances (G', k) of size $f(F)k^{g(F)}$ for some functions f and g. The degree g of the polynomial grows very quickly; it is not even known to be computable. Fomin et al. left open whether Planar-F-Minor-Free Deletion has kernels whose size is uniformly polynomial, i.e., of the form $f(F)k^c$ for some universal constant c that does not depend on F. In this talk we discuss to what extent provably effective and efficient preprocessing is possible for F-Minor-Free Deletion. In particular, we show that not all Planar-F-Minor-Free Deletion problems admit uniformly polynomial kernels but also that there exist problems that do admit uniformly polynomial kernels.

Hypertree Decompositions

• G. Gottlob – University of Oxford
• Nicola Leone – University of Calabria
• Francesco Scarcello – University of Calabria

Abstract: One of the best-known methods for decomposing graphs is the method of tree-decompositions introduced by Robertson and Seymour. Many NP-hard problems become polynomially solvable if restricted to instances whose underlying graph structure has bounded treewidth. The notion of treewidth can be straightforwardly extended to hypergraphs by simply considering the treewidth of their primal graphs or, alteratively, of their incidence graphs. However, doing so comes along with a loss of information on the structure of a hypergraph with the effect that many polynomially solvable problems cannot be recognized as such because the treewidth of the underlying hypergraphs is unbounded. In particular, the treewidth of the class of acyclic hypergraphs is unbounded. In this talk, I will describe more appropriate measures for hypergraph acyclicity, and, in particular, the method of hypertree decompositions and the associated concept of hypertree width. After giving general results on hypertree decompositions, I will report on game-theoretic characterizations of hypergraph decomposition methods, give a survey on more recent results, and state some open problems.

On the structure of 1-perfectly orientable graphs

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Abstract: We study the class of 1-perfectly orientable graphs. Following the terminology of Kammer and Tholey [1], we say that an orientation of a graph is *1-perfect* if the out-neighborhood of every vertex induces a tournament, and that a graph is *1-perfectly orientable* (1-p.o. for short) if it has a 1-perfect orientation. The notion of 1-p.o. graphs was introduced by Skrien [2] (under the name $\{B_2\}$-graphs), where the problem of characterizing 1-p.o. graphs was posed. By definition, 1-p.o. graphs are exactly the graphs that admit an orientation that is an out-tournament. (A simple arc reversal argument shows that that 1-p.o. graphs are exactly the graphs that admit an orientation that is an in-tournament. Such orientations were called fraternal orientations in several papers.)

1-p.o. graphs form a common generalization of chordal graphs and circular arc graphs. While they can be recognized in polynomial time via a reduction to 2-SAT [3], little is known about their structure. We prove several results related to the structure of 1-p.o. graphs. First, we give a characterization of 1-p.o. graphs in terms of edge clique covers, similar to a known characterization of squared graphs due to Mukhopadhyay. We exhibit several examples of 1-p.o. and non-1-p.o. graphs. The examples of non-1-p.o. graphs include two infinite families: the complements of even cycles of length at least 6, and the complements of odd cycles augmented by a component consisting of a single edge. We identify several graph transformations preserving the class of 1-p.o. graphs. In particular, we show that the class of 1-p.o. graphs is closed under taking induced minors. We also study the behavior of 1-p.o. graphs under some operations that in general do not preserve the class, such as pasting along a clique and the join. The result for the join motivates the problem of characterizing the 1-p.o. co-bipartite graphs. We show that all the presented examples of non-1-p.o. graphs are minimal forbidden induced minors for the class of 1-p.o. graphs. As our main results we obtain complete characterizations of 1-p.o. graphs within the classes of complements of forests and of cographs.

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A $(2-\epsilon)$-Hall's theorem with an application to space complexity.

• Ilario Bonacina – Sapienza University of Rome
• Nicola Galesi – Sapienza University of Rome
• Tony Huynh – Sapienza University of Rome
• Paul Wollan – Sapienza University of Rome

Abstract: Let G be a bipartite graph with bipartition (L, R) and left-degree at most 3. A $(2,4)$-*matching* is a set of vertex disjoint paths, each of length 2 or 4 and each beginning and ending in R. We prove a variant of Hall's theorem for $(2,4)$-matchings. That is, if every subset A of L satisfies $|N_G(A)| \geq (2-\epsilon)|A|$ for a fixed $\epsilon < \frac{1}{23}$, then G has a $(2,4)$-matching covering all the vertices of L.

Using our $(2-\epsilon)$-Hall's theorem, we then give an application in the theory of space complexity. Specifically, we prove a $\Omega(n^2/\log^2 n)$ lower bound for the *total space* needed in Resolution to refute a random 3-CNF formula ϕ in n variables. Previously, no lower bound for refuting any family of 3-CNFs was known for the *total space* in resolution or for the *monomial space* in algebraic systems.

In this talk, no knowledge of space complexity will be assumed.

The Erdős-Pósa property of odd and long cycles through prescribed vertices

• Felix Joos – Universität Ulm

Abstract: A result by Erdős and Pósa says that for every graph G and every integer k, the graph G has k disjoint cycles or a set X of vertices of size $O(k \log k)$ such that $G - X$ is a forest. This result is the origin for the notion *Erdős-Pósa property*, which is defined as follows: a family $\mathcal{H}$ of graphs is said to have it if there is a function $f : \mathbb{N} \to \mathbb{N}$ so that any graph contains k disjoint subgraphs that are isomorphic to graphs in $\mathcal{H}$, or it contains a vertex set of size $f(k)$ meeting all such subgraphs.

For a vertex set S, let an S-cycle be a cycle that contains at least one vertex of S.

All cycles of length at least ℓ and, stretching the definition a bit, S-cycles are just two of many examples having the Erdős-Pósa property. Others include:

- the family of cycles of length 0 mod m for any integer $m \geq 2$,
- the family of cycles of length not equal to 0 mod m for any odd integer $m \geq 3$,
- the family of graphs that can be contracted to a specific planar graph,
- and the family of all (directed) cycles in a digraph.

In this talk we present a result that brings together two lines of research by showing that the class of all S-cycles of length at least ℓ has the Erdős-Pósa property. Moreover, we show that odd S-cycles also have it if we strict ourselves to graphs with high connectivity.

The first result is joint work with Henning Bruhn and Oliver Schaudt.

Upper Bounds on the Size of Obstructions for linear rank-width and linear branch-width of representable matroids

• O-joung Kwon– Department of Mathematical Sciences, KAIST
• Mamadou Moustapha Kanté – Université Blaise Pascal, LIMOS, CNRS

Abstract: We prove that the size of the pivot-minor obstructions for linear rank-width k is bounded by $2^{2^{O(k)}}$. Our techniques are similar to the ones used by Lagergren in [1] to bound the sizes of graph minor obstructions for path-width. Our basic tools are the algebraic operations introduced by Courcelle and Kanté in [2], then generalised to edge-coloured graphs by Kanté and Rao in [3], and an analogue of the Tutte linking Theorem for rank-width. The proof ideas are as follows:

1. encode each linear layout of width k of an obstruction G in a compact way using the algebraic operations,
2. define a quasi-order $\lesssim$ on graphs (using the encodings) such that (1) if H is a pivot-minor of G, then $H \lesssim G$, (2) if $\mathrm{lrwd}(G \otimes H) \leq k$ and $G' \lesssim G$, then $\mathrm{lrwd}(G' \otimes H) \leq k$,
3. prove that the maximal chain with respect to $\lesssim$ is bounded.

We then prove that for every $\mathbf{F}$-representable matroid $\mathcal{M}$ one can associate a bipartite graph $\mathcal{B}(\mathcal{M})$ whose adjacency matrix over $\mathbf{F}$ is skew-symmetric and vice-versa, and such that

1. $\mathrm{lbwd}(\mathcal{M}) = \mathrm{lrwd}(\mathcal{B}(\mathcal{M})) + 1$,
2. if $\mathcal{N}$ is a matroid minor of $\mathcal{M}$, then $\mathcal{B}(\mathcal{N})$ is a pivot-minor of $\mathcal{B}(\mathcal{M})$.

As a consequence, the size of obstructions for linear branch-width k on $\mathbf{F}$-representable matroids is bounded by $|\mathbf{F}|^{|\mathbf{F}|^{O(k)}}$, which is finite whenever $\mathbf{F}$ is finite.

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Algorithmic Applications of Tree-Cut Width

• Robert Ganian and Stefan Szeider Vienna University of Technology
• Eun Jung Kim – CNRS-Paris Dauphine University

Abstract: Wollan [1] has recently introduced the graph parameter *tree-cut width*, which plays a similar role with respect to immersions as the graph parameter *treewidth* plays with respect to minors. In this paper we provide the first algorithmic applications of tree-cut width to hard combinatorial problems. Tree-cut width is known to be lower-bounded by a function of treewidth, but it can be much larger and hence has the potential to facilitate the efficient solution of problems which are not believed to be fixed-parameter tractable (FPT) when parameterized by treewidth.

We briefly outline the methodology used to obtain our algorithmic results. As a first step, we develop the notion of *nice tree-cut decompositions*[1] and show that any tree-cut decomposition can be transformed into a nice one in polynomial time. These nice tree-cut decompositions are of independent interest, since they provide a means of simplifying the complex structure of tree-cut decompositions. Secondly, we introduce a general three-stage dynamic framework for the design of FPT algorithms on nice tree-cut decompositions and apply it to our problems. The crucial part of this framework is the computation of the "joins." We show that the children of any node in a nice tree-cut decomposition can be partitioned into (i) a bounded number of children with complex connections to the remainder of the graph, and (ii) a potentially large set of children with only simple connections to the remainder of the graph. We then process these by a combination of branching techniques applied to (i) and integer linear programming applied to (ii). The specifics of these procedures differ from problem to problem. We provide FPT algorithms for the showcase problems Capacitated Vertex Cover, Capacitated Dominating Set and Imbalance parameterized by the tree-cut width of an input graph G.

On the other hand, we show that List Coloring, Precoloring Extension and Boolean CSP (the latter parameterized by the tree-cut width of the incidence graph) are W[1]-hard and hence unlikely to be fixed parameter tractable when parameterized by tree-cut width.

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[1] We call them "nice" as they serve a similar purpose as the nice tree decompositions [2], although the definitions are completely unrelated.

Width Parameters for Matroids

• D. Král' – University of Warwick

Abstract: Many results on graph decompositions are motivated by their applications in algorithm design. It is natural to investigate to what extent such algorithmic results can be extended to matroids, a generalization of the notion of graphs. The first result in this direction was obtained by Hliněný [2] who proved the analogue of the celebrated result of Courcelle on testing monadic second order properties of graphs with bounded tree-width for matroids representable over finite fields.

In this talk, we survey width parameters for matroids with the related algorithmic applications. We start with mentioning classical results related to the notion of matroid branch-width, which can be viewed as the most appropriate matroid analogue of graph tree-width. We will then survey results on extending the result of Hliněný to matroids non-representable over finite fields [3, 4] and results on testing first order properties of matroids [1]. At the end of the talk, we will mention the role played by matroid branch-depth, the analogue of graph tree-depth, in relation to matroid limits [5].

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The Directed Grid Theorem

• Ken-ichi Kawarabayashi – National Institut of Informatics, Tokyo
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Abstract: The grid theorem, originally proved by Robertson and Seymour in Graph Minors V [5] in 1986, is one of the fundamental results in the study of graph minors. It has found numerous applications in algorithmic graph structure theory, for instance in bidimensionality theory, and it is the basis for several other structure theorems developed in the graph minors project.

In the mid-90s, Reed [4] and Johnson, Robertson, Seymour and Thomas [1], independently, conjectured an analogous theorem for directed graphs, i.e. the existence of a function $f : \mathbb{N} - > \mathbb{N}$ such that every digraph of directed tree-width at least $f(k)$ contains a directed grid of order k. In an unpublished manuscript from 2001 [2], Johnson, Robertson, Seymour and Thomas give a proof of this conjecture for planar digraphs. A proof of the full conjecture was announced by Kawarabayashi and Kreutzer in 2014 [3].

In this talk we will give an introduction to directed tree width and present the main ideas of the proof of the directed grid theorem.

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Characterizing the linear rank-width of distance-hereditary graphs via split decompositions

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• O-joung Kwon – Department of Mathematical Sciences, KAIST, South Korea.

Abstract: *Linear rank-width* is the linearized variant of rank-width, similar to path-width, which can be seen as the linearized variant of tree-width. While path-width is a well-studied notion, much less is known about linear rank-width. A graph G is *distance-hereditary*, if for any two vertices u and v of G, the distance between u and v in any connected, induced subgraph of G that contains both u and v, is the same as the distance between u and v in G. Distance-hereditary graphs are exactly the graphs of rank-width at most 1 [4].

We present a characterization of the linear rank-width of distance-hereditary graphs. The characterization is similar to the known characterization of path-width on trees [1,3], and we develop modifications of canonical split decompositions to obtain our result. Using the characterization, we show that the linear rank-width of every n-vertex distance-hereditary graph can be computed in time $\mathcal{O}(n^2 \cdot \log(n))$, and a linear layout witnessing the linear rank-width can be computed with the same time complexity.

We prove three structural results related to linear rank-width of distance-hereditary graphs. First, we provide a set of distance-hereditary graphs that contains the set of distance-hereditary vertex-minor obstructions for bounded linear rank-width. It generalizes the constructions given by Jeong, Kwon, and Oum [2]. Second, we prove that for any fixed tree T, if a distance-hereditary graph of linear rank-width at least $3 \cdot 2^{5|V(T)|} - 2$, then it contains a vertex-minor isomorphic to T. Finally, we characterize graphs of linear rank-width at most 1 in terms of canonical split decompositions and give a linear time algorithm to recognize this class.

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Erdős-Pósa Property for Topological Minors

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- Luke Postle – Department of Combinatorics and Optimization, University of Waterloo, Canada
- Paul Wollan – Department of Computer Science, University of Rome "La Sapienza", Italy

Abstract: A family $\mathcal{F}$ of graphs has the *Erdős-Pósa property* if there exists a function f such that for every integer k, every graph either contains k disjoint members of $\mathcal{F}$ or contains $f(k)$ vertices that intersect in every subgraph isomorphic to a member of $\mathcal{F}$.

Robertson and Seymour [1] proved that for every graph H, the set $\mathcal{M}(H)$ of graphs which contain H as a minor has the Erdős-Pósa property if and only if H is planar. Let $\mathcal{TM}(H)$ be the set of graphs containing H as a topological minor. In the same paper, Robertson and Seymour posed the problem of characterizing for which H does $\mathcal{TM}(H)$ have the Erdős-Pósa property. We will provide such a characterization in this talk.

Note that such a characterization is expected to be complicated as Thomassen [2] showed that there exists a tree T such that $\mathcal{TM}(T)$ does not have the Erdős-Pósa property. Our characterization requires a couple of definitions to be formally stated. Roughly speaking, for a connected graph H, $\mathcal{TM}(H)$ has the Erdős-Pósa property if and only if the following hold.

1. H can be drawn in the plane such that every vertex of degree at least four is incident with the infinite face.

2. Every "partition" of H does not contain three pairwise incomparable parts with respect to the "rooted topological minor containment."

3. For every "partition" of H, the maximal parts with respect to the "rooted topological minor containment" are "symmetric."

This characterization can be generalized to graphs with more than one components.

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Tree Decompositions and Graph algorithms

• D. Lokshtanov – University of Bergen

Abstract: A central concept in graph theory is the notion of tree decompositions - these are decompositions that allow us to split a graph up into "nice" pieces by "small" cuts. It is possible to solve many algorithmic problems on graphs by decomposing the graph into "nice" pieces, finding a solution in each of the pieces, and then gluing these solutions together to form a solution to the entire graph. Examples of this approach include algorithms for deciding whether a given input graph is planar, the k-Disjoint paths algorithm of Robertson and Seymour, as well as many algorithms on graphs of bounded tree-width.

In this talk we will look at a way to compare two tree decompositions of the same graph and decide which of the two is "better". It turns out that for every cut size k, every graph G has a tree decomposition with (approximately) this cut size, such that this tree-decomposition is "better than" every other tree-decomposition of the same graph with cut size at most k. We will discuss some consequences of this result, as well as possible improvements and research directions.

The Parameterized Complexity of Graph Cyclability

• Petr A. Golovach, Marcin Kamiński, Spyridon Maniatis, Dimitrios M. Thilikos
• Spyridon Maniatis – University of Athens

Abstract: The cyclability of a graph is the maximum integer k for which every k vertices lie on a cycle. The algorithmic version of the problem, given a graph G and a non-negative integer k, decide whether the cyclability of G is at least k, is NP-hard. We prove that this problem, parameterized by k, is co-W[1]-hard. We give an FPT algorithm for planar graphs that runs in time $2^{2^{O(k^2 \log k)}} \cdot n^2$. Our algorithm is based on a series of graph theoretical results on cyclic linkages in planar graphs.

Optimal parameterized algorithms for planar facility location problems using Voronoi diagrams and sphere cut decompositions

• Dániel Marx – MTA SZTAKI, Hungarian Academy of Sciences
• Michał Pilipczuk – University of Warsaw

Abstract: We study a general family of facility location problems defined on planar graphs and on the 2-dimensional plane. In these problems, a subset of k objects has to be selected, satisfying certain packing (disjointness) and covering constraints. Our main result is showing that, for each of these problems, the $n^{O(k)}$ time brute force algorithm of selecting k objects can be improved to $n^{O(\sqrt{k})}$ time. The algorithm is based on the idea of focusing on the Voronoi diagram of a hypothetical solution of k objects and defining subproblems that correspond to the possible separators of a sphere cut decomposition of the Voronoi diagram (similar techniques were used before for the design of geometric QPTASs, but not for exact algorithms and for planar graphs).

The following list is an exemplary selection of concrete consequences of our main result. We can solve each of the following problems in time $n^{O(\sqrt{k})}$, where n is the total size of the input:

- d-Scattered Set: find k vertices in an edge-weighted planar graph that pairwise are at distance at least d from each other (d is part of the input).
- d-Dominating Set (or (k,d)-Center): find k vertices in an edge-weighted planar graph such that every vertex of the graph is at distance at most d from at least one selected vertex (d is part of the input).
- Given a set $\mathcal{D}$ of connected vertex sets in a planar graph G, find a set of k pairwise disjoint vertex sets in $\mathcal{D}$.
- Given a set $\mathcal{D}$ of disks in the plane (of possibly different radii), find a set of k pairwise disjoint disks in $\mathcal{D}$.
- Given a set $\mathcal{D}$ of simple polygons in the plane, find a set of k pairwise disjoint polygons in $\mathcal{D}$.
- Given a set $\mathcal{D}$ of disks in the plane (of possibly different radii) and a set $\mathcal{P}$ of points, find a set of k disks in $\mathcal{D}$ that together cover the maximum number of points in $\mathcal{P}$.
- Given a set $\mathcal{D}$ of axis-parallel squares in the plane (of possibly different sizes) and a set $\mathcal{P}$ of points, find a set of k squares in $\mathcal{D}$ that together cover the maximum number of points in $\mathcal{P}$.

It is known that, assuming the Exponential Time Hypothesis (ETH), there is no $f(k)n^{o(\sqrt{k})}$ time algorithm for any computable function f for any of these problems. Furthermore, we give evidence that packing problems have $n^{O(\sqrt{k})}$ time algorithms for a much more general class of objects than covering problems have. For example, we show that assuming ETH, the problem where a set $\mathcal{D}$ of axis-parallel rectangles and a set $\mathcal{P}$ of points are given and the task is to select k rectangles that together cover the entire point set does not admit an $f(k)n^{o(k)}$ time algorithm for any computable function f.

Saturation in the Hypercube

• J. Noel – University of Oxford
• A. Scott – University of Oxford
• N. Morrison – University of Oxford

Abstract: Let Q_d denote the hypercube of dimension d. Given $d \geq m$, a spanning subgraph G of Q_d is said to be (Q_d, Q_m)-saturated if it does not contain Q_m as a subgraph but adding any edge of $E(Q_d)\backslash E(G)$ creates a copy of Q_m in G. We say G is weakly (Q_d, Q_m)-saturated if the edges of $E(Q_d)\backslash E(G)$ can be added to G one at a time so that each additional edge creates a new copy of Q_m.

In this talk we answer two questions of Johnson and Pinto [1]. First we show that for fixed $m \geq 2$ the minimum number of edges in a (Q_d, Q_m)-saturated graph is $\Theta(2^d)$. We also determine the minimum number of edges in a weakly (Q_d, Q_m)-saturated graph for all $d \geq m \geq 1$.

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Subdivisions in 4-connected graphs of large tree-width

• I. Muzi – University of Rome "La Sapienza"
• P. Wollan – University of Rome "La Sapienza"

Abstract: The grid theorem of Robertson and Seymour [1] proves that graphs of sufficiently large treewidth contain a $r \times r$ grid as a minor. The same does not hold true for grid subdivisions, as any graph G of maximum degree 3 constitutes a counterexample regardless of its treewidth. By restricting the problem to 4-connected graphs, we prove that graphs of sufficiently large treewidth contain either a large grid or a graph obtained by adding an apex vertex to a 3-regular graph of large treewidth. Using analogous techniques we prove that nonplanar graphs of sufficiently large treewidth contain K_5 as a subdivision. This problem is connected to a well known conjecture posed independently by Seymour (1975) [2] and Kelmans (1979) [3] that states that every 5-connected nonplanar graph contains K_5 as a subdivision. This is joint work with Paul Wollan.

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Constructive algorithm for path-width and branch-width of matroids and rank-width of graphs

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- Sang-il Oum – KAIST

Abstract: We present, for a constant k, an explicit and constructive algoritm that decides whether a given input matroid represented over a fixed finite field has *branch-width* (or *path-width*) at most k and if so, find a branch-decomposition (or a path-decomposition) of width at most k. In addition, as a corollary, we obtain an explicit algorithm to decide whether an input graph has *rank-width* at most k and if so, find a rank-decomposition of width at most k.

No such algorithms were known; all known algorithms are indirect and based on the finiteness of forbidden minors (or vertex-minors) and use dynamic programming to test forbidden minors (or vertex-minors) [2].

Our approach is based on the dynamic programming combined with the idea of Bodlaender and Kloks [1] for their work on tree-width of graphs.

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Amalgams and χ-boundedness

• Irena Penev– ENS de Lyon

Abstract: A class of graphs is *hereditary* if it is closed under isomorphism and induced subgraphs. A hereditary class $\mathcal{G}$ is *χ-bounded* if there exists a non-decreasing function $f : \mathbb{N} \to \mathbb{N}$ (called a *χ-bounding function* for $\mathcal{G}$) such that every graph G in $\mathcal{G}$ satisfies $\chi(G) \leq f(\omega(G))$, where $\chi(G)$ is the chromatic number of G, and $\omega(G)$ is the clique number (i.e. the maximum size of a clique) of G. For many hereditary classes of graphs, there is a decomposition theorem of the following form: every graph in the class either belongs to some class of well-understood basic graphs, or it admits one of several decompositions. This raises the following question: which graph decompositions preserve χ-boundedness? Formally, we say that a graph decomposition D *preserves χ-boundedness* if for all hereditary classes $\mathcal{G}$ and $\mathcal{G}^*$ such that $\mathcal{G}$ is χ-bounded and every graph in $\mathcal{G}^*$ either belongs to $\mathcal{G}$ or admits the decomposition D, we have that $\mathcal{G}^*$ is χ-bounded (however, the optimal χ-bounding functions for $\mathcal{G}$ and $\mathcal{G}^*$ need not be the same). This can be generalized to several decompositions: we say that graph decompositions $D_1, \dots, D_k$ *together preserve χ-boundedness* if for all hereditary classes $\mathcal{G}$ and $\mathcal{G}^*$ such that $\mathcal{G}$ is χ-bounded and every graph in $\mathcal{G}^*$ either belongs to $\mathcal{G}$ or admits at least one of $D_1, \dots, D_k$, we have that $\mathcal{G}^*$ is χ-bounded. The fact that each of $D_1, \dots, D_k$ individually preserves χ-boundedness does not imply that $D_1, \dots, D_k$ together preserve it (this essentially follows from the fact that the preservation of χ-boundedness does not entail the preservation of the optimal χ-bounding function).

Our main result is that proper homogeneous sets, clique-cutsets, and amalgams together preserve χ-boundedness. This generalizes two earlier results: that proper homogeneous sets and clique-cutsets together preserve χ-boundedness (due to Chudnovsky, Penev, Scott, and Trotignon), and that 1-joins preserve χ-boundedness (due to Dvořák and Král'). As an application of this result, as well as of a decomposition theorem for "cap-free" graphs (due to Conforti, Cornuéjols, Kapoor, and Vušković), we show that the class of graphs that do not contain any subdivision of the "house" (i.e. the complement of the four-edge path) as an induced subgraph is χ-bounded.

Fixed-parameter tractable canonization and isomorphism test for graphs of bounded treewidth

• Daniel Lokshtanov – University of Bergen, Norway
• Marcin Pilipczuk – University of Warwick, UK
• Michał Pilipczuk – University of Warsaw, Poland
• Saket Saurabh – Institute of Mathematical Sciences, India and University of Bergen, Norway

Abstract: We give a fixed-parameter tractable algorithm that, given a parameter k and two graphs G_1, G_2, either concludes that one of these graphs has treewidth at least k, or determines whether G_1 and G_2 are isomorphic. The running time of the algorithm on an n-vertex graph is $2^{\mathcal{O}(k^5 \log k)} \cdot n^5$, and this is the first fixed-parameter algorithm for Graph Isomorphism parameterized by treewidth.

Our algorithm in fact solves the more general *canonization* problem. We namely design a procedure working in $2^{\mathcal{O}(k^5 \log k)} \cdot n^5$ time that, for a given graph G on n vertices, either concludes that the treewidth of G is at least k, or:

- finds in an isomorphic-invariant way a graph $\mathfrak{c}(G)$ that is isomorphic to G;
- finds an isomorphism-invariant *construction term* — an algebraic expression that encodes G together with a tree decomposition of G of width $\mathcal{O}(k^4)$.

Hence, the isomorphism test reduces to verifying whether the computed isomorphic copies or the construction terms for G_1 and G_2 are equal.

At the heart of our result lies an isomorphic-invariant approximation algorithm for treewidth, based on the well-known constant approximation algorithm of Robertson and Seymour. That is, we show how to modify the Robertson-Seymour algorithm so that it does not make any choices depending on the representation of the graph in the memory (like, e.g., "take an arbitrary vertex"), at the cost of worse approximation guarantee.

The work, available at arXiv (1404.0818), has been presented at FOCS 2014. The talk is meant as a follow-up to the invited talk of Daniel Lokshtanov, where the result will be introduced in a survey manner. The talk aims at providing all important and novel parts of the proof in bigger detail.

Kernelization of Dominating Set in sparse graph classes

• Pål Grønås Drange – University of Bergen
• Markus S. Dregi – University of Bergen
• Fedor V. Fomin – University of Bergen
• Stephan Kreutzer – Technische Universität Berlin
• Daniel Lokshtanov – University of Bergen
• Marcin Pilipczuk – University of Warwick
• Michał Pilipczuk – University of Warsaw
• Felix Reidl – RWTH Aachen University
• Saket Saurabh – University of Bergen and IMSc Chennai
• Fernando Sánchez Villaamil – RWTH Aachen University
• Somnath Sikdar – RWTH Aachen University

Abstract: In this work we show that for every graph class $\mathcal{G}$ of bounded expansion there exists a polynomial-time algorithm that, given a graph $G \in \mathcal{G}$ and integer k, outputs a subset of vertices $S \subseteq V(G)$ of size linear in k such that G has a dominating set of size at most k if and only if $G[S]$ does. In the language of Parameterized Complexity, we thus give the first linear kernel for the Dominating Set problem on graph classes of bounded expansion. At the cost of having a slightly super-linear size of the kernel, we can also handle the more general case when class $\mathcal{G}$ is nowhere dense.

In the prior work, linear kernels for Dominating Set were consecutively given for planar [1], bounded genus [2], apex-minor-free [3], H-minor-free [4], and H-topological-minor-free graphs [5]. However, all these results exploit topological features of the considered graph classes, in particular the concept of bidimensionality, as well as use deep decomposition theorems for graphs excluding (topological) minors. Our approach for bounded expansion graphs avoids all these arguments and uses only basic tools from the theory of sparse graphs. Thus, while subsuming all the previous results, the new approach yields a simpler and cleaner analysis.

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Multigraphs without large bonds are wqo by contraction

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• Jean-Florent Raymond – LIRMM – Université Montpellier 2, France and Institute of Computer Science, University of Warsaw, Poland.
• Théophile Trunck – LIP, ÉNS de Lyon, France.

Abstract: A *well-quasi-order* (*wqo* for short) is a quasi-order which contains no infinite decreasing sequence, nor infinite collection of pairwise incomparable elements. One of the most significant results in this field is the theorem by Robertson and Seymour which states that graphs are well-quasi-ordered by the minor relation [6].

Nonetheless, most of graph containment relations do not well-quasi-order the class of all graphs. For example, graphs are not well-quasi-ordered by (induced) subgraphs or topological minors. This initiated two antipodal lines of research for such relations: a quest for subclasses that are well-quasi-ordered (see for instance [1, 2, 4]), and a study of infinite antichains [3] (which are obstructions of being well-quasi-order).

We show that a class of multigraphs is well-quasi-ordered by edge contraction iff for some $p, k \in \mathbb{N}$ none of its members have more than p connected components or a bond of size more than k. (A *bond* is a minimal non-empty edge cut.) Our proof relies on a decomposition theorem by Tutte [7] and on a result by Oporowski et al. on typical subgraphs of 3-connected graphs [5]. We also characterize canonical antichains for this relation and show that they are fundamental.

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FPT algorithm for a generalized cut problem and some applications

• Ignasi Sau – CNRS, LIRMM, Montpellier, France

Abstract: An r-*allocation* of a set S is an r-tuple $\mathcal{V} = (V_1, \ldots, V_r)$ of possibly empty sets that are pairwise disjoint and whose union is the set S. We refer to the elements of $\mathcal{V}$ as the *parts* of $\mathcal{V}$ and we denote by $\mathcal{V}^{(i)}$ the i-th part of $\mathcal{V}$, i.e., $\mathcal{V}^{(i)} = V_i$. We define the following parameterized problem:

List Allocation
Input: A tuple $I = (G, r, \lambda, \alpha)$, where G is a graph, $r \in \mathbb{Z}_{\geq 1}$, $\lambda : V(G) \to 2^{[r]}$, and $\alpha : \binom{[r]}{2} \to \mathbb{Z}_{\geq 0}$.
Parameter: $k = \sum \alpha$.
Question: Find an r-allocation $\mathcal{V}$ of $V(G)$ such that

1. $\forall \{i, j\} \in \binom{[r]}{2}$, $|\delta(\mathcal{V}^{(i)}, \mathcal{V}^{(j)})| = \alpha(i, j)$ and
2. $\forall v \in V(G), \forall i \in [r]$, if $v \in \mathcal{V}^{(i)}$ then $i \in \lambda(v)$,

or correctly report that such an r-allocation does not exist. (Here, $|\delta(\mathcal{V}^{(i)}, \mathcal{V}^{(j)})|$ denotes the number of edges in G with an endpoint in $\mathcal{V}^{(i)}$ and the other in $\mathcal{V}^{(j)}$.) Using, among others, the techniques introduced by Chitnis *et al.* [1], we are able to prove the following theorem, where $n = |V(G)|$.

Theorem 1. *The* List Allocation *problem can be solved in time* $2^{O(k^2 \log k)} \cdot n^4 \cdot \log n$.

Besides being a natural and quite general cut problem by itself, the relevance of List Allocation is best demonstrated by the following three corollaries of Theorem 1, which we obtain by reducing in FPT time each corresponding problem to particular cases of List Allocation.

1. Our first application concerns a generalization of Digraph Homomorphism where, given two directed graphs G and H where G is simple and H may have loops but not multiple directed edges, we are also given a list $\lambda : V(G) \to 2^{V(H)}$ of allowed images for every vertex in G and a function α bounding the maximum number of arcs in G mapped to each arc of H. The objective is to decide whether there exists a homomorphism from G to H respecting the constraints imposed by λ and α. We call this problem Arc-Bounded List Digraph Homomorphism, and we consider as parameter k the sum of the values taken by the function α over all the arcs of H.

   Corollary 1. Arc-Bounded List Digraph Homomorphism *can be solved in time* $f(k) \cdot n^{O(1)}$.

2. We also consider a parameterization of a special graph partitioning problem.

   Min-Max Graph Partitioning

   Input: An undirected graph G, $w, r \in \mathbb{Z}_{\geq 0}$, and a set $T \subseteq V(G)$, where $|T| = r$.

   Parameter: $k = w \cdot r$.

   Question: Find a partition $\{\mathcal{P}_1, \ldots, \mathcal{P}_r\}$ of $V(G)$ such that for every $i \in [r]$, it holds that $|\mathcal{P}_i \cap T| = 1$ and $|\delta(\mathcal{P}_i, V(G) \setminus \mathcal{P}_i)| \leq w$, or correctly report that such a partition does not exist.

Corollary 2. Min-Max Graph Partitioning *can be solved in time* $f(k) \cdot n^{O(1)}$.

3. Our last application deals with *tree-cut width*, a graph invariant recently introduced by Wollan [2] and that has proved of fundamental importance in the structure of graphs not admitting a fixed graph as an immersion. We prove that following result.

   Corollary 3. *There exists an algorithm that, given a graph* G *and a* $k \in \mathbb{Z}_{\geq 0}$, *in time* $2^{O(k^2 \cdot \log k)} \cdot n^5 \cdot \log n$ *either outputs a tree-cut decomposition of* G *with width at most* $2k$, *or correctly reports that no tree-cut decomposition of* G *with width at most* k *exists.*

This is joint work with EunJung Kim, Sang-Il Oum, Christophe Paul, and Dimitrios M. Thilikos.

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Colouring graphs with no odd holes, and other stories

• Paul Seymour – Princeton

Abstract: The chromatic number $\chi(G)$ of a graph G is always at least the size of its largest clique (denoted by $\omega(G)$), and there are graphs G with $\omega(G) = 2$ and $\chi(G)$ arbitrarily large.

On the other hand, the perfect graph theorem asserts that if neither G nor its complement has an odd hole, then $\chi(G) = \omega(G)$. (A "hole" is an induced cycle of length at least four, and "odd holes" are holes of odd length.) What happens in between?

With Alex Scott, we recently proved the following, a 1985 conjecture of Gyárfás:

For graphs G with no odd hole, $\chi(G)$ is bounded by a function of $\omega(G)$.

Gyárfás also made the stronger conjecture that for every integer k and for all graphs G with no odd hole of length more than k, $\chi(G)$ is bounded by a function of k and $\omega(G)$. This is far from settled, and indeed the following much weaker statement is not settled: for every integer k, every triangle-free graph with no hole of length at least k has chromatic number bounded by a function of k. We give a partial result towards the latter:

For all k, every triangle-free graph with no hole of length at least k admits a tree-decomposition into bags with chromatic number bounded by a function of k.

Both results have quite pretty proofs, which will more-or-less be given in full.

Solving #SAT and MaxSAT by dynamic programming

• Sigve Hortemo Sæther
• Martin Vatshelle
• Jan Arne Telle, all at University of Bergen, Norway

Abstract: In this paper we look at dynamic programming algorithms for propositional model counting, also called #SAT, and MaxSAT. We focus on the minimal information that any efficient dynamic programming approach to these problems must maintain, and develop an algorithm that uses only this information.

A subset of clauses of a CNF formula F is called projection satisfiable if there is some complete assignment satisfying these clauses only. The ps-value of F is the number of projection satisfiable subsets of clauses. We relate the ps-value of F to the mim-value of F, which is the size of a maximum induced matching, a set of edges incident to no other edges, in the incidence graph of F. We show that the ps-value of F is upper bounded by the number of clauses of F raised to the power of its mim-value, plus one. Families of CNF formulas with small mim-value, and thus small ps-value, are themselves of algorithmic interest, but in this paper we focus on even larger families of CNF formulas.

Applying the notion of branch decompositions to CNF formulas and using ps-value as cut function, we define the ps-width of a formula. A crucial property of such decompositions is that a formula with ps-value exponential, in formula size, may have ps-width polynomial. For a formula given with a branch decomposition of polynomial ps-width we show dynamic programming algorithms, working along the branch decomposition, solving weighted MaxSAT and #SAT in polynomial time.

Combining with results of 'Belmonte and Vatshelle, Graph classes with structured neighborhoods and algorithmic applications, Theor. Comput. Sci. 511: 54-65 (2013)' we relate ps-width of a formula to mim-width, tree-width and clique-width of its incidence graph. We show that our algorithms extend all previous results for MaxSAT and #SAT achieved by dynamic programming along structural decompositions of the incidence graph of the input formula.

For certain classes of formulas we get polynomial-time algorithms assuming only the formula as input. For example, we get $O(m^2(m+n)s)$ algorithms for formulas F of m clauses and n variables and total size s, whenever F has a total ordering of its variables and clauses such that for any variable x occurring in clause C, if x appears before C then any variable between them also occurs in C, and if C appears before x then x occurs also in any clause between them. We show that the class of incidence graphs of such formulas does not have bounded clique-width.

Induced Cycles Modulo 3

• Stéphan Thomassé– ENS de Lyon

Abstract: Studying the length of induced cycles modulo 3 in a graph is definitively an exotic goal. The aim of this talk is to provide some motivation for it. In particular, this notion plays an important role when studying the stable set complex of a graph. After a brief introduction to the subject, I will sketch the proof of our main theorem, obtained in collaboration with Pierre Charbit and Marthe Bonamy : Every graph with high chromatic number contains an induced cycle of length 0 mod 3.

Large Induced Subgraphs Via Triangulations and CMSO

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Abstract: Consider the following optimization problem. Let φ be a Counting Monadic Second Order Logic formula and t be an integer. Given a graph $G = (V, E)$, the task is to find two vertex subsets $X \subseteq F \subseteq V$ such that the induced subgraph $G[F]$ has treewidth at most t, the structure $(G[F], X)$ models φ and X is of maximum size under these constraints. Note that our generic problem encompasses many classical optimization problems like Feedback Vertex Set, Longest Induced Path, Maximum Induced Matching, Independent $\mathcal{H}$-packing, etc.

Using the theory of potential maximal cliques, we provide an algorithm for this problem with running time $\mathcal{O}(|\Pi_G| \cdot n^{t+4})$ where Π_G is the set of potential maximal cliques of G. The hidden constant depends on t and φ.

As a consequence, the generic problem can be solved in polynomial time for classes of graphs with polynomially many minimal separators, and in time $\mathcal{O}(1.7347^n)$ for arbitrary graphs.

Rencontres internationales sur les méthodes de décomposition de graphes

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